Assignment 3

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Contents

Hello,this is a comment from Ismael

Theoretical exercises

1

a:

test Show that $* = Cov(z_t, \varepsilon_{yt}) \neq 0$.

- Recall the formula for covariance: $Cov(z_t, \varepsilon_{yt}) = E(z_t \varepsilon_{yt}) - E(z_t)E(\varepsilon_{yt})$. Because $\varepsilon_{yt} \sim WN(0, \sigma_y^2)$, we obtain: $* = E(z_t \varepsilon_{yt})$.
- Next, expand the the expression for y_t in the expression for z_t: $* = E[(-b_{21}[(b_{12}z_t + \gamma_{11}y_{t-1} + \gamma_{12}z_{t-1} + \varepsilon_{yt}] + \gamma_{21}y_{t-1} + \gamma_{22}z_{t-1} + \varepsilon_{zt})\varepsilon_{yt}]$.
- Now distribute ε_{yt} over the system: $* = E(-b_{21}[(b_{12}z_t + \gamma_{11}y_{t-1} + \gamma_{12}z_{t-1} + \varepsilon_{yt}]\varepsilon_{yt} + \gamma_{21}y_{t-1}\varepsilon_{yt} + \gamma_{22}z_{t-1}\varepsilon_{yt} + \varepsilon_{zt}\varepsilon_{yt})$
- Expand the expectation operator to a sum: $* = E(-b_{21}[(b_{12}z_t + \gamma_{11}y_{t-1} + \gamma_{12}z_{t-1} + \varepsilon_{yt}]\varepsilon_{yt}) + E(\gamma_{21}y_{t-1}\varepsilon_{yt}) + E(\gamma_{22}z_{t-1}\varepsilon_{yt}) + E(\varepsilon_{zt}\varepsilon_{yt})$.
- Exploit intertemporal independence and that ε_{yt} and ε_{zt} are independent: $* = E(-b_{21}[(b_{12}z_t + \gamma_{11}y_{t-1} + \gamma_{12}z_{t-1} + \varepsilon_{yt}]\varepsilon_{yt})$
- Distibute ε_{yt}: $* = -b_{21}E([(b_{12}z_t\varepsilon_{yt} + \gamma_{11}y_{t-1}\varepsilon_{yt} + \gamma_{12}z_{t-1}\varepsilon_{yt} + \varepsilon_{yt}\varepsilon_{yt}])$
- Expand the expectation: $* = -b_{21}[E(b_{12}z_t\varepsilon_{yt}) + E(\gamma_{11}y_{t-1}\varepsilon_{yt}) + E(\gamma_{12}z_{t-1}\varepsilon_{yt}) + E(\varepsilon_{yt}^2)]$
- What remains after exploiting independence is $* = -b_{21}E(\varepsilon_{yt}^2) = -b_{21}\sigma_y^2 \neq 0$ QED.

The implications on estimation are that estimates will be inefficient and baised.

b

Firstly, we express (1) in the following matrix form:

$$BX_t = \Gamma_0 + \Gamma_1 X_{t-1} + \varepsilon_t$$

Where

$$B = \begin{bmatrix} 1 & b_{12} \\ b_{21} & 1 \end{bmatrix}$$

,

$$X_t = \begin{bmatrix} y_t \\ z_t \end{bmatrix}$$

$$\Gamma_0 = \begin{bmatrix} b_{10} \\ b_{20} \end{bmatrix}$$

$$\Gamma_1 = \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix}$$

$$\Gamma_0 = \begin{bmatrix} b_{10} \\ b_{20} \end{bmatrix}$$

2

Empirical exercises

Do exercises 10a-10g in the textbook (p.340)

- Remark 1: It is possible that the values you obtain for the F-statistics, p-values and correlations are different than those reported since the sample is extended. However, the main conclusions should be the same.
- Remark 2: Exercise d. is optional and so is the part on the forecast error variance in e. (but you could use the command fevd in STATA to answer these questions).
- Remark 3: You find the appropriate specifications for the variables st , Δlip, and Δur described in the text to exercise 9 (p.339).

```
## -- Attaching packages --------------------------------------- tidyverse 1.2.1 --

## v ggplot2 3.1.0     v purrr   0.2.5
## v tibble  2.0.1     v dplyr   0.7.8
## v tidyr   0.8.2     v stringr 1.3.1
## v readr   1.3.1     v forcats 0.3.0

## -- Conflicts ------------------------------------------ tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()    masks stats::lag()

## Loading required package: MASS

##
## Attaching package: 'MASS'

## The following object is masked from 'package:dplyr':
##
##     select

## Loading required package: strucchange

## Loading required package: zoo

##
## Attaching package: 'zoo'
```

```
## The following objects are masked from 'package:base':
##
##     as.Date, as.Date.numeric
```

```
## Loading required package: sandwich
```

```
##
## Attaching package: 'strucchange'
```

```
## The following object is masked from 'package:stringr':
##
##     boundary
```

```
## Loading required package: urca
```

```
## Loading required package: lmtest
```

```
##
## Please cite as:
```

```
##  Hlavac, Marek (2018). stargazer: Well-Formatted Regression and Summary Statistics Tables.
```

```
##  R package version 5.2.2. https://CRAN.R-project.org/package=stargazer
```

10:

Estimate the three-VAR beginning in 1961Q1 and use the ordering such that Δlip_t is causally prior to Δur_t and that Δur_t is causally prior to s_t.

We begin by defining the variables we are going to include in our analysis.

Do we need to check for stationarity here? Might be enough to assume it.

The lag length is already determined to be 3.

```
##
## VAR Estimation Results:
## =========================
## Endogenous variables: dlip, dur, s
## Deterministic variables: none
## Sample size: 231
## Log Likelihood: 612.866
## Roots of the characteristic polynomial:
## 0.9173 0.7824 0.6453 0.463 0.463 0.4479 0.4479 0.1831 0.1831
## Call:
## VAR(y = tmp, p = 3, type = "none")
##
##
## Estimation results for equation dlip:
## =====================================
## dlip = dlip.l1 + dur.l1 + s.l1 + dlip.l2 + dur.l2 + s.l2 + dlip.l3 + dur.l3 + s.l3
##
##           Estimate Std. Error t value Pr(>|t|)
## dlip.l1  0.5404560  0.0897942   6.019 7.19e-09 ***
## dur.l1  -0.0063116  0.0045486  -1.388   0.1666
## s.l1     0.0009883  0.0015925   0.621   0.5355
## dlip.l2 -0.0650182  0.0970909  -0.670   0.5038
## dur.l2   0.0062905  0.0050051   1.257   0.2101
## s.l2     0.0010944  0.0023105   0.474   0.6362
```

```
## dlip.l3  0.1584778  0.0882214   1.796   0.0738 .
## dur.l3   0.0044926  0.0044997   0.998   0.3192
## s.l3    -0.0002954  0.0016058  -0.184   0.8542
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.01211 on 222 degrees of freedom
## Multiple R-Squared: 0.4889,  Adjusted R-squared: 0.4682
## F-statistic:  23.6 on 9 and 222 DF,  p-value: < 2.2e-16
##
##
## Estimation results for equation dur:
## ====================================
## dur = dlip.l1 + dur.l1 + s.l1 + dlip.l2 + dur.l2 + s.l2 + dlip.l3 + dur.l3 + s.l3
##
##          Estimate Std. Error t value Pr(>|t|)
## dlip.l1 -4.056028   1.779886  -2.279   0.0236 *
## dur.l1   0.522127   0.090161   5.791 2.37e-08 ***
## s.l1     0.006923   0.031567   0.219   0.8266
## dlip.l2  2.175020   1.924519   1.130   0.2596
## dur.l2   0.056733   0.099211   0.572   0.5680
## s.l2    -0.009138   0.045798  -0.200   0.8420
## dlip.l3  1.033190   1.748709   0.591   0.5552
## dur.l3   0.028446   0.089192   0.319   0.7501
## s.l3    -0.016687   0.031829  -0.524   0.6006
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.2401 on 222 degrees of freedom
## Multiple R-Squared: 0.4751,  Adjusted R-squared: 0.4538
## F-statistic: 22.32 on 9 and 222 DF,  p-value: < 2.2e-16
##
##
## Estimation results for equation s:
## ==================================
## s = dlip.l1 + dur.l1 + s.l1 + dlip.l2 + dur.l2 + s.l2 + dlip.l3 + dur.l3 + s.l3
##
##         Estimate Std. Error t value Pr(>|t|)
## dlip.l1  0.50432    3.78403   0.133  0.89409
## dur.l1   0.58848    0.19168   3.070  0.00241 **
## s.l1     1.08636    0.06711  16.188  < 2e-16 ***
## dlip.l2  2.10670    4.09152   0.515  0.60714
## dur.l2  -0.24687    0.21092  -1.170  0.24307
## s.l2    -0.31910    0.09737  -3.277  0.00122 **
## dlip.l3  0.35224    3.71775   0.095  0.92460
## dur.l3   0.30594    0.18962   1.613  0.10808
## s.l3     0.18490    0.06767   2.732  0.00679 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.5104 on 222 degrees of freedom
```

```
## Multiple R-Squared: 0.9281,  Adjusted R-squared: 0.9252
## F-statistic: 318.3 on 9 and 222 DF,  p-value: < 2.2e-16
##
##
##
## Covariance matrix of residuals:
##           dlip      dur        s
## dlip  0.0001467 -0.001979 -0.001128
## dur  -0.0019789  0.056898  0.026343
## s    -0.0011285  0.026343  0.258211
##
## Correlation matrix of residuals:
##         dlip     dur       s
## dlip  1.0000 -0.6849 -0.1833
## dur  -0.6849  1.0000  0.2173
## s    -0.1833  0.2173  1.0000
```

10 a:

If you perform a test to determine whether s_t Granger causes Δlip_t, you should find that the F-statistic is 2.44 with a prob-value of 0.065. How do you interpret this result?

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
Res.Df	2	225.500	2.121	224	224.8	226.2	227
Df	1	−3.000		−3.000	−3.000	−3.000	−3.000
F	1	2.774		2.774	2.774	2.774	2.774
Pr(>F)	1	0.042		0.042	0.042	0.042	0.042

10 b:

Verify that s_t Granger causes $\Delta unemp_t$. You should find that the F statistic is 5.93 with a prob value of less than 0.001.

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
Res.Df	2	225.500	2.121	224	224.8	226.2	227
Df	1	−3.000		−3.000	−3.000	−3.000	−3.000
F	1	4.450		4.450	4.450	4.450	4.450
Pr(>F)	1	0.005		0.005	0.005	0.005	0.005

10 c:

It turns out that the correlation coefficient between e_{1t} and e_{2t} is -0.72. The correlation between e_{1t} and e_{3t} is -0.11 and between e_{2t} and e_{3t} is 0.10. Explain why the ordering of a Choleski composition is likely to be important for obtaining the impulse responses.

10 e:

Now estimate the model using the levels of lip_t and ur_t. Do you now find a lag length of 5 appropriate?

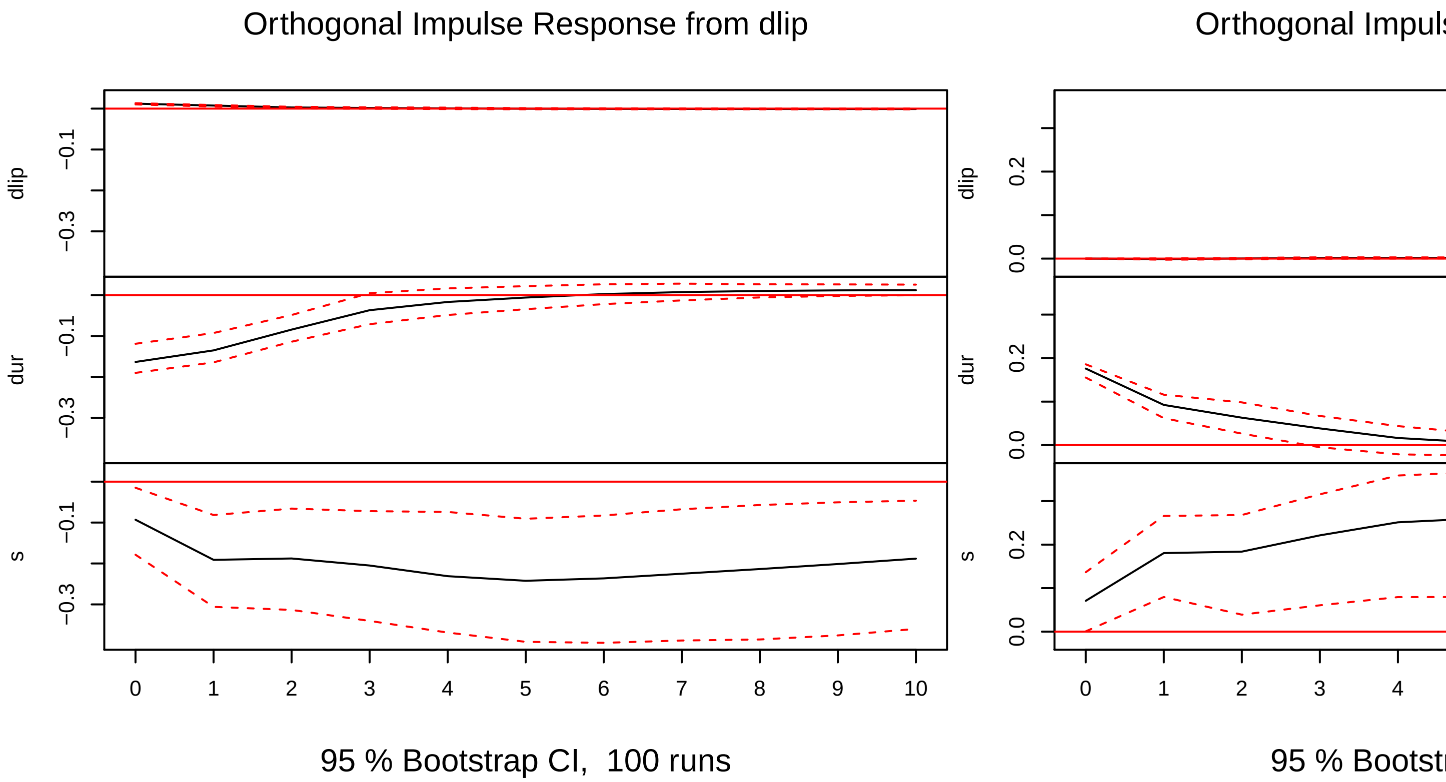
```
##
## VAR Estimation Results:
## =========================
## Endogenous variables: lip, urate, s
## Deterministic variables: none
## Sample size: 229
## Log Likelihood: 639.41
## Roots of the characteristic polynomial:
## 1.001 0.9148 0.8094 0.7407 0.7407 0.633 0.633 0.5668 0.5668 0.5233 0.5233 0.5183 0.476 0.476 0.1885
## Call:
## VAR(y = tmp, p = 5, type = "none")
##
##
## Estimation results for equation lip:
## ====================================
## lip = lip.l1 + urate.l1 + s.l1 + lip.l2 + urate.l2 + s.l2 + lip.l3 + urate.l3 + s.l3 + lip.l4 + urate
##
##           Estimate Std. Error t value Pr(>|t|)
## lip.l1    1.5636597  0.0912021  17.145  < 2e-16 ***
## urate.l1 -0.0047452  0.0047465  -1.000    0.319
## s.l1      0.0008271  0.0016555   0.500    0.618
## lip.l2   -0.6324525  0.1546058  -4.091 6.09e-05 ***
## urate.l2  0.0099261  0.0079278   1.252    0.212
## s.l2      0.0014317  0.0023774   0.602    0.548
## lip.l3    0.2598367  0.1582817   1.642    0.102
## urate.l3 -0.0008371  0.0080801  -0.104    0.918
## s.l3     -0.0019767  0.0023775  -0.831    0.407
## lip.l4   -0.2638230  0.1544323  -1.708    0.089 .
## urate.l4 -0.0033497  0.0079157  -0.423    0.673
## s.l4      0.0005087  0.0023427   0.217    0.828
## lip.l5    0.0720440  0.0908467   0.793    0.429
## urate.l5 -0.0005302  0.0045609  -0.116    0.908
## s.l5      0.0010022  0.0016599   0.604    0.547
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.01187 on 214 degrees of freedom
## Multiple R-Squared:     1,   Adjusted R-squared:     1
## F-statistic: 1.875e+06 on 15 and 214 DF,  p-value: < 2.2e-16
##
##
## Estimation results for equation urate:
## ======================================
## urate = lip.l1 + urate.l1 + s.l1 + lip.l2 + urate.l2 + s.l2 + lip.l3 + urate.l3 + s.l3 + lip.l4 + ura
##
##          Estimate Std. Error t value Pr(>|t|)
## lip.l1   -5.813422   1.786661  -3.254  0.00132 **
## urate.l1  1.393686   0.092984  14.988  < 2e-16 ***
## s.l1      0.014813   0.032432   0.457  0.64833
## lip.l2    7.828040   3.028747   2.585  0.01041 *
## urate.l2 -0.368202   0.155307  -2.371  0.01864 *
## s.l2     -0.033366   0.046575  -0.716  0.47452
## lip.l3   -2.860759   3.100758  -0.923  0.35725
```

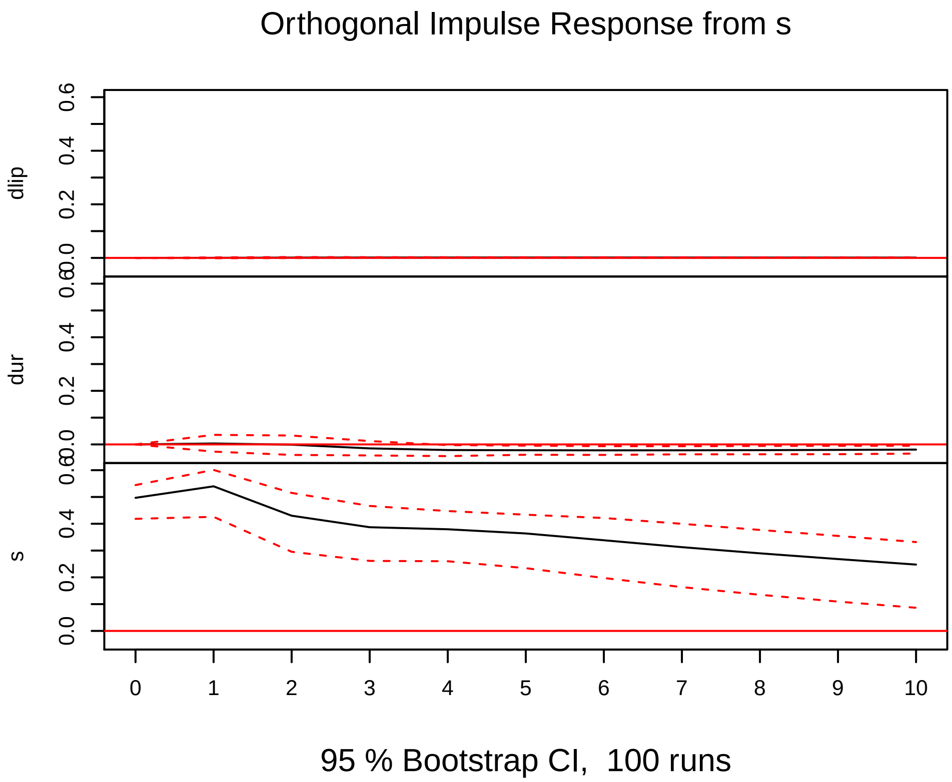
```
## urate.l3  0.068186   0.158290   0.431  0.66707
## s.l3      0.020262   0.046576   0.435  0.66397
## lip.l4    4.406660   3.025348   1.457  0.14670
## urate.l4 -0.112506   0.155069  -0.726  0.46893
## s.l4     -0.035293   0.045895  -0.769  0.44275
## lip.l5   -3.524396   1.779697  -1.980  0.04895 *
## urate.l5  0.005204   0.089349   0.058  0.95361
## s.l5     -0.011518   0.032519  -0.354  0.72354
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.2326 on 214 degrees of freedom
## Multiple R-Squared: 0.9987,  Adjusted R-squared: 0.9986
## F-statistic: 1.092e+04 on 15 and 214 DF,  p-value: < 2.2e-16
##
##
## Estimation results for equation s:
## ==================================
## s = lip.l1 + urate.l1 + s.l1 + lip.l2 + urate.l2 + s.l2 + lip.l3 + urate.l3 + s.l3 + lip.l4 + urate.l
##
##          Estimate Std. Error t value Pr(>|t|)
## lip.l1   -2.15155    3.84245  -0.560 0.576104
## urate.l1  0.47192    0.19997   2.360 0.019180 *
## s.l1      1.06067    0.06975  15.207  < 2e-16 ***
## lip.l2    3.41963    6.51372   0.525 0.600134
## urate.l2 -0.74449    0.33401  -2.229 0.026856 *
## s.l2     -0.36958    0.10016  -3.690 0.000285 ***
## lip.l3   -2.71398    6.66858  -0.407 0.684430
## urate.l3  0.60294    0.34042   1.771 0.077961 .
## s.l3      0.28900    0.10017   2.885 0.004313 **
## lip.l4    1.00773    6.50641   0.155 0.877060
## urate.l4 -0.35483    0.33350  -1.064 0.288548
## s.l4     -0.20263    0.09870  -2.053 0.041292 *
## lip.l5    0.41580    3.82747   0.109 0.913593
## urate.l5  0.07923    0.19216   0.412 0.680521
## s.l5      0.06935    0.06994   0.992 0.322475
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.5002 on 214 degrees of freedom
## Multiple R-Squared: 0.9331,  Adjusted R-squared: 0.9284
## F-statistic: 198.9 on 15 and 214 DF,  p-value: < 2.2e-16
##
##
##
## Covariance matrix of residuals:
##             lip     urate       s
## lip    0.000141 -0.001938 -0.00114
## urate -0.001938  0.054103  0.02373
## s     -0.001140  0.023729  0.25023
##
## Correlation matrix of residuals:
```

```
##            lip   urate       s
## lip    1.0000 -0.7018 -0.1920
## urate -0.7018  1.0000  0.2039
## s     -0.1920  0.2039  1.0000
```

10 f:

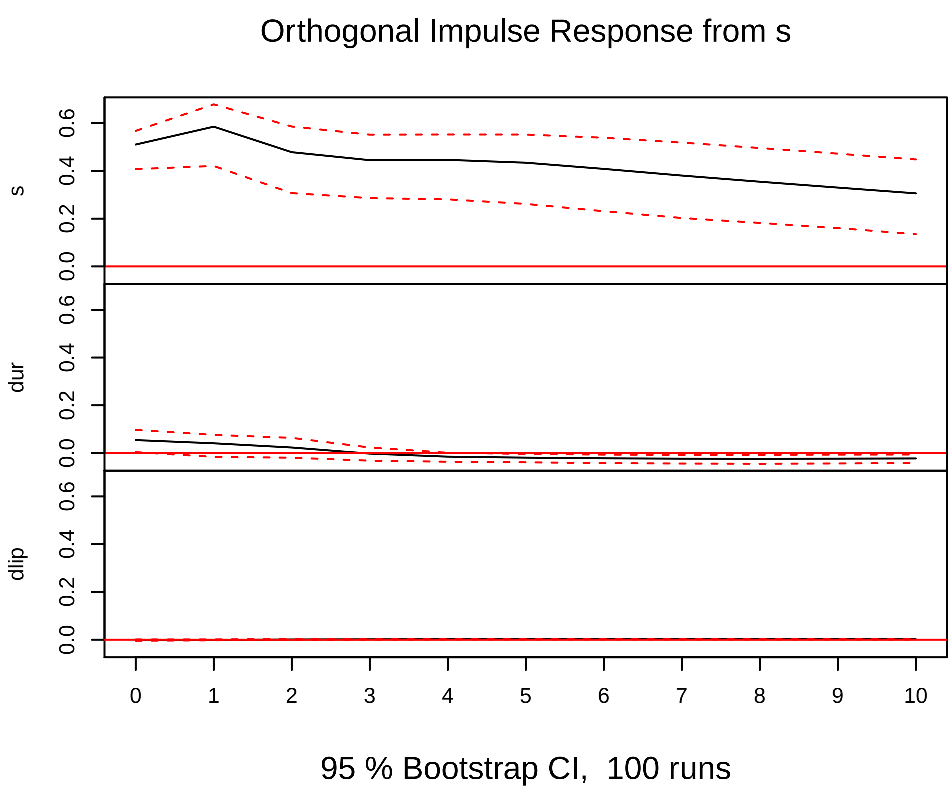
Obtain the impulse response function from the model using $\Delta lip_t, \Delta ur_t$ and s_t. Show that a positive shock to the industrial production induces a decline in the unemployment rate that lasts six quarters. Then, Δur_t overshoots its long run level before returning to zero.

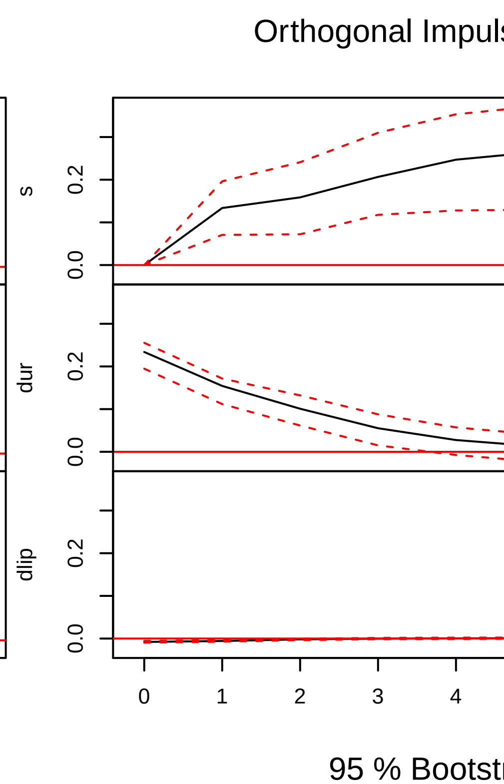




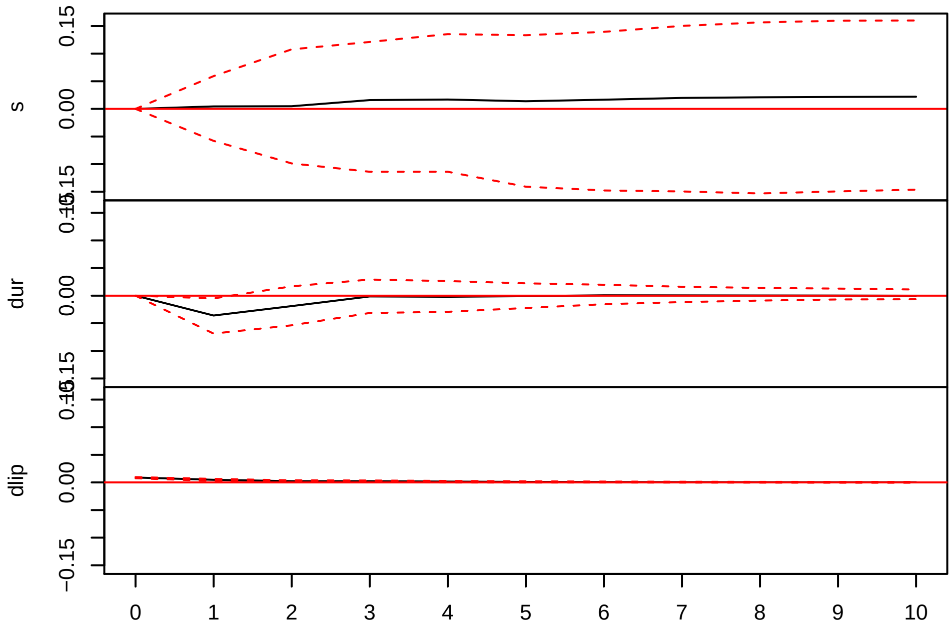
10 g:

Reverse the ordering and explain why the results depend on whether or not Δlip_t proceeds Δur_t





Orthogonal Impulse Response from dlip



95 % Bootstrap CI, 100 runs