Problem set Industrial Organization & Digitalization

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Contents

1 Monopoly and double marginalisation

1.1 Should Moon enter?

First, Moon's profit function is the following:

$$\begin{align}
\Pi &= p(q)q - c(q)q \tag{1}\\
&= (226 - q)q - (12)q - 6000 \tag{2}
\end{align}$$

This function is concave, hence I can use the first order condition to obtain a profit maximising quantity:

$$\begin{align}
&\frac{\partial \Pi}{\partial q} = [foc] = 0 \implies \tag{3}\\
&q = 107 \implies \tag{4}\\
&p = 119. \tag{5}
\end{align}$$

Using this quantity, I can calculate the profit from entering into the market by plugging in the value in Equation (2). I get $\Pi = 5449 > 0$ and conclude that yes, Moon should indeed enter the market and earn 5449 Euros.

1.2 Astra and Moon

Set up Astra's profit function:

$$\begin{align}
\Pi_A &= p(q)q - c(q) \tag{6}\\
&(226 - q)q - tq \tag{7}
\end{align}$$

Astra's first order condition gives us:

$$\begin{align}
&\frac{\partial \Pi}{\partial q} = [foc] = 0 \implies \tag{8}\\
&\text{Quantity: } q(t) = \frac{226 - t}{2} \implies \tag{9}\\
&\text{Price: } p(q(t)) = 226 - \frac{226 - t}{2} = \frac{226 + t}{2} \implies \tag{10}\\
&\text{Profit: } \Pi_A(t) = \frac{226 + t}{2}\frac{226 - t}{2} - t\frac{226 - t}{2} = \tag{11}\\
&(226^2 - t^2)/4 - t\frac{226 - t}{2}. \tag{12}
\end{align}$$

1.3 Optimal wholesale price for Moon

This question amounts to finding an optimal t and then plugging it into Equations 10 and 12.

First, find the profit of Moon as a function of t:

$$\Pi_M = t * q(t) - c(q(t)) = \tag{13}$$

$$= t * \frac{226-t}{2} - 2\frac{226-t}{2} - 6000 \tag{14}$$

$$= (226-t)(t/2-1) - 6000 \tag{15}$$

And the first order condition:

$$\frac{\partial \Pi_M}{\partial t} = -(t/2-1) + (226-t)/2 = 0 \implies \tag{16}$$

$$t = 114,\ q(t) = 56, \Pi_M = 272 \tag{17}$$

$$\text{For Astra: } q(t) = 56, p(t) = 170, \Pi_A = 3136. \tag{18}$$

The expressions in (17) show that Moon's quantity drastically decrease, price marginally decrease and this leads to a great reduction in profit. This all happened despite the impressive cost savings. The intuition behind this result is that now there is a second firm that is also a monopolist that generates another dead weight loss in the supply chain. The economist thus had a point.

1.4 Fixed cost savings

Note that the difference between the profits with and without outsorucing amount to the following: $\Pi_1 - \Pi_2 = 5449 - 272 = 5177$. If there are cost savings that amount to more than 5177 Euros, outsourcing becomes profitable.

2 Durable goods monopoly

(a) If the monopolist can commit to an output plan, it would prefer to limit competition with its future self and ensure that the marginal consumer's valuation is where the first period price is. I.e. there is no consumer that would have bought in the first period that doesn't do so because it expects a smaller price in the second period. Hence, the monopolist commits itself to not lower the price in the future ($p_2 \geq p_1$) and, consequently, no one buys in the second period, $x_2 = 0$. The key insight is that the monopolist won't lose high paying customers in period 1 to itself because they expect the price to go down. The commitment is thus to not decrease price in the second period, making it unattractive to wait with purchasing and thereby foregoing the utility of using the product in one period.

The indifferent consumer, located at $1 - x_1$, will value the use of the product in two periods equal to the price: $(1+\delta)(1-x_1) = p_1$.

$$\text{Profit function: } \Pi(x_1,x_2) = x_1 * p_1 = x_1(1+\delta)(1-x_1) \tag{19}$$

$$\frac{\partial \Pi}{\partial x_1} = (1+\delta)(1-x_1) - x_1(1+\delta) = \tag{20}$$

$$= (1+\delta)(1-2x_1) = [foc] = 0 \tag{21}$$

$$\implies (x_1^m, x_2^m) = (1/2, 0), p_1^m = \frac{1+\delta}{2}, \tag{22}$$

$$p_2^m \geq p_1^m, \tag{23}$$

$$\Pi^m = \frac{1+\delta}{4}. \tag{24}$$

(b) Arriving to the second period, having sold at $x_1 = 1/2$ units, the monopolist acknowledges there are customers that can still be served. Now the indifferent customer will lie at $1 - x_1 - x_2$ and value the product at $1 - x_1 - x_2 = p_2$.

$$\Pi_2 = x_2 * p_2 = x_2(1 - x_1 - x_2) = x_2(1/2 - x_2) \tag{25}$$

$$\frac{\partial \Pi_2}{\partial x_2} = 1/2 - 2x_2 = [foc] = 0 \tag{26}$$

$$\implies x_2 = 1/4 \neq 0. \tag{27}$$

The profit maximising output in period 2, taking $x_1 = 1/2$ as given is $x_2 = 1/4$ which is not what the monopolist commited itself to produce. There is therefore an incentive to deviate from the original plan. Without a credible commitment device, the marginal consumer will realise that the monopolist intends to lower price in the second period and buy the good for less in the second period so the plan in the previous exercise is unstable.

(c) First, the following will hold at $t = 2$, taking x_1^* as a given

$$p_2 = 1 - x_1^* - x_2 \tag{28}$$

$$\Pi_2 = p_2 x_2 = (1 - x_1^* - x_2)x_2 \tag{29}$$

$$\frac{\partial \Pi_2}{\partial x_2} = 1 - x_1^* - 2x_2 = [foc] = 0 \text{ (optimize w.r.t } x_2\text{, taking } x_1^* \text{ as given)} \tag{30}$$

$$\implies x_2^* = \frac{1-x_1^*}{2} \text{ which means that period 2 profit becomes:} \tag{31}$$

$$\Pi_2 = (1 - x_1^* - (\frac{1-x_1^*}{2}))\frac{1-x_1^*}{2} = \frac{1-x_1^*}{2} - \frac{x_1^* - x_1^{2*}}{2} - \frac{(1-x_1^*)^2}{4} = \tag{32}$$

$$= \frac{1}{4} - \frac{x_1^*}{2} + \frac{x_1^{2*}}{4} = \frac{(1-x_1^*)^2}{4} \tag{33}$$

In order to find x_1^*, we first have to find the marginal consumer in period 1 that is indifferent between buying in period 1 and period 2. This consumer will be located at $1 - x_1^*$ and receive this utility from using the product in both periods and compare the utility to the utility of keeping the first period price and buying in the second period.

$$(1+\delta)(1-x_1) = p_1 + \delta(1 - x_1 - p_2) \tag{34}$$

$$(1+\delta)(1-x_1) = p_1 + \delta(1 - x_1 - (1 - x_1 - \frac{1-x_1}{2})) \tag{35}$$

$$\text{Solving for } p_1 \text{ yields} \tag{36}$$

$$p_1 = (1 + \frac{\delta}{2})(1 - x_1) \tag{37}$$

Next, I use the expression for p_1 to express the profit function in terms of x_1.

$$\Pi(x_1) = x_1(1 + \frac{\delta}{2})(1 - x_1) + \delta\Pi_2(x_1) \tag{38}$$

$$\frac{\partial \Pi}{\partial x_1} = 1 - (1 + \delta/2)(1 - 2x_1) - \delta(1 - x_1^*) = [foc] = 0 \tag{39}$$

$$\implies x_1^* = \frac{2}{\delta + 4}, x_2^* = \frac{\delta + 2}{2(\delta + 4)} \tag{40}$$

For prices and profits, we can plug in x_1^* and get the following:

$$p_1^* = \frac{(\delta+2)^2}{2(\delta+4)} \tag{41}$$

$$\Pi = \frac{(\delta+2)^2}{4(\delta+4)} \tag{42}$$

For the comparison:

$$(x_1^c, x_2^c) = (\frac{2}{\delta+4}, \frac{\delta+2}{2(\delta+4)}), \tag{43}$$

$$(x_1^a, x_2^a) = (1/2, 0) \tag{44}$$

$$(p_1^c, p_2^c) = (\frac{(\delta+2)^2}{2(\delta+4)}, 1/2 - \frac{\delta+2}{4(\delta+4)}) \tag{45}$$

$$(p_1^a, p_2^a) = (\frac{1+\delta}{2}, > \frac{1+\delta}{2}) \tag{46}$$

$$\Pi_c = \frac{(\delta+2)^2}{4(\delta+4)} \leq \Pi_a = \frac{1+\delta}{4}, \delta \in [0,1] \text{ with equality when } \delta = 0. \tag{47}$$

In words, total production is larger under the optimal time consistent plan than under the optimal plan when commitment is possible, rendering prices to be lower and profits to be less.

This result, known as the Coase conjecture, can be explained by the intuition that customers anticipate lower future prices and may abstain from purchasing at a high price, which may be lower than their valuation, in the first period knowing that the monopolist has an incentive to lower the price in the second period. Because of the loss of consumers in the first period, the price in the first period will be lower to retain some customers that value the good at a relatively high price. In essence, the monopolist is competing with itself charging a lower price in the future!

(d) The idea is that the leasing agreement takes away the "durable" part of "durable goods", making the problem of durability a non issue. The monopolist will rent out in the first period, and then again in the second period and will no longer face competition from goods produced in the first period because these goods were taken off the market. In each period, the monopolist solves a standard monopolist problem:

$$\Pi_i = x_i p_i = x_i(1 - x_i), i = \{1, 2\} \tag{48}$$

$$\frac{\partial \Pi}{\partial x_i} = 1 - 2x_i = [foc] = 0 \tag{49}$$

$$\implies x_i = 1/2, p_i = 1/2. \tag{50}$$

$$\Pi = \Pi_1 + \delta\Pi_2 = 1/4 + \delta/4 = \frac{1+\delta}{4} = \Pi_a. \tag{51}$$

Equation (51) shows that profits from being a monopolist that can commit itself is equal to the profit of a monopolist that can lease instead of sell goods. Leasing is thus a substitute for commitment.

3 Complementarity and monotone comparative statics

3.1 Part 1

(a) Supermodular games provide an appropriate framework for modelling strategic interaction in the presence of complementary games because of several reasons. First, the framework is widely applicable to strategic interaction, making the framework relevant. Second, it typically simplifies analysis by doing away with many mathematical convenience assumptions, thereby extending the realism and robustness of the solution by focusing the analysis on the "drivers of the results". Finally, it is emphasised that supermodularity can easily incorporate more complex situations and guarantees existence of a pure Nash Equilibrium (which, additionally, can be ordered no matter the strategy space whenever there are multiple Nash Equilibria).

TABLE 1

Slope of Reaction Curves	Investment Makes Incumbent: Tough	Investment Makes Incumbent: Soft
Upward	Case IV	Case I
	A: Puppy Dog *D*: Top Dog	*A*: Fat Cat *D*: Lean and Hungry
Downward	Case III	Case II
	A: Top Dog *A*: Top Dog	*A*: Lean and Hungry *A*: Lean and Hungry

Note: *A* = Accommodate entry; *D* = Deter entry.

Figure 1: Taxonomy table of behavior of incumbent in an entry game.

(b) The full taxonomy is presented in Figure (1), bluntly copy pasted from Fudenberg and Tirole (1984). The paper considers a two period environment in which there is an incumbent who is able to act strategically and invest in order to deter entry in the second period, a special case of what is considered in Vives (2005) and more directly related to what is asked for in the question. The main tradeoff is whether the incumbent should overinvest (relative to what it would have done had it not considered entry) in order to gain market recognition or lower costs making the firm more competitive in the second period; or whether it should underinvest to have a credible threat of behaving aggressively in the second period, thereby making the case for tougher competition in the second period and possibly dettering entry.

A distinction is whether investments makes the incumbent soft or tough in the second period. An investment is said to make the incumbent tough if it induces the incumbent to increase quantity or lower price in the second period which is the case if investments goes into productive technology/machines or if the investment is coupled with price competition in the second period.

On the contrary, an investment is said to make an incumbent soft if it induces it to keep prices high in the second period. Case one refers to this situation where investments goes into advertisement that makes the incumbent recognised by consumers in the second period, enabling it to charge a higher price. Case two refers to when investment goes into cost lowering technology and there is a possibility to invest in R&D in the second period which also pays off in terms of lower costs if there is a unique innovator. Since the probability of payoff decreases with the other firms R&D expenditures (probability of being the unique innovator decreases), the reaction curve is downward sloping in the other firm's expenditure. The incumbent thus has the incentive in the first period to create a setting in which it wants to spend on R&D in the second period so that the incumbent don't invest too much and grab all the profit. This is achieved by having a large potential gain in R&D expenditure which occurs if its first period costs are relatively high which is achieved by not investing.

3.2 Part 2

(a) Lemma 1 in the lecture notes for lecture 3 tells us we can do a cross partial test to check for increasing differences. We want a to test for strictly increasing differences which is similar but with strict inequalities rather than weak inequalities. This can be proved by changing the weak inequalities in the proof under Lemma 1 to strict inequalities.

$$\Pi(q,p) = pq(p) - c(q(p)) \tag{52}$$

$$\frac{\partial \Pi(q,p)}{\partial q \partial p} = q(p) + pq'(q) - c'(q(p))q'(p) = \underbrace{q(p)}_{>0\ *} + q'(p)\underbrace{[p - c'(q(p))]}_{=0^{**}} > 0 \tag{53}$$

- * Is motivated by the fact that the firm is "competitive", i.e. it produces something as otherwise the market wouldn't exist if not even competitive firms produced anything.
- ** Is simply the price equals marginal cost condition that a rational firm would follow.

(b) For monotonicity of supply, a nonnegative cross partial derivative is enough. The statement essentially means that there is an upward sloping supply curve over the full domain. To see why, as p changes exogenously, the profit maximising quantity must change in the same direction because of the non negativity of the cross partial derivative. Owing to positive relationship between the two, the supply curve is upward sloping. Example, the price of a good increases thereby increasing profits holding quantity constant $\frac{\partial \Pi}{\partial p} > 0$, since the cross partial is nonnegative $\frac{\partial \Pi}{\partial q} \geq 0$ and the best response of a price increase is to increase quantity at any part of the curve, meaning the curve is monotonic.

Strictly speaking, (1) is a sufficient but not a required condition since the supply curve could be monotonic without differentiability as a discrete (read: non differentiable) supply curve can still be monotonic. It is thus wrong to say that (1) is "needed".

3.3 Part 3, monotone comparative statics

(a) That the solution is increasing in $y, y \in \{0,1\}$ means that $U(q,0) = \Pi(q) < U(q,1) = W(q)$, or in other words, that overall welfare is larger than the profit of the producing firm (which is included in overall welfare). Hence there is someone other than the producer benefiting from the production, such as consumers. This can be expressed as $W(q) - \Pi(q) = C(q) > 0$.

(b) According to the definition, $U : q \times y \to \mathbb{R}$ in (q, y) has strictly increasing differences if:

$$U(q', y') - U(q, y') > U(q', y) - U(q, y), \forall q' > q, y' > y. \tag{54}$$

Which in our case can be specified further as:

$$W(q^H) - W(q^L) > \Pi(q^H) - \Pi(q^L) \tag{55}$$

In words this means that the overall welfare increases faster than profits when quantity produced increases. This is fairly intuitive as consumers benefit positively from consuming the goods produced. However, this wouldn't necessarily be the case when negative externalities are present.

(c) The inequality still holds if we assume the following:

$$v(q^H) - v(q^L) > p(q^H)q^H - p(q^L)q^L \tag{56}$$

Which we get by:

$$W(q^H) - W(q^L) > \Pi(q^H) - \Pi(q^L) \iff \tag{57}$$
$$v(q^H) - c(q^H) - v(q^L) + c(q^L) > p(q^H)q^H - c(q^H) - p(q^L)q^L + c(q^L) \iff \tag{58}$$
$$v(q^H) - v(q^L) > p(q^H)q^H - p(q^L)q^L. \tag{59}$$

The result is fairly general since revenue is part of value created. The inequality doesn't hold when the product causes harm as something would have to be subtracted from the overall value created by the firm in terms of revenue. No assumptions need to be imposed on the cost function since it cancels out anyway.

4 Vertical differentiation

(a) First, find the consumer who is indifferent between buying from firms A and B.

$$\text{Firm indifference: } U(A) = U(B) \iff \tag{60}$$
$$\theta^* s_A - p_A = \theta^* s_B - p_B \iff \theta^* = \frac{P_A - P_B}{S_A - S_B} \tag{61}$$
$$\tag{62}$$

Consumer's whose valuation of quality $\theta > \theta^*$ buy the high quality product and the demand for firm A looks like the following:
[1]

$$D(S_A, S_B, P_A, P_B) = 0 \qquad \text{if } P_A > \overline{\theta}(S_A - S_B) + P_B \tag{63}$$
$$D(S_A, S_B, P_A, P_B) = (\overline{\theta} - \frac{P_A - P_B}{S_A - S_B}) \qquad \text{if else} \tag{64}$$
$$D(S_A, S_B, P_A, P_B) = (\overline{\theta} - \underline{\theta}) \qquad \text{if } P_A < \underline{\theta}(S_A - S_B) + P_B \tag{65}$$
$$\tag{66}$$

And for the low quality firm, we get something similar assuming $P_B/S_B \leq \underline{\theta}$. The reason I assume this is because this assures that everyone in the market purchases because the following condition will be satsified: $U_{min} = \underline{\theta} s_B - P_B \geq 0$.

$$D_B(S_A, S_B, P_A, P_B) = 0 \qquad \text{if } P_B > P_A - \underline{\theta}(S_A - S_B) \tag{67}$$
$$D_B(S_A, S_B, P_A, P_B) = (\frac{P_A - P_B}{S_A - S_B} - \underline{\theta}) \qquad \text{if else} \tag{68}$$
$$D_B(S_A, S_B, P_A, P_B) = (\overline{\theta} - \underline{\theta}) \qquad \text{if } P_B < P_A - \overline{\theta}(S_A - S_B) \tag{69}$$

Case where some consumers don't buy:

In the case when $P_B/S_B > \underline{\theta}$, we instead get the following demand:

$$D(S_A, S_B, P_A, P_B) = \frac{P_A - P_B}{S_A - S_B} - P_B/S_B. \tag{70}$$

For the inner case where both firms sell. Note that it is no longer $\underline{\theta}$ that determines the marginal consumer to the left, but rather the firms' own price-quality relation. The intuition is that the price is too high relative to quality and what consumers are willing to pay so that some consumers are deterred from purchasing based on the price.

I provide Figure (2) which conveys how the demand for B depends on whether we have that the intersection with the horizontal axis (P_B/S_B) is to the right or left of θ. If we have θ', then this is the lower bound, otherwise we might have θ'' and it is instead the horizontal axis intersection that limits the demand for firm B.

(b) I find a Nash equilibrium by writing down the best response functions, considering interior solutions (i.e. mid rows of the demand functions) and

[1] These inequalities are obtained by comparing U(A) with U(B) for the extreme cases of the quality preference parameter θ. Example: where everyone buys from A has the condition: $\underline{\theta} S_A - P_A > \underline{\theta} S_B - P_B \implies P_A < \underline{\theta}(S_A - S_B) + P_B$.

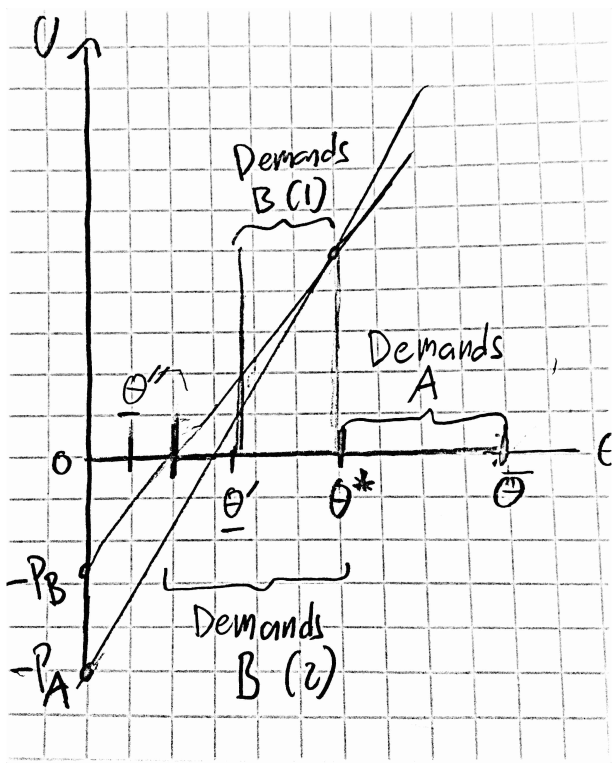

Figure 2: Demand specification depends on slope of the curves

can therefore use the first order conditions. In this case, qualities are given and it is enough to find a best price response for both firms in terms of the other firm's price. I take the derivative of the profit functions which are simply prices multiplied by demand (mid rows above).

$$\frac{\partial \Pi_A(P_A, P_B^*)}{\partial P_A} = \overline{\theta} - \frac{2P_A - P_B^*}{S_A - S_B} = [foc] = 0 \tag{71}$$

$$\frac{\partial \Pi_B(P_A^*, P_B)}{\partial P_B} = \frac{P_A^* - 2P_B}{S_A - S_B} - \underline{\theta} = [foc] = 0 \tag{72}$$

The system is linear in their respective arguments and has two equations in two unknowns and the solution:

$$P_A = \frac{(S_A - S_B)(2\overline{\theta} - \underline{\theta})}{3} \tag{73}$$

$$P_B = \frac{(S_A - S_B)(\overline{\theta} - 2\underline{\theta})}{3} \tag{74}$$

Since both equations (71) and (72) are linear, there is a unique solution. Furthermore, the boundary points can't be equilibrium points because whenever we are at a boundary point, the profit of one of the firms would be zero and it could profitably deviate from there.

Plugging in the prices we obtain the profits:

$$\Pi_A = P_A * D_A = \frac{(S_A - S_B)(2\overline{\theta} - \underline{\theta})}{3} * (\overline{\theta} - \frac{P_A - P_B}{S_A - S_B}) = \tag{75}$$

$$= \frac{(S_A - S_B)(2\overline{\theta} - \underline{\theta})}{3} * \frac{2\overline{\theta} - \underline{\theta}}{3} = \frac{(S_A - S_B)(2\overline{\theta} - \underline{\theta})^2}{9}, \tag{76}$$

$$\Pi_B = P_B * D_B = \frac{(S_A - S_B)(\overline{\theta} - 2\underline{\theta})}{3} (\frac{P_A - P_B}{S_A - S_B} - \underline{\theta}) \tag{77}$$

$$= \frac{(S_A - S_B)(\overline{\theta} - 2\underline{\theta})^2}{9} \tag{78}$$

Case where some consumers don't buy:

Now, consider the case where $P_B/S_B > \underline{\theta}$, the profit function for firm B now takes the form:

$$\Pi_B = P_B(\theta^*(P_B) - P_B/S_B) \tag{79}$$

And has the associated derivative and first order condition:

$$\frac{\partial \Pi_B}{\partial P_B} = (P_A - 2P_B)/(S_A - S_B) - 2P_B/S_B = [foc] = 0 \tag{80}$$

Combine this with Equation (71) and obtain an alternative solution that is rather messy.

From this point, I refrain from conducting the analysis twice and assume that al consumers buy, $P_B/S_B \leq \underline{\theta}$.

(c) Using backward induction, I solve for prices taking qualities as given. This was already done in equations (73) and (74). I plug these values into the profit expressions:

$$\Pi_A(S_A, S_B^*) = (S_A - S_B^*)\frac{(2\overline{\theta} - \underline{\theta})^2}{3^2} \tag{81}$$

$$\Pi_B(S_A^*, S_B) = (S_A^* - S_B)\frac{(\overline{\theta} - 2\underline{\theta})^2}{3^2} \tag{82}$$

No matter what B does, firm A has a profit function that is strictly increasing in S_A, so $S_A^* = \overline{s}$. On the contrary, B has a profit function that is strictly decreasing in S_B, so $S_B^* = \underline{s}$. Because firms can increase quality without cost, it seems counterintuitive that don't produce high quality products. However, this serves a purpose because the differentiation allows the firms to relax the price competition and instead cater to different segments on the market. If B were to increase the quality, it knows the price will reflect the fiercer competition.

(d) I start by solving for equilibrium qualities, taking price as given.

$$\frac{\partial \Pi_A(S_A, S_B^*)}{\partial S_A} = \frac{P_A(P_A - P_B)}{(S_A - S_B^*)^2} \geq 0 \tag{83}$$

$$\frac{\partial \Pi_B(S_A^*, S_B)}{\partial S_B} = \frac{P_B(P_A - P_B)}{(S_A^* - S_B)^2} \geq 0 \tag{84}$$

Because the quality of the product of firm A is higher than the quality of the product of firm B, I assume that $P_A \geq P_B \geq 0$ with equality if $P_A = P_B$. We see that the profit is increasing in quality when price is taken as given. This leads to the equilibrium qualities $(S_A, S_B) = (\overline{s}, \overline{s} - \varepsilon), \quad \varepsilon > 0$.

The reason I include ε is because of the condition $S_A > S_B$ which makes the profit derivatives well defined. In the first period, firms take qualities as given and choose prices. Plugging in the difference ε in Equations (73) and (74) yields:

$$P_A = lim_{\varepsilon \to 0} \frac{\varepsilon(2\overline{\theta} - \underline{\theta})}{3} = 0 \tag{85}$$

$$P_B = lim_{\varepsilon \to 0} \frac{\varepsilon(\overline{\theta} - 2\underline{\theta})}{3} = 0 \tag{86}$$

And thus, profits will also tend to zero (demand is bounded by $\overline{\theta}\overline{s} \leq 2\overline{s}$). The intuition is that the firms don't take price war into account when they set qualities as they did in the previous exercise (because price is already given when they set quality), they merely improve the quality so as to attract more customers knowing that price can't be changed. Anticipating this behaviour, the firms recognize the products will not be differentiated and act as Bertrand duopolists in the first period and price at marginal cost.

5 Problem 5, BBPD

(a) The problem set up means that consumers buy the good in the first period if their valuation is higher than the price charged in the first period, $v \geq p_1$. Because of the uniform distribution, the mass that did not buy in the first period is thus equal to $1 - (1 - p_1) = p_1$ in a similar way that the total mass is 1. In the second period, the total demand by consumers in the two segments is limited by the total masses, 1 & p_1, as upper bounds. The lower bound for the consumer that didn't buy in the first period is p_2, and the lower bound for the consumer that bought in the first period will be p_1 (if $p_1 \leq \hat{p_2}$) or $\hat{p_2}$ if $p_1 < \hat{p_2}$. See the profit equation (89).

Starting form behind, I solve the problem using backward induction supposing first that $\hat{p_2} > p_1$ so that everyone that buys at $\hat{p_2}$ can be identified as a high paying customer and that it is the second period price, $\hat{p_2}$ that is limiting demand. This turns out to be contradictory, however, therefore we must have that $\hat{p_2} = p_1$ so that p_1 constitutes the limit of demand. [2]

[2] The case where $\hat{p_2} < p_1$ can't be optimal, the firm would not be able to identify customers with a value $v \in [\hat{p_2}, p_1)$ since only those who bought at p_1 are identified. Instead, the firm would have to resort to charging the lower price p_2 and so it can't be a profit maximising policy to have $\hat{p_2} < p_1$.

$$\text{Assume: } \hat{p_2} > p_1 \tag{87}$$
$$\Pi_2 = p_2 q_2 + \hat{p_2}\hat{q_2} = p_2(p_1 - p_2) + \hat{p_2}(1 - \hat{p_2}) \tag{88}$$
$$\frac{\partial \Pi_2}{\partial \hat{p_2}} = 1 - 2\hat{p_2} = [f.o.c] = 0 \implies \tag{89}$$
$$\hat{p_2} = 1/2. \tag{90}$$
$$\frac{\partial \Pi_2}{\partial p_2} = p_1 - 2p_2 = [f.o.c] = 0 \implies \tag{91}$$
$$p_2 = p_1/2 \tag{92}$$
$$\Pi = p_1 q_1 + \delta \Pi_2 = p_1(1 - p_1) + \delta \Pi_2 \tag{93}$$
$$\frac{\partial \Pi}{\partial p_1} = 1 - 2p_1 + \delta(p_1/2) = [foc] = 0 \implies \tag{94}$$
$$p_1 = \frac{1}{2 - \delta/2}. \tag{95}$$

Note that $\hat{p_2} \leq p_1$, which contradicts our assumption. It must thus be the case that $\hat{p_2} = p_1$. Solving for p_1 then gives us:

$$\Pi = p_1(1 - p_1) + \delta\Big[(p_1/2)(p_1/2) + p_1(1 - p_1)\Big] \tag{96}$$
$$\frac{\partial \Pi}{\partial p_1} = 1 - 2p_1 + \delta\Big[p_1/2 + 1 - 2p_1\Big] = [foc] = 0 \iff \tag{97}$$
$$p_1 = \frac{1 + \delta}{2 + \frac{3\delta}{2}} \tag{98}$$

The second period price remains $p_1/2$ as it is unaffected by the change we did.

The firm's optimal price policy is thus $\{\frac{1+\delta}{2+\frac{3\delta}{2}}, \frac{1+\delta}{4+\frac{3}{\delta}}, \frac{1+\delta}{2+\frac{3\delta}{2}}\}$.

(b) In this scenario, there are three possible cases that determine the demand function.

$$\text{First: } \hat{p_2} > v^* : \Pi_2' = p_2(v^* - p_2) + \hat{p_2}(1 - \hat{p_2}) \tag{99}$$
$$\text{Second: } \hat{p_2} = v^* : \Pi_2'' = p_2(v^* - p_2) + v^*(1 - v^*) \tag{100}$$
$$\text{Third: } \hat{p_2} < v^* : \Pi_2''' = p_2(v^* - p_2) + \hat{p_2}(1 - v^*) \tag{101}$$

The first profit case is similar to the previous one, everyone who did not buy in the first period face the price p_2, and the number of customers paying the high price is limited by their willingness to pay the high price. The third case is motivated by the fact that even if some consumers value the good at $v_i \in [\hat{p_2}, v^*]$ they are not recognised since only customers with a valuation $\geq v^*$ bought in the first period for the same reasons I assumed $\hat{p_2} \geq p_1$ above, so the third case can't be optimal – it is reducing price for no chance at compensating with higher quantity. It is easy to see that $\Pi_2'' > \Pi_2'''$ because the only difference is what the high price is and in the second case it is higher by construction.

Additionally, assuming $v^* > 1/2$[3], then we get that $\Pi_2'' > \Pi_2'$ because $\hat{p_2}(1 - \hat{p_2})$ is maximised at $\hat{p_2} = 1/2$ and so any increase in $\hat{p_2}$ from $1/2$ will lower the product. In our case, the closest we can get to $1/2$ is when the firm sets $\hat{p_2} = v^*$, and so we are in case 2. Optimal prices in terms of v^* are then:

$$\frac{\partial \Pi_2''}{\partial p_2} = 0 \implies p_2 = \frac{v^*}{2} \tag{102}$$

$$\text{By the paragraph above: } \hat{p_2} = v^*. \tag{103}$$

Next, the following expression identifies the consumer who is indifferent between buying in both periods and just the last period:

$$v^*(1+\delta) - p_1 - \delta\hat{p_2} = \delta(v^* - p_2) \iff \tag{104}$$

$$\text{[Plug in values for } p_2 \text{ and } \hat{p_2}\text{]: } v^* - p_1 = \delta(v^*/2) \iff \tag{105}$$

$$v^* = \frac{p_1}{1-\delta/2} = \frac{2p_1}{2-\delta}. \tag{106}$$

Finally, select the optimal value of p_1 : which gives us what we need to solve for everything else:

$$\Pi = p_1 q_1(p_1) + \delta \Pi_2(p_1) \iff \tag{107}$$

$$\Pi = p_1(1 - v^*) + \delta\Big[(\frac{v^*}{2})^2 + v^*(1 - v^*)\Big] \iff \tag{108}$$

$$\Pi = p_1 + v^*(\delta - p_1) - \delta\frac{3v^*}{4} \iff \tag{109}$$

$$\Pi = p_1 + \frac{2p_1}{2-\delta}(\delta - p_1) - \delta\frac{3}{4}(\frac{2p_1}{2-\delta})^2 \tag{110}$$

Optimizing with respect to p_1 yields:

$$\frac{\partial \Pi}{\partial p_1} = 1 + \frac{2\delta}{2-\delta} - \frac{4p}{2-\delta} - 2\delta\frac{3}{4}\frac{2p_1}{2-\delta}\frac{2}{2-\delta} = [foc] = 0 \tag{111}$$

[3]The intuition behind why this is a reasonable assumption is that the marginal consumer in period 1 is going to value the good higher now than in (a) that they anticipate they will get a lower price if they wait.

Solving for p_1 gives me the following solution:

$$p_1^* = \frac{4-\delta^2}{2(4+\delta)} \tag{112}$$

$$\hat{p_2^*} = v^* = \frac{2+\delta}{4+\delta} \tag{113}$$

$$p_2 = \hat{p_2^*}/2 = \frac{2+\delta}{2(4+\delta)} \tag{114}$$

We see that $v^* > 1/2 \quad \forall \delta \in [0,1]$ as assumed.

(c)

$$\Pi = p(1-p) + \delta p(1-p) \tag{115}$$

$$\frac{\partial \Pi}{\partial p} = 1 - 2p + \delta(1-2p) = [foc] = 0 \tag{116}$$

$$p = 1/2. \tag{117}$$

The monopolist chooses the monopoly price for both periods. The overall profit is $\Pi = \frac{1+\delta}{4}$.

For the comparison, I plot the profit functions for various values of δ (3). If the firm was able to commit, it would do so as profits under a fixed price policy are higher than when the firm is able to price discriminate. This happens because consumers anticipate that they will be "ripped" of in the future if they reveal information today and so refrain from consuming if their valuation is not high enough.

(d) Conceptually, the solution for this exercise is simple; compare welfare (or just consumer surplus) under policies allowing BBPD and not allowing BBPD for scenarios (a) and (b).

In order to make this comparison, we first need the benchmark. I.e. what would welfare (or consumer surplus) look like without BBPD. The benchmark for comparison is given by the prices that optimise the firm's profit such that prices are allowed to vary over time (unlike (c)), but not based on consumer types (as in (a) and (b)), i.e. BBPD is not allowed. However, the prices will simply be $p_1 = p_2 = 1/2$ as in (c) because the firm sets prices as a monopolist in both periods (which we already know the behaviour of from above). In the last period, the firm can't use information from the first period and so sets prices as a monopolist ($p_2 = 1/2$). Recognizining this, consumers don't have an incentive to hide information in the first period and behaves "normally" so the monopolist can charge $p_1 = 1/2$.

The benchmark profit is thus the same as in (c) and overall welfare for the benchmark can be expressed in terms of p:

Figure 3: Image for 5 (c)

- $F(s,p) = p(1-p) + \delta(p(1-p)) + \frac{(1-p)^2}{2} + \delta\frac{(1-p)^2}{2}$

If we don't consider profits as being part of social welfare, the corresponding function becomes:

- $F_C(s,p) = \frac{(1-p)^2}{2} + \delta\frac{(1-p)^2}{2}$

i.e. the consumer surplus in both periods.

Because of the numerous values that has to be plugged in the various expressions, I solve this exercise numerically rather than analytically. This has the advantage of visually showing an exact solution for all values of δ, is less painful to debug in case algebra was made wrong so is less error prone. The results are presented in Figure (4).

(a) First, I define the profits and consumer surpluses for both periods. The profits are straightforward and the consumer surplus is defined as the area between the price and the demand curve. In our case, we have surpluses stemming from two periods where, for the second period, we have two prices facing different consumers. The high price yields a triangle starting at 1 extending down to $1-p_1$ and has the same base because of the unit demand, linearity and the slope of the line. Second there is another smaller triangle with height and base $\hat{p}_2 - p_2$ facing consumers with the lower valuation. This area equals (using the derived values for $\hat{p}_2 = 1/2$ and $p_2 = p_1/2$ what I specify below along with the other "building blocks" of solving this exercise.

- $\Pi_1 = p_1(1-p_1)$
- $\Pi_2 = p_2(p_1 - p_2) + \hat{p}_2(1-\hat{p}_2) = (p_1/2)^2 + p_1(1-p_1)$

- $CS_1 = \frac{(1-p_1)^2}{2}$
- $CS_2 = \frac{(p_1/2)^2}{2} + \frac{(1-p_1)^2}{2}$

The welfare function under the paradigm of (a) can be expressed as:

$$F(s', p_1) = \Pi_1 + \delta\Pi_2 + CS_1 + \delta CS_2 \tag{118}$$

The results are plugged into `R` and displayed in Figure (4). Overall welfare (blue curve) is positively affected by allowing BBPD. In essence, this occurs because the monopolist can replicate the benchmark scenario *and* sell additional units at a lower cost, so a main driver of the difference is the profit gain by selling these additional units at a lower cost.

The recommendation is to allow for BBPD as this allows the firm to exploit the extra information and make large profits. Not only can the monopolist sell at the monopoly price twice, it can also sell some extra units to consumers who do not want to pay the monopoly price.

On the other hand, the reason the profits can be larger is because the monopolist is to some extent ripping the consumers off, so their overall surplus is lower. If regulators care only about consumer welfare and consumers are deemed naive (red curve), then the recommendation is to not allow BBPD.

(b) Profits come from Equation (108).

- $\Pi_1 = p_1(1 - v^*)$
- $\Pi_2 = v^*(1 - v^*) + (\frac{v}{2})^2$
- $CS_1 = \frac{(1-p_1)(1-v^*)}{2}$
- $CS_2 = \frac{(1-v^*)^2}{2} + \frac{(v^*-p_2)^2}{2}$

Where, again we can use the relationship between the prices obtained in (b):

- $p_1^* = \frac{4-\delta^2}{2(4+\delta)}$
- $\hat{p_2^*} = v^* = \frac{2+\delta}{4+\delta}$
- $p_2 = \hat{p_2^*}/2 = \frac{2+\delta}{2(4+\delta)}$

The welfare function under (b) is:

$$F(s', p_1) = \Pi_1 + \delta\Pi_2 + CS_1 + \delta CS_2 \tag{119}$$

With this information, I can plug all the expressions into `R` and see how welfare varies with δ. The results are presented in Figure (4).

It is noted that overall welfare is affected negatively by enacting BBPD when consumers are forward looking (purple line). This occurs because the consumers realise that, depending on their personal valuation, they

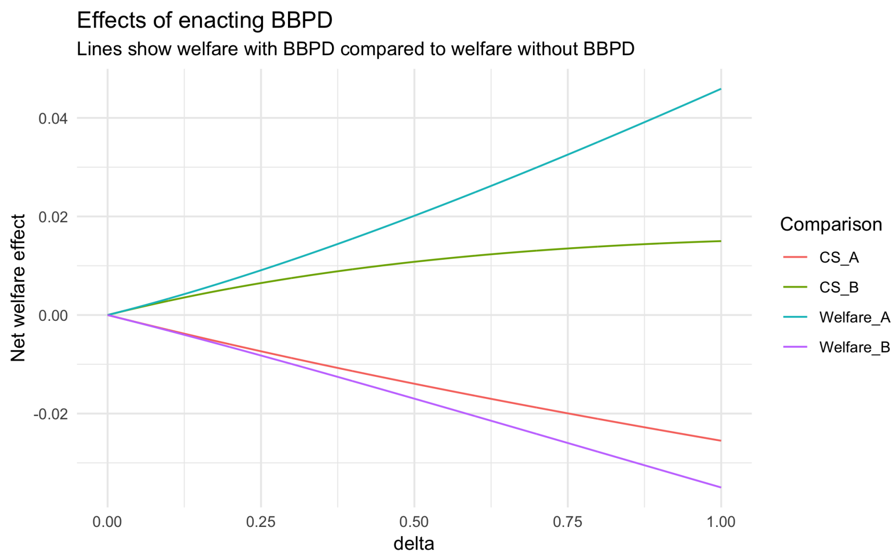

Figure 4: Net effects of enacting BBPD for various discounting values

will be able to get a lower price in the second period if they don't buy in the first period and so buy less, rendering the monopolist profits to become lower. Compared with the baseline scenario, some consumers that would have bought twice only buy only once when BBPD is enacted. On the other hand, consumers are better off because of their opportunity to delay purchases and receive the lower future prices (green line). If regulators care about consumer welfare and the consumers are deemed to be forward looking and sophisticated, the recommendation is to allow behaviour based price discrimination.

Furthermore, we see that the effects are magnified when the future is discounted less (δ closer to 1). This happens because more customers find it worthwhile to delay their purchases to the second period and face the lower price, resulting in less profits and a higher valuation of (less discounted) consumer surplus in the second period.

References

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