TENSOR TECHNIQUES FOR CMES

THE AUTHOR

The first and more basic is the finite state projection (FSP)[1]. When the CME describes a chemical reaction network that has a finite copy numbers (the system is closed and does not create species), the FSP method provides an exact analytical solution. When the system is allowed to have a large or even infinite variation of copy numbers, the FSP method truncates the state-space of the CME given a precision. This precision is quantified by the mass of probability that "leaves" the state-space due to jumps to out of it. The algorithm tries to expand the state-space until it reaches the precision, but at the cost of computing the whole probability at each trial. Another methods as sliding windows[2] that are based on the probability of jumping out of the state-space in a given interval of time. The difference is that the state-space is "slid" according to such probabilities and iteratively.

More recent advances try to address the curse of dimensionality using tensor decomposition techniques[3]. In special, the called "tensor-train" decomposition, found by Oseledets[4] (cf. a more comprehensive review[5]). The main idea is to decompose the tensors obtained from the CME operator and the state-space probabilities into product of lower-rank tensors, achieving a lower complexity for the evaluation of the evolution and a lower storage cost[6]. The technique has been used for solving the chemical master equation[7]

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