Assignment 4

Filip Mellgren, Ismael Moreno Martinez, Athina Swahn

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Part I

1 a

Rewriting the equation in the following way:

$$-\alpha_s(r_{Lt-1} - \beta r_{St-1} - \mu) = -\Delta r_{St} + \sum_{i=1}^{2} a_{i,11}\Delta r_{St-1} + \Delta a_{i,12}\Delta r_{Lt-i} + \epsilon_{St}$$

It is clear from the equation above that all RHS variables are I(0). Therefore, the linear combination $r_{Lt-1} - \beta r_{St-1}$ must be a stationary process. That would mean that r_{St} and $r_{Lt} must have a stochastic trend in common. Hence, the cointergration vector is

$$B = \begin{bmatrix} 1 \\ -\beta \end{bmatrix}$$

The long run equilibrium is defined by $r_{Lt} = \beta r_{St}$

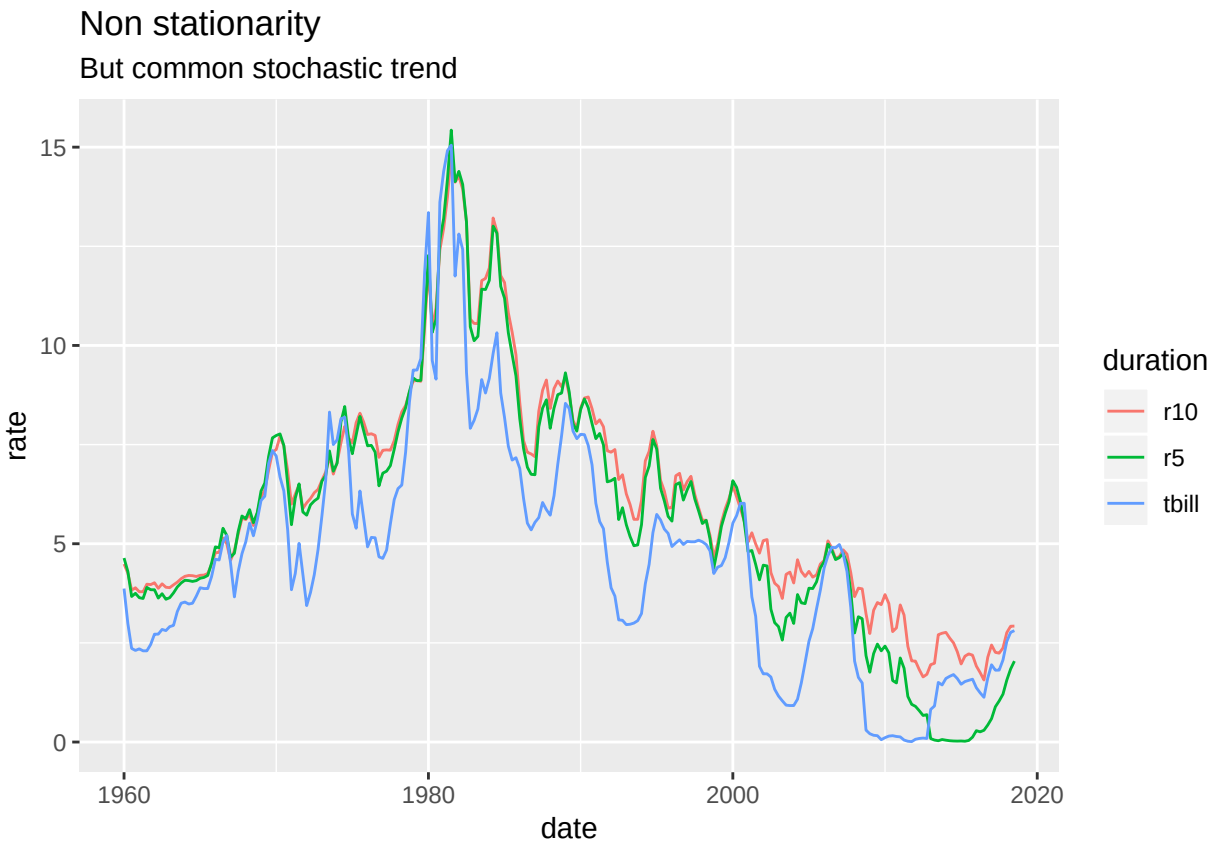
1 b

r_{Lt} does not Granger cause r_{St} $H_0 : \Delta a_{i,12} = \alpha_S = 0$

1 c

assuming $\alpha_s, \alpha_L > 0$ (i) $r_{Lt-1} > \beta r_{St-1} + \mu$ (ii) $r_{Lt-1} < \beta r_{St-1} + \mu$ assuming $\alpha_S > 0$ and $\alpha_L = 0$ (iii) $r_{Lt-1} > \beta r_{St-1} + \mu$ (iv) $r_{Lt-1} < \beta r_{St-1}$ # Part II Do exercise 4 (but not 4f) in the textbook (pp.402-403). Remark: It is possible that the values you obtain differ from those reported in the text to the exercise since the sample is extended. However, the main conclusions should be the same.

```
## -- Attaching packages --------------------------- tidyverse 1.2.1 --
## v ggplot2 3.1.0     v purrr   0.2.5
## v tibble  2.0.1     v dplyr   0.7.8
## v tidyr   0.8.2     v stringr 1.3.1
## v readr   1.3.1     v forcats 0.3.0
## -- Conflicts ------------------------------ tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()    masks stats::lag()
```



4a

Pretest to show that all variables act as unit root processes using ADF with lag length equal to the longest lag length with significicant at the 5% level, including an intercept but not a time trend.

```
##
## ###############################################
## # Augmented Dickey-Fuller Test Unit Root Test #
## ###############################################
##
## Test regression drift
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 + 1 + z.diff.lag)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -3.3429 -0.2288  0.0241  0.3020  3.2625
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.13340    0.08227   1.621  0.10636
## z.lag.1     -0.02805    0.01484  -1.890  0.06011 .
## z.diff.lag1  0.38160    0.06547   5.829 1.99e-08 ***
## z.diff.lag2 -0.34302    0.07012  -4.892 1.94e-06 ***
```

```
## z.diff.lag3  0.39176    0.07312   5.357 2.14e-07 ***
## z.diff.lag4 -0.11781    0.07713  -1.527  0.12810
## z.diff.lag5  0.19271    0.07357   2.619  0.00943 **
## z.diff.lag6 -0.05730    0.07031  -0.815  0.41600
## z.diff.lag7 -0.21134    0.06613  -3.196  0.00160 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.6244 on 218 degrees of freedom
## Multiple R-squared:  0.2843, Adjusted R-squared:  0.258
## F-statistic: 10.83 on 8 and 218 DF,  p-value: 8.21e-13
##
##
## Value of test-statistic is: -1.8898 1.7859
##
## Critical values for test statistics:
##       1pct  5pct 10pct
## tau2 -3.46 -2.88 -2.57
## phi1  6.52  4.63  3.81
##
## ################################################
## # Augmented Dickey-Fuller Test Unit Root Test #
## ################################################
##
## Test regression drift
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 + 1 + z.diff.lag)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.32015 -0.27607  0.01406  0.28401  1.65735
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.07523    0.07204   1.044  0.29747
## z.lag.1     -0.01415    0.01096  -1.291  0.19801
## z.diff.lag1  0.30050    0.06678   4.500  1.1e-05 ***
## z.diff.lag2 -0.20130    0.06966  -2.890  0.00424 **
## z.diff.lag3  0.21191    0.06982   3.035  0.00269 **
## z.diff.lag4 -0.03547    0.06951  -0.510  0.61034
## z.diff.lag5 -0.07397    0.06705  -1.103  0.27115
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5216 on 222 degrees of freedom
## Multiple R-squared:  0.1281, Adjusted R-squared:  0.1045
## F-statistic: 5.435 on 6 and 222 DF,  p-value: 2.897e-05
##
##
## Value of test-statistic is: -1.2911 0.8508
##
```

```
## Critical values for test statistics:
##       1pct  5pct 10pct
## tau2 -3.46 -2.88 -2.57
## phi1  6.52  4.63  3.81
```

```
##
## ###############################################
## # Augmented Dickey-Fuller Test Unit Root Test #
## ###############################################
##
## Test regression drift
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 + 1 + z.diff.lag)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.98401 -0.26287  0.00455  0.24402  1.41340
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.0606603  0.0740927   0.819   0.4138
## z.lag.1     -0.0106172  0.0108784  -0.976   0.3302
## z.diff.lag1  0.2801263  0.0671550   4.171 4.37e-05 ***
## z.diff.lag2 -0.1392781  0.0697716  -1.996   0.0472 *
## z.diff.lag3  0.1405834  0.0694938   2.023   0.0443 *
## z.diff.lag4 -0.0006824  0.0701232  -0.010   0.9922
## z.diff.lag5 -0.1616127  0.0695722  -2.323   0.0211 *
## z.diff.lag6  0.0345299  0.0695810   0.496   0.6202
## z.diff.lag7 -0.1304329  0.0674188  -1.935   0.0543 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4487 on 218 degrees of freedom
## Multiple R-squared:  0.1322, Adjusted R-squared:  0.1004
## F-statistic: 4.152 on 8 and 218 DF,  p-value: 0.0001224
##
##
## Value of test-statistic is: -0.976 0.4936
##
## Critical values for test statistics:
##       1pct  5pct 10pct
## tau2 -3.46 -2.88 -2.57
## phi1  6.52  4.63  3.81
```

4b

Use the Engle-Granger procedure to estimate cointergrating relationships.

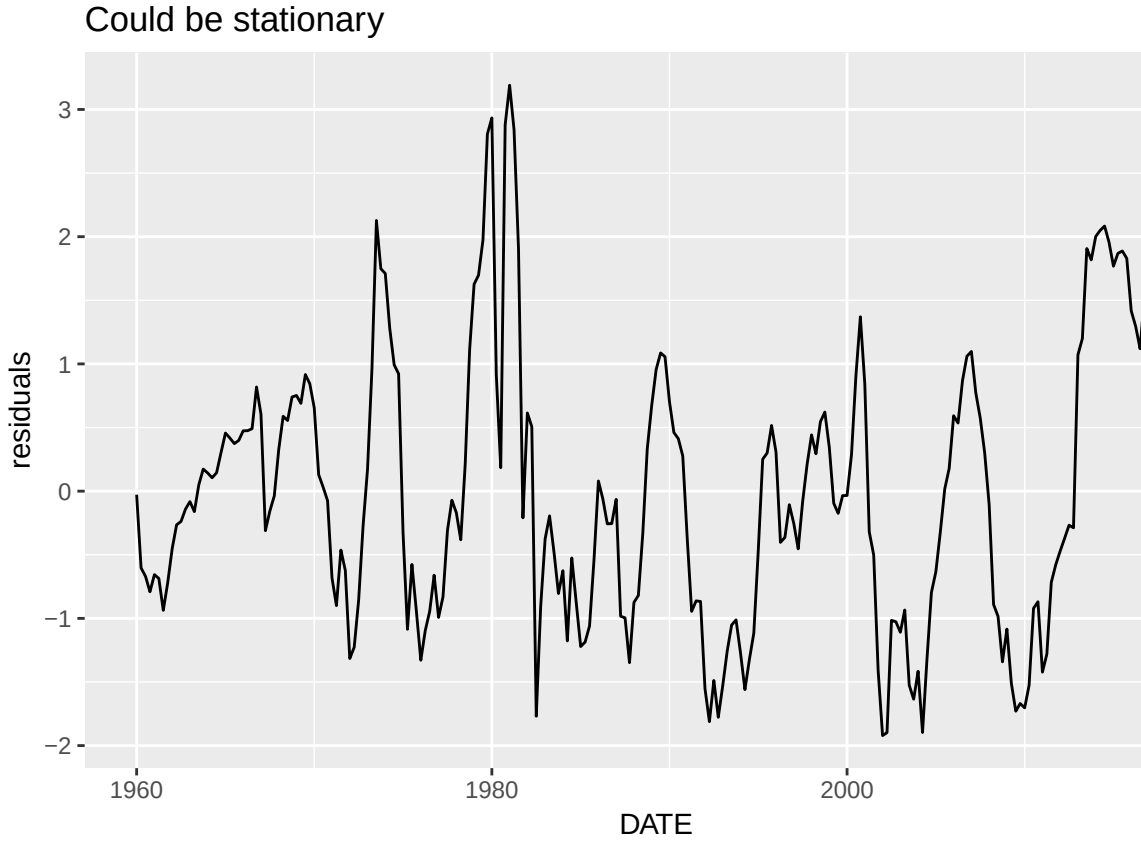
We begin by running the regression:

$Tbill_t = a_0 + a_1 R5_t + a_2 R10_t$

```
##
```

```
## Call:
## lm(formula = tbill ~ r5 + r10, data = df)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.9214 -0.8683 -0.1057  0.6633  3.1906
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.07301    0.23512  -0.311    0.756
## r5           0.98553    0.13691   7.198  8.4e-12 ***
## r10         -0.13409    0.15578  -0.861    0.390
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.082 on 232 degrees of freedom
## Multiple R-squared:  0.8725, Adjusted R-squared:  0.8714
## F-statistic: 793.5 on 2 and 232 DF,  p-value: < 2.2e-16
```

The estimates differ from those in the book. However, when we estimated the same regression using the data used in the book, we obtianed similar values. We proceed by testing if the residuals are serially conintegrated using ADF-tests.



We plot the residual series:

Next, we perform ADF test on residuals. We select lag length using the AIC criteria, with a maximal lag length of 10.

```
##
## ###############################################
## # Augmented Dickey-Fuller Test Unit Root Test #
```

```
## ###############################################
##
## Test regression none
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 - 1 + z.diff.lag)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.72942 -0.19718  0.02017  0.23527  2.48827
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## z.lag.1     -0.1743042  0.0395451  -4.408 1.65e-05 ***
## z.diff.lag1  0.2940184  0.0688267   4.272 2.92e-05 ***
## z.diff.lag2 -0.0973810  0.0716596  -1.359 0.175597
## z.diff.lag3  0.2434532  0.0703014   3.463 0.000645 ***
## z.diff.lag4  0.0005642  0.0705065   0.008 0.993623
## z.diff.lag5  0.1734476  0.0705443   2.459 0.014738 *
## z.diff.lag6 -0.0444056  0.0706463  -0.629 0.530306
## z.diff.lag7 -0.0374750  0.0705873  -0.531 0.596037
## z.diff.lag8  0.1411249  0.0680282   2.075 0.039228 *
## z.diff.lag9  0.1243237  0.0683708   1.818 0.070405 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4585 on 214 degrees of freedom
## Multiple R-squared:  0.2005, Adjusted R-squared:  0.1632
## F-statistic: 5.368 on 10 and 214 DF,  p-value: 4.48e-07
##
##
## Value of test-statistic is: -4.4077
##
## Critical values for test statistics:
##       1pct  5pct 10pct
## tau1 -2.58 -1.95 -1.62
```

We need Engle - Granger critical values to evaluate the test statistic,-4.408.

4c, similar to above

4d

Use the Johansen procedure

4e

Check to determine whether the individual interest pairs are cointegrated. In particular, is R5 with cointegrated with R10.