UNIVERZITET U BEOGRADU

ELEKTROTEHNIČKI FAKULTET

POZITIVNOST MTP FUNKCIJA

Treći projektni zadatak

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SADRŽAJ

1. POSTAVKA PROJEKTNOG ZADATKA

Za pogodno izabranu MTP funkciju $f : (0, c) \rightarrow R$ dokazati MTP nejednakost $f(x) > 0$ nad $(0, c)$, određujući pozitivnu nanižnu polinomsku aproksimaciju $P(x) > 0$ nad $(0, c)$. Pod pogodno izabranom MTP funkcijom podrazumevamo da je izbor funkcije takav da je grafički - vizuelno pozitivna nad posmatranim intervalom i da se sastoji od bar dva sabirka od kojih je bar jedan sa pozitivnim i bar jedan sa negativnim koeficijentom.

Teorijska osnova za izradu ovog projektnog zadatka data je u materijalima sa predavanja profersora Maleševića. [1]

2. PREGLED REŠENJA PROJEKTNOG ZADATKA

Programski jezik u kome je rađena implementacija ovog projektnog zadatka je Python (Python 3.9). U izradi projekta su korišćene biblioteke sympy i math, kao i prvi deo rešenja drugog projektnog zadatka, odnosno implementacija Šturmovog algoritma. Implementacija rešenja data je na sledećim slikama:

projekat-3.py

```python
import sympy as sym
import math as m

# KOD IZ DRUGOG PROJEKTNOG ZADATKA

# ispis polinoma
def ispisiPolinom(polinom):
    return str(polinom).replace('x**', 'x^')

# trazenje prvog izvoda polinoma
def nadjiPrviIzvod(polinom):
    return sym.diff(polinom, x, 1)

# trazenje najveceg zajednickog delioca polinoma
def izracunaj_GCD(polinom1, polinom2):
    return sym.gcd(polinom1, polinom2)

# implementacija sturmovog algoritma
def sturmova_teorema(polinom1, polinom2):
    sturm_niz = []
    pol0, rem0 = sym.div(polinom1, polinom2)
    pol1 = sym.diff(pol0, x, 1)
    sturm_niz.append(pol0)
    sturm_niz.append(pol1)
    while sym.degree(pol0) > 1:
        pol, rem = sym.div(pol0, pol1)
        sturm_niz.append(-rem)
        pol0 = pol1
        pol1 = -rem
    return sturm_niz
```

projekat-3.py – Deo 1

projekat-3.py

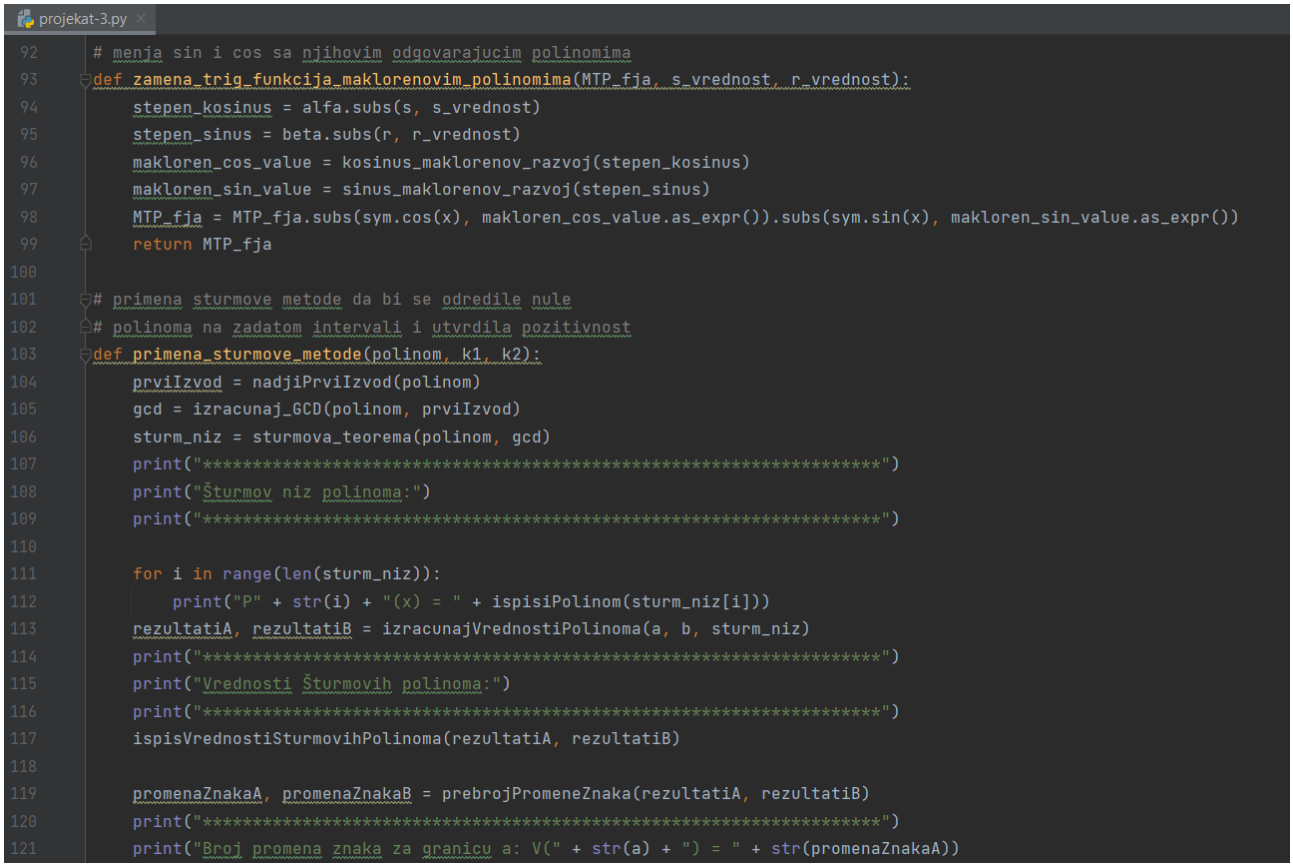
```
31
32  # uvrstavanje granica intervala [a, b] u sturmove polinome
33  def izracunajVrednostiPolinoma(a, b, sturm_niz):
34      rezultatiA = []
35      rezultatiB = []
36      for polinom in sturm_niz:
37          vrednost1 = polinom.subs(x, a)
38          vrednost2 = polinom.subs(x, b)
39          rezultatiA.append(vrednost1)
40          rezultatiB.append(vrednost2)
41      return rezultatiA, rezultatiB
42
43  # ispis vrednosti sturmovih polinoma sa a i b granicama intervala
44  def ispisVrednostiSturmovihPolinoma(rezultatiA, rezultatiB):
45      for i in range(len(rezultatiA)):
46          val1 = rezultatiA[i]
47          val2 = rezultatiB[i]
48          print("P" + str(i) + "(" + str(a) + ") = " + str(val1) + "  P" + str(i) + "(" + str(b) + ") = " + str(val2))
49
50  # prebrojavanje promena znaka za nizove vrednosti sa a i b granicama intervala
51  def prebrojPromeneZnaka(rezultatiA, rezultatiB):
52      promenaZnakaA = 0
53      promenaZnakaB = 0
54      duzina = len(rezultatiA)
55      for i in range(0, duzina - 1):
56          if (rezultatiA[i] < 0 and rezultatiA[i + 1] >= 0) or (rezultatiA[i] >= 0 and rezultatiA[i + 1] < 0):
57              promenaZnakaA += 1
58          if (rezultatiB[i] < 0 and rezultatiB[i + 1] >= 0) or (rezultatiB[i] >= 0 and rezultatiB[i + 1] < 0):
59              promenaZnakaB += 1
60      return promenaZnakaA, promenaZnakaB
```

projekat-3.py – Deo 2

projekat-3.py

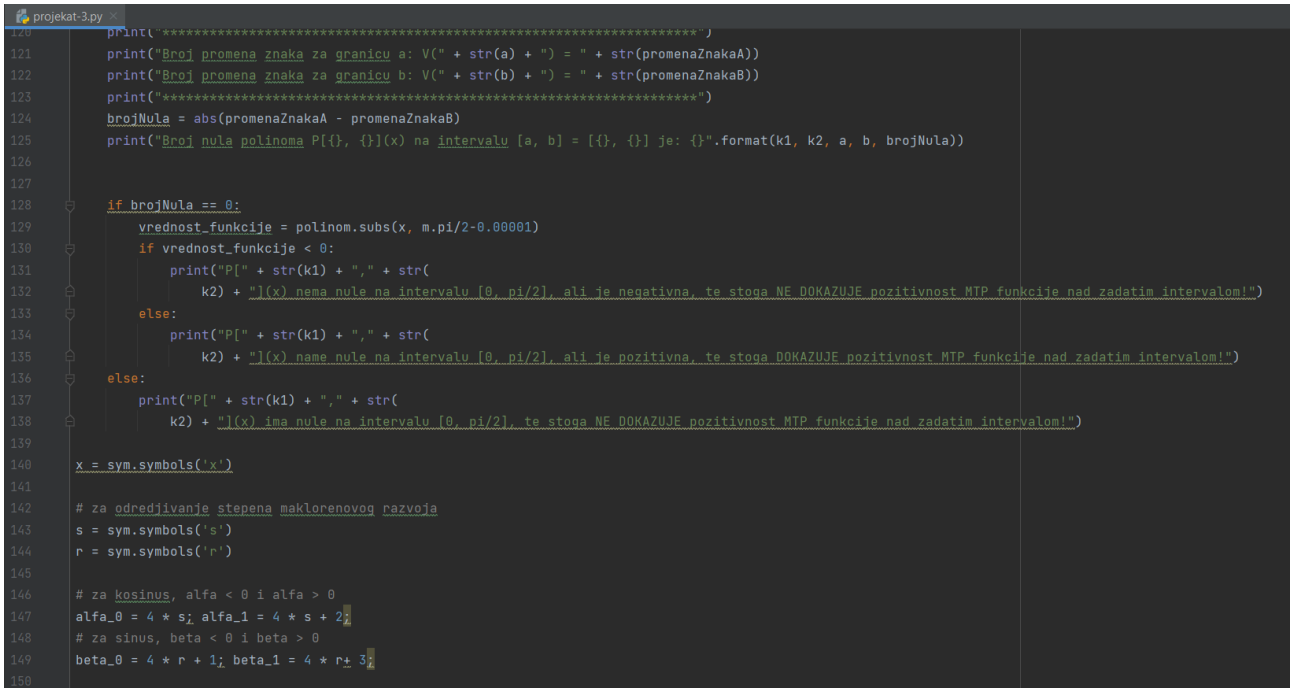
```
61
62  # KOD ZA TRECI PROJEKTNI ZADATAK
63
64  # ispis funkcije
65  def ispisiFunkciju(funkcija):
66      return str(funkcija).replace('**', '^')
67
68  # maklorenov razvoj za funkciju kosinusa do zadatog stepena
69  def kosinus_maklorenov_razvoj(stepen):
70      broj_clanova = stepen / 2 + 1
71      maklorenov_razvoj = sym.Poly(1, x)
72      for k in range(1, broj_clanova):
73          znak = (-1) ** k
74          brojilac = znak * x ** (2 * k)
75          imenilac = m.factorial(2 * k)
76          clan = sym.Poly(brojilac / imenilac)
77          maklorenov_razvoj = maklorenov_razvoj.add(clan)
78      return maklorenov_razvoj
79
80  # maklorenov razvoj za funkciju sinusa do zadatog stepena
81  def sinus_maklorenov_razvoj(stepen):
82      broj_clanova = (stepen - 1) / 2 + 1
83      maklorenov_razvoj = sym.Poly(x, x)
84      for k in range(1, broj_clanova):
85          znak = (-1) ** k
86          brojilac = znak * x ** (2 * k + 1)
87          imenilac = m.factorial(2 * k + 1)
88          clan = sym.Poly(brojilac / imenilac)
89          maklorenov_razvoj = maklorenov_razvoj.add(clan)
90      return maklorenov_razvoj
91
```

projekat-3.py – Deo 3



```
projekat-3.py
92  # menja sin i cos sa njihovim odgovarajucim polinomima
93  def zamena_trig_funkcija_maklorenovim_polinomima(MTP_fja, s_vrednost, r_vrednost):
94      stepen_kosinus = alfa.subs(s, s_vrednost)
95      stepen_sinus = beta.subs(r, r_vrednost)
96      makloren_cos_value = kosinus_maklorenov_razvoj(stepen_kosinus)
97      makloren_sin_value = sinus_maklorenov_razvoj(stepen_sinus)
98      MTP_fja = MTP_fja.subs(sym.cos(x), makloren_cos_value.as_expr()).subs(sym.sin(x), makloren_sin_value.as_expr())
99      return MTP_fja
100
101 # primena sturmove metode da bi se odredile nule
102 # polinoma na zadatom intervali i utvrdila pozitivnost
103 def primena_sturmove_metode(polinom, k1, k2):
104     prviIzvod = nadjiPrviIzvod(polinom)
105     gcd = izracunaj_GCD(polinom, prviIzvod)
106     sturm_niz = sturmova_teorema(polinom, gcd)
107     print("*****************************************************************")
108     print("Šturmov niz polinoma:")
109     print("*****************************************************************")
110
111     for i in range(len(sturm_niz)):
112         print("P" + str(i) + "(x) = " + ispisiPolinom(sturm_niz[i]))
113     rezultatiA, rezultatiB = izracunajVrednostiPolinoma(a, b, sturm_niz)
114     print("*****************************************************************")
115     print("Vrednosti Šturmovih polinoma:")
116     print("*****************************************************************")
117     ispisVrednostiSturmovihPolinoma(rezultatiA, rezultatiB)
118
119     promenaZnakaA, promenaZnakaB = prebrojPromeneZnaka(rezultatiA, rezultatiB)
120     print("*****************************************************************")
121     print("Broj promena znaka za granicu a: V(" + str(a) + ") = " + str(promenaZnakaA))
```

projekat-3.py – Deo 4



```
projekat-3.py
120     print("*****************************************************************")
121     print("Broj promena znaka za granicu a: V(" + str(a) + ") = " + str(promenaZnakaA))
122     print("Broj promena znaka za granicu b: V(" + str(b) + ") = " + str(promenaZnakaB))
123     print("*****************************************************************")
124     brojNula = abs(promenaZnakaA - promenaZnakaB)
125     print("Broj nula polinoma P[{}, {}](x) na intervalu [a, b] = [{}, {}] je: {}".format(k1, k2, a, b, brojNula))
126
127
128     if brojNula == 0:
129         vrednost_funkcije = polinom.subs(x, m.pi/2-0.00001)
130         if vrednost_funkcije < 0:
131             print("P[" + str(k1) + "," + str(
132                 k2) + "](x) nema nule na intervalu [0, pi/2], ali je negativna, te stoga NE DOKAZUJE pozitivnost MTP funkcije nad zadatim intervalom!")
133         else:
134             print("P[" + str(k1) + "," + str(
135                 k2) + "](x) name nule na intervalu [0, pi/2], ali je pozitivna, te stoga DOKAZUJE pozitivnost MTP funkcije nad zadatim intervalom!")
136     else:
137         print("P[" + str(k1) + "," + str(
138             k2) + "](x) ima nule na intervalu [0, pi/2], te stoga NE DOKAZUJE pozitivnost MTP funkcije nad zadatim intervalom!")
139
140 x = sym.symbols('x')
141
142 # za odredjivanje stepena maklorenovog razvoja
143 s = sym.symbols('s')
144 r = sym.symbols('r')
145
146 # za kosinus, alfa < 0 i alfa > 0
147 alfa_0 = 4 * s; alfa_1 = 4 * s + 2;
148 # za sinus, beta < 0 i beta > 0
149 beta_0 = 4 * r + 1; beta_1 = 4 * r+ 3;
150
```

projekat-3.py – Deo 5

projekat-3.py

```
150
151    # PRIMER:
152    MTP_funkcija = x ** 3 * sym.sin(x) - x * sym.cos(x) ** 3 + x - 3 / 2 * x ** 3 + 3 / 32 * x ** 4
153    # koriscena poredjenja
154    alfa = alfa_0
155    beta = beta_1
156    # granice intervala [0-PI/2]
157    a = 0.0001; b = m.pi / 2
158
159    print("MTP funkcija za koju utvrdjujemo pozitivnost: " + ispisiFunkciju(MTP_funkcija))
160
161    # odredjivanje koeficijenata k1 i k2 za koje je polinom
162    # i funkcija pozitivna na zadatom intervalu
163    P0 = zamena_trig_funkcija_maklorenovim_polinomima(MTP_funkcija, 0, 0)
164    P1 = zamena_trig_funkcija_maklorenovim_polinomima(MTP_funkcija, 0, 1)
165    P2 = zamena_trig_funkcija_maklorenovim_polinomima(MTP_funkcija, 1, 0)
166    P3 = zamena_trig_funkcija_maklorenovim_polinomima(MTP_funkcija, 1, 1)
167
168    print("")
169    print("P0(x) = P[0,0](x) = " + ispisiPolinom(P0))
170    primena_sturmove_metode(P0, 0, 0)
171    print("**********************************************************************\n")
172    print("P1(x) = P[0,1](x) = " + ispisiPolinom(P1))
173    primena_sturmove_metode(P1, 0, 1)
174    print("**********************************************************************\n")
175    print("P2(x) = P[1,0](x) = " + ispisiPolinom(P2))
176    primena_sturmove_metode(P2, 1, 0)
177    print("**********************************************************************\n")
178    print("P3(x) = P[1,1](x) = " + ispisiPolinom(P3))
179    primena_sturmove_metode(P3, 1, 1)
180    print("**********************************************************************")
```

projekat-3.py – Deo 6

3. TESTIRANJE REŠENJA PROJEKTNOG ZADATKA

U nastavku će biti izloženo testiranje rešenja projektnog zadatka sa tri različita seta ulaznih podataka.

1. Test primer 1(primer sa predavanja):

```
C:\Users\Korisnik\AppData\Local\Programs\Python\Python39\python.exe C:/Users/Korisnik/Desktop/OPNA-DZ3/projekat-3.py
MTP funkcija za koju utvrdjujemo pozitivnost: 0.09375*x^4 + x^3*sin(x) - 1.5*x^3 - x*cos(x)^3 + x

P0(x) = P[0,0](x) = 0.09375*x^4 + x^3*(-x^3/6 + x) - 1.5*x^3
**********************************************************************
Šturmov niz polinoma:
**********************************************************************
P0(x) = -0.166666666666667*x^4 + 1.09375*x^2 - 1.5*x
P1(x) = -0.666666666666667*x^3 + 2.1875*x - 1.5
P2(x) = -0.546875*x^2 + 1.125*x
P3(x) = 0.633724489795918*x + 1.5
P4(x) = 5.72669395367387
**********************************************************************
Vrednosti Šturmovih polinoma:
**********************************************************************
P0(0.0001) = -0.000149989062500017  P0(1.5707963267948966) = -0.672152568373665
P1(0.0001) = -1.49978125000067  P1(1.5707963267948966) = -0.647739425161149
P2(0.0001) = 0.000112494531250000  P2(1.5707963267948966) = 0.417785890932823
P3(0.0001) = 1.50006337244898  P3(1.5707963267948966) = 2.49545210077140
P4(0.0001) = 5.72669395367387  P4(1.5707963267948966) = 5.72669395367387
**********************************************************************
Broj promena znaka za granicu a: V(0.0001) = 1
Broj promena znaka za granicu b: V(1.5707963267948966) = 1
**********************************************************************
Broj nula polinoma P[0, 0](x) na intervalu [a, b] = [0.0001, 1.5707963267948966] je: 0
P[0,0](x) nema nule na intervalu [0, pi/2], ali je negativna, te stoga NE DOKAZUJE pozitivnost MTP funkcije nad zadatim intervalom!
**********************************************************************
```

Test primer 1 – ispitivanje P[0,0]

```
P1(x) = P[0,1](x) = 0.09375*x^4 + x^3*(-x^7/5040 + x^5/120 - x^3/6 + x) - 1.5*x^3
**********************************************************************
Šturmov niz polinoma:
**********************************************************************
P0(x) = -0.000198412698412698*x^8 + 0.00833333333333333*x^6 - 0.166666666666667*x^4 + 1.09375*x^2 - 1.5*x
P1(x) = -0.00158730158730159*x^7 + 0.05*x^5 - 0.666666666666667*x^3 + 2.1875*x - 1.5
P2(x) = -0.00208333333333333*x^6 + 0.0833333333333333*x^4 - 0.8203125*x^2 + 1.3125*x
P3(x) = 0.0134920634920635*x^5 + 0.0416666666666665*x^3 + 1.0*x^2 - 2.1875*x + 1.5
P4(x) = -0.0897671568627451*x^4 - 0.154411764705882*x^3 + 1.15808823529412*x^2 - 1.54411764705882*x
P5(x) = -0.255649357204704*x^3 - 0.468508660555161*x^2 + 1.78828742909061*x - 1.5
P6(x) = -0.511654968176454*x^2 + 0.946783909171182*x + 0.0592462492321491
P7(x) = 1.6090276725249 - 0.016369638889228*x
P8(x) = 4850.28121310413
**********************************************************************
Vrednosti Šturmovih polinoma:
**********************************************************************
P0(0.0001) = -0.000149989062500017  P0(1.5707963267948966) = -0.554325765992263
P1(0.0001) = -1.49978125000067  P1(1.5707963267948966) = -0.207037700766696
P2(0.0001) = 0.000131241796875008  P2(1.5707963267948966) = 0.513674008508215
P3(0.0001) = 1.49978126000004  P3(1.5707963267948966) = 0.821801316564573
P4(0.0001) = -0.000154400183977950  P4(1.5707963267948966) = -0.713001451726683
P5(0.0001) = -1.49982117594243  P5(1.5707963267948966) = -0.837805297500633
P6(0.0001) = 0.0593409225065165  P6(1.5707963267948966) = 0.283992904588364
P7(0.0001) = 1.60902603556101  P7(1.5707963267948966) = 1.58331430388674
P8(0.0001) = 4850.28121310413  P8(1.5707963267948966) = 4850.28121310413
**********************************************************************
Broj promena znaka za granicu a: V(0.0001) = 3
Broj promena znaka za granicu b: V(1.5707963267948966) = 3
**********************************************************************
Broj nula polinoma P[0, 1](x) na intervalu [a, b] = [0.0001, 1.5707963267948966] je: 0
P[0,1](x) nema nule na intervalu [0, pi/2], ali je negativna, te stoga NE DOKAZUJE pozitivnost MTP funkcije nad zadatim intervalom!
**********************************************************************
```

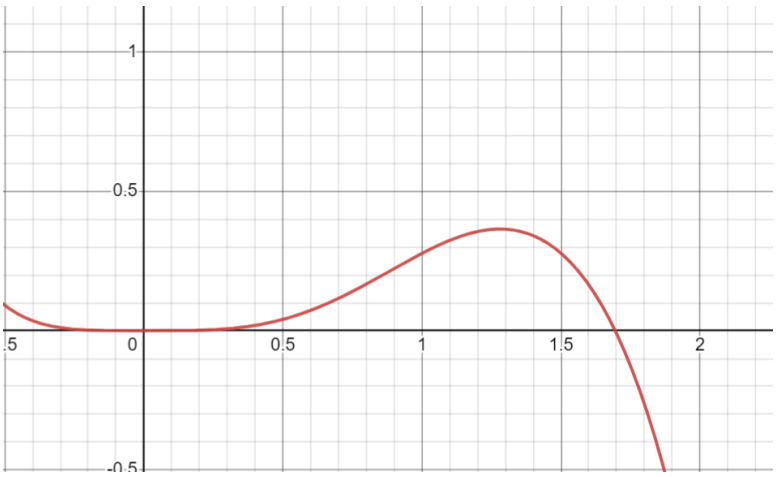
Test primer 1 – ispitivanje P[0,1]

```
P2(x) = P[1,0](x) = 0.09375*x^4 + x^3*(-x^3/6 + x) - 1.5*x^3 - x*(x^4/24 - x^2/2 + 1)**3 + x
**************************************************************************
Šturmov niz polinoma:
**************************************************************************
P0(x) = -7.2337962962963e-5*x^10 + 0.00260416666666667*x^8 - 0.0364583333333333*x^6 + 0.25*x^4 - 0.166666666666667*x^3 - 0.875*x^2 + 1.09375*x
P1(x) = -0.00072337962962963*x^9 + 0.0208333333333333*x^7 - 0.21875*x^5 + 1.0*x^3 - 0.5*x^2 - 1.75*x + 1.09375
P2(x) = -0.000520833333333333*x^8 + 0.0145833333333333*x^6 - 0.15*x^4 + 0.116666666666667*x^3 + 0.7*x^2 - 0.984375*x
P3(x) = -0.000578703703703699*x^7 + 0.0104166666666667*x^5 + 0.162037037037037*x^4 - 0.0277777777777777*x^3 - 0.8671875*x^2 + 1.75*x - 1.09375
P4(x) = -0.00520833333333327*x^6 + 0.145833333333334*x^5 + 0.125*x^4 - 0.897135416666673*x^3 + 0.875000000000012*x^2 - 7.21644966006352e-15*x
P5(x) = 0.45717592592594*x^5 + 0.127170138888896*x^4 - 2.66608796296304*x^3 + 3.58940972222232*x^2 - 1.75000000000002*x + 1.09375
P6(x) = -0.0536581839306923*x^4 - 0.00265363999158796*x^3 + 0.301287682387631*x^2 - 0.576233976926761*x + 0.352358458680492
P7(x) = 0.1042418737776972*x^3 + 0.733089247278539*x^2 - 0.129272783835378*x - 1.78037134426302
P8(x) = 2.40037518851583*x^2 + 1.02799819744564*x - 6.75197806501617
P9(x) = 0.130890213543531*x - 0.156147809746992
P10(x) = 2.10946216457649
**************************************************************************
Vrednosti Šturmovih polinoma:
**************************************************************************
P0(0.0001) = 0.000109366249833358   P0(1.5707963267948966) = -0.0226241077091411
P1(0.0001) = 1.09357499500100   P1(1.5707963267948966) = -0.655522602749865
P2(0.0001) = -9.84304998833483e-5   P2(1.5707963267948966) = -0.0803451419439107
P3(0.0001) = -1.09357500867190   P3(1.5707963267948966) = 0.480236694636935
P4(0.0001) = 8.74910287636326e-9   P4(1.5707963267948966) = 0.759263774278954
P5(0.0001) = 1.09357503589143   P5(1.5707963267948966) = 2.01443379730940
P6(0.0001) = 0.352300838295673   P6(1.5707963267948966) = -0.146349816303874
P7(0.0001) = -1.78038426421041   P7(1.5707963267948966) = 0.229411704566917
P8(0.0001) = -6.75187524119267   P8(1.5707963267948966) = 0.785486108693591
P9(0.0001) = -0.156134720725637   P9(1.5707963267948966) = 0.0494540569005870
P10(0.0001) = 2.10946216457649   P10(1.5707963267948966) = 2.10946216457649
**************************************************************************
Broj promena znaka za granicu a: V(0.0001) = 4
Broj promena znaka za granicu b: V(1.5707963267948966) = 3
**************************************************************************
Broj nula polinoma P[1, 0](x) na intervalu [a, b] = [0.0001, 1.5707963267948966] je: 1
P[1,0](x) ima nule na intervalu [0, pi/2], te stoga NE DOKAZUJE pozitivnost MTP funkcije nad zadatim intervalom!
**************************************************************************
```

Test primer 1 – ispitivanje P[1,0]

```
P3(x) = P[1,1](x) = 0.09375*x^4 + x^3*(-x^7/5040 + x^5/120 - x^3/6 + x) - 1.5*x^3 - x*(x^4/24 - x^2/2 + 1)**3 + x
**************************************************************************
Šturmov niz polinoma:
**************************************************************************
P0(x) = -7.2337962962963e-5*x^10 + 0.00260416666666667*x^8 - 0.00019841269841269*x^7 - 0.0364583333333333*x^6 + 0.00833333333333333*x^5 + 0.25*x^4 - 0.166666666666667*x^3 - 0.875*x^2 + 1.09375*x
P1(x) = -0.00072337962962963*x^9 + 0.0208333333333333*x^7 - 0.00138888888888889*x^6 - 0.21875*x^5 + 0.0416666666666667*x^4 + 1.0*x^3 - 0.5*x^2 - 1.75*x + 1.09375
P2(x) = -0.000520833333333333*x^8 + 5.952380952380952e-5*x^7 + 0.0145833333333333*x^6 - 0.00416666666666667*x^5 - 0.15*x^4 + 0.116666666666667*x^3 + 0.7*x^2 - 0.984375*x
P3(x) = -0.000569255479969761*x^7 - 0.00208333333333333*x^6 + 0.00975529100529*x^5 + 0.0965608465608466*x^4 - 0.00925925925925916*x^3 - 0.756076388888889*x^2 + 1.59375*x - 1.09375
P4(x) = 0.00153591894652886*x^6 + 0.0588287262616004*x^5 - 0.191897091992223*x^4 - 0.776457200755559*x^3 + 3.368920424708654*x^2 - 5.519570836418 2*x + 3.77672829755003
P5(x) = 0.816692693711262*x^5 - 2.272626761134766*x^4 - 11.20860207785 43*x^3 + 46.0566976157 85*x^2 - 73.8614807241908*x + 49.5846680693732
P6(x) = -0.0047797266987464*x^4 - 0.00297226513303905*x^3 + 0.0507984220947582*x^2 - 0.0941748827311 2*x + 0.0544916144600 26
P7(x) = 0.795839364222781*x^3 - 0.414709771987287*x^2 + 9.76684874140465*x - 17.8855422461216
P8(x) = -0.106337682596458*x^2 + 0.134500077185102*x + 0.0673895611362704
P9(x) = 17.5072307406969 - 11.0287888129215*x
P10(x) = -0.0129387489752660
**************************************************************************
Vrednosti Šturmovih polinoma:
**************************************************************************
P0(0.0001) = 0.000109366249833358   P0(1.5707963267948966) = 0.0523867644017071
P1(0.0001) = 1.09357499500100   P1(1.5707963267948966) = -0.422716575612170
P2(0.0001) = -9.84304998833483e-5   P2(1.5707963267948966) = -0.118786928826399
P3(0.0001) = -1.09359063256077   P3(1.5707963267948966) = 0.144708630500385
P4(0.0001) = 3.77617637415482   P4(1.5707963267948966) = -0.172920759768166
P5(0.0001) = 49.5772823818566   P5(1.5707963267948966) = -2.26423574253450
P6(0.0001) = 0.0544821974797342   P6(1.5707963267948966) = -0.00871702430353827
P7(0.0001) = -17.8845655653938   P7(1.5707963267948966) = -0.467062388096117
P8(0.0001) = 0.0674030100806124   P8(1.5707963267948966) = 0.0162840723126698
P9(0.0001) = 17.5061278618156   P9(1.5707963267948966) = 0.183249784363213
P10(0.0001) = -0.0129387489752660   P10(1.5707963267948966) = -0.0129387489752660
**************************************************************************
Broj promena znaka za granicu a: V(0.0001) = 5
Broj promena znaka za granicu b: V(1.5707963267948966) = 5
**************************************************************************
Broj nula polinoma P[1, 1](x) na intervalu [a, b] = [0.0001, 1.5707963267948966] je: 0
P[1,1](x) name nule na intervalu [0, pi/2], ali je pozitivna, te stoga DOKAZUJE pozitivnost MTP funkcije nad zadatim intervalom!
**************************************************************************
```

Test primer 1 – ispitivanje P[1,1] – dokazana pozitivnost MTP funkcije



Test primer 1 – grafik MTP funkcije

2. Test primer 2:

```
# za kosinus, alfa < 0 i alfa > 0
alfa_0 = 4 * s; alfa_1 = 4 * s + 2;
# za sinus, beta < 0 i beta > 0
beta_0 = 4 * r + 1; beta_1 = 4 * r+ 3;

# PRIMER:
MTP_funkcija = x**4 * sym.cos(x)**4 + x**3 - 0.5 * x**2 + 2 * x * sym.sin(x) + 1
# koriscena poredjenja
alfa = alfa_1
beta = beta_1
# granice intervala [0-PI/2]
a = 0.0001; b = m.pi / 2

```

Test primer 2 – ulazna MTP funkcija

```
C:\Users\Korisnik\AppData\Local\Programs\Python\Python39\python.exe C:/Users/Korisnik/Desktop/OPNA-DZ3/projekat-3.py
MTP funkcija za koju utvrdjujemo pozitivnost: x^4*cos(x)^4 + x^3 - 0.5*x^2 + 2*x*sin(x) + 1

P0(x) = P[0,0](x) = x^4*(1 - x^2/2)**4 + x^3 - 0.5*x^2 + 2*x*(-x^3/6 + x) + 1
**********************************************************************
Šturmov niz polinoma:
**********************************************************************
P0(x) = 0.0625*x^12 - 0.5*x^10 + 1.5*x^8 - 2.0*x^6 + 0.666666666666667*x^4 + 1.0*x^3 + 1.5*x^2 + 1.0
P1(x) = 0.75*x^11 - 5.0*x^9 + 12.0*x^7 - 12.0*x^5 + 2.66666666666667*x^3 + 3.0*x^2 + 3.0*x
P2(x) = 0.0833333333333334*x^10 - 0.5*x^8 + 1.0*x^6 - 0.444444444444444*x^4 - 0.75*x^3 - 1.25*x^2 - 1.0
P3(x) = 0.500000000000002*x^9 - 3.0*x^7 + 8.0*x^5 - 6.75*x^4 - 13.9166666666667*x^3 - 3.0*x^2 - 12.0*x
P4(x) = 9.43689570931383e-16*x^8 + 0.333333333333329*x^6 - 1.125*x^5 - 1.87499999999999*x^4 + 0.250000000000002*x^3 - 0.749999999999993*x^2 + 1.0
P5(x) = 176611750092962.0*x^7 - 596064656563742.0*x^6 - 993441094272911.0*x^5 + 1324588125697290*x^4 - 397376437709145.0*x^3 + 3.0*x^2 + 529835250278896.0*x
P6(x) = -0.333333333333346*x^6 + 1.12499999999998*x^5 + 1.87499999999999*x^4 - 0.250000000000009*x^3 + 0.749999999999996*x^2 + 9.55485690568033e-15*x - 1.0
P7(x) = 132.953124999995*x^5 + 149.703124999994*x^4 - 23.90625*x^3 + 54.6562499999975*x^2 - 28.5624999999992*x - 83.6249999999969
P8(x) = -0.125717666858223*x^4 - 0.156804791221527*x^3 - 0.0616134714856499*x^2 - 0.112657155662573*x + 0.0563220230918103
P9(x) = 68.9518220234847*x^3 + 56.5812920983679*x^2 - 45.4520044376667*x + 90.8496426192279
P10(x) = 0.100466554792941*x^2 - 0.0176261656159009*x - 0.126999620780175
P11(x) = -53.7590452697253*x - 177.665923375309
P12(x) = -1.02855563646198
**********************************************************************
Vrednosti Šturmovih polinoma:
**********************************************************************
P0(0.0001) = 1.00000001500100  P0(1.5707963267948966) = 6.56569028107653
P1(0.0001) = 0.000300030002666667  P1(1.5707963267948966) = 7.48136971119510
P2(0.0001) = -1.00000001250075  P2(1.5707963267948966) = -5.58638127593156
P3(0.0001) = -0.00120003001391734  P3(1.5707963267948966) = -86.4569744059767
P4(0.0001) = 0.999999992500250  P4(1.5707963267948966) = -17.0480016945198
P5(0.0001) = 52983524630.5264  P5(1.5707963267948966) = -1.41884255510608e+16
P6(0.0001) = -0.999999992500250  P6(1.5707963267948966) = 17.0480016945194
P7(0.0001) = -83.6278557034583  P7(1.5707963267948966) = 2096.56142313271
P8(0.0001) = 0.0563107567599525  P8(1.5707963267948966) = -1.64578389169526
P9(0.0001) = 90.8450979846660  P9(1.5707963267948966) = 426.304952290289
P10(0.0001) = -0.127001382392071  P10(1.5707963267948966) = 0.0932045508515629
P11(0.0001) = -177.671299279836  P11(1.5707963267948966) = -262.110434216994
P12(0.0001) = -1.02855563646198  P12(1.5707963267948966) = -1.02855563646198
**********************************************************************
Broj promena znaka za granicu a: V(0.0001) = 5
Broj promena znaka za granicu b: V(1.5707963267948966) = 5
**********************************************************************
Broj nula polinoma P[0, 0](x) na intervalu [a, b] = [0.0001, 1.5707963267948966] je: 0
P[0,0](x) name nule na intervalu [0, pi/2], ali je pozitivna, te stoga DOKAZUJE pozitivnost MTP funkcije nad zadatim intervalom!
**********************************************************************
```

Test primer 2 – ispitivanje P[0,0] – dokazana pozitivnost MTP funkcije

```
P1(x) = P[0,1](x) = x^4*(1 - x^2/2)**4 + x^3 - 0.5*x^2 + 2*x*(-x^7/5040 + x^5/120 - x^3/6 + x) + 1
**********************************************************************
Šturmov niz polinoma:
**********************************************************************
P0(x) = 0.0625*x^12 - 0.5*x^10 + 1.49960317460317*x^8 - 1.983333333333333*x^6 + 0.666666666666667*x^4 + 1.0*x^3 + 1.5*x^2 + 1.0
P1(x) = 0.75*x^11 - 5.0*x^9 + 11.9968253968254*x^7 - 11.9*x^5 + 2.66666666666667*x^3 + 3.0*x^2 + 3.0*x
P2(x) = 0.0833333333333334*x^10 - 0.499867724867725*x^8 + 0.991666666666667*x^6 - 0.444444444444444*x^4 - 0.75*x^3 - 1.25*x^2 - 1.0
P3(x) = 0.501190476190478*x^9 - 3.07182539682254*x^7 + 7.9*x^5 - 6.75*x^4 - 13.9166666666667*x^3 - 3.0*x^2 - 12.0*x
P4(x) = -0.0108870917065684*x^8 + 0.321872525732381*x^6 - 1.12232779097387*x^5 - 1.86949063077329*x^4 + 0.251187648456058*x^3 - 0.745249406175767*x^2 + 1.0
P5(x) = -11.74567121222*x^7 + 51.666690716*x^6 + 78.1625523072983*x^5 - 4.813497445962*x^4 + 48.2244427770263*x^3 + 3.0*x^2 - 34.0352947966901*x
P6(x) = -0.0387656876148149*x^6 + 1.43655405202342*x^5 + 1.8945641759672*x^4 - 0.0517840554698333*x^3 + 0.725933743302793*x^2 - 0.138770247795007*x - 1.0
P7(x) = 14710.961903998*x^5 + 18736.3441667409*x^4 - 340.689859669919*x^3 + 7138.26294718395*x^2 - 1642.12496862549*x - 9895.26827976584
P8(x) = -0.00114268116422211*x^4 - 0.00143884209600861*x^3 - 0.000583652745921315*x^2 - 0.00102219926676381*x + 0.000497119125560497
P9(x) = 8122.36082641503*x^3 + 6130.18889599961*x^2 - 4567.65341808904*x + 9802.78419521634
P10(x) = 0.00079119939202291 8*x^2 - 3.27330341119889e-5*x - 0.00119280155359564
P11(x) = -7945.02516992917*x - 19551.1740667945
P12(x) = -0.00367891254539449
**********************************************************************
Vrednosti Šturmovih polinoma:
**********************************************************************
P0(0.0001) = 1.00000001500100   P0(1.5707963267948966) = 6.80134388583934
P1(0.0001) = 0.000300030002666667   P1(1.5707963267948966) = 8.36277315998399
P2(0.0001) = -1.00000001250075   P2(1.5707963267948966) = -5.70665943904587
P3(0.0001) = -0.00120003001391734   P3(1.5707963267948966) = -89.0387703260879
P4(0.0001) = 0.999999992547757   P4(1.5707963267948966) = -17.5482672496447
P5(0.0001) = -0.00340349943144505   P5(1.5707963267948966) = 1357.99033462348
P6(0.0001) = -1.00001386976549   P6(1.5707963267948966) = 25.0623270204148
P7(0.0001) = -9895.43242088041   P7(1.5707963267948966) = 258568.568754740
P8(0.0001) = 0.000497016899795854   P8(1.5707963267948966) = -0.0150820160181854
P9(0.0001) = 9802.32749118454   P9(1.5707963267948966) = 49234.0866960083
P10(0.0001) = -0.00119280481898706   P10(1.5707963267948966) = 0.000707987767068552
P11(0.0001) = -19551.9685693115   P11(1.5707963267948966) = -32031.1904200123
P12(0.0001) = -0.00367891254539449   P12(1.5707963267948966) = -0.00367891254539449
**********************************************************************
Broj promena znaka za granicu a: V(0.0001) = 5
Broj promena znaka za granicu b: V(1.5707963267948966) = 5
**********************************************************************
Broj nula polinoma P[0, 1](x) na intervalu [a, b] = [0.0001, 1.5707963267948966] je: 0
P[0,1](x) name nule na intervalu [0, pi/2], ali je pozitivna, te stoga DOKAZUJE pozitivnost MTP funkcije nad zadatim intervalom!
**********************************************************************
```

Test primer 2 – ispitivanje P[0,1] – dokazana pozitivnost MTP funkcije

```
P2(x) = P[1,0](x) = x^4*(-x^6/720 + x^4/24 - x^2/2 + 1)**4 + x^3 - 0.5*x^2 + 2*x*(-x^3/6 + x) + 1
**********************************************************************
Šturmov niz polinoma:
**********************************************************************
P0(x) = 3.72108862978205e-12*x^28 - 4.46530635573845e-10*x^26 + 2.54522462277092e-8*x^24 - 8.94847393689986e-7*x^22
  + 2.13396990740741e-5*x^20 - 0.000358796296296296*x^18 + 0.00429398148148148*x^16 - 0.0361111111111111*x^14
  + 0.20625*x^12 - 0.755555555555556*x^10 + 1.66666666666667*x^8 - 2.0*x^6 + 0.666666666666667*x^4 + 1.0*x^3 + 1.5*x^2 + 1.0
P1(x) = 1.04190481633897e-10*x^27 - 1.160979652492e-8*x^25 + 6.10853909465021e-7*x^23 - 1.96866426611797e-5*x^21
  + 0.000426793981481481*x^19 - 0.00645833333333333*x^17 + 0.0687037037037037*x^15 - 0.505555555555556*x^13
  + 2.475*x^11 - 7.55555555555556*x^9 + 13.3333333333333*x^7 - 12.0*x^5 + 2.66666666666667*x^3 + 3.0*x^2 + 3.0*x
P2(x) = 3.18950453981318e-11*x^26 - 3.63603517538703e-9*x^24 + 1.91753012933569e-7*x^22 - 6.09705687830688e-6*x^20
  + 0.000128141534391534*x^18 - 0.00184027777777778*x^16 + 0.0180555555555556*x^14 - 0.117857142857143*x^12
  + 0.485714285714286*x^10 - 1.19047619047619*x^8 + 1.57142857142857*x^6 - 0.571428571428571*x^4 - 0.892857142857143*x^3
  - 1.39285714285714*x^2 - 1.0
P3(x) = -2.67918381344308e-10*x^25 + 1.55392661179702e-8*x^23 - 2.30409807956114e-7*x^21 - 8.19830246913573e-6*x^19
  + 0.000446759259259259*x^17 - 0.00972222222222222*x^15 + 0.120555555555556*x^13 - 0.888333333333333*x^11
  + 3.66666666666667*x^9 - 8.2*x^7 + 10.1333333333333*x^5 - 2.91666666666667*x^4 - 7.21666666666667*x^3 - 3.0*x^2 - 6.26666666666667*x
P4(x) = 1.78612254229534e-9*x^24 - 1.64323273891174e-7*x^22 + 7.0730452674897e-6*x^20 - 0.000181327160493827*x^18
  + 0.00299768518518518*x^16 - 0.0324074074074074*x^14 + 0.223611111111111*x^12 - 0.922222222222221*x^10 + 2.16666666666667*x^8
  - 2.77777777777778*x^6 + 0.347222222222222*x^5 + 1.43055555555555*x^4 + 1.25*x^3 + 2.13888888888889*x^2 + 1.0
P5(x) = 9.10922496570655e-9*x^23 - 8.3054698216737e-7*x^21 + 3.53973765432105e-5*x^19 - 0.000896412037037048*x^17
  + 0.0145833333333335*x^15 - 0.154097222222223*x^13 + 1.02666666666667*x^11 - 3.99166666666667*x^9 + 8.61666666666668*x^7
  - 0.0520833333333346*x^6 - 10.3479166666667*x^5 + 2.72916666666666*x^4 + 6.89583333333333*x^3 + 3.0*x^2 + 6.11666666666666*x
P6(x) = 1.47092444659777e-9*x^22 - 1.32383200193779e-7*x^20 + 5.56009440813769e-6*x^18 - 0.000138208061002257*x^16
  + 0.00219226579520788*x^14 - 0.0223039215686339*x^12 + 0.139542483660157*x^10 - 0.477124183006594*x^8 - 0.0102124183006535*x^7
  + 0.748774509803992*x^6 + 0.187908496732006*x^5 - 0.0784313725490691*x^4 - 0.661764705882374*x^3 - 0.939542483660174*x^2 - 1.0
P7(x) = 1.07167352539046e-8*x^21 - 9.64506172851334e-7*x^19 + 4.05092592596917e-5*x^17 - 0.00100694444445325*x^15
  + 0.0159722222223288*x^13 - 0.162500000000753*x^11 + 1.03690476190753*x^9 - 0.0632440476189801*x^8 - 3.97961309524256*x^7
  + 1.21577380952246*x^6 + 9.86220238095259*x^5 - 6.82738095237675*x^4 - 12.7142857142798*x^3 - 3.0*x^2 - 12.3095238095173*x
P8(x) = -9.343832199399e-21*x^20 - 7.40560905784435e-18*x^18 + 4.27636441882595e-16*x^16 - 1.10055194124659e-14*x^14
  + 1.54626311754669e-13*x^12 + 0.00277777777762652*x^10 - 0.00868055555554484*x^9 - 0.0690972222216618*x^8 + 0.177083333331266*x^7
  + 0.604861111095805*x^6 - 1.12499999998884*x^5 - 1.66666666664613*x^4 + 0.250000000004664*x^3 - 0.749999999980006*x^2 + 1.0
P9(x) = 9.45823229862019e-6*x^19 - 0.000530978954645734*x^17 + 0.0136295210659266*x^15 - 0.193318006157648*x^13
  - 3185920764.46009*x^11 + 9956002393.8074*x^10 + 79249779048.2206*x^9 - 203102448833.744*x^8 - 693734246787.528*x^7
  + 1290297910239.35*x^6 + 1911552459601.2*x^5 - 286732868943.715*x^4 + 860198606825.361*x^3 + 3.0*x^2 - 1146931475768.46*x
```

Test primer 2 – ispitivanje P[1,0] – 1

```
P9(x) = 9.45823229862019e-6*x^19 - 0.000530978954645734*x^17 + 0.0136295210659266*x^15 - 0.193318006157648*x^13
  - 3185920764.46009*x^11 + 9956002393.8074*x^10 + 79249779048.2206*x^9 - 203102448833.744*x^8 - 693734246787.528*x^7
  + 1290297910239.35*x^6 + 1911552459601.2*x^5 - 286732868943.715*x^4 + 860198606825.361*x^3 + 3.0*x^2 - 1146931475768.46*x
P10(x) = 7.93016566599044e-18*x^18 - 4.41101109885809e-16*x^16 + 1.11964991805271e-14*x^14 + 3.1473859619146e-6*x^12
  - 9.835581619001080e-6*x^11 - 0.00285606900620736*x^10 + 0.0088812014204761*x^9 + 0.0697825655438171*x^8 - 0.178358024709091*x^7
  - 0.606749542766643*x^6 + 1.12528326473947*x^5 + 1.66581687239425*x^4 - 0.250000000004667*x^3 + 0.751133058982517*x^2 - 1.0
P11(x) = 4.88190444519429e-6*x^17 - 0.000275564207887999*x^15 + 3753857.15104482*x^13 - 11730803.5749506*x^12
  - 220485262.774487*x^11 + 636521099.862551*x^10 + 3979215541.0972*x^9 - 9623451364.82733*x^8 - 29930095709.1241*x^7
  + 51816613962.8794*x^6 + 75251298312.2144*x^5 - 11439725966.8695*x^4 + 35670566437.6829*x^3 - 3.0*x^2 - 45758903851.6104*x
P12(x) = -6.52539421761118e-18*x^16 + 6.09776561048674e-6*x^14 - 1.90555175319186e-5*x^13 - 0.000361303650661507*x^12
  + 0.001043800046734662*x^11 + 0.00931990680062546*x^10 - 0.0245135359854853*x^9 - 0.118401013536168*x^8 + 0.262528933334885*x^7
  + 0.728987753058565*x^6 - 1.14386595579974*x^5 - 1.60787360365022*x^4 + 0.249999999999794*x^3 - 0.825463823197824*x^2 + 1.0
P13(x) = -4561978.6399185*x^15 + 14256183.2499687*x^14 + 266551635.422712*x^13 - 769177440.663019*x^12
  - 6752104119.791*x^11 + 17703020742.9873*x^10 + 84601246423.5946*x^9 - 186784784505.537*x^8 - 515454356703.565*x^7
  + 803954563700.84*x^6 + 1127662277665.44*x^5 - 175595121369.045*x^4 + 581891434175.328*x^3 + 3.0*x^2 - 702380485492.664*x
P14(x) = -6.09776561004174e-6*x^14 + 1.90555175320099e-5*x^13 + 0.000361303650648411*x^12 - 0.001043800046735148*x^11
  - 0.009319906800425 32*x^10 + 0.0245135359855963*x^9 + 0.118401013534596*x^8 - 0.262528933336039*x^7 - 0.728987753053358*x^6
  + 1.14386595580453*x^5 + 1.60787360365027*x^4 - 0.249999999997193*x^3 + 0.825463823196825*x^2 - 3.13961188803269e-12*x - 1.0
P15(x) = 3753857.14541881*x^13 - 11730803.5737388*x^12 - 220485262.754781*x^11 + 636521099.797866*x^10 + 3979215542.02961*x^9
  - 9623451363.35424*x^8 - 29930095722.7415*x^7 + 51816613947.6325*x^6 + 75251298373.8021*x^5 - 11439725902.0125*x^4
  + 35670566434.5482*x^3 + 28.1562024356321*x^2 - 45758903861.0162*x - 40.5893860091839
P16(x) = -3.14738615869856e-6*x^12 + 9.835581572964 01e-6*x^11 + 0.00285606900882081*x^10 - 0.00888120141892227*x^9
  - 0.069782565565408*x^8 + 0.178358024690779*x^7 + 0.606749542847945*x^6 - 1.12528326465917*x^5 - 1.66581687239553*x^4
  + 0.250000000045342*x^3 - 0.75113305899674*x^2 - 5.13663166710767e-11*x + 1.0
P17(x) = -3185920724.3109*x^11 + 9956002441.40245*x^10 + 79249777464.6909*x^9 - 203102450597.065*x^8 - 693734226440.386*x^7
  + 1290297932023.35*x^6 + 1911552362938.46*x^5 - 286732971496.288*x^4 + 860198611560.438*x^3 - 47841.4561030389*x^2
  - 1146931460822.77*x + 63777.0518384946
P18(x) = -0.002777777777757921*x^10 + 0.00868055555561158*x^9 + 0.0690972222182607*x^8 - 0.177083333334237*x^7
  - 0.604861111082265*x^6 + 1.12500000000911*x^5 + 1.6666666666428*x^4 - 0.249999999998443*x^3 + 0.749999999979538*x^2
  - 1.11771545348345e-11*x - 1.0
P19(x) = 0.0548741386008942*x^9 - 0.0383108285768685*x^8 - 1.18864020639921*x^7 + 0.745827162158093*x^6 + 9.58993531478336*x^5
  - 3.37645566085121*x^4 - 14.5283001985699*x^3 - 3.2011550290481*x^2 - 11.1374504590432*x - 1.26806623546145
P20(x) = -0.0136336390892642*x^8 - 0.00669423778851705*x^7 + 0.211034085443549*x^6 + 0.224032148900106*x^5 - 1.34602501548556*x^4
  - 1.37274030509486*x^3 - 0.579470694295662*x^2 - 1.30403283851077*x + 0.844219464307756
P21(x) = 0.307206949054805*x^7 - 0.637466052635528*x^6 - 3.10002660515807*x^5 + 2.45914581775931*x^4 + 10.2902863518619*x^3
  + 5.67625341361964*x^2 + 1.49806886183894*x + 5.30874333836181
P22(x) = -0.000862699693335786*x^6 + 0.0198618955136831*x^5 + 0.609302836846155*x^4 - 0.0510200702794241*x^3
  - 0.133421158005134*x^2 + 0.897835112669942*x - 1.44877618545686
P23(x) = -362.033086478772*x^5 - 4529.41163584869*x^4 + 417.807332551111*x^3 + 669.865956994495*x^2 - 6183.02919236579*x + 10801.8995123351
P24(x) = -0.224779085922015*x^4 + 0.0172384586182546*x^3 + 0.0619664973194899*x^2 - 0.348546646479 17*x + 0.534124831959264
P25(x) = 31.4898746775051*x^3 + 25.0689283689362*x^2 - 23.1425838797459*x + 26.9592302008368
```

Test primer 2 – ispitivanje P[1,0] – 2

```
P25(x) = 31.4898746775051*x^3 + 25.0689283689362*x^2 - 23.1425838797459*x + 26.9592302008368
P26(x) = 0.25940950400526*x^2 + 0.011928145040114*x - 0.366167097217266
P27(x) = -20.2205217410039*x - 60.3011840760827
P28(x) = -1.90528896879192
**********************************************************************
Vrednosti Šturmovih polinoma:
**********************************************************************
P0(0.0001) = 1.00000001500100   P0(1.5707963267948966) = 6.54753017224151
P1(0.0001) = 0.000300030002666667   P1(1.5707963267948966) = 6.94687951867142
P2(0.0001) = -1.00000001392946   P2(1.5707963267948966) = -6.15781114257668
P3(0.0001) = -0.000626696673883625   P3(1.5707963267948966) = -38.54425878899345
P4(0.0001) = 1.00000002139014   P4(1.5707963267948966) = 13.3655706814172
P5(0.0001) = 0.000611696673562772   P5(1.5707963267948966) = 35.3950703901512
P6(0.0001) = -1.00000000939609   P6(1.5707963267948966) = -2.46391449406541
P7(0.0001) = -0.00123098239366670   P7(1.5707963267948966) = -59.3633339223995
P8(0.0001) = 0.999999992500250   P8(1.5707963267948966) = -10.3347903705202
P9(0.0001) = -114693146.716676   P9(1.5707963267948966) = 18619114400894.6
P10(0.0001) = -0.999999992488919   P10(1.5707963267948966) = 10.3058972830937
P11(0.0001) = -4575890.34949164   P11(1.5707963267948966) = 688712776991.630
P12(0.0001) = 0.999999991745612   P12(1.5707963267948966) = -8.54857667269777
P13(0.0001) = -70238047.9673925   P13(1.5707963267948966) = 9357357435201.02
P14(0.0001) = -0.999999991745612   P14(1.5707963267948966) = 8.54857667276063
P15(0.0001) = -4575930.93981793   P15(1.5707963267948966) = 688712777538.778
P16(0.0001) = 0.999999992488914   P16(1.5707963267948966) = -10.3058972820187
P17(0.0001) = -114629368.170747   P17(1.5707963267948966) = 18619113499676.5
P18(0.0001) = -0.999999992500251   P18(1.5707963267948966) = 10.3347903708360
P19(0.0001) = -1.26918001253343   P19(1.5707963267948966) = -26.8850238752025
P20(0.0001) = 0.844089055227825   P20(1.5707963267948966) = -11.4998202308886
P21(0.0001) = 5.30889324727711   P21(1.5707963267948966) = 44.5490076142240
P22(0.0001) = -1.44868640327985   P22(1.5707963267948966) = 3.32105318906284
P23(0.0001) = 10801.2812161150   P23(1.5707963267948966) = -26675.7518050266
P24(0.0001) = 0.534089977914298   P24(1.5707963267948966) = -1.16213260689316
P25(0.0001) = 26.9569161931696   P25(1.5707963267948966) = 174.510016749987
P26(0.0001) = -0.366165901808667   P26(1.5707963267948966) = 0.292636884800902
P27(0.0001) = -60.3032061282568   P27(1.5707963267948966) = -92.0635053527280
P28(0.0001) = -1.90528896879192   P28(1.5707963267948966) = -1.90528896879192
**********************************************************************
Broj promena znaka za granicu a: V(0.0001) = 13
Broj promena znaka za granicu b: V(1.5707963267948966) = 13
**********************************************************************
Broj nula polinoma P[1, 0](x) na intervalu [a, b] = [0.0001, 1.5707963267948966] je: 0
P[1,0](x) name nule na intervalu [0, pi/2], ali je pozitivna, te stoga DOKAZUJE pozitivnost MTP funkcije nad zadatim intervalom!
**********************************************************************
```

Test primer 2 – ispitivanje P[1,0] – dokazana pozitivnost MTP funkcije

```
P3(x) = P[1,1](x) = x^4*(-x^6/720 + x^4/24 - x^2/2 + 1)**4 + x^3 - 0.5*x^2 + 2*x*(-x^7/5040 + x^5/120 - x^3/6 + x) + 1
**************************************************************************
Šturmov niz polinoma:
**************************************************************************
P0(x) = 3.72108862978205e-12*x^28 - 4.46530635573845e-10*x^26 + 2.54522462277092e-8*x^24 - 8.94847393689986e-7*x^22
  + 2.13396990740741e-5*x^20 - 0.000358796296296296*x^18 + 0.00429398148148148*x^16 - 0.0361111111111111*x^14
  + 0.20625*x^12 - 0.755555555555556*x^10 + 1.66626984126984*x^8 - 1.98333333333333*x^6 + 0.666666666666667*x^4 + 1.0*x^3 + 1.5*x^2 + 1.0
P1(x) = 1.04190481633897e-10*x^27 - 1.160979652492e-8*x^25 + 6.10853909465021e-7*x^23 - 1.96866426611797e-5*x^21
  + 0.000426793981481481*x^19 - 0.00645833333333333*x^17 + 0.0687037037037037*x^15 - 0.505555555555556*x^13 + 2.475*x^11
  - 7.55555555555556*x^9 + 13.3301587301587*x^7 - 11.9*x^5 + 2.66666666666667*x^3 + 3.0*x^2 + 3.0*x
P2(x) = 3.18950453981318e-11*x^26 - 3.63603517538703e-9*x^24 + 1.91753012933569e-7*x^22 - 6.09705687830688e-6*x^20
  + 0.000128141534391534*x^18 - 0.00184027777777778*x^16 + 0.0180555555555556*x^14 - 0.117857142857143*x^12 + 0.485714285714286*x^10
  - 1.19019274376417*x^8 + 1.55833333333333*x^6 - 0.571428571428571*x^4 - 0.892857142857143*x^3 - 1.39285714285714*x^2 - 1.0
P3(x) = -2.67918381344308e-10*x^25 + 1.55392661179702e-8*x^23 - 2.30409807956114e-7*x^21 - 8.19830246913573e-6*x^19
  + 0.000446759259259259*x^17 - 0.00972222222222222*x^15 + 0.120555555555556*x^13 - 0.888333333333333*x^11 + 3.66759259259259*x^9
  - 8.23960317460318*x^7 + 10.0333333333333*x^5 - 2.91666666666667*x^4 - 7.21666666666667*x^3 - 3.0*x^2 - 6.26666666666667*x
P4(x) = 1.78612254229534e-9*x^24 - 1.64323273891174e-7*x^22 + 7.0730452674897e-6*x^20 - 0.000181327160493827*x^18
  + 0.00299768518518518*x^16 - 0.0324074074074074*x^14 + 0.223611111111111*x^12 - 0.922332451499117*x^10 + 2.17109788359788*x^8
  - 2.75277777777778*x^6 + 0.347222222222222*x^5 + 1.43055555555555*x^4 + 1.25*x^3 + 2.13888888888889*x^2 + 1.0
P5(x) = 9.10922496570655e-9*x^23 - 8.3054698216737e-7*x^21 + 3.53973765432105e-5*x^19 - 0.000896412037037048*x^17
  + 0.0145833333333335*x^15 - 0.154097222222223*x^13 + 1.0266832010582*x^11 - 3.99325727513228*x^9 + 8.65251984126985*x^7
  - 0.0520833333333346*x^6 - 10.2479166666667*x^5 + 2.72916666666666*x^4 + 6.89583333333333*x^3 + 3.0*x^2 + 6.11666666666666*x
P6(x) = 1.47092444659777e-9*x^22 - 1.32383200193779e-7*x^20 + 5.56009440813769e-6*x^18 - 0.000138208061002257*x^16
  + 0.00219226579520788*x^14 - 0.0223006795310782*x^12 + 0.139340828924188*x^10 - 0.474525365701895*x^8 - 0.0102124183006535*x^7
  + 0.743382352941246*x^6 + 0.187908496732006*x^5 - 0.0784313725490691*x^4 - 0.661764705882374*x^3 - 0.939542483660174*x^2 - 1.0
P7(x) = 1.07167352539046e-8*x^21 - 9.64506172851334e-7*x^19 + 4.05092592596917e-5*x^17 - 0.00100694444445325*x^15
  + 0.0159922996977633*x^13 - 0.16376535336432*x^11 + 1.05458947468151*x^9 - 0.0632440476189801*x^8 - 4.04885912698856*x^7
  + 1.21577380952246*x^6 + 9.76220238095259*x^5 - 6.82738095237675*x^4 - 12.7142857142798*x^3 - 3.0*x^2 - 12.3095238095173*x
P8(x) = -9.343832199399e-21*x^20 - 7.40560905784435e-18*x^18 + 4.27636441882595e-16*x^16 + 2.75573191135022e-6*x^14
  - 0.000176917989261293*x^12 + 0.00540674603046501*x^10 - 0.00868055555554484*x^9 - 0.0812003968196808*x^8 + 0.177083333331266*x^7
  + 0.596527777762627*x^6 - 1.12499999998884*x^5 - 1.66666666664613*x^4 + 0.250000000004664*x^3 - 0.749999999980006*x^2 + 1.0
P9(x) = 9.45823229862019e-6*x^19 - 0.000530978954645734*x^17 - 3160635.66693413*x^15 + 202912810.499629*x^13
  - 6201167203.72929*x^11 + 9956002393.8074*x^10 + 93131290957.3261*x^9 - 203102448833.744*x^8 - 684176484489.465*x^7
  + 1290297910239.35*x^6 + 1911552459601.3*x^5 - 286732868943.715*x^4 + 860198606825.361*x^3 + 3.0*x^2 - 1146931475768.46*x
P10(x) = 7.93016566599044e-18*x^18 + 3.12240642206383e-9*x^16 - 2.95619043177518e-6*x^14 + 0.000183044151525305*x^12
  - 9.83558161900108e-6*x^11 - 0.00549875087108894*x^10 + 0.00888120142047614*x^9 + 0.0818762979885258*x^8 - 0.178358024709091*x^7
  - 0.598416209433466*x^6 + 1.12528326473947*x^5 + 1.66581687239425*x^4 - 0.250000000004667*x^3 + 0.751133058982517*x^2 - 1.0
P11(x) = 3724.06463183841*x^17 - 365184.221369036*x^15 + 15402188.0703204*x^13 - 11730803.5749506*x^12 - 357140060.145974*x^11
  + 636521099.862551*x^10 + 4521781972.49475*x^9 - 9623451364.82733*x^8 - 29548771510.5398*x^7 + 51816613962.8794*x^6
  + 75251298312.1143*x^5 - 11439725966.8695*x^4 + 35670566437.6829*x^3 - 3.0*x^2 - 45758903851.6104*x
```

Test primer 2 – ispitivanje P[1,1] – 1

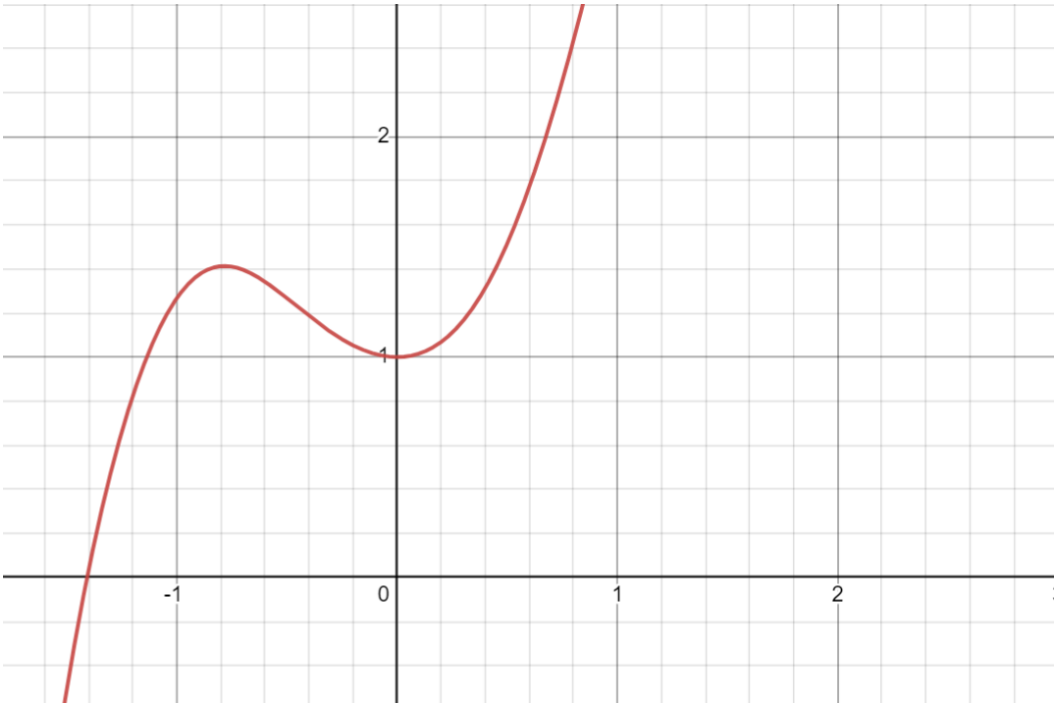
```
P12(x) = -3.12240719970108e-9*x^16 + 2.95619046457319e-6*x^14 - 2.49800218152034e-14*x^13 - 0.000183044152285812*x^12
  + 9.83558297443346e-6*x^11 + 0.00549875088071778*x^10 - 0.00888120144096864*x^9 - 0.0818762980514448*x^8 + 0.178358024819431*x^7
  + 0.598416209593709*x^6 - 1.12528326476383*x^5 - 1.66581687231829*x^4 + 0.250000000004667*x^3 - 0.751133059079958*x^2 + 1.0
P13(x) = -3160635.33065766*x^15 + 0.0297934285295835*x^14 + 202912788.162663*x^13 - 0.367631360888481*x^12
  - 6201166516.95002*x^11 + 9956001290.46181*x^10 + 93131280635.2422*x^9 - 203102426316.402*x^8 - 684176408690.198*x^7
  + 1290297767373.36*x^6 + 1911552247975.55*x^5 - 286732837197.254*x^4 + 860198511558.313*x^3 + 3.0*x^2 - 1146931348782.62*x
P14(x) = -2.75573192241715e-6*x^14 + 2.65064403785853e-14*x^13 + 0.000176917989361777*x^12 - 3.5421914450735e-13*x^11
  - 0.00540674603009639*x^10 + 0.00868055555552144*x^9 + 0.0812003968079176*x^8 - 0.177083333331061*x^7 - 0.596527777770732*x^6
  + 1.12500000000047*x^5 + 1.66666666665906*x^4 - 0.249999999996654*x^3 + 0.749999999955337*x^2 - 1.06806741422362e-11*x - 1.0
P15(x) = -0.000279366964217586*x^13 + 0.000376736132912704*x^12 + 0.133478164594746*x^11 - 0.540698234615776*x^10
  - 0.844207775319154*x^9 + 13.6745543312533*x^8 - 20.2333981964118*x^7 - 125.358837079831*x^6 + 232.667478969065*x^5
  + 373.260068577195*x^4 - 78.6677242753508*x^3 + 150.124998990444*x^2 + 1.290283200769991*x - 220.499998666855
P16(x) = 0.00114474889896188*x^12 - 0.00355800015550397*x^11 - 0.0101131763982895*x^10 + 0.114978160670662*x^9
  - 0.0988847690471698*x^8 - 1.32863057263094*x^7 + 1.22405569245467*x^6 + 5.65190649701702*x^5 + 2.5225268161885*x^4
  + 0.684407769009686*x^3 + 1.25972345257945*x^2 - 2.1578927030125*x - 1.9331396031073
P17(x) = -0.129482285105743*x^11 + 0.516981410869554*x^10 + 0.818967134443807*x^9 - 13.307850452005*x^8
  + 20.5052033508302*x^7 + 123.453912864857*x^6 - 235.710062981773*x^5 - 374.510289371921*x^4 + 78.0664076179972*x^3
  - 150.139319210126*x^2 + 0.108103498024857*x + 221.33010678133
P18(x) = -0.00117034148925863*x^10 - 0.00372859914452543*x^9 + 0.02167298079199*x^8 + 0.0768173239054403*x^7
  - 0.105621967131395*x^6 - 0.497501308210207*x^5 - 0.283850701660409*x^4 + 0.0324507619351945*x^3 - 0.0865142036278621*x^2
  + 0.200274614685991*x + 0.202226850309522
P19(x) = 4.54015221460776*x^9 + 4.59368319613213*x^8 - 93.2001147072259*x^7 - 94.6093362304428*x^6 + 599.42765938394*x^5
  + 603.538319617291*x^4 - 113.410824688965*x^3 + 241.007635773926*x^2 - 136.795085048091*x - 381.9412145965
P20(x) = -0.000222703479239885*x^8 - 0.000196749482120562*x^7 + 0.00412635801922696*x^6 + 0.00598368266761179*x^5
  - 0.0251585221670029*x^4 - 0.0310173474608733*x^3 - 0.0132923626258718*x^2 - 0.0251546954797945*x + 0.01182624439269
P21(x) = 9.5927151039726*x^7 - 38.1727110815543*x^6 - 102.187250659853*x^5 + 94.618338131222*x^4 + 465.544486049589*x^3
  + 306.585286728374*x^2 - 38.4906420835562*x + 351.001064625196
P22(x) = 0.00255549339842 64*x^6 + 0.00335602943010049*x^5 + 0.00366861536361493*x^4 - 0.0286576395533011*x^3
  - 0.0204257844013351*x^2 + 0.0213512665670904*x - 0.0514522758814028
P23(x) = 49.2835191825418*x^5 - 275.077351873193*x^4 + 27.1284656912376*x^3 + 179.365038261932*x^2 - 578.838441825168*x + 671.210580407111
P24(x) = -0.100606152313373*x^4 + 0.0476570651660608*x^3 + 0.0545369896553812*x^2 - 0.193490365621262*x + 0.291419997497154
P25(x) = 65.4008974135828*x^3 + 51.8790680012454*x^2 - 48.0603736552829*x + 57.966252277556
P26(x) = 0.120503575161853*x^2 + 0.0106540112327431*x - 0.178447097114251
P27(x) = -44.7126770115371*x - 126.228486008501
P28(x) = -0.751878147769259
**************************************************************************
```

Test primer 2 – ispitivanje P[1,1] – 2

```
**********************************************************************
Vrednosti Šturmovih polinoma:
**********************************************************************
P0(0.0001) = 1.00000001500100   P0(1.5707963267948966) = 6.78318377700431
P1(0.0001) = 0.000300030002666667   P1(1.5707963267948966) = 7.82828296746025
P2(0.0001) = -1.00000001392946   P2(1.5707963267948966) = -6.34401812949013
P3(0.0001) = -0.000626696673883625   P3(1.5707963267948966) = -40.3811401922095
P4(0.0001) = 1.00000002139014   P4(1.5707963267948966) = 13.8952736873130
P5(0.0001) = 0.000611696673562772   P5(1.5707963267948966) = 37.1071434620333
P6(0.0001) = -1.00000000939609   P6(1.5707963267948966) = -2.46630022691871
P7(0.0001) = -0.00123098239366670   P7(1.5707963267948966) = -61.0986147289181
P8(0.0001) = 0.999999992500250   P8(1.5707963267948966) = -10.7065303046796
P9(0.0001) = -114693146.716676   P9(1.5707963267948966) = 19288839486101.2
P10(0.0001) = -0.999999992488919   P10(1.5707963267948966) = 10.6765979398190
P11(0.0001) = -4575890.34949164   P11(1.5707963267948966) = 713485610614.650
P12(0.0001) = 0.999999992488919   P12(1.5707963267948966) = -10.6765979374325
P13(0.0001) = -114693134.018092   P13(1.5707963267948966) = 19288837351980.9
P14(0.0001) = -0.999999992500251   P14(1.5707963267948966) = 10.7065303057008
P15(0.0001) = -220.499868137363   P15(1.5707963267948966) = 2411.37628106679
P16(0.0001) = -1.93335537977968   P16(1.5707963267948966) = 58.7335623448370
P17(0.0001) = 221.330116090365   P17(1.5707963267948966) = -2459.11197111088
P18(0.0001) = 0.202246876905881   P18(1.5707963267948966) = -5.35149178627651
P19(0.0001) = -381.954891695042   P19(1.5707963267948966) = 5779.31360939338
P20(0.0001) = 0.0118237287901874   P20(1.5707963267948966) = -0.227556939241660
P21(0.0001) = 350.997218627307   P21(1.5707963267948966) = 2103.10442014010
P22(0.0001) = -0.0514501409590326   P22(1.5707963267948966) = -0.0865664780364924
P23(0.0001) = 671.152698356606   P23(1.5707963267948966) = -893.702842628142
P24(0.0001) = 0.291400649006009   P24(1.5707963267948966) = -0.305737926850579
P25(0.0001) = 57.9614467590466   P25(1.5707963267948966) = 363.959453386006
P26(0.0001) = -0.178446030508092   P26(1.5707963267948966) = 0.135618838536881
P27(0.0001) = -126.232957276202   P27(1.5707963267948966) = -196.462994819390
P28(0.0001) = -0.751878147769259   P28(1.5707963267948966) = -0.751878147769259
**********************************************************************
Broj promena znaka za granicu a: V(0.0001) = 13
Broj promena znaka za granicu b: V(1.5707963267948966) = 13
**********************************************************************
Broj nula polinoma P[1, 1](x) na intervalu [a, b] = [0.0001, 1.5707963267948966] je: 0
P[1,1](x) name nule na intervalu [0, pi/2], ali je pozitivna, te stoga DOKAZUJE pozitivnost MTP funkcije nad zadatim intervalom!
**********************************************************************

Process finished with exit code 0
```

Test primer 2 – ispitivanje P[1,1] – dokazana pozitivnost MTP funkcije



Test primer 2 – grafik MTP funkcije

3. Test primer 3:

```
146    # za kosinus, alfa < 0 i alfa > 0
147    alfa_0 = 4 * s; alfa_1 = 4 * s + 2;
148    # za sinus, beta < 0 i beta > 0
149    beta_0 = 4 * r + 1; beta_1 = 4 * r+ 3;
150
151    # PRIMER:
152    MTP_funkcija = x**3 * sym.cos(x)**2 + x**2 - x * sym.sin(x) + 0.5
153    # koriscena poredjenja
154    alfa = alfa_1
155    beta = beta_0
156    # granice intervala [0-PI/2]
157    a = 0.0001; b = m.pi / 2
```

Test primer 3 – ulazna MTP funkcija

```
C:\Users\Korisnik\AppData\Local\Programs\Python\Python39\python.exe C:/Users/Korisnik/Desktop/OPNA-DZ3/projekat-3.py
MTP funkcija za koju utvrdjujemo pozitivnost: x^3*cos(x)^2 + x^2 - x*sin(x) + 0.5

P0(x) = P[0,0](x) = x^3*(1 - x^2/2)**2 + 0.5
**********************************************************************
Šturmov niz polinoma:
**********************************************************************
P0(x) = 0.25*x^7 - 1.0*x^5 + 1.0*x^3 + 0.5
P1(x) = 1.75*x^6 - 5.0*x^4 + 3.0*x^2
P2(x) = 0.285714285714286*x^5 - 0.571428571428571*x^3 - 0.5
P3(x) = 1.5*x^4 - 3.0*x^2 - 3.0625*x
P4(x) = 2.22044604925031e-16*x^3 - 0.583333333333333*x^2 + 0.5
P5(x) = -1.03524799018639e+31*x^2 + 3.37769972052788e+15*x + 8.87355420159761e+30
P6(x) = 1.4791141972894e-31*x - 5.55111512312578e-17
P7(x) = 1.45814748460538e+60
**********************************************************************
Vrednosti Šturmovih polinoma:
**********************************************************************
P0(0.0001) = 0.500000000001000   P0(1.5707963267948966) = 0.711679645998979
P1(0.0001) = 2.99999995000000e-8   P1(1.5707963267948966) = 3.24984811451599
P2(0.0001) = -0.500000000000571   P2(1.5707963267948966) = 0.0175845655614553
P3(0.0001) = -0.000306279999999850   P3(1.5707963267948966) = -3.08066476718865
P4(0.0001) = 0.499999994166667   P4(1.5707963267948966) = -0.939317308492196
P5(0.0001) = 8.87355409807281e+30   P5(1.5707963267948966) = -1.667016609880086e+31
P6(0.0001) = -5.55111512312578e-17   P6(1.5707963267948966) = -5.55111512312576e-17
P7(0.0001) = 1.45814748460538e+60   P7(1.5707963267948966) = 1.45814748460538e+60
**********************************************************************
Broj promena znaka za granicu a: V(0.0001) = 4
Broj promena znaka za granicu b: V(1.5707963267948966) = 2
**********************************************************************
Broj nula polinoma P[0, 0](x) na intervalu [a, b] = [0.0001, 1.5707963267948966] je: 2
P[0,0](x) ima nule na intervalu [0, pi/2], te stoga NE DOKAZUJE pozitivnost MTP funkcije nad zadatim intervalom!
**********************************************************************
```

Test primer 3 – ispitivanje P[0,0]

```
Broj nula polinoma P[0, 0](x) na intervalu [a, b] = [0.0001, 1.5707963267948966] je: 2
P[0,0](x) ima nule na intervalu [0, pi/2], te stoga NE DOKAZUJE pozitivnost MTP funkcije nad zadatim intervalom!
**********************************************************************

P1(x) = P[0,1](x) = x^3*(1 - x^2/2)**2 + x^2 - x*(x^5/120 - x^3/6 + x) + 0.5
**********************************************************************
Šturmov niz polinoma:
**********************************************************************
P0(x) = 0.25*x^7 - 0.008333333333333333*x^6 - 1.0*x^5 + 0.1666666666666667*x^4 + 1.0*x^3 + 0.5
P1(x) = 1.75*x^6 - 0.05*x^5 - 5.0*x^4 + 0.6666666666666667*x^3 + 3.0*x^2
P2(x) = 0.285748299319728*x^5 - 0.0680272108843537*x^4 - 0.5718820861678*x^3 - 0.00204081632653061*x^2 - 0.5
P3(x) = 1.41035978191569*x^4 - 1.41289390074037*x^3 - 3.00261838164073*x^2 - 3.06213546006428*x - 0.641503501979354
P4(x) = -0.25509506545243*x^3 - 1.08298372444271*x^2 - 0.603797840254215*x + 0.400735819877557
P5(x) = -25.0770999130965*x^2 - 16.6699553890529*x + 12.267077098862
P6(x) = 0.121396402962604*x + 0.0460809146244898
P7(x) = -14.9815009925654
**********************************************************************
Vrednosti Šturmovih polinoma:
**********************************************************************
P0(0.0001) = 0.500000000001000   P0(1.5707963267948966) = 1.60117679302305
P1(0.0001) = 3.00006661666662e-8   P1(1.5707963267948966) = 5.35554874706397
P2(0.0001) = -0.500000000020980   P2(1.5707963267948966) = -0.403037694911933
P3(0.0001) = -0.641809745552957   P3(1.5707963267948966) = -9.74986440993045
P4(0.0001) = 0.400675429263439   P4(1.5707963267948966) = -4.20855636538709
P5(0.0001) = 12.2654098525521   P5(1.5707963267948966) = -75.7932915113107
P6(0.0001) = 0.0460930542647861   P6(1.5707963267948966) = 0.236769938484261
P7(0.0001) = -14.9815009925654   P7(1.5707963267948966) = -14.9815009925654
**********************************************************************
Broj promena znaka za granicu a: V(0.0001) = 3
Broj promena znaka za granicu b: V(1.5707963267948966) = 3
**********************************************************************
Broj nula polinoma P[0, 1](x) na intervalu [a, b] = [0.0001, 1.5707963267948966] je: 0
P[0,1](x) name nule na intervalu [0, pi/2], ali je pozitivna, te stoga DOKAZUJE pozitivnost MTP funkcije nad zadatim intervalom!
**********************************************************************
```

Test primer 3 – ispitivanje P[0,1] – dokazana pozitivnost MTP funkcije

```
P2(x) = P[1,0](x) = x^3*(-x^6/720 + x^4/24 - x^2/2 + 1)**2 + 0.5
**********************************************************************
Šturmov niz polinoma:
**********************************************************************
P0(x) = 1.92901234567901e-6*x^15 - 0.000115740740740741*x^13 + 0.003125*x^11 - 0.0444444444444444*x^9 + 0.3333333333333333*x^7 - 1.0*x^5 + 1.0*x^3 + 0.5
P1(x) = 2.89351851851852e-5*x^14 - 0.00150462962962963*x^12 + 0.034375*x^10 - 0.4*x^8 + 2.33333333333333*x^6 - 5.0*x^4 + 3.0*x^2
P2(x) = 1.54320987654321e-5*x^13 - 0.000833333333333333*x^11 + 0.0177777777777778*x^9 - 0.177777777777778*x^7 + 0.666666666666667*x^5 - 0.8*x^3 - 0.5
P3(x) = -5.787037037037698e-5*x^12 - 0.00104166666666669*x^10 + 0.0666666666666665*x^8 - 1.08333333333333*x^6 + 3.5*x^4 - 3.0*x^2 - 0.937499999999999*x
P4(x) = 0.00111111111111112*x^11 - 0.0355555555555558*x^9 + 0.4666666666667*x^7 - 1.60000000000001*x^5 + 1.60000000000001*x^3 + 0.250000000000002*x^2 + 0.5
P5(x) = 0.00289351851851852*x^10 - 0.0909722222222221*x^8 + 1.16666666666667*x^6 - 3.58333333333333*x^4 - 0.0130208333333332*x^3 + 3.0*x^2 + 0.911458333333333*x
P6(x) = 0.000622222222222225*x^9 - 0.0186666666666666*x^7 + 0.223999999999995*x^5 - 0.005*x^4 - 0.448*x^3 + 0.1*x^2 - 0.5
P7(x) = 0.00416666666666748*x^8 - 0.125000000000007*x^6 - 0.0232514880952238*x^5 + 1.50000000000001*x^4 + 0.478050595238094*x^3 - 3.0*x^2 - 3.23660714285713*x
P8(x) = 2.39391839684799e-15*x^7 - 0.00347222222222154*x^6 - 4.046762947587e-14*x^5 + 0.0763888888888751*x^4 + 8.5154105988774 95e-14*x^3 - 0.583333333333324*x^2 + 0.5
P9(x) = -8.765669626953 43e+21*x^6 + 30795460589.9937*x^5 + 1.9284473179297 8e+23*x^4 - 800331628802.833*x^3 - 1.4726324973282 3e+24*x^2 + 870260797562.924*x + 1.26225642628154e+24
P10(x) = 9.21231765117098e-27*x^5 - 1.89427591448911e-25*x^3 + 1.28206936493967e-24*x
P11(x) = -1.26013068759487e+22*x^4 + 167102277839.14*x^3 + 2.52722707258118e+23*x^2 + 3415513759572.47*x - 1.26225642628154e+24
P12(x) = 4.67200380670417e-27*x^3 - 4.94693747619375e-36*x^2 - 3.5928355624457e-25*x + 1.22367951283605e-35
P13(x) = 7.16335224848714e+23*x^2 + 976813201624025.0*x + 1.26225642628154e+24
P14(x) = 3.67516108022084e-25*x - 3.21795284317446e-35
P15(x) = -1.26225642628154e+24
**********************************************************************
Vrednosti Šturmovih polinoma:
**********************************************************************
P0(0.0001) = 0.500000000001000   P0(1.5707963267948966) = 0.500003101291957
P1(0.0001) = 2.99999995000000e-8   P1(1.5707963267948966) = 0.00697125508148844
P2(0.0001) = -0.500000000000800   P2(1.5707963267948966) = -0.499273073166948
P3(0.0001) = -9.37799999996500e-5   P3(1.5707963267948966) = -1.47745183520318
P4(0.0001) = 0.500000002501600   P4(1.5707963267948966) = 1.11814665070123
P5(0.0001) = 9.11758333199542e-5   P5(1.5707963267948966) = 1.38597367625842
P6(0.0001) = -0.499999999000448   P6(1.5707963267948966) = -0.282147024575988
P7(0.0001) = -0.000323690713807513   P7(1.5707963267948966) = -3.44696470034728
P8(0.0001) = 0.499999994166667   P8(1.5707963267948966) = -0.526415245915165
P9(0.0001) = 1.26225641155522e+24   P9(1.5707963267948966) = -1.32894205409844e+24
P10(0.0001) = 1.28206936304540e-28   P10(1.5707963267948966) = 1.36778776471732e-24
P11(0.0001) = -1.26225642375431e+24   P11(1.5707963267948966) = -7.15405755862211e+23
P12(0.0001) = -3.59283433829899e-29   P12(1.5707963267948966) = -5.46253610091487e-25
P13(0.0001) = 1.26225643344499e+24   P13(1.5707963267948966) = 3.02974274977147e+24
P14(0.0001) = 3.67515786222800e-29   P14(1.5707963267948966) = 5.77292952486866e-25
P15(0.0001) = -1.26225642628154e+24   P15(1.5707963267948966) = -1.26225642628154e+24
**********************************************************************
Broj promena znaka za granicu a: V(0.0001) = 7
Broj promena znaka za granicu b: V(1.5707963267948966) = 7
**********************************************************************
Broj nula polinoma P[1, 0](x) na intervalu [a, b] = [0.0001, 1.5707963267948966] je: 0
P[1,0](x) name nule na intervalu [0, pi/2], ali je pozitivna, te stoga DOKAZUJE pozitivnost MTP funkcije nad zadatim intervalom!
**********************************************************************
```

Test primer 3 – ispitivanje P[1,0] – dokazana pozitivnost MTP funkcije

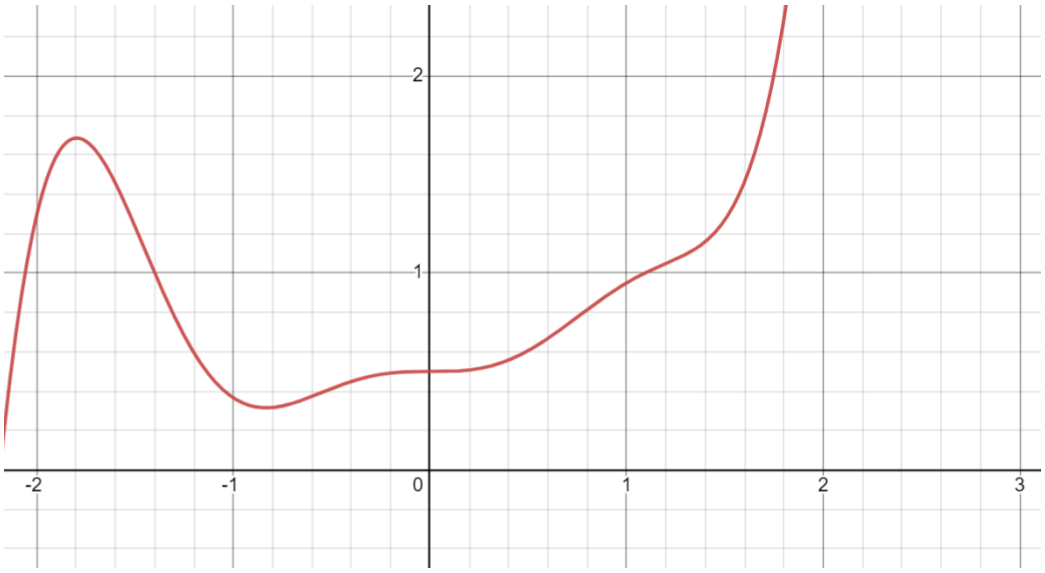
```
P3(x) = P[1,1](x) = x^3*(-x^6/720 + x^4/24 - x^2/2 + 1)**2 + x^2 - x*(x^5/120 - x^3/6 + x) + 0.5
**********************************************************************
Šturmov niz polinoma:
**********************************************************************
P0(x) = 1.929012345679O1e-6*x^15 - 0.000115740740740741*x^13 + 0.003125*x^11 - 0.0444444444444444*x^9 + 0.3333333333333333*x^7
 - 0.008333333333333333*x^6 - 1.0*x^5 + 0.166666666666667*x^4 + 1.0*x^3 + 0.5
P1(x) = 2.893518518518852e-5*x^14 - 0.001504629629629629*x^12 + 0.034375*x^10 - 0.4*x^8 + 2.333333333333333*x^6 - 0.05*x^5 - 5.0*x^4 + 0.6666666666666667*x^3 + 3.0*x^2
P2(x) = 1.543209876543210e-5*x^13 - 0.000833333333333333*x^11 + 0.0177777777777778*x^9 - 0.177777777777778*x^7 + 0.005*x^6
 + 0.6666666666666667*x^5 - 0.1222222222222222*x^4 - 0.8*x^3 - 0.5
P3(x) = -5.787037037036980e-5*x^12 - 0.00104166666666669*x^10 + 0.06666666666666669*x^8 + 0.009374999999999999*x^7 - 1.083333333333333*x^6
 - 0.1791666666666667*x^5 + 3.5*x^4 - 0.6666666666666667*x^3 - 3.0*x^2 - 0.937499999999999*x
P4(x) = 0.001111111111111112*x^11 - 0.03555555555555558*x^9 - 0.00250000000000002*x^8 + 0.46666666666667*x^7 + 0.04277777777777782*x^6
 - 1.60000000000001*x^5 + 0.300000000000002*x^4 + 1.60000000000001*x^3 + 0.250000000000002*x^2 + 0.5
P5(x) = 0.002893518518518852*x^10 + 0.000130208333333332*x^9 - 0.0909722222222221*x^8 - 0.0116030092592592*x^7 + 1.16666666666667*x^6
 + 0.1635416666666667*x^5 - 3.583333333333333*x^4 + 0.6536458333333333*x^3 + 3.0*x^2 + 0.911458333333333*x
P6(x) = 0.000619972222222225*x^9 - 0.000383555555555561*x^8 - 0.01846616666666666*x^7 - 0.000137777777777685*x^6 + 0.221173999999999*x^5
 + 0.01292*x^4 - 0.459295*x^3 + 0.0481600000000003*x^2 - 0.01575*x - 0.5
P7(x) = 0.003599359148314655*x^8 - 0.04623793439162870*x^7 - 0.134835782605006*x^6 + 0.581832229672439*x^5 + 1.47974241389958*x^4
 - 1.85151509390057*x^3 - 2.92433511137026*x^2 - 3.29383029848242*x - 1.54872182620315
P8(x) = -0.102141578541043*x^7 - 0.183625629929616*x^6 + 1.25911668576741*x^5 + 2.78468999700338*x^4 - 3.94393236612591*x^3
 - 6.77452566109812*x^2 - 7.18823271758002*x - 2.76180388551181
P9(x) = -0.004291517788595170*x^6 - 0.0302125103030252*x^5 + 0.0962370380947524*x^4 + 0.0550320731384999*x^3 - 0.318256586143928*x^2 - 0.318231602121093*x + 0.123532351529222
P10(x) = 4.801043722016010*x^5 - 13.48246203091120*x^4 - 10.49723066863090*x^3 + 38.90951080982610*x^2 + 49.83443835625510*x - 12.65143600256690
P11(x) = 0.031833701902948500*x^4 + 0.002596059658771490*x^3 - 0.068813617027828*x^2 - 0.109157547912085*x - 0.012160401799358
P12(x) = -1.012423213350580*x^3 - 25.38141721665050*x^2 - 4.094607326492960*x + 17.95126879555151
P13(x) = -19.74494220590390*x^2 - 3.672470738017160*x + 14.11670100310520
P14(x) = 0.132636383384670*x + 0.060615571876613900
P15(x) = -11.67122799175000
**********************************************************************
```

Test primer 3 – ispitivanje P[1,1] – 1

```
**********************************************************************
Vrednosti Šturmovih polinoma:
**********************************************************************
P0(0.0001) = 0.500000000001000   P0(1.5707963267948966) = 1.38950024831603
P1(0.0001) = 3.00006661666662e-8   P1(1.5707963267948966) = 2.11267188762947
P2(0.0001) = -0.500000000000800   P2(1.5707963267948966) = -1.16826176559528
P3(0.0001) = -9.37800006663166e-5   P3(1.5707963267948966) = -5.55348680662695
P4(0.0001) = 0.500000002501600   P4(1.5707963267948966) = 3.49450087939625
P5(0.0001) = 9.11758339866208e-5   P5(1.5707963267948966) = 5.26759362197398
P6(0.0001) = -0.500001574518859   P6(1.5707963267948966) = -0.408187279605078
P7(0.0001) = -1.54905123847820   P7(1.5707963267948966) = -9.52442534101872
P8(0.0001) = -2.76252277653277   P8(1.5707963267948966) = -22.2284044453486
P9(0.0001) = 0.123500525186499   P9(1.5707963267948966) = -0.715812854072449
P10(0.0001) = -12.6464521696466   P10(1.5707963267948966) = 84.7794674308828
P11(0.0001) = -0.0121713182422828   P11(1.5707963267948966) = -0.149547955549126
P12(0.0001) = 17.9508590809673   P12(1.5707963267948966) = -55.0305964032805
P13(0.0001) = 14.1163335585819   P13(1.5707963267948966) = -40.3706946660950
P14(0.0001) = 0.0606288355149524   P14(1.5707963267948966) = 0.268960315696612
P15(0.0001) = -11.6712279917500   P15(1.5707963267948966) = -11.6712279917500
**********************************************************************
Broj promena znaka za granicu a: V(0.0001) = 7
Broj promena znaka za granicu b: V(1.5707963267948966) = 7
**********************************************************************
Broj nula polinoma P[1, 1](x) na intervalu [a, b] = [0.0001, 1.5707963267948966] je: 0
P[1,1](x) name nule na intervalu [0, pi/2], ali je pozitivna, te stoga DOKAZUJE pozitivnost MTP funkcije nad zadatim intervalom!
**********************************************************************

Process finished with exit code 0
```

Test primer 3 – ispitivanje P[1,1] – dokazana pozitivnost MTP funkcije



Test primer 3 – grafik MTP funkcije

SPISAK SLIKA

LITERATURA

[1] Branko J. Malešević, Pozitivnost MTP funkcija (2023).pdf