Session Types Course [Exercise Class 2]

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Exercise 1. Check if each of the following processes reduces to $\mathbf{0}$ and provide its type (when defined).

1. $(\nu xy)\big(x \triangleleft \{\ell : x[].\mathbf{0}\} \mid y[u].\mathbf{0}\big)$
2. $(\nu xy)\big(x \triangleleft \{\ell : x[].\mathbf{0}\} \mid y[\ell].y().\mathbf{0}\big)$
3. $(\nu xy)\big(x \triangleleft \{\ell : x[].\mathbf{0}\} \mid y \triangleright \{\ell : y().\mathbf{0}\}\big)$
4. $(\nu xy)\big(x \triangleleft \{\ell_1 : x[].\mathbf{0}\} \mid y \triangleright \{\ell_2 : y().\mathbf{0}\}\big)$
5. $(\nu xy)\big(x \triangleleft \{\ell_1 : x[].\mathbf{0}\ ,\ \ell_2 : x().\mathbf{0}\} \mid y \triangleright \{\ell_1 : y().\mathbf{0}\}\big)$
6. $(\nu xy)\big(x \triangleleft \{\ell_1 : x[].\mathbf{0}\} \mid y \triangleright \{\ell_1 : y().\mathbf{0}\ ,\ \ell_2 : y().\mathbf{0}\}\big)$
7. $(\nu xy)\big(x \triangleleft \{\ell_1 : x[].\mathbf{0}\ ,\ \ell_2 : x().\mathbf{0}\} \mid y \triangleright \{\ell_1 : y().\mathbf{0}, \ell_2 : y[].\mathbf{0}\}\big)$

Exercise 2. Consider the following processes.

$$\mathsf{T}(x) = x \triangleleft \{\ell_{true} : x[].\mathbf{0}\} \qquad \mathsf{F}(x) = x \triangleleft \left\{\ell_{false} : x[].\mathbf{0}\right\}$$

$$\mathsf{Cond}(x, P, Q) = x \triangleright \left\{\ell_{true} : x().P\ ,\ \ell_{false} : x().Q\right\}$$

1. Show that $T(x)$ and $F(x)$ have the same type.
2. Show that

   $$(\nu xy)\,(\mathsf{T}(x) \mid \mathsf{Cond}(y, P, Q)) \to^2 P \qquad \text{and} \qquad (\nu xy)\,(\mathsf{F}(x) \mid \mathsf{Cond}(y, P, Q)) \to^2 Q$$

3. Define a process $\mathsf{Neg}(u, v)$ such that

   $$(\nu xy)\,(\mathsf{T}(x) \mid \mathsf{Neg}(y, z)) \to^2 \mathsf{F}(z) \qquad \text{and} \qquad (\nu xy)\,(\mathsf{F}(x) \mid \mathsf{Neg}(y, z)) \to^2 \mathsf{T}(z)$$

4. Define a process $\mathsf{And}(u, v, w)$ such that

   $$(\nu x_1 y_1)(\nu x_2 y_2)\big(P(x_1) \mid Q(x_2) \mid \mathsf{And}(y_1, y_2, z)\big)$$

   reduces in two steps to $\mathsf{T}(z)$ if $P(x) = Q(x) = \mathsf{T}(x)$ and to $\mathsf{F}(z)$ if $P(x), Q(x) \in \{\mathsf{T}(x), \mathsf{F}(x)\}$ with $P(x)$ or $Q(x)$ equal to $\mathsf{F}(x)$.

   [Beware that in defining $\mathsf{And}(x, y, z)$ we cannot do a sequential evaluation because of linearity]

Exercise 3.

- Give a typable process P_1 such that $P_1 \to P_1'$ with P_1' reducible;
- Give a typable process P_2 such that $P_2 \to P_2'$ with P_2' is not reducible;
- Give a process P_3 such that $P_3 \to P_3'$ with P_3' is a runtime error;
- Give a typable process P_4 such that $P_4 \to P_4'$ with P_4' is stuck (note: it cannot be a runtime error!);
- Give a runtime error process P_5 which is not stuck. Is it true that any P_5' such that $P_5 \to P_5'$ is a runtime error?

[Tip: have a look at the Exercises 2 and 4 from Class 1 and Exercise 1 above]

Processes			Structural Equivalence (Processes)
			$P \mid \mathbf{0} \equiv P$
P, Q	$:=$ $\mathbf{0}$	inact	$P \mid Q \equiv Q \mid P$
	$\mid$ $x[].P$	close	$P \mid (Q \mid R) \equiv (P \mid Q) \mid R$
	$\mid$ $x().P$	wait	$(\nu xy)\mathbf{0} \equiv \mathbf{0}$
	$\mid$ $x[y].P$	send (y through x)	$(\nu x_1 x_2)(\nu y_1 y_2)P \equiv (\nu y_1 y_2)(\nu x_1 x_2)P$
	$\mid$ $x(y).P$	receive (y on x)	$((\nu xy)P_1) \mid P_2 \equiv (\nu xy)\,(P_1 \mid P_2)$
	$\mid$ $(\nu xy)P$	nu	$\alpha.((\nu xy)P) \equiv (\nu xy)(\alpha.P)$
	$\mid$ $P \mid Q$	parallel	$\boxed{\text{with } x, y \notin \mathsf{fv}(P_2),\ \alpha. \in \{z[].,z().,z[w].,z(w).\}}$
			plus the standard α-equivalence

Operational Semantics (Processes)

Close:	$(\nu xy)\big(x[].P \mid y().Q\big) \to P \mid Q$	
Com:	$(\nu xy)\big(x[a].P \mid y(b).Q\big) \to (\nu xy)\big(P \mid Q[a/b]\big)$	
Par:	$P \mid Q \to P' \mid Q$	if $P \to P'$
Res:	$(\nu xy)P \to (\nu xy)P'$	if $P \to P'$
Struct:	$P \to Q$	if $P \equiv P' \to Q' \equiv Q$
Choice:	$(\nu xy)\big(x \triangleright \{\ell_i : P_i : \ell_i \in L\} \mid x \triangleleft \ell : Q\big) \to (\nu xy)\big(P_k \mid Q\big)$	if $\ell = \ell_k \in L$

Figure 1: Syntax and semantics for processes

Types	Duality (for Types)
$T, U ::= \mathsf{close} \mid \mathsf{wait} \mid [T] \blacktriangleleft U \mid (T) \blacktriangleleft U \mid \oplus\{\ell : T\}^{\ell \in L} \mid \&\{\ell : T\}^{\ell \in L}$	$\mathsf{close} \perp \mathsf{wait}$ $[T] \blacktriangleleft U \perp (T) \blacktriangleleft V$ if $U \perp V$ $\&\{\ell : T_\ell\}^{\ell \in L} \perp \oplus\{\ell : T_\ell\}^{\ell \in L}$ if $U_\ell \perp V_\ell$ for each $\ell \in L$

Typing Rules

$$\textsf{T-Inact}\ \frac{}{\vdash \mathbf{0}} \qquad \textsf{T-Par}\ \frac{\Gamma_1 \vdash P \qquad \Gamma_2 \vdash Q}{\Gamma_1, \Gamma_2 \vdash P \mid Q} \qquad \textsf{T-Resr}\ \frac{\Gamma, x : T, y : U \vdash P \qquad T \perp U}{\Gamma \vdash (\nu xy)P}$$

$$\textsf{T-Close}\ \frac{\Gamma \vdash P}{\Gamma, x : \mathsf{close} \vdash x[].P} \qquad \textsf{T-Wait}\ \frac{\Gamma \vdash P}{\Gamma, x : \mathsf{wait} \vdash x().P}$$

$$\textsf{T-Send}\ \frac{\Gamma, x : U, y : T \vdash P}{\Gamma, x : [T] \blacktriangleleft U \vdash x[y].P} \qquad \textsf{T-Recv}\ \frac{\Gamma, x : U \vdash P}{\Gamma, x : (T) \blacktriangleleft U \vdash x(y).P}$$

$$\textsf{T-Bra}\ \frac{\Gamma, x : T_1 \vdash P_1 \quad \cdots \quad \Gamma, x : T_n \vdash P_n}{\Gamma, x : \&\{\ell_i : T_i\}^{\ell_i \in L} \vdash x \triangleright \{\ell_i : P_i\}} \qquad \textsf{T-Sel}\ \frac{\Gamma, x : T_k \vdash P_k}{\Gamma, x : \oplus\{\ell_i : T_i\}^{\ell_i \in L} \vdash x \triangleleft \ell_i : P_i}\ \ell_k \in L$$

Figure 2: Types