Session Types Course [Exercise Class 1]

Sonia Marin & Matteo Acclavio

Notation: we use the symbol $\mathbf{0}$ to denote the process **inact** *(because I'm lazy)*

Definition A ***prefix*** is a process of one of the following forms

$$x[].P \qquad x().P \qquad x[y].P \qquad x(y).P \qquad x \triangleright \{\ell_i : P_i\} \qquad x \triangleleft \ell_i : P_i$$

A process P is in ***canonical form*** if $P = (\nu x_1 y_1) \ldots (\nu x_n y_n) (P_1 \mid \cdots \mid P_m)$ with P_i (called ***thread***) a prefix, and x_i or y_i occurring in a P_j for all i and some j in $\{1, \ldots, m\}$.

Exercise 1. Prove that each process P is structurally equivalent to a process P' in canonical form.

Definition Let P and Q be processes. We say that P ***reduces*** to Q if there are $P_1, \ldots, P_n$ such that $P \equiv P_1 \to \cdots \to P_n \equiv Q$ and that P is ***reducible*** if it reduces to a process Q.

Exercise 2. Check if the following processes reduce to $\mathbf{0}$.

1. $x[u].x[].\mathbf{0} \mid y(v).y().\mathbf{0}$
2. $(\nu xy)\big(x[u].x[].\mathbf{0} \mid y(v).y().\mathbf{0}\big)$
3. $(\nu xy)\big(x[u].x(w).x[].\mathbf{0} \mid y(v).y[z].y().\mathbf{0}\big)$
4. $(\nu xy)\big(x[u].x(w).\mathbf{0} \mid y(v).y[z].\mathbf{0}\big)$
5. $(\nu xy)\big(x[u].x[].\mathbf{0} \mid y[v].y[].\mathbf{0}\big)$
6. $(\nu xy)\big(x[u].x(w).x[].\mathbf{0} \mid y(v).y(z).y[].\mathbf{0}\big)$
7. $(\nu xy)\big(x(a).y[b].y().x[].\mathbf{0}\big)$
8. $(\nu xy)\big(x(a).x().\mathbf{0}\big)$
9. $(\nu xy)\big(x[u].x[].\mathbf{0} \mid y(v).y().\mathbf{0} \mid a[b].a().\mathbf{0} \mid c(d).c[].\mathbf{0}\big)$
10. $(\nu x_1 y_1)(\nu x_2 y_2)\big(x_1[a].x_2(b).x_1[].x_2[].\mathbf{0} \mid y_2[c].y_1(d).y_1[].y_2[].\mathbf{0}\big)$

Recall: a process is ***linear*** if each name occurs in at most a thread.

Exercise 3. Let P be a process. Prove that if P is linear, then if $P \to Q_1$ and $P \to Q_2$ with Q_1 and Q_2 irreducible, then $Q_1 \equiv Q_2$.

Definition A process P is ***typable*** if there is a typing derivation of a judgment of the form $\Gamma \vdash P$.

Exercise 4. Which of the processes in Exercise 2 is typable?

Processes			Structural Equivalence (Processes)
$P, Q \quad :=$	$\mathbf{0}$	inact	$P \mid \mathbf{0} \equiv P$
$\mid$	$x[].P$	close	$P \mid Q \equiv Q \mid P$
$\mid$	$x().P$	wait	$P \mid (Q \mid R) \equiv (P \mid Q) \mid R$
$\mid$	$x[y].P$	send (y through x)	$(\nu xy)\mathbf{0} \equiv \mathbf{0}$
$\mid$	$x(y).P$	receive (y on x)	$(\nu x_1 x_2)(\nu y_1 y_2)P \equiv (\nu y_1 y_2)(\nu x_1 x_2)P$
$\mid$	$(\nu xy)P$	nu	$((\nu xy)P_1) \mid P_2 \equiv (\nu xy)\,(P_1 \mid P_2)$
$\mid$	$P \mid Q$	parallel	$\alpha.((\nu xy)P) \equiv (\nu xy)(\alpha.P)$
			$\boxed{\text{with } x, y \notin \mathsf{fv}(P_2),\ \alpha. \in \{z[].,z().,z[w].,z(w).\}}$

Operational Semantics (Processes)

Close:	$(\nu xy)\big(x[].P \mid y().Q\big) \to (\nu xy)\big(P \mid Q\big)$	
Com:	$(\nu xy)\big(x[a].P \mid y(b).Q\big) \to (\nu xy)\big(P \mid Q[a/b]\big)$	
Par:	$P \mid Q \to P' \mid Q$	if $P \to P'$
Res:	$(\nu xy)P \to (\nu xy)P'$	if $P \to P'$
Struct:	$P \to Q$	if $P \equiv P' \to Q' \equiv Q$

Figure 1: Syntax and semantics for processes

Types		Duality (for Types)
$T, U :=$	$:= \ \mathsf{close}$	
	$\mid \ \mathsf{wait}$	$\mathsf{close} \perp \mathsf{wait}$
	$\mid \ [T] \blacktriangleleft U$	$\dfrac{}{[T] \blacktriangleleft U \perp (T) \blacktriangleleft V} \quad \text{if } U \perp V$
	$\mid \ (T) \blacktriangleleft U$	

Typing Rules

$$\text{T-Inact}\ \frac{}{\vdash \mathbf{0}} \qquad \text{T-Par}\ \frac{\Gamma_1 \vdash P \qquad \Gamma_2 \vdash Q}{\Gamma_1, \Gamma_2 \vdash P \mid Q} \qquad \text{T-Resr}\ \frac{\Gamma, x : T, y : U \vdash P \qquad T \perp U}{\Gamma \vdash (\nu xy)P}$$

$$\text{T-Close}\ \frac{\Gamma \vdash P}{\Gamma, x : \mathsf{close} \vdash x[].P} \qquad \text{T-Wait}\ \frac{\Gamma \vdash P}{\Gamma, x : \mathsf{wait} \vdash x().P}$$

$$\text{T-Send}\ \frac{\Gamma, x : U, y : T}{\Gamma, x : [T] \blacktriangleleft U \vdash x[y].P} \qquad \text{T-Recv}\ \frac{\Gamma, x : U}{\Gamma, x : (T) \blacktriangleleft U \vdash x(y).P}$$

Figure 2: Types