Assignment 3

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Theoretical exercises

1

a:

test Show that $* = Cov(z_t, \varepsilon_{yt}) \neq 0$.

- Recall the formula for covariance: $Cov(z_t, \varepsilon_{yt}) = E(z_t\varepsilon_{yt}) - E(z_t)E(\varepsilon_{yt})$. Because $\varepsilon_{yt} \sim WN(0, \sigma_y^2)$, we obtain: $* = E(z_t\varepsilon_{yt})$.
- Next, expand the the expression for y_t in the expression for z_t: $* = E[(-b_{21}[(b_{12}z_t + \gamma_{11}y_{t-1} + \gamma_{12}z_{t-1} + \varepsilon_{yt}] + \gamma_{21}y_{t-1} + \gamma_{22}z_{t-1} + \varepsilon_{zt})\varepsilon_{yt}]$.
- Now distribute ε_{yt} over the system: $* = E(-b_{21}[(b_{12}z_t + \gamma_{11}y_{t-1} + \gamma_{12}z_{t-1} + \varepsilon_{yt}]\varepsilon_{yt} + \gamma_{21}y_{t-1}\varepsilon_{yt} + \gamma_{22}z_{t-1}\varepsilon_{yt} + \varepsilon_{zt}\varepsilon_{yt})$
- Expand the expectation operator to a sum: $* = E(-b_{21}[(b_{12}z_t + \gamma_{11}y_{t-1} + \gamma_{12}z_{t-1} + \varepsilon_{yt}]\varepsilon_{yt}) + E(\gamma_{21}y_{t-1}\varepsilon_{yt}) + E(\gamma_{22}z_{t-1}\varepsilon_{yt}) + E(\varepsilon_{zt}\varepsilon_{yt})$.
- Exploit intertemporal independence and that ε_{yt} and ε_{zt} are independent: $* = E(-b_{21}[(b_{12}z_t + \gamma_{11}y_{t-1} + \gamma_{12}z_{t-1} + \varepsilon_{yt}]\varepsilon_{yt})$
- Distibute ε_{yt}: $* = -b_{21}E([(b_{12}z_t\varepsilon_{yt} + \gamma_{11}y_{t-1}\varepsilon_{yt} + \gamma_{12}z_{t-1}\varepsilon_{yt} + \varepsilon_{yt}\varepsilon_{yt}])$
- Expand the expectation: $* = -b_{21}[E(b_{12}z_t\varepsilon_{yt}) + E(\gamma_{11}y_{t-1}\varepsilon_{yt}) + E(\gamma_{12}z_{t-1}\varepsilon_{yt}) + E(\varepsilon_{yt}^2)]$
- What remains after exploiting independence is $* = -b_{21}E(\varepsilon_{yt}^2) = -b_{21}\sigma_y^2 \neq 0$ QED.

The implications on estimation are that estimates will be inefficient and baised.

b

Firstly, we express (1) in the following matrix form:

$$BX_t = \Gamma_1 X_{t-1} + \varepsilon_t$$

Where

$$B = \begin{bmatrix} 1 & b_{12} \\ b_{21} & 1 \end{bmatrix}$$

,

$$X_t = \begin{bmatrix} y_t \\ z_t \end{bmatrix}$$

$$\Gamma_1 = \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix}$$

$$\varepsilon_t = \begin{bmatrix} \varepsilon_{y,t} \\ \varepsilon_{z,t} \end{bmatrix}$$

Multiplying both sides by the inverse of B makes us obtain the VAR model in standard form:

$$x_t = A_1 x_{t-1} + e_t$$

where $A_1 = B^{-1}\Gamma_1$ and $e_t = B^{-1}\varepsilon_t$

c(i)

In this particular case,

$$B = \begin{bmatrix} 1 & b_{12} \\ 0 & 1 \end{bmatrix}$$

. We also know that $BA_1 = \Gamma_1$, therefore we can express Γ_1 as:

$$\Gamma_1 = \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix} = \begin{bmatrix} 0.6 - 0.1b_{12} & 0.2 - 0.8b_{12} \\ -0.1 & -0.8 \end{bmatrix}$$

Also, we know $e_t = B\varepsilon_t$, then:

$$e_t = \begin{bmatrix} e_{1,t} \\ e_{2,t} \end{bmatrix} = \begin{bmatrix} \varepsilon_{y,t} - \varepsilon_{z,t}b_{12} \\ -\varepsilon_{z,t} \end{bmatrix}$$

From where we can get the following matrix of covariances expressed as variances of the structural errors and parameter b_{12}:

$$\Sigma_e = \begin{bmatrix} \sigma_y^2 + (b_{12}^2 + b_{12})\sigma_z^2 & b_{12}\sigma_z^2 \\ b_{12}\sigma_z^2 & \sigma_z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 2 \end{bmatrix}$$

With some algebra we find that $b_{12} = 0.25$, $\sigma_y^2 = 3/8$, $\sigma_z^2 = 2$ and

$$\Gamma_1 = \begin{bmatrix} 0.575 & 0 \\ -0.1 & -0.8 \end{bmatrix}$$

c(ii)

First, we define

$$B = \begin{bmatrix} 1 & b \\ b & 1 \end{bmatrix}$$

, where $b = b_{12} = b_{21}$. Additionally, as explained in p.317 of the book, the covariance matrix of the reduce form can be expressed as:

$$\Sigma_e = B^{-1}\Sigma_\varepsilon(B^{-1})^T$$

Given B is a symetric matrix, the following expression holds:

$$B\Sigma_e B = BB^{-1}\Sigma_\varepsilon(B^{-1})^T B = \Sigma_\varepsilon$$

Where the extremes' expressions are equivalent to:

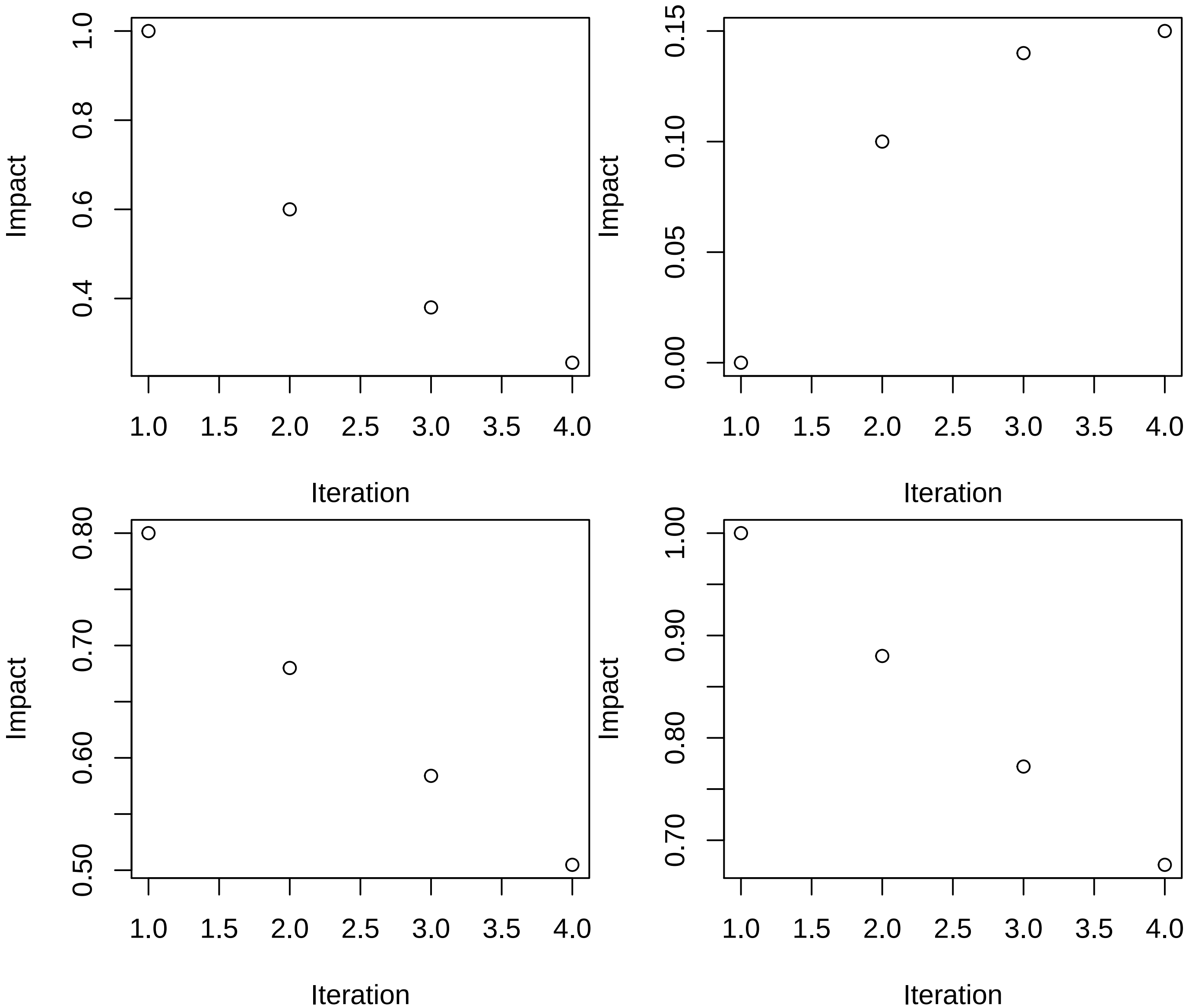
$$\begin{bmatrix} 2b^2 - b + 1 & -0.5b^2 + 3b - 0.5 \\ -0.5b^2 + 3b - 0.5 & b^2 - b + 2 \end{bmatrix} = \begin{bmatrix} \sigma_y^2 & 0 \\ 0 & \sigma_z^2 \end{bmatrix}$$

Solving for b in $-0.5b^2 + 3b - 0.5 = 0$ we get the following two sets of solutions:

$$\theta_i = (b_1, \sigma_y^2, \sigma_z^2, \gamma_{11}, \gamma_{12}, \gamma_{21}, \gamma_{22})_i = (0.172, 0.887, 1.858, 0.583, 0.063, 0.003, -0.766)$$

$$\theta_{ii} = (b_1, \sigma_y^2, \sigma_z^2, \gamma_{11}, \gamma_{12}, \gamma_{21}, \gamma_{22})_{ii} = (5.828, 63.112, 30.142, 0.0172, -4.463, 3.397, 0.366)$$

d



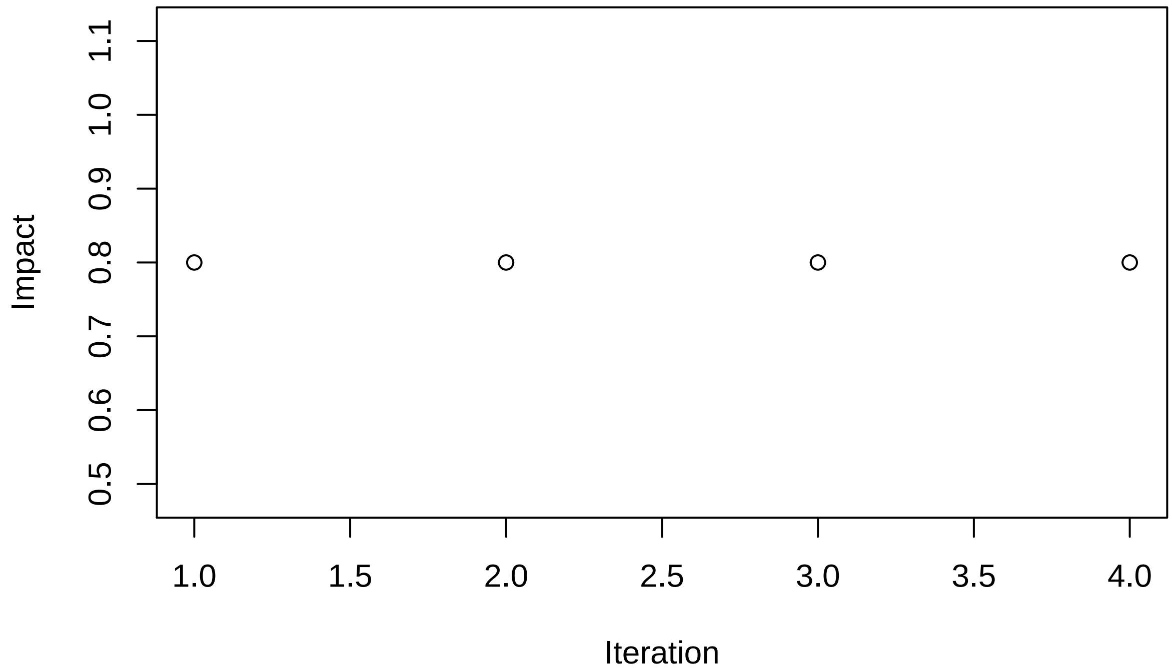
There's a unit root if solutions of the system $det(I - zA_1) = 0$ lie on the unit circle. A unit root means $z = (1, 1)$, then

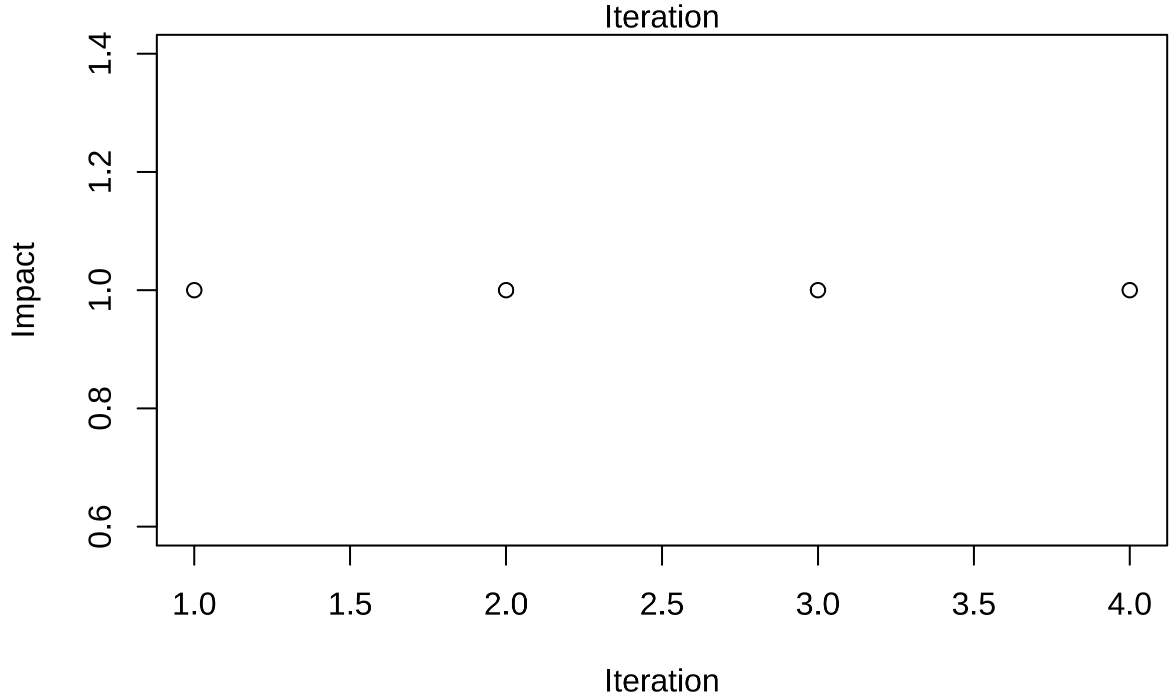
$$I - A_1 = \begin{bmatrix} 1 - a_{11} & -a_{12} \\ -a_{21} & 1 - a_{22} \end{bmatrix}$$

For simplicity, take $A_1 = I$, then the determinant:

$$det(I - A_1) = 0$$

.





The shock will never fade as the impacts seem to not return to zero and remain constant. ## 2

Empirical exercises

Do exercises 10a-10g in the textbook (p.340)

- Remark 1: It is possible that the values you obtain for the F-statistics, p-values and correlations are different than those reported since the sample is extended. However, the main conclusions should be the same.
- Remark 2: Exercise d. is optional and so is the part on the forecast error variance in e. (but you could use the command fevd in STATA to answer these questions).
- Remark 3: You find the appropriate specifications for the variables st , Δlip, and Δur described in the text to exercise 9 (p.339).

10:

Estimate the three-VAR beginning in 1961Q1 and use the ordering such that Δlip_t is causally prior to Δur_t and that Δur_t is causally prior to s_t.

We begin by defining the variables we are going to include in our analysis. We create $dlip = log(indprod_t) - log(indprod_{t-1}), dur = urate_t - urate_{t-1}$ and $s = r10 - tbill$.

In the context of chapter 5, we assume that staionarity holds. Additionally, it is provided for us that the appropriate lag length is 3. The result of the var estimation is as follows:

```
##
## VAR Estimation Results:
## =========================
## Endogenous variables: s, dur, dlip
## Deterministic variables: none
## Sample size: 231
## Log Likelihood: 612.866
## Roots of the characteristic polynomial:
## 0.9173 0.7824 0.6453 0.463 0.463 0.4479 0.4479 0.1831 0.1831
## Call:
## VAR(y = ., p = 3, type = "none")
##
##
## Estimation results for equation s:
## ==================================
## s = s.l1 + dur.l1 + dlip.l1 + s.l2 + dur.l2 + dlip.l2 + s.l3 + dur.l3 + dlip.l3
##
##         Estimate Std. Error t value Pr(>|t|)
## s.l1     1.08636    0.06711  16.188  < 2e-16 ***
## dur.l1   0.58848    0.19168   3.070  0.00241 **
## dlip.l1  0.50432    3.78403   0.133  0.89409
## s.l2    -0.31910    0.09737  -3.277  0.00122 **
## dur.l2  -0.24687    0.21092  -1.170  0.24307
## dlip.l2  2.10670    4.09152   0.515  0.60714
## s.l3     0.18490    0.06767   2.732  0.00679 **
## dur.l3   0.30594    0.18962   1.613  0.10808
## dlip.l3  0.35224    3.71775   0.095  0.92460
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.5104 on 222 degrees of freedom
## Multiple R-Squared: 0.9281,  Adjusted R-squared: 0.9252
## F-statistic: 318.3 on 9 and 222 DF,  p-value: < 2.2e-16
##
```

```
##
## Estimation results for equation dur:
## ====================================
## dur = s.l1 + dur.l1 + dlip.l1 + s.l2 + dur.l2 + dlip.l2 + s.l3 + dur.l3 + dlip.l3
##
##          Estimate Std. Error t value Pr(>|t|)
## s.l1     0.006923   0.031567   0.219   0.8266
## dur.l1   0.522127   0.090161   5.791 2.37e-08 ***
## dlip.l1 -4.056028   1.779886  -2.279   0.0236 *
## s.l2    -0.009138   0.045798  -0.200   0.8420
## dur.l2   0.056733   0.099211   0.572   0.5680
## dlip.l2  2.175020   1.924519   1.130   0.2596
## s.l3    -0.016687   0.031829  -0.524   0.6006
## dur.l3   0.028446   0.089192   0.319   0.7501
## dlip.l3  1.033190   1.748709   0.591   0.5552
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.2401 on 222 degrees of freedom
## Multiple R-Squared: 0.4751,  Adjusted R-squared: 0.4538
## F-statistic: 22.32 on 9 and 222 DF,  p-value: < 2.2e-16
##
##
## Estimation results for equation dlip:
## =====================================
## dlip = s.l1 + dur.l1 + dlip.l1 + s.l2 + dur.l2 + dlip.l2 + s.l3 + dur.l3 + dlip.l3
##
##           Estimate Std. Error t value Pr(>|t|)
## s.l1     0.0009883  0.0015925   0.621   0.5355
## dur.l1  -0.0063116  0.0045486  -1.388   0.1666
## dlip.l1  0.5404560  0.0897942   6.019 7.19e-09 ***
## s.l2     0.0010944  0.0023105   0.474   0.6362
## dur.l2   0.0062905  0.0050051   1.257   0.2101
## dlip.l2 -0.0650182  0.0970909  -0.670   0.5038
## s.l3    -0.0002954  0.0016058  -0.184   0.8542
## dur.l3   0.0044926  0.0044997   0.998   0.3192
## dlip.l3  0.1584778  0.0882214   1.796   0.0738 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.01211 on 222 degrees of freedom
## Multiple R-Squared: 0.4889,  Adjusted R-squared: 0.4682
## F-statistic:  23.6 on 9 and 222 DF,  p-value: < 2.2e-16
##
##
##
## Covariance matrix of residuals:
##              s       dur       dlip
## s     0.258211  0.026343 -0.0011285
## dur   0.026343  0.056898 -0.0019789
## dlip -0.001128 -0.001979  0.0001467
##
```

```
## Correlation matrix of residuals:
##            s     dur    dlip
## s     1.0000  0.2173 -0.1833
## dur   0.2173  1.0000 -0.6849
## dlip -0.1833 -0.6849  1.0000
```

We also check for serial correlation using the adjusted Portmanteau test with 8 lags:

```
##
##  Portmanteau Test (adjusted)
##
## data:  Residuals of VAR object var
## Chi-squared = 101.67, df = 45, p-value = 2.864e-06
```

The null of no serial correlation is rejected. Hence, there is still serial correlation in the data. Anyway, we specified the model with the number of lags given in the book, so we proceed without further changes.

10 a:

If you perform a test to determine whether s_t Granger causes Δlip_t, you should find that the F-statistic is 2.44 with a prob-value of 0.065. How do you interpret this result?

Statistic	N	Mean	St. Dev.
Res.Df	2	225.500	2.121
Df	1	−3.000	
F	1	2.774	
Pr(>F)	1	0.042	

The p-value is borderline significant. Assuming the null that no lag of s predicts *dlip* does not hold, then the meaning is that there is a lag of s that does predict *dlip*. Hence, s granger causes *dlip*. However, it is not clear cut given the p-value.

10 b:

Verify that s_t Granger causes $\Delta unemp_t$. You should find that the F statistic is 5.93 with a prob value of less than 0.001.

Statistic	N	Mean	St. Dev.
Res.Df	2	225.500	2.121
Df	1	−3.000	
F	1	4.450	
Pr(>F)	1	0.005	

10 c:

It turns out that the correlation coefficient between e_{1t} and e_{2t} is -0.72. The correlation between e_{1t} and e_{3t} is -0.11 and between e_{2t} and e_{3t} is 0.10. Explain why the ordering of a Choleski composition is likely to be important for obtaining the impulse responses.

The ordering of the Cholesky decomposition is important as it determines which variables are causally prior to each other. Knowing this relationship in turn allows for computing how random shocks propagate themselves in the system.

10 e:

Now estimate the model using the levels of lip_t and ur_t. Do you now find a lag length of 5 appropriate?

```
##
## VAR Estimation Results:
## =========================
## Endogenous variables: s, dur, dlip
## Deterministic variables: none
## Sample size: 229
## Log Likelihood: 634.837
## Roots of the characteristic polynomial:
## 0.9312 0.838 0.7113 0.6921 0.6921 0.6647 0.6647 0.6472 0.6472 0.6283 0.6283 0.597 0.597 0.4849 0.4849
## Call:
## VAR(y = ., p = 5, type = "none")
##
##
## Estimation results for equation s:
## ==================================
## s = s.l1 + dur.l1 + dlip.l1 + s.l2 + dur.l2 + dlip.l2 + s.l3 + dur.l3 + dlip.l3 + s.l4 + dur.l4 + dli
##
##         Estimate Std. Error t value Pr(>|t|)
## s.l1     1.11837    0.06839  16.353  < 2e-16 ***
## dur.l1   0.68432    0.19940   3.432 0.000719 ***
## dlip.l1  0.52247    3.83414   0.136 0.891737
## s.l2    -0.41224    0.10160  -4.057 6.96e-05 ***
## dur.l2  -0.21959    0.21194  -1.036 0.301337
## dlip.l2  4.31175    4.17523   1.033 0.302911
## s.l3     0.35337    0.10223   3.457 0.000659 ***
## dur.l3   0.32927    0.21304   1.546 0.123689
## dlip.l3 -1.77888    4.11297  -0.433 0.665811
## s.l4    -0.24039    0.10036  -2.395 0.017467 *
## dur.l4  -0.23391    0.21536  -1.086 0.278633
## dlip.l4  3.28703    4.13872   0.794 0.427950
## s.l5     0.14193    0.06857   2.070 0.039678 *
## dur.l5   0.14418    0.19029   0.758 0.449472
## dlip.l5 -5.51291    3.83101  -1.439 0.151606
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.5044 on 214 degrees of freedom
## Multiple R-Squared: 0.9319,  Adjusted R-squared: 0.9272
## F-statistic: 195.4 on 15 and 214 DF,  p-value: < 2.2e-16
##
##
## Estimation results for equation dur:
## ====================================
## dur = s.l1 + dur.l1 + dlip.l1 + s.l2 + dur.l2 + dlip.l2 + s.l3 + dur.l3 + dlip.l3 + s.l4 + dur.l4 + d
##
##          Estimate Std. Error t value Pr(>|t|)
## s.l1     0.023731   0.031893   0.744  0.45765
## dur.l1   0.451130   0.092993   4.851 2.36e-06 ***
## dlip.l1 -5.096861   1.788062  -2.850  0.00479 **
```

```
## s.l2    -0.042737   0.047383  -0.902  0.36810
## dur.l2   0.049793   0.098839   0.504  0.61494
## dlip.l2  2.100112   1.947130   1.079  0.28199
## s.l3     0.029039   0.047673   0.609  0.54309
## dur.l3   0.110662   0.099352   1.114  0.26660
## dlip.l3 -0.541665   1.918096  -0.282  0.77791
## s.l4    -0.044488   0.046802  -0.951  0.34290
## dur.l4  -0.073315   0.100434  -0.730  0.46620
## dlip.l4  2.558021   1.930105   1.325  0.18648
## s.l5     0.006982   0.031979   0.218  0.82737
## dur.l5   0.154565   0.088744   1.742  0.08300 .
## dlip.l5  3.509294   1.786606   1.964  0.05080 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.2352 on 214 degrees of freedom
## Multiple R-Squared: 0.5078,  Adjusted R-squared: 0.4733
## F-statistic: 14.72 on 15 and 214 DF,  p-value: < 2.2e-16
##
##
## Estimation results for equation dlip:
## =====================================
## dlip = s.l1 + dur.l1 + dlip.l1 + s.l2 + dur.l2 + dlip.l2 + s.l3 + dur.l3 + dlip.l3 + s.l4 + dur.l4 +
##
##           Estimate Std. Error t value Pr(>|t|)
## s.l1     0.0008782  0.0016122   0.545   0.5865
## dur.l1  -0.0056531  0.0047009  -1.203   0.2305
## dlip.l1  0.5565343  0.0903890   6.157 3.61e-09 ***
## s.l2     0.0015771  0.0023953   0.658   0.5110
## dur.l2   0.0048147  0.0049965   0.964   0.3363
## dlip.l2 -0.0693264  0.0984300  -0.704   0.4820
## s.l3    -0.0021046  0.0024100  -0.873   0.3835
## dur.l3   0.0041926  0.0050224   0.835   0.4048
## dlip.l3  0.1924347  0.0969624   1.985   0.0485 *
## s.l4     0.0006620  0.0023659   0.280   0.7799
## dur.l4   0.0020471  0.0050771   0.403   0.6872
## dlip.l4 -0.0596774  0.0975694  -0.612   0.5414
## s.l5     0.0008779  0.0016166   0.543   0.5877
## dur.l5  -0.0022074  0.0044862  -0.492   0.6232
## dlip.l5 -0.0358022  0.0903153  -0.396   0.6922
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.01189 on 214 degrees of freedom
## Multiple R-Squared: 0.5119,  Adjusted R-squared: 0.4777
## F-statistic: 14.96 on 15 and 214 DF,  p-value: < 2.2e-16
##
##
##
## Covariance matrix of residuals:
##             s       dur       dlip
## s     0.25167  0.026099 -0.0011302
```

```
## dur   0.02610  0.054949 -0.0019559
## dlip -0.00113 -0.001956  0.0001414
##
## Correlation matrix of residuals:
##            s     dur    dlip
## s     1.0000  0.2219 -0.1895
## dur   0.2219  1.0000 -0.7017
## dlip -0.1895 -0.7017  1.0000
```

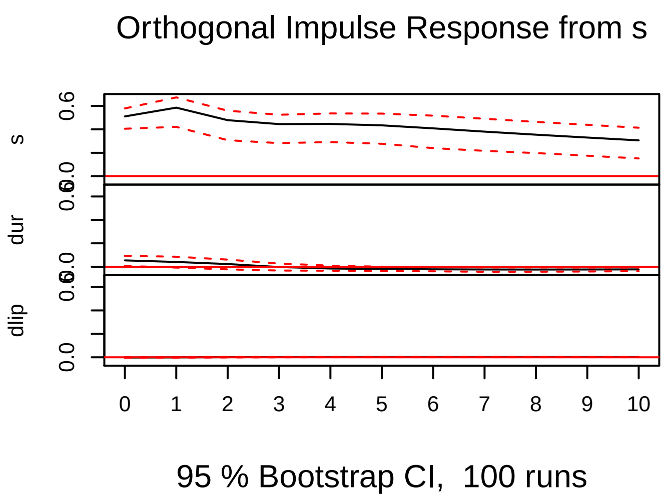
We do not think a lag length of 5 is appropriate as the AIC suggests a lag length of 1 is sufficient. Next, we check for serial correlation:

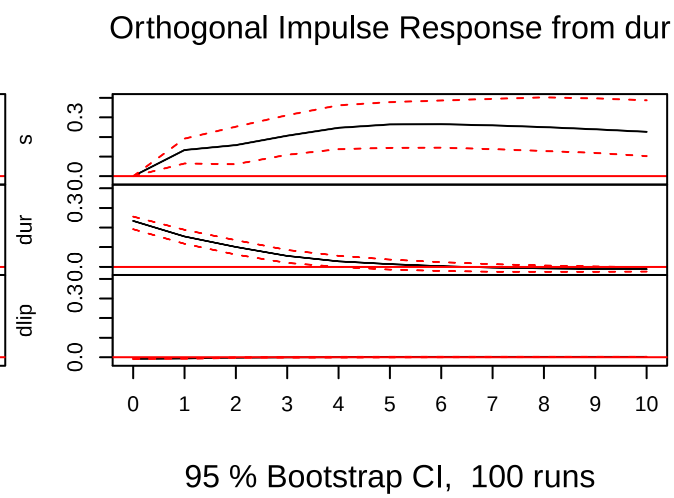
```
##
##  Portmanteau Test (adjusted)
##
## data:  Residuals of VAR object var_e
## Chi-squared = 70.315, df = 27, p-value = 1.011e-05
```

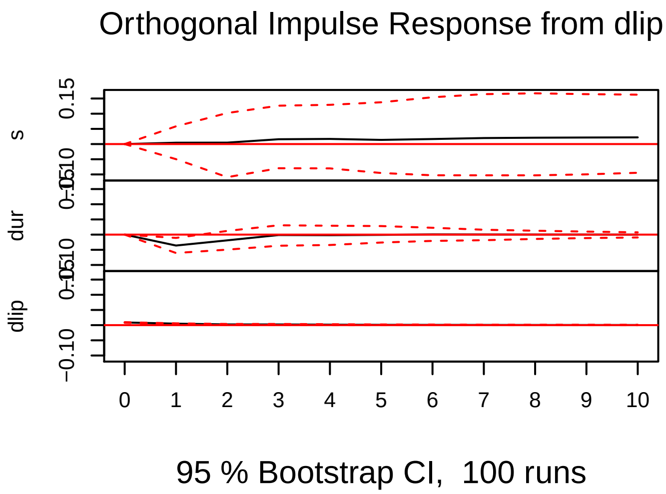
We find that there is serial correlation in the specified model using the adjusted portmanteau test.

10 f:

Obtain the impulse response function from the model using $\Delta lip_t, \Delta ur_t$ and s_t. Show that a positive shock to the industrial production induces a decline in the unemployment rate that lasts six quarters. Then, Δur_t overshoots its long run level before returning to zero.

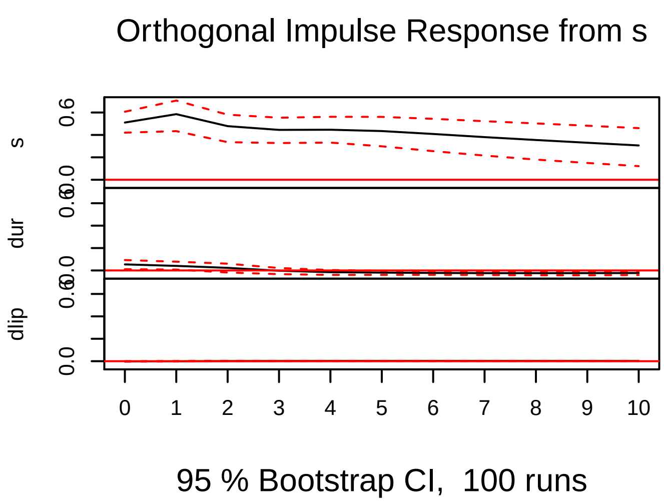


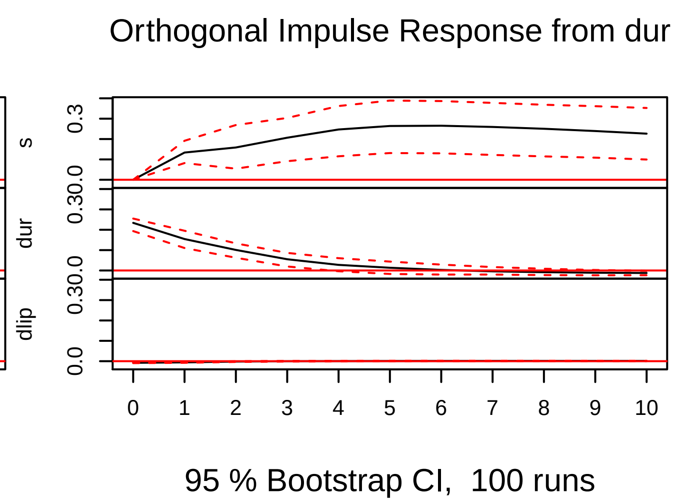


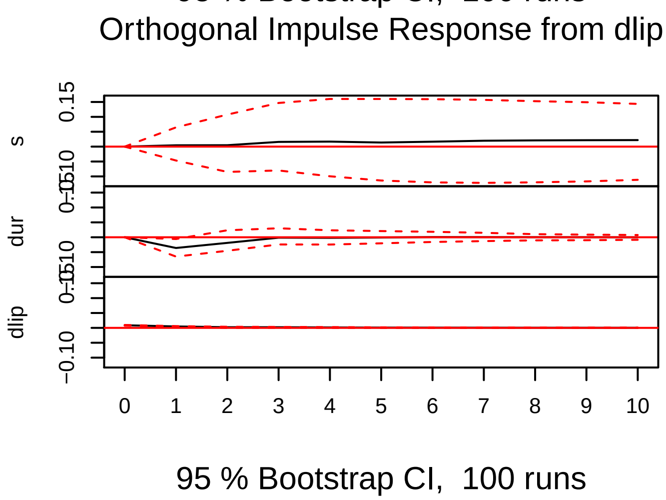


10 g:

Reverse the ordering and explain why the results depend on whether or not Δlip_t proceeds Δur_t







If Δlip_t proceeds Δur_t, then a contemporary effect on Δlip_t affects Δur_t, but not vice versa. On the other hand, if the reverse holds, then shocks to Δlip_t will be delayed until next period before any noticeable change occurs to Δur_t.