# Assignment 3

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*2019-02-23*

## Contents

Hello,this is a comment from Ismael

## Theoretical exercises

### 1

**a:**

test Show that $* = Cov(z_t, \varepsilon_{yt}) \neq 0$.

- Recall the formula for covariance: $Cov(z_t, \varepsilon_{yt}) = E(z_t\varepsilon_{yt}) - E(z_t)E(\varepsilon_{yt})$. Because $\varepsilon_{yt} \sim WN(0, \sigma_y^2)$, we obtain: $* = E(z_t\varepsilon_{yt})$.

- Next, expand the the expression for $y_t$ in the expression for $z_t$: $* = E[(-b_{21}[(b_{12}z_t + \gamma_{11}y_{t-1} + \gamma_{12}z_{t-1} + \varepsilon_{yt}] + \gamma_{21}y_{t-1} + \gamma_{22}z_{t-1} + \varepsilon_{zt})\varepsilon_{yt}]$.

- Now distribute $\varepsilon_{yt}$ over the system: $* = E(-b_{21}[(b_{12}z_t + \gamma_{11}y_{t-1} + \gamma_{12}z_{t-1} + \varepsilon_{yt}]\varepsilon_{yt} + \gamma_{21}y_{t-1}\varepsilon_{yt} + \gamma_{22}z_{t-1}\varepsilon_{yt} + \varepsilon_{zt}\varepsilon_{yt})$

- Expand the expectation operator to a sum: $* = E(-b_{21}[(b_{12}z_t + \gamma_{11}y_{t-1} + \gamma_{12}z_{t-1} + \varepsilon_{yt}]\varepsilon_{yt}) + E(\gamma_{21}y_{t-1}\varepsilon_{yt}) + E(\gamma_{22}z_{t-1}\varepsilon_{yt}) + E(\varepsilon_{zt}\varepsilon_{yt})$.

- Exploit intertemporal independence and that $\varepsilon_{yt}$ and $\varepsilon_{zt}$ are independent: $* = E(-b_{21}[(b_{12}z_t + \gamma_{11}y_{t-1} + \gamma_{12}z_{t-1} + \varepsilon_{yt}]\varepsilon_{yt})$

- Distibute $\varepsilon_{yt}$: $* = -b_{21}E([(b_{12}z_t\varepsilon_{yt} + \gamma_{11}y_{t-1}\varepsilon_{yt} + \gamma_{12}z_{t-1}\varepsilon_{yt} + \varepsilon_{yt}\varepsilon_{yt}])$

- Expand the expectation: $* = -b_{21}[E(b_{12}z_t\varepsilon_{yt}) + E(\gamma_{11}y_{t-1}\varepsilon_{yt}) + E(\gamma_{12}z_{t-1}\varepsilon_{yt}) + E(\varepsilon_{yt}^2)]$

- What remains after exploiting independence is $* = -b_{21}E(\varepsilon_{yt}^2) = -b_{21}\sigma_y^2 \neq 0$ QED.

The implications on estimation are that estimates will be inefficient and baised.

**b**

Firstly, we express (1) in the following matrix form:

$$BX_t = \Gamma_0 + \Gamma_1 X_{t-1} + \varepsilon_t$$

Where

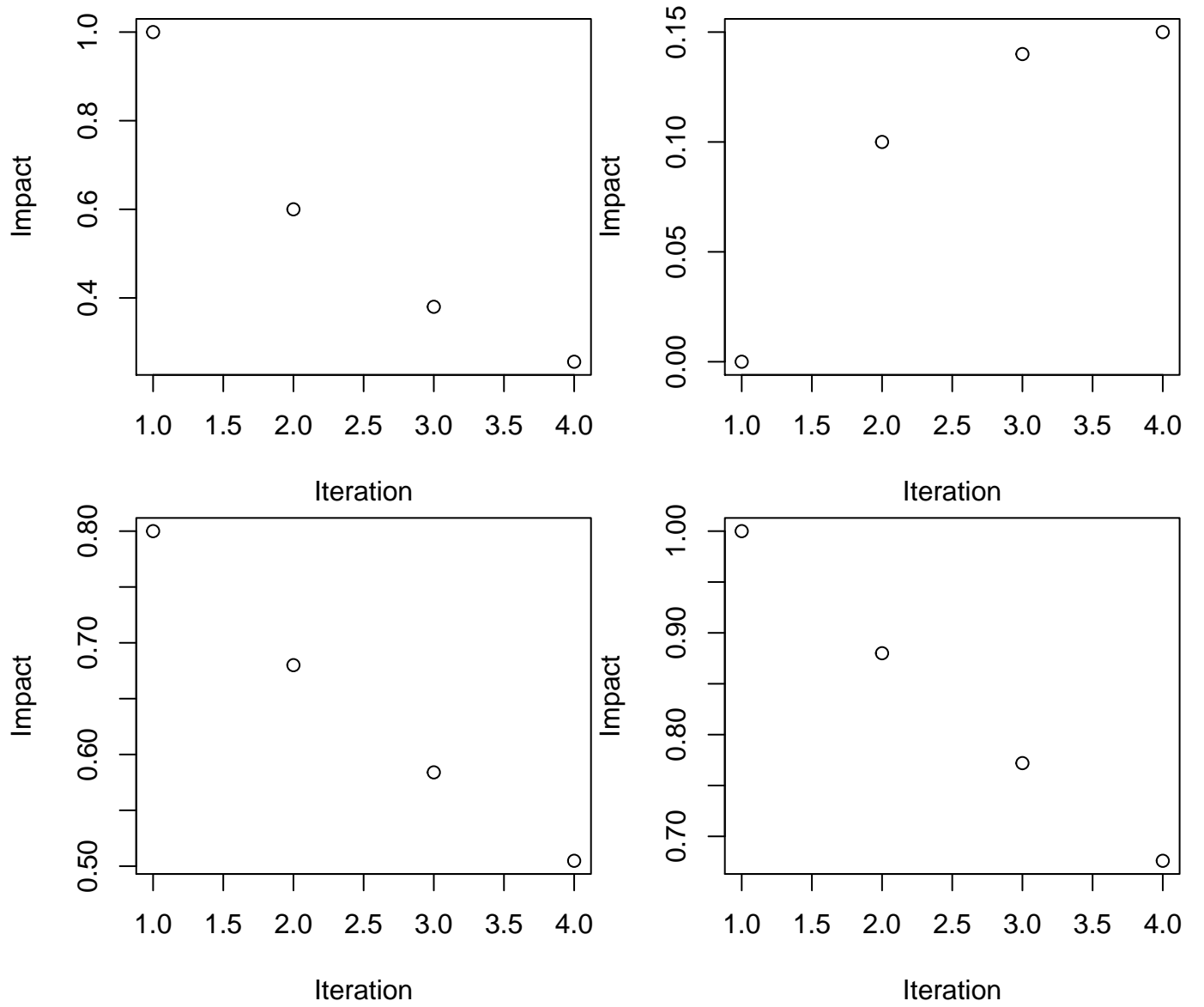$$B = \begin{bmatrix} 1 & b_{12} \\ b_{21} & 1 \end{bmatrix}$$

,

$$X_t = \begin{bmatrix} y_t \\ z_t \end{bmatrix}$$

$$\Gamma_0 = \begin{bmatrix} b_{10} \\ b_{20} \end{bmatrix}$$

$$\Gamma_1 = \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix}$$

$$\Gamma_0 = \begin{bmatrix} b_{10} \\ b_{20} \end{bmatrix}$$

**d**

There's a unit root if solutions of the system $det(I - zA_1) = 0$ lie on the unit circle. Take $z = 1$, then

$$I - A_1 = \begin{bmatrix} 1 - a_{11} & -a_{12} \\ -a_{21} & 1 - a_{22} \end{bmatrix}$$

# 2

# Empirical exercises

Do exercises 10a-10g in the textbook (p.340)

- Remark 1: It is possible that the values you obtain for the F-statistics, p-values and correlations are different than those reported since the sample is extended. However, the main conclusions should be the same.

- Remark 2: Exercise d. is optional and so is the part on the forecast error variance in e. (but you could use the command fevd in STATA to answer these questions).

- Remark 3: You find the appropriate specifications for the variables st , $\Delta$lip, and $\Delta$ur described in the text to exercise 9 (p.339).

**10:**

Estimate the three-VAR beginning in 1961Q1 and use the ordering such that $\Delta lip_t$ is causally prior to $\Delta ur_t$ and that $\Delta ur_t$ is causally prior to $s_t$.

We begin by defining the variables we are going to include in our analysis. We create $dlip = log(indprod_t) - log(indprod_{t-1})$, $dur = urate_t - urate_{t-1}$ and $s = r10 - tbill$.

In the context of chapter 5, we assume that staionarity holds. Additionally, it is provided for us that the appropriate lag length is 3. The result of the var estimation is as follows:

```
##
## VAR Estimation Results:
## =========================
## Endogenous variables: s, dur, dlip
## Deterministic variables: none
## Sample size: 231
## Log Likelihood: 612.866
## Roots of the characteristic polynomial:
## 0.9173 0.7824 0.6453 0.463 0.463 0.4479 0.4479 0.1831 0.1831
## Call:
## VAR(y = ., p = 3, type = "none")
##
##
## Estimation results for equation s:
## ===================================
## s = s.l1 + dur.l1 + dlip.l1 + s.l2 + dur.l2 + dlip.l2 + s.l3 + dur.l3 + dlip.l3
##
##           Estimate Std. Error t value Pr(>|t|)
## s.l1       1.08636    0.06711  16.188  < 2e-16 ***
## dur.l1     0.58848    0.19168   3.070  0.00241 **
## dlip.l1    0.50432    3.78403   0.133  0.89409
## s.l2      -0.31910    0.09737  -3.277  0.00122 **
## dur.l2    -0.24687    0.21092  -1.170  0.24307
```

```
## dlip.l2   2.10670     4.09152    0.515  0.60714
## s.l3       0.18490     0.06767    2.732  0.00679 **
## dur.l3     0.30594     0.18962    1.613  0.10808
## dlip.l3    0.35224     3.71775    0.095  0.92460
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.5104 on 222 degrees of freedom
## Multiple R-Squared: 0.9281,  Adjusted R-squared: 0.9252
## F-statistic: 318.3 on 9 and 222 DF,  p-value: < 2.2e-16
##
##
## Estimation results for equation dur:
## ====================================
## dur = s.l1 + dur.l1 + dlip.l1 + s.l2 + dur.l2 + dlip.l2 + s.l3 + dur.l3 + dlip.l3
##
##           Estimate Std. Error t value Pr(>|t|)
## s.l1      0.006923   0.031567    0.219   0.8266
## dur.l1    0.522127   0.090161    5.791 2.37e-08 ***
## dlip.l1  -4.056028   1.779886   -2.279   0.0236 *
## s.l2     -0.009138   0.045798   -0.200   0.8420
## dur.l2    0.056733   0.099211    0.572   0.5680
## dlip.l2   2.175020   1.924519    1.130   0.2596
## s.l3     -0.016687   0.031829   -0.524   0.6006
## dur.l3    0.028446   0.089192    0.319   0.7501
## dlip.l3   1.033190   1.748709    0.591   0.5552
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.2401 on 222 degrees of freedom
## Multiple R-Squared: 0.4751,  Adjusted R-squared: 0.4538
## F-statistic: 22.32 on 9 and 222 DF,  p-value: < 2.2e-16
##
##
## Estimation results for equation dlip:
## =====================================
## dlip = s.l1 + dur.l1 + dlip.l1 + s.l2 + dur.l2 + dlip.l2 + s.l3 + dur.l3 + dlip.l3
##
##            Estimate Std. Error t value Pr(>|t|)
## s.l1      0.0009883  0.0015925    0.621   0.5355
## dur.l1   -0.0063116  0.0045486   -1.388   0.1666
## dlip.l1   0.5404560  0.0897942    6.019 7.19e-09 ***
## s.l2      0.0010944  0.0023105    0.474   0.6362
## dur.l2    0.0062905  0.0050051    1.257   0.2101
## dlip.l2  -0.0650182  0.0970909   -0.670   0.5038
## s.l3     -0.0002954  0.0016058   -0.184   0.8542
## dur.l3    0.0044926  0.0044997    0.998   0.3192
## dlip.l3   0.1584778  0.0882214    1.796   0.0738 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
```

```
## Residual standard error: 0.01211 on 222 degrees of freedom
## Multiple R-Squared: 0.4889,  Adjusted R-squared: 0.4682
## F-statistic:  23.6 on 9 and 222 DF,  p-value: < 2.2e-16
##
##
##
## Covariance matrix of residuals:
##             s        dur        dlip
## s      0.258211  0.026343 -0.0011285
## dur    0.026343  0.056898 -0.0019789
## dlip  -0.001128 -0.001979  0.0001467
##
## Correlation matrix of residuals:
##             s       dur      dlip
## s      1.0000   0.2173  -0.1833
## dur    0.2173   1.0000  -0.6849
## dlip  -0.1833  -0.6849   1.0000
```

**10 a:**

If you perform a test to determine whether $s_t$ Granger causes $\Delta lip_t$, you should find that the F-statistic is 2.44 with a prob-value of 0.065. How do you interpret this result?

| Statistic | N | Mean | St. Dev. |
|---|---|---|---|
| Res.Df | 2 | 225.500 | 2.121 |
| Df | 1 | −3.000 | |
| F | 1 | 2.774 | |
| Pr(>F) | 1 | 0.042 | |

The p-value is borderline significant. Assuming the null that no lag of $s$ predicts *dlip* does not hold, then the meaning is that there is a lag of $s$ that does predict *dlip*. Hence, $s$ granger causes *dlip*. However, it is not clear cut given the p-value.

**10 b:**

Verify that $s_t$ Granger causes $\Delta unemp_t$. You should find that the F statistic is 5.93 with a prob value of less than 0.001.

| Statistic | N | Mean | St. Dev. |
|---|---|---|---|
| Res.Df | 2 | 225.500 | 2.121 |
| Df | 1 | −3.000 | |
| F | 1 | 4.450 | |
| Pr(>F) | 1 | 0.005 | |

**10 c:**

It turns out that the correlation coefficient between $e_{1t}$ and $e_{2t}$ is -0.72. The correlation between $e_{1t}$ and $e_{3t}$ is -0.11 and between $e_{2t}$ and $e_{3t}$ is 0.10. Explain why the ordering of a Choleski composition is likely to be important for obtaining the impulse responses.

**10 e:**

Now estimate the model using the levels of $lip_t$ and $ur_t$. Do you now find a lag length of 5 appropriate?

```
##
## VAR Estimation Results:
## =========================
## Endogenous variables: s, dur, dlip
## Deterministic variables: none
## Sample size: 229
## Log Likelihood: 634.837
## Roots of the characteristic polynomial:
## 0.9312 0.838 0.7113 0.6921 0.6921 0.6647 0.6647 0.6472 0.6472 0.6283 0.6283 0.597 0.597 0.4849 0.4849
## Call:
## VAR(y = ., p = 5, type = "none")
##
##
## Estimation results for equation s:
## ===================================
## s = s.l1 + dur.l1 + dlip.l1 + s.l2 + dur.l2 + dlip.l2 + s.l3 + dur.l3 + dlip.l3 + s.l4 + dur.l4 + dl
##
##          Estimate Std. Error t value Pr(>|t|)
## s.l1      1.11837    0.06839  16.353  < 2e-16 ***
## dur.l1    0.68432    0.19940   3.432 0.000719 ***
## dlip.l1   0.52247    3.83414   0.136 0.891737
## s.l2     -0.41224    0.10160  -4.057 6.96e-05 ***
## dur.l2   -0.21959    0.21194  -1.036 0.301337
## dlip.l2   4.31175    4.17523   1.033 0.302911
## s.l3      0.35337    0.10223   3.457 0.000659 ***
## dur.l3    0.32927    0.21304   1.546 0.123689
## dlip.l3  -1.77888    4.11297  -0.433 0.665811
## s.l4     -0.24039    0.10036  -2.395 0.017467 *
## dur.l4   -0.23391    0.21536  -1.086 0.278633
## dlip.l4   3.28703    4.13872   0.794 0.427950
## s.l5      0.14193    0.06857   2.070 0.039678 *
## dur.l5    0.14418    0.19029   0.758 0.449472
## dlip.l5  -5.51291    3.83101  -1.439 0.151606
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.5044 on 214 degrees of freedom
## Multiple R-Squared: 0.9319,  Adjusted R-squared: 0.9272
## F-statistic: 195.4 on 15 and 214 DF,  p-value: < 2.2e-16
##
##
## Estimation results for equation dur:
## ====================================
## dur = s.l1 + dur.l1 + dlip.l1 + s.l2 + dur.l2 + dlip.l2 + s.l3 + dur.l3 + dlip.l3 + s.l4 + dur.l4 + c
##
##          Estimate Std. Error t value Pr(>|t|)
## s.l1     0.023731   0.031893   0.744  0.45765
## dur.l1   0.451130   0.092993   4.851 2.36e-06 ***
## dlip.l1 -5.096861   1.788062  -2.850  0.00479 **
```

```
## s.l2     -0.042737    0.047383   -0.902   0.36810
## dur.l2    0.049793    0.098839    0.504   0.61494
## dlip.l2   2.100112    1.947130    1.079   0.28199
## s.l3      0.029039    0.047673    0.609   0.54309
## dur.l3    0.110662    0.099352    1.114   0.26660
## dlip.l3  -0.541665    1.918096   -0.282   0.77791
## s.l4     -0.044488    0.046802   -0.951   0.34290
## dur.l4   -0.073315    0.100434   -0.730   0.46620
## dlip.l4   2.558021    1.930105    1.325   0.18648
## s.l5      0.006982    0.031979    0.218   0.82737
## dur.l5    0.154565    0.088744    1.742   0.08300 .
## dlip.l5   3.509294    1.786606    1.964   0.05080 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.2352 on 214 degrees of freedom
## Multiple R-Squared: 0.5078,  Adjusted R-squared: 0.4733
## F-statistic: 14.72 on 15 and 214 DF,  p-value: < 2.2e-16
##
##
## Estimation results for equation dlip:
## ======================================
## dlip = s.l1 + dur.l1 + dlip.l1 + s.l2 + dur.l2 + dlip.l2 + s.l3 + dur.l3 + dlip.l3 + s.l4 + dur.l4 +
##
##            Estimate Std. Error t value Pr(>|t|)
## s.l1      0.0008782  0.0016122   0.545   0.5865
## dur.l1   -0.0056531  0.0047009  -1.203   0.2305
## dlip.l1   0.5565343  0.0903890   6.157 3.61e-09 ***
## s.l2      0.0015771  0.0023953   0.658   0.5110
## dur.l2    0.0048147  0.0049965   0.964   0.3363
## dlip.l2  -0.0693264  0.0984300  -0.704   0.4820
## s.l3     -0.0021046  0.0024100  -0.873   0.3835
## dur.l3    0.0041926  0.0050224   0.835   0.4048
## dlip.l3   0.1924347  0.0969624   1.985   0.0485 *
## s.l4      0.0006620  0.0023659   0.280   0.7799
## dur.l4    0.0020471  0.0050771   0.403   0.6872
## dlip.l4  -0.0596774  0.0975694  -0.612   0.5414
## s.l5      0.0008779  0.0016166   0.543   0.5877
## dur.l5   -0.0022074  0.0044862  -0.492   0.6232
## dlip.l5  -0.0358022  0.0903153  -0.396   0.6922
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.01189 on 214 degrees of freedom
## Multiple R-Squared: 0.5119,  Adjusted R-squared: 0.4777
## F-statistic: 14.96 on 15 and 214 DF,  p-value: < 2.2e-16
##
##
##
## Covariance matrix of residuals:
##             s        dur        dlip
## s     0.25167  0.026099 -0.0011302
```
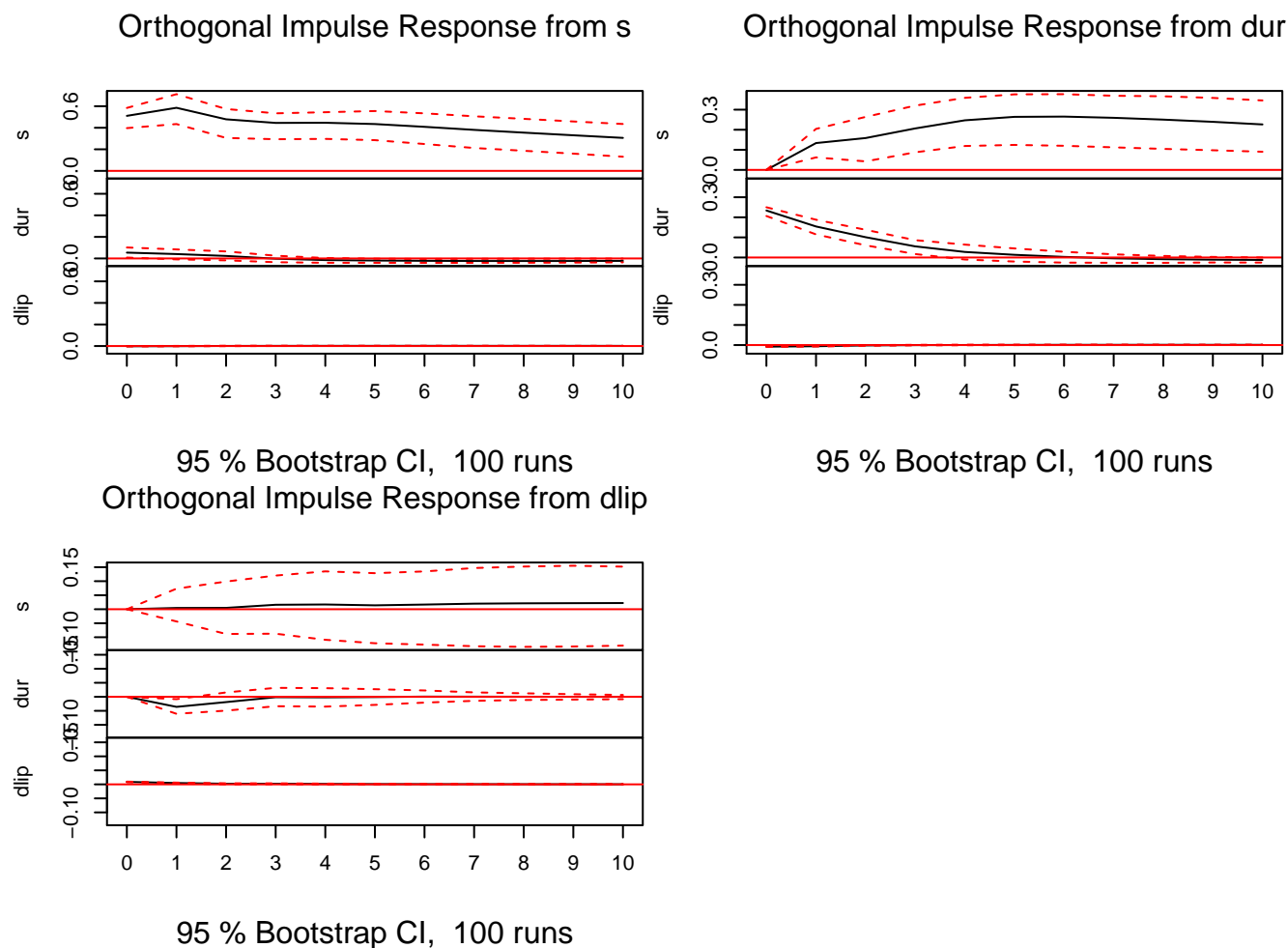
7

```
## dur    0.02610   0.054949 -0.0019559
## dlip -0.00113  -0.001956  0.0001414
##
## Correlation matrix of residuals:
##            s      dur     dlip
## s     1.0000   0.2219  -0.1895
## dur   0.2219   1.0000  -0.7017
## dlip -0.1895  -0.7017   1.0000
```

We do not think a lag length of 5 is appropriate as the AIC suggests a lag length of 1 is sufficient.

**10 f:**

Obtain the impulse response function from the model using $\Delta lip_t, \Delta ur_t$ and $s_t$. Show that a positive shock to the industrial production induces a decline in the unemployment rate that lasts six quarters. Then, $\Delta ur_t$ overshoots its long run level before returning to zero.
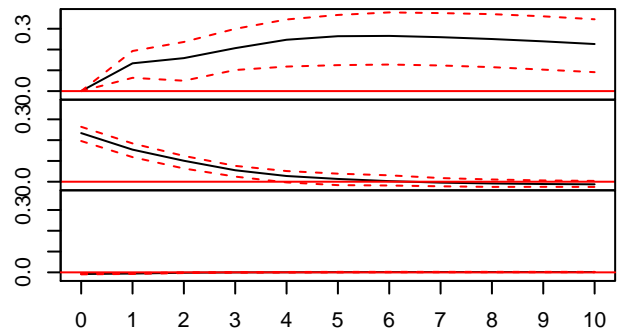


Orthogonal Impulse Response from s

95 % Bootstrap CI,  100 runs



Orthogonal Impulse Response from dur

95 % Bootstrap CI,  100 runs



Orthogonal Impulse Response from dlip

95 % Bootstrap CI,  100 runs

**10 g:**

Reverse the ordering and explain why the results depend on whether or not $\Delta lip_t$ proceeds $\Delta ur_t$

## Orthogonal Impulse Response from s
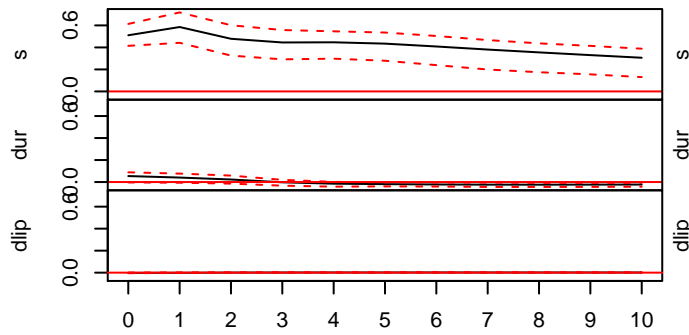


95 % Bootstrap CI,  100 runs
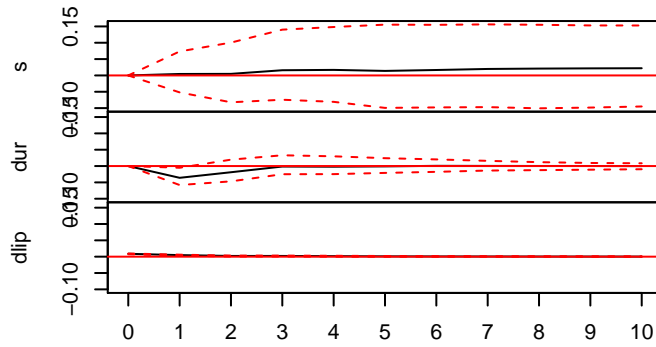
## Orthogonal Impulse Response from dur



95 % Bootstrap CI,  100 runs

## Orthogonal Impulse Response from dlip



95 % Bootstrap CI,  100 runs

If $\Delta lip_t$ proceeds $\Delta ur_t$, then a contemporary effect on $\Delta lip_t$ affects $\Delta ur_t$, but not vice versa. On the other hand, if the reverse holds, then shocks to $\Delta lip_t$ will be delayed until next period before any noticeable change occurs to $\Delta ur_t$.