Kalman Filters

MATLAB function examples

```
close all
clearvars
```

List of functions for the exam:

- [kalman_sys, K_bar, P_bar] = kalman(sys, V1, V2, V12, 'delayed')
- [P_bar, K_bar_transposed, eigens] = idare(F', H', V1, V2, V12, eye(nx))

System without exogenous input u(t)

Let us define a system:

```
% Define system matrices
F = [0.5 1; 0.5 0];
H = [0.2 0.6];
V1 = [1 0; 0 10];
V2 = [5];
V12 = [0.25; 0];

% Kalman functions:
% Function to compute the DRE solution
[P_bar, K_bar_transpose, eigens] = idare(F', H', V1, V2, V12, eye(size(F, 1)))
```

```
P_bar = 2×2
    8.5137    1.7534
    1.7534   11.9398
K_bar_transpose = 1×2
    0.9088    0.1369
eigens = 2×1
    0.6230
   -0.3869
```

```
% Function to get the kalman predictor ('delayed') or filter ('current') as
% a discrete time system
[kalman_sys, K_bar, P_bar] = kalman(ss(F, eye(size(F, 1)), H, zeros(size(H, 1)), -1), V1, V2, V
```

```
kalman_sys =

  A =
             x1_e      x2_e
   x1_e    0.3182    0.4547
   x2_e    0.4726  -0.08215

  B =
             y1
   x1_e  0.9088
   x2_e  0.1369

  C =
         x1_e   x2_e
```

```
    y1_e   0.2   0.6
    x1_e     1     0
    x2_e     0     1

  D =
          y1
    y1_e   0
    x1_e   0
    x2_e   0

Input groups:
       Name        Channels
    Measurement       1

Output groups:
         Name         Channels
    OutputEstimate       1
    StateEstimate       2,3

Sample time: unspecified
Discrete-time state-space model.
K_bar = 2×1
    0.9088
    0.1369
P_bar = 2×2
    8.5137    1.7534
    1.7534   11.9398
```

System with exogenous input u(t)

If the system has also G matrix (hence has an input u(t)):

```
% Define system matrices
F = [0.5 1; 0.5 0];
G = [1; 0];
H = [0.2 0.6];
D = 0;
V1 = [1 0; 0 10];
V2 = [5];
V12 = [0.25; 0];

% Kalman functions:
% Function to compute the DRE solution
[P_bar, K_bar_transpose, eigens] = idare(F', H', V1, V2, V12, eye(size(F, 1)))
```

```
P_bar = 2×2
    8.5137    1.7534
    1.7534   11.9398
K_bar_transpose = 1×2
    0.9088    0.1369
eigens = 2×1
    0.6230
   -0.3869
```

```
% Function to get the kalman predictor ('delayed') or filter ('current') as
% a discrete time system
[kalman_sys, K_bar, P_bar] = kalman(ss(F, [G, eye(size(F, 1))], H, [D, zeros(size(H))], -1), V1
```

```
kalman_sys =

  A =
             x1_e      x2_e
   x1_e    0.3182    0.4547
   x2_e    0.4726  -0.08215

  B =
             u1      y1
   x1_e       1  0.9088
   x2_e       0  0.1369

  C =
         x1_e  x2_e
   y1_e   0.2   0.6
   x1_e     1     0
   x2_e     0     1

  D =
         u1  y1
   y1_e   0   0
   x1_e   0   0
   x2_e   0   0

Input groups:
       Name        Channels
    KnownInput        1
    Measurement       2

Output groups:
         Name         Channels
    OutputEstimate       1
    StateEstimate       2,3

Sample time: unspecified
Discrete-time state-space model.
K_bar = 2×1
    0.9088
    0.1369
P_bar = 2×2
    8.5137    1.7534
    1.7534   11.9398
```

Possible questions:

- given the following discrete time system ($F = \dots$, $G = \dots$, $H = \dots$, $D = \dots$, $V_1 = \dots$, $V_2 = \dots$ and $V_{12} = \dots$). Compute the asymptotic state prediction error variance using MATLAB command *idare*.
- given the following discrete time system ($F = \dots$, $G = \dots$, $H = \dots$, $D = \dots$, $V_1 = \dots$, $V_2 = \dots$ and $V_{12} = \dots$). Compute the asymptotic Kalman predictor gain using MATLAB command *kalman*.
- given the following discrete time system ($F = \dots$, $H = \dots$, $V_1 = \dots$, $V_2 = \dots$ and $V_{12} = \dots$). Compute the eigenvalues of the asymptotic Kalman predictor using MATLAB command *idare*.
- given the following discrete time system ($F = \dots$, $H = \dots$, $V_1 = \dots$, $V_2 = \dots$ and $V_{12} = \dots$). Compute the asymptotic Kalman predictor as a discrete time MATLAB system using MATLAB command *kalman*.