Problem Set 3

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Shortish Answer

1. Suppose that we have a test scaled with the Rasch model whose first 3 items have known difficulties -1, 0, and 1.5. An examinee with ability theta got the first item right, the second item right, and the third item wrong. Can you write the likelihood of observing this sequence of item responses as a function of theta?

The likelihood of getting each item right in the Rash model is given by:

$$\frac{\epsilon^{\theta-b}}{1+\epsilon^{\theta-b}}$$

Therefore, each item's likelihood, considering its difficulty is as follows: Item dif -1: $\frac{\epsilon^{\theta+1}}{1+\epsilon^{\theta+1}}$

Item dif 0: $\frac{\epsilon^{\theta-0}}{1+\epsilon^{\theta-0}}$

Item dif 1.5: $1 - \frac{\epsilon^{\theta-1.5}}{1+\epsilon^{\theta-1.5}}$

The chance of getting the specific sequence is the product of the probabilities of each item in the sequence. Hence, a 1-1-0 sequence has the probability equal to:

$$\sum \left(\frac{\epsilon^{\theta+1}}{1+\epsilon^{\theta+1}} \right) \times \left(\frac{\epsilon^{\theta-0}}{1+\epsilon^{\theta-0}} \right) \times \left(1 - \frac{\epsilon^{\theta-1.5}}{1+\epsilon^{\theta-1.5}} \right)$$
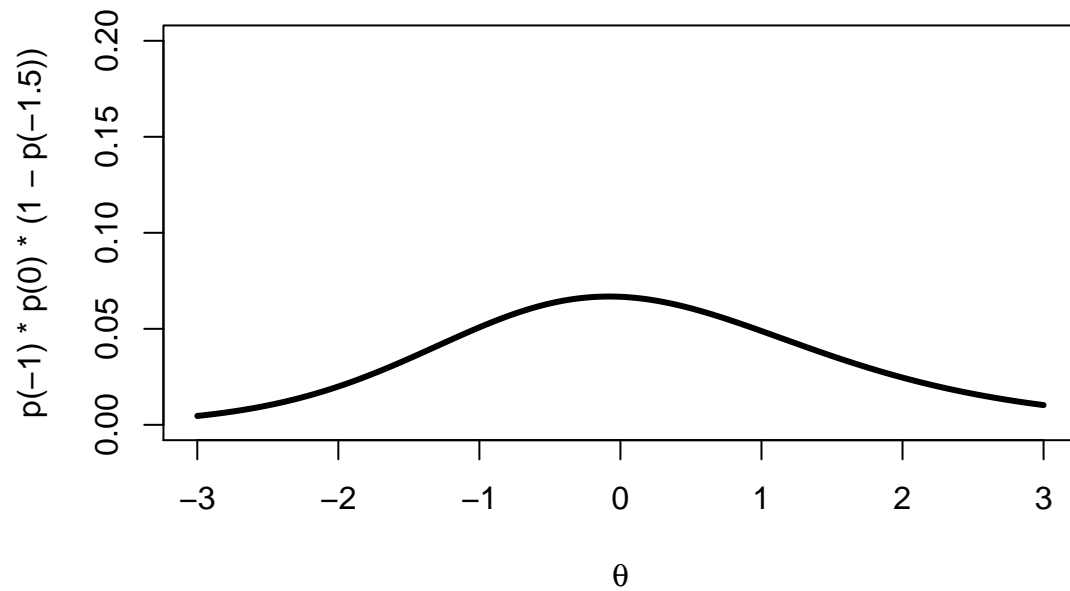
2. Can you plot this as a function of theta?

Figure 1: Probabilities as a function of theta

16 3. If theta=0.5, what is the likelihood of that response sequence?

```
th<-0.5
p<-function(b) exp(th-b)/(1+exp(th-b))
round(p(-1)*p(0)*(1-p(-1.5)),3)

## [1] 0.061
```

17 4. If theta=0.5, what is the most likely response sequence given the known item difficulties?

18 The probabilities of getting each item right is 0.82, 0.62 and 0.27, respectively for item difficulties of
19 -1, 0, 1.5. Therefore, the most likely sequence is exactly 1-1-0.

20 5. At what value of theta does a response sequence of 1-1-0 (that is: they got the first and second items
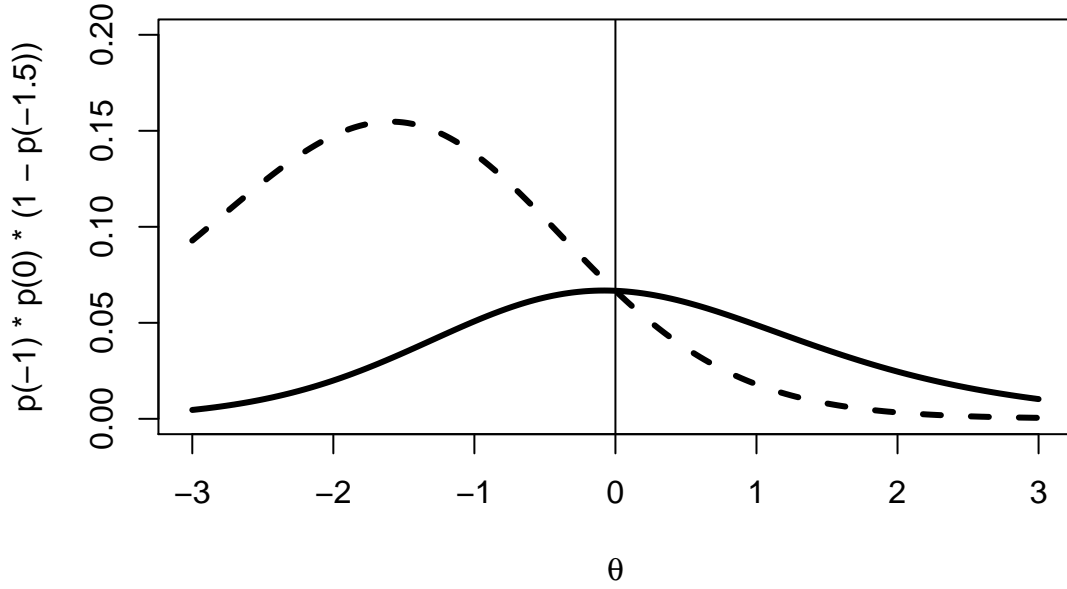21 right and the third item wrong) become more likely than a response sequence of 1-0-0?

Figure 2: Probabilities as a function of theta

Considering that the only difference between the two sequeces is either the student got the second item right or wrong and that this specific item has difficulty $b = 0$, then getting 1-1-0 becames more likely at $\theta = 0$.

6. Returning to questions 1 and 2, can you plot the "test information" as a function of theta (see Eqn 2-6 in Lord).

To get the test information we need to sum over the all items information. In order to get the item information, we need to take the derivative with respect to θ of the probability of getting each item right. The "test information" is as follows:

$$\sum \frac{\left(\dfrac{\epsilon^{\theta+b}}{\left(\epsilon^{\theta}+\epsilon^{b}\right)^{2}}\right)^{2}}{\left(\dfrac{\epsilon^{\theta-b}}{1+\epsilon^{\theta-b}}\right)\left(1-\dfrac{\epsilon^{\theta-b}}{1+\epsilon^{\theta-b}}\right)}$$
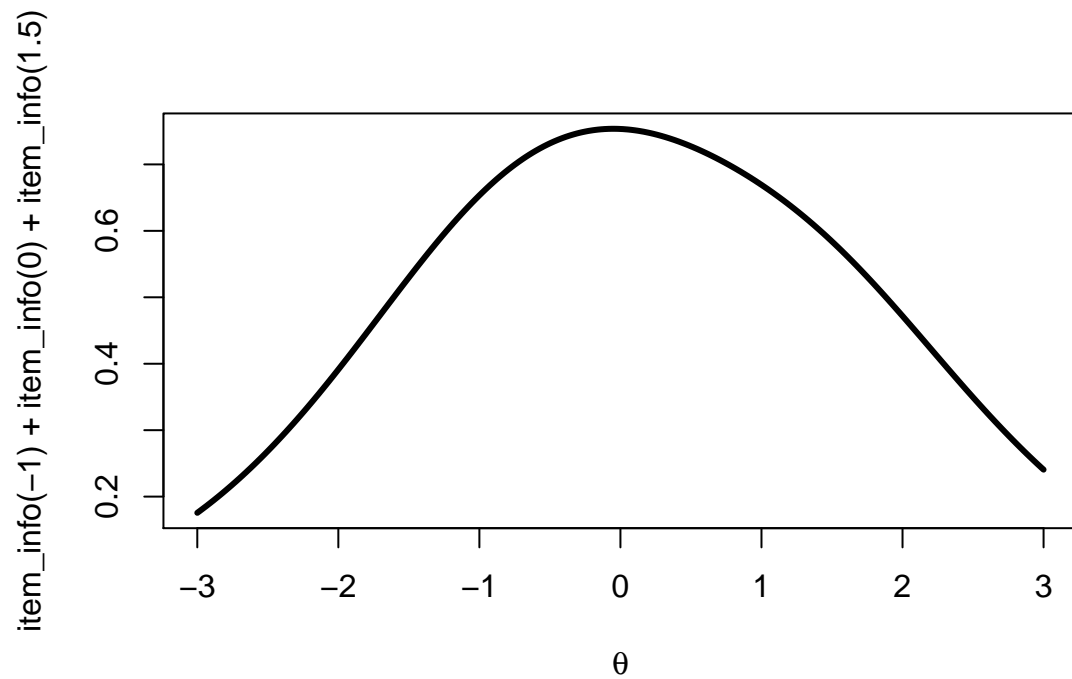
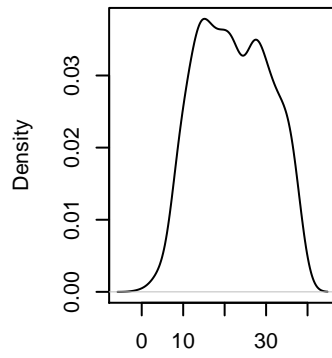Using this formula, we get firgure 3.

Figure 3: Test information curve

7. Where is the function in #6 maximized? What do you think this implies?

   As we are dealing with a Rash model, the maximum information will be at the average of the difficulties of the items in the test. In our case, the maximum is at 0.167.

8. For an item response dataset of your choosing, consider the relationship between theta and the SE across the three IRT models for dichotomous items. How much of a difference does the choice of model have on the size of the error estimate?
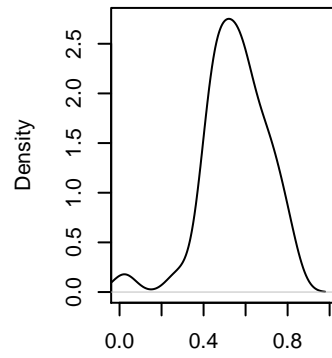
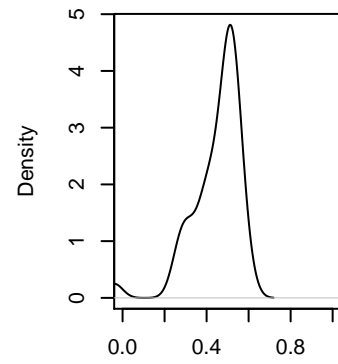# Consulting Excercise

density.default(x = score)   **density.default(x = pv)**   **density.default(x = discriminat**

N = 1000   Bandwidth = 1.946   N = 40   Bandwidth = 0.0558   N = 40   Bandwidth = 0.04016

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