MESOSCOPIC THERMODYNAMICS OF SINGLE-PARTICLE ENZYMATIC REACTIONS

FILIPE P. DE FARIAS¹, FRANCESCO CORONA², MICHELA MULAS¹

In this work, we consider the framework of mesoscopic non-equilibrium thermodynamics^{5,7,9} to evaluate the stochastic energetics of enzymatic reactions described in terms of classic Michaelis-Menten/Briggs-Haldane theories. We consider a system comprising of a single enzyme which can exist in one of two states, either in free form or complexed with the substrate, and that the formation of product is reversible. Moreover, we assume that the kinetics of the reaction network can be expressed using elementary reactions and that the system is coupled to a heat reservoir. Because the interactions with the heat bath are assumed to be uncontrolled and intrinsically stochastic, each reaction will be described as a random event characterised by a single parameter, the rate constant. It is thus expected that a different evolution of the system is observed each time the experiment is repeated. Such a description leads to model the system as a continuous-time Markov process with discrete state-space, a Markovian jump process. The resulting stochastic kinetics of such a system are represented with two complementary formulations of the dynamics: either in terms of the fluctuating number of molecular species that follow the Gillespie algorithm,³ or in terms of the probability distribution that satisfies the chemical master equation.⁴

We discuss how thermodynamic observables of relevance, like the stochastic heat and the stochastic work, are defined at the level of single trajectories and compute their value for this system. We establish the stochastic counterpart of the first and second law of thermodynamics and establish the fundamental fluctuation relations. The analysis relies on irreversible nature of non-equilibrium processes and the definition of non-equilibrium entropy and entropy production. Free energy is computed as proportional to the Kullback-Leibler divergence between the system's probability distribution and its equilibrium distribution.

References

 $^{^{1}}$ Post-graduate Programme in Teleinformatics Engineering, Federal University of Ceará, Brazil.

² School of Chemical Engineering, Aalto University, Finland.

¹ M. Esposito and C. Van den Broeck. Second law and landauer principle far from equilibrium. *Europhysics Letters*, 95(4):40004, aug 2011.

² Massimiliano Esposito. Introduction to stochastic thermodynamics. https://enabla.com/set/144, 2023. Nordita Winter School 2023.

³ Daniel T Gillespie. A general method for numerically simulating the stochastic time evolution of coupled chemical reactions. *Journal of Computational Physics*, 22(4):403–434, 1976.

⁴ Daniel T. Gillespie. A rigorous derivation of the chemical master equation. *Physica A: Statistical Mechanics and its Applications*, 188(1):404–425, 1992.

⁵ P. Mazur. Fluctuations and non-equilibrium thermodynamics. *Physica A: Statistical Mechanics and its Applications*, 261(3):451–457, 1998.

- ⁶ Luca Peliti and Simone Pigolotti. *Stochastic Thermodynamics: An Introduction*. Princeton University Press, 2021.
- ⁷ I. Prigogine and P. Mazur. Sur l'extension de la thermodynamique aux phénomènes irreversibles liés aux degrés de liberté internes. *Physica*, 19(1):241–254, 1953.
- ⁸ Hong Qian and Hao Ge. Stochastic Chemical Reaction Systems in Biology. Springer International Publishing, 2021.
- ⁹ D. Reguera, J. M. Rubí, and J. M. G. Vilar. The mesoscopic dynamics of thermodynamic systems. The Journal of Physical Chemistry B, 109(46):21502–21515, 2005. PMID: 16853792.
- ¹⁰ N.G. Van Kampen. Stochastic processes in physics and chemistry. North-Holland personal library. Elsevier, 2007.

 $^{^1}_2$ Post-graduate Programme in Teleinformatics Engineering, Federal University of Ceará, Brazil. 2 School of Chemical Engineering, Aalto University, Finland.