Politecnico di Torino
1859
Dipartimento di Scienze Matematiche "G. L. Lagrange"
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Homework Spectral Clustering

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Problem Overview

Algorithm 1

function Spectral clustering

Input

x_i: set of points ($x_i \in R^n$, i=1,..,N)

Output

c_j: set of clusters (j=1,..,K)

Define a similarity function

Compute the similarity graph

Construct the Laplacian matrix of the graph

Compute the smallest eigenvalues according to some criterion

Construct the matrix U that contains as columns eigenvectors corresponding to the smallest eigenvalues

Consider the rows of U as a new set of points y_i (i=1,..,N) and compute Kmeans on them

Associate the clusters obtained for y_i to the corresponding x_i (1 to 1 correspondence)

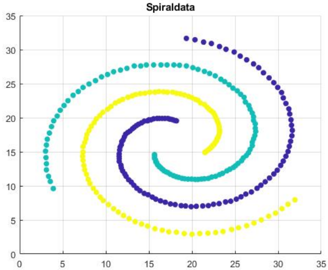
end function

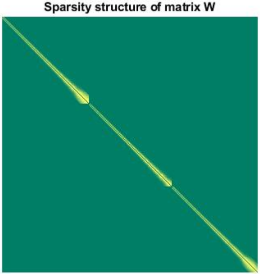
Proposed Approach

For this project we used the following elements:

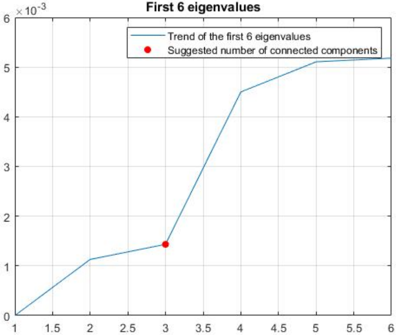
- Circle and Spiral datasets,
- Similarity function $s_{ij} = exp\left(-\frac{\left\|x_i - x_j\right\|^2}{2\sigma^2}\right)$ with $\sigma = 1$,
- K-nearest-neighborhood similarity graph,
- Elbow method for computation of the smallest eigenvalues,
- Function KNN_similarity_graph implemented by us.

Spiral dataset example

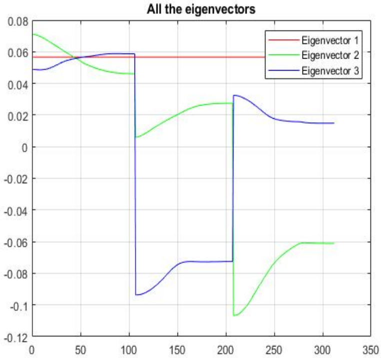


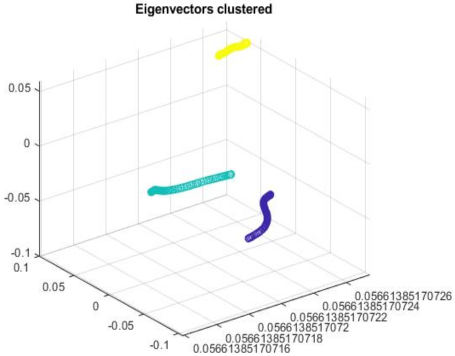


1. Plot of Spiral dataset
2. Spy of matrix W
3. Elbow method

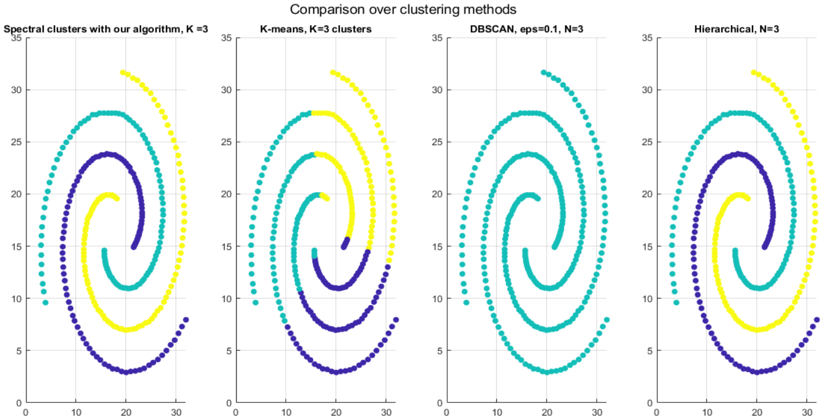


Eigenvector analysis

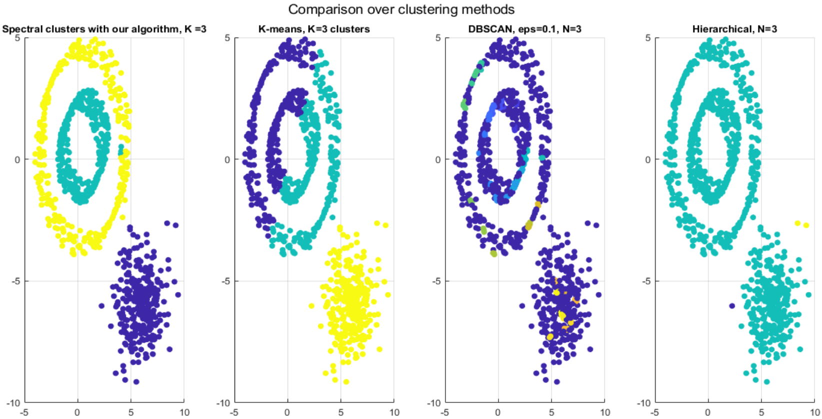




Results for Spiral 2-D dataset



Results for Circle 2-D dataset

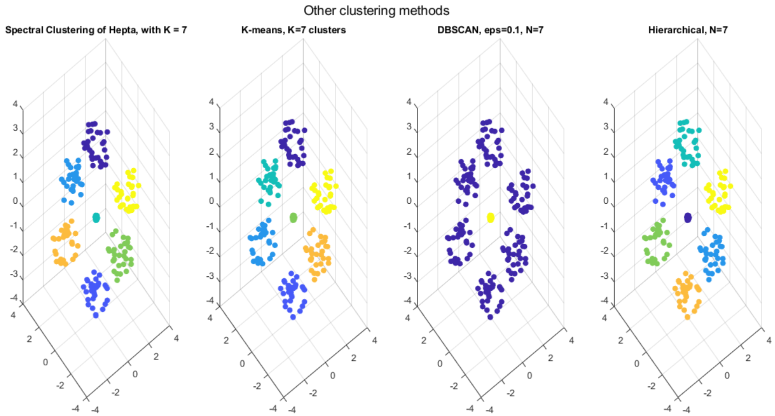


Results for 3-D datasets

Generally we can say that our algorithm about Spectral Clustering is accurate, while the suggestion function for the eigenvalue elbow method can be improved.

	Atom	**EngyTime**	**GolfBall**	**Hepta**	**Target**	**Tetra**	**TwoDiamonds**
SC functions with L	yes	yes	yes	yes	no	yes	yes
SC functions with L_s	yes	yes	yes	yes	no	yes	yes
KNN	10	2	10	10	10	10	10
comp_conn	4	3	3	9	6	6	4
rescaling	yes	no	no	no	yes	no	no
real number of clusters	2	2	1	7	6	4	2
suggested number of clusters	incorrect	correct	correct	correct	correct	correct	incorrect

Results for Hepta 3-D dataset



Normalized symmetric Laplacian matrix L_s: important results

- Zero is eigenvalue for both the matrices L_s and L and has the same multiplicity in both cases.
- The eigenvectors corresponding to the zero eigenvalues have the form $v_0 = D^{\frac{1}{2}} \mathbb{1}_{comp\_conn}$, where $\mathbb{1}_{comp\_conn}$ is the vector with the value 1 on the indices corresponding to one connected component and zero elsewhere.
- The relation between eigenvalues of the two matrices is:

$$\lambda_s = \lambda_L \left[\frac{1 - \mathcal{R}_{D^{-\frac{1}{2}} A D^{-\frac{1}{2}}}(v)}{\mathcal{R}_D(x) - \mathcal{R}_A(x)} \right] \tag{1}$$

where x is the corresponding eigenvector of L, v is the corresponding eigenvector of L_s and $\mathcal{R}_Q(y)$ is the Rayleigh coefficient of matrix Q w.r.t. the normalized vector y.

Grazie per l'attenzione

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