

Stochastic thermodynamic analysis of the Michaelis-Menten kinetics

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Introduction

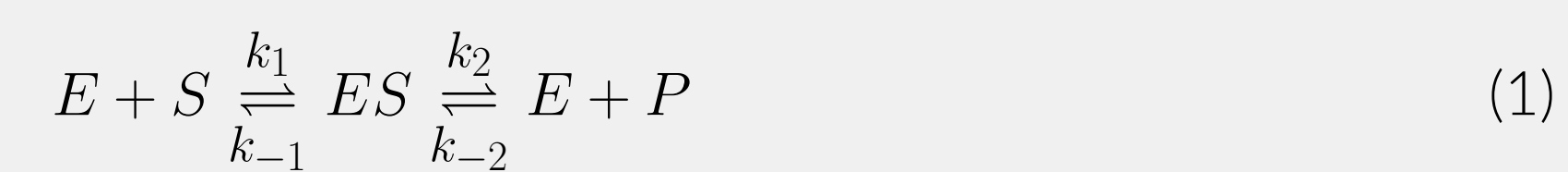
Stochastic thermodynamics (ST) deals with the interaction of mesoscopic, nonequilibrium physical systems with heat reservoirs in equilibrium.¹ Such interactions are assumed to be the source of the randomness in the dynamics of the system, assigning to it a probability $p_x(t)$ of being in the state x at time t .

- We will use the Michaelis-Menten kinetics as case of study for the ST.

Michaelis-Menten kinetics (MM)

The system (MM) is composed by a single molecule of enzyme E . We assume the enzyme processes a single molecule of substrate S per time. Then the system can be in two states: free enzyme E and complexed ES .

- The reaction network that models the kinetics is:



- The substrate S and the product P are **chemostated**.

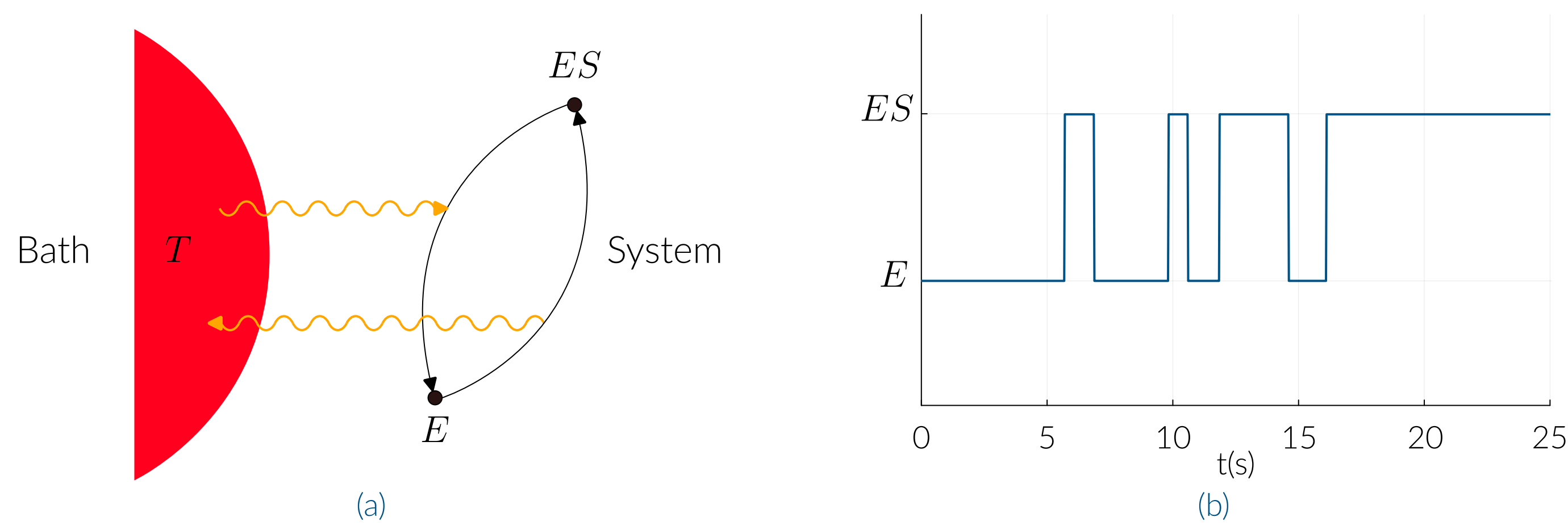


Figure 1. In (b) the representation of the MM[Massimiliano REF?] and in (b) a single realization of the system.

The system is kept in contact with a heat bath with temperature T .

- The changes in state of the system are due to energy exchanges with the bath.

Master Equation

How the probability $p_x(t)$ of the system being in $x \in \{E, ES\}$ change with time, is given by a **master equation**.² It reads:

$$\frac{dp_x(t)}{dt} = \sum_x W_{x'x} p_x(t) - W_{xx'} p_{x'}(t) \quad (2)$$

- The $W_{x'x}$ is the **probability transition rate** from the state x' to x .
- $W_{x'x}$ forms a **stochastic matrix** W dependent on the kinetics of the chemical reactions.
- Integrating or sampling (2) allow us to obtain the probabilities $p_x(t)$.

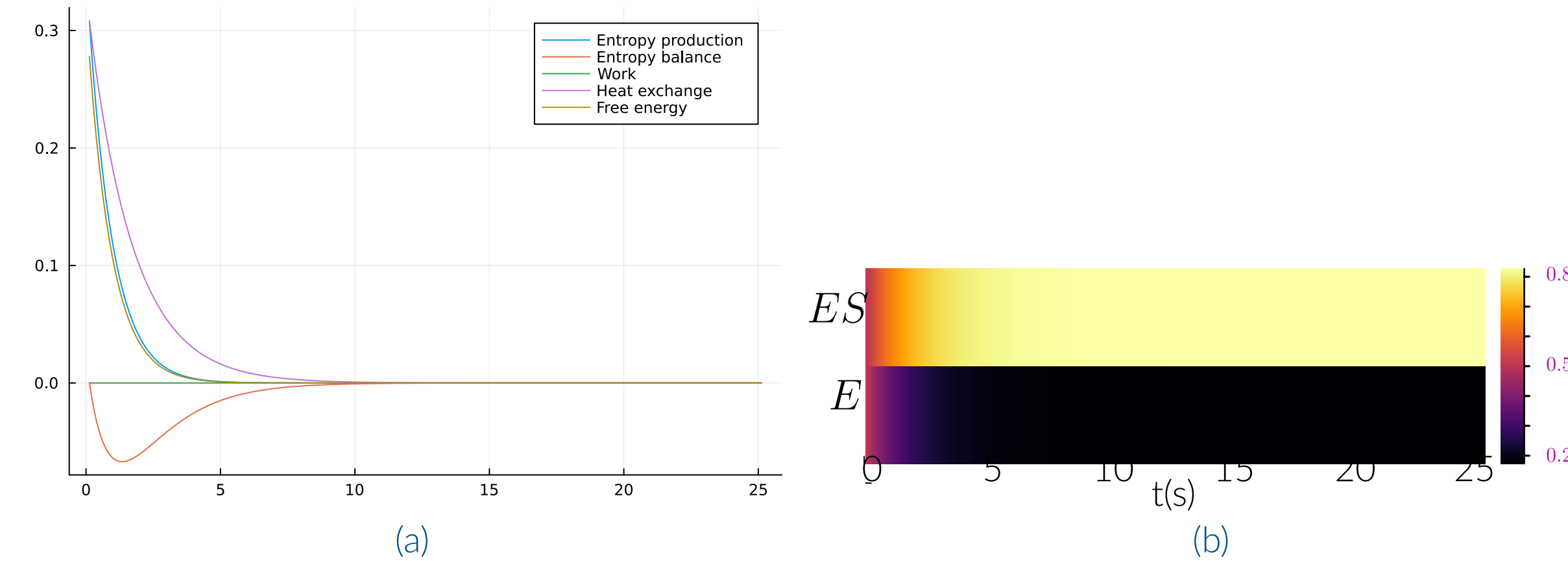


Figure 2. In (b) the representation of the MM[Massimiliano REF?] and in (b) a single realization of the system.

Stochastic Thermodynamics

The classical thermodynamics is defined assumed an **equilibrium** situation of the system.

- In the ST, the equilibrium is held by the bath, the system is allowed to be in nonequilibrium.
- In such case, ST gives that the system has nonnegative average entropy production rate \dot{S}^{sys} :

$$\frac{\dot{S}^{sys}}{k_B} = \frac{1}{2} \sum_{x \neq x'} [W_{x'x} p_x(t) - W_{xx'} p_{x'}(t)] \ln \frac{p_x(t)}{p_{x'}(t)}. \quad (3)$$

This expression can be separated in two parts:

$$\frac{\dot{S}^{tot}}{k_B} = \frac{1}{2} \sum_{x \neq x'} [W_{x'x} p_x(t) - W_{xx'} p_{x'}(t)] \ln \frac{W_{x'x} p_x(t)}{W_{xx'} p_{x'}(t)} \quad \frac{\dot{S}^{bath}}{k_B} = \frac{1}{2} \sum_{x \neq x'} [W_{x'x} p_x(t) - W_{xx'} p_{x'}(t)] \ln \frac{W_{x'x}}{W_{xx'}} \quad (4)$$

The term $\frac{\dot{S}^{bath}}{k_B}$ is the average heat absorbed by the bath when the system jumps between the states, while $\frac{\dot{S}^{tot}}{k_B}$ is the total entropy change (or balance) of both system and bath.

- If p_x^{eq} is the probability of the system when in equilibrium, ST gives us the **free energy**

$$\frac{\dot{F}(t)}{k_B T} = \sum_{x \neq x'} [W_{x'x} p_x(t) - W_{xx'} p_{x'}(t)] \ln \frac{p_x(t)}{p_{x'}^{eq}} = w - T \dot{S}^{tot}. \quad (5)$$

The average work w that can be done in the system is then defined in terms of p_x^{eq}

$$w(t) = \frac{1}{2} \sum_{x \neq x'} [W_{x'x} p_x(t) - W_{xx'} p_{x'}(t)] \ln \frac{W_{x'x} p_x^{eq}}{W_{xx'} p_{x'}^{eq}} \quad (6)$$

References

- ¹ Luca Peliti and Simone Pigolotti. *Stochastic Thermodynamics: An Introduction*. Princeton University Press, 2021.
- ² N.G. VAN KAMPEN. *Stochastic processes in physics and chemistry*. North-Holland personal library. Elsevier, 2007.