Solutions to the IO theory problem set 2022

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Bertrand / Cournot

a) Find the Bertrand equilibrium

First note that the demand facing each firm is identical and that costs are the same, we therefore look for a price vector where $p_1 = p_2$. In Bertrand competition, firms take each others prices as given. We therefore have that $\partial p_i / \partial p_j = 0$ in what follows.

Firm i make profits:

$$\Pi_i(p_i) = (p_i - c) q_i(p_i)$$

The first order condition, MR = MC, gives us:

$$\frac{\partial \Pi_i}{\partial p_i} = q_i(p_i) + (p_i - c)\frac{\partial q_i(p_i)}{\partial p_i} = 0$$

With $\frac{\partial q_i(p_i)}{\partial p_i} = -\frac{\beta(1+\gamma/2)}{2}$.

We now need to solve for p_i:

$$\frac{1}{2}\left[\alpha - \beta\,(1+\gamma/2)p_i + \beta\frac{\gamma}{2}p_j\right] = (p_i - c)\left(\frac{\beta(1+\gamma/2)}{2}\right)$$

Divide by β and use symmetry to set $p_i = p_j = p$ and we find:

$$p = \frac{\alpha/\beta + c(1+\gamma/2)}{2+\gamma/2}$$

b) Find the Cournot equilibrium

The key difference in this exercise is that firms will now take each others' quantities as given when making their decision,. We can solve the problem in a similar was as before using a symmetry argument. The objective function is now:

$$\Pi_i(q_i) = (p_i(q_i) - c) q_i$$

First order condition:

$$p_i(q_i) - c = -\frac{\partial p_i(q_i)}{\partial q_i} q_i$$

With

$$\frac{\partial p_i(q_i)}{\partial q_i} = -\frac{1}{\beta}\frac{2+\gamma}{1+\gamma}$$

We therefore solve:

$$\frac{\alpha}{\beta} - \frac{1}{\beta}\frac{2+\gamma}{1+\gamma}\left(q_1 + \frac{\gamma}{2+\gamma}q_2\right) - c = \frac{1}{\beta}\frac{2+\gamma}{1+\gamma}q_1$$

Setting $q_1 = q_2 = q$. I find that:

$$q = \frac{(\alpha - \beta c)(1+\gamma)}{4+3\gamma}$$

Comparisons

Comparing Bertrand and Cournot

In the table, we see differences between Bertrand and Cournot. Prices, and profits are higher under Cournot than Bertrand, whereas quantities are lower. The reaction functions are notably different. Under Bertrand, the strategic variable is prices and reaction functions slope upward. That is, prices are strategic complements–if the rival firm increases prices, it pays to increase own prices as well. Under Cournot, the scenario is the opposite and the strategic variable (quantity) is a strategic substitute.

As pointed out earlier, the key difference is what firms take as given when setting their strategic variable. Cournot has an element of commiting output to it.

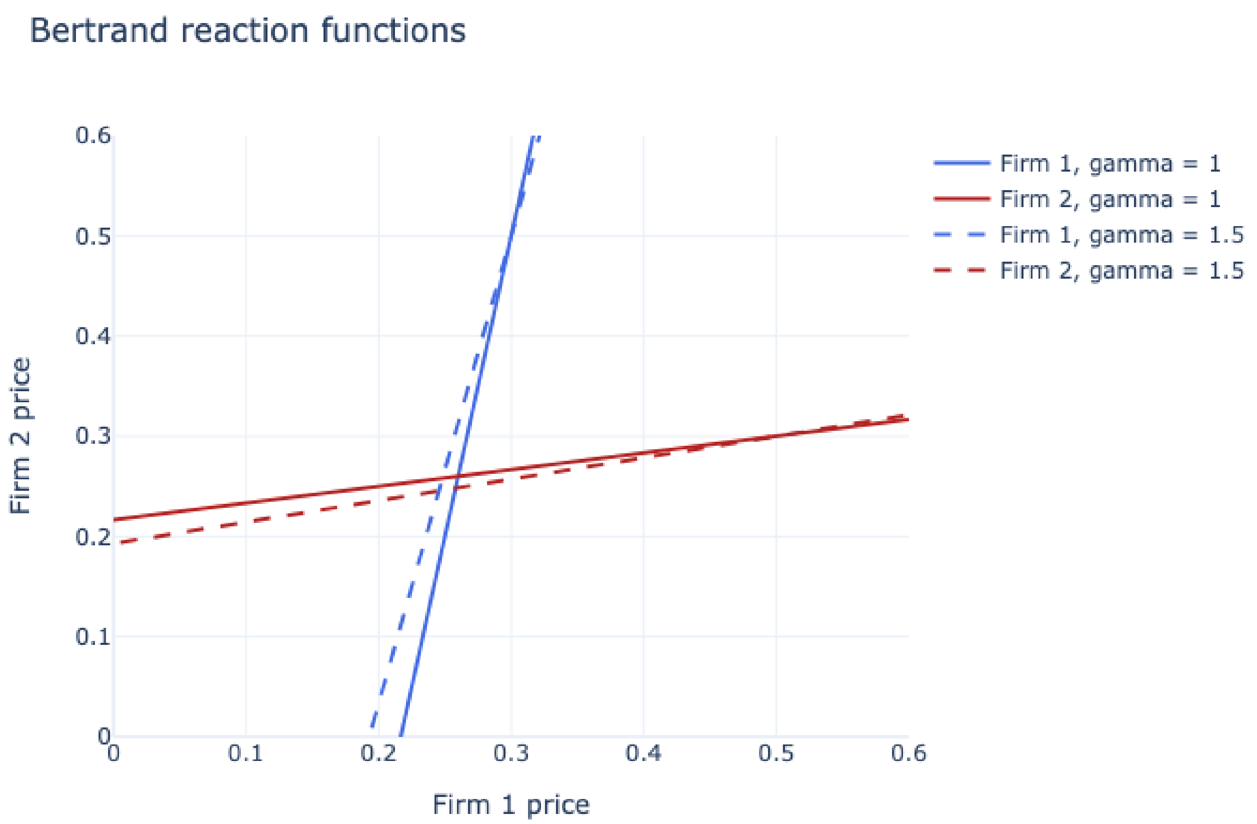
Comparing different values of γ

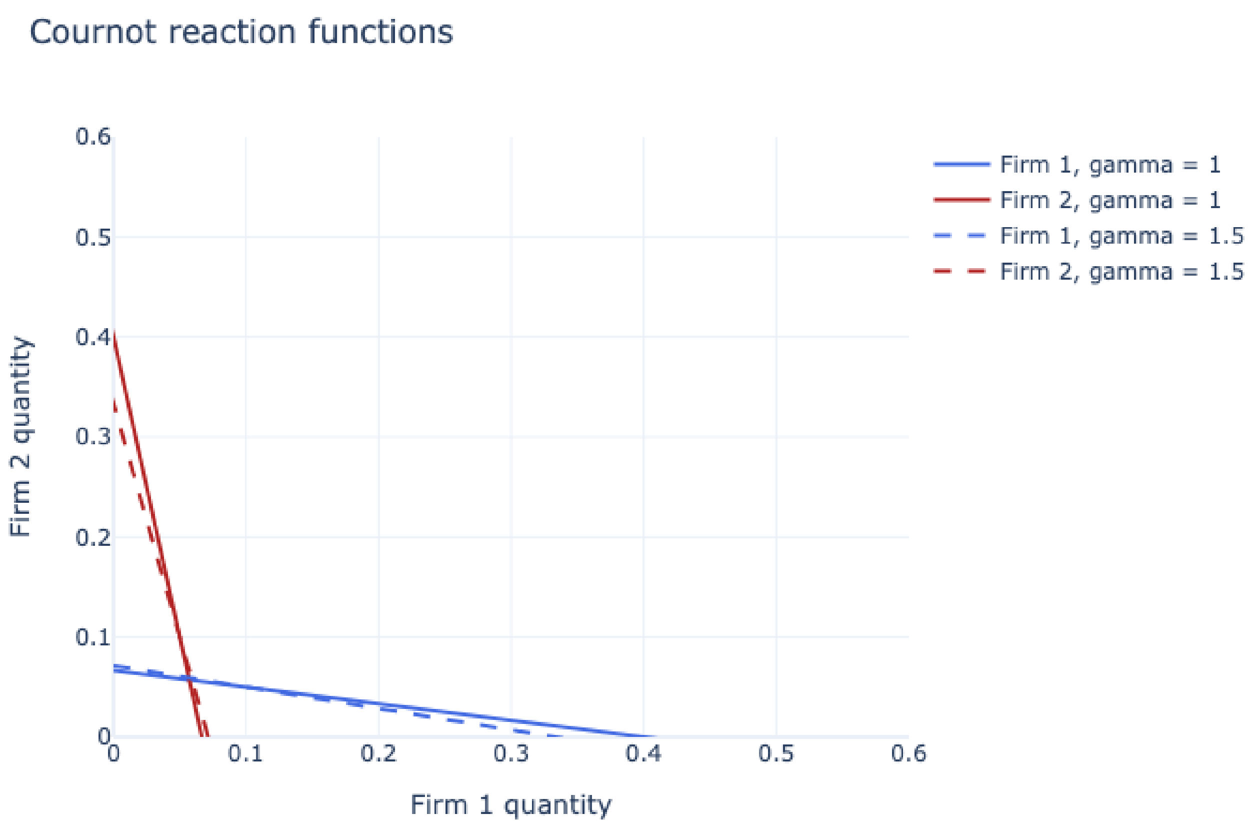
γ is a parameter that measures the degree of product differentiation, at zero, the rival's price (or quantity) is not affecting demand at all, so the products must be completely unrelated (highly differentiated). A higher values of γ increases the importance of the rival's price, so the products are more similar.

Intuitively then, we would expect fiercer competition with high values of γ. In the figures, a high value of γ is shown in the dashed lines and in the Bertrand figure, we see that reaction functions are steeper–i.e. firms react more to each others' prices. We also see that the resulting equilbirium prices are lower when γ is higher.

Similarly, under Cournot competition, I find that quantities are higher and prices are lower, so competition increases with γ.

$\gamma = 1$	p	q	profit	cost
Bert	0.26026	0.0599349	0.00960519	0.1
Cour	0.271772	0.0570571	0.00980079	0.1
$\gamma = 1.5$	p	q	profit	cost
Bert	0.245245	0.0636887	0.00925048	0.1
Cour	0.263764	0.0590591	0.00967173	0.1





Linear city

Find equilibrium profits with price competition

Depending on the consumers outside option, there can be several cases where either all consumers are served by the market and there is firm competition, not all consumers are served by the market and firms have local monopolies, and not all firms are served by the market but the firms compete regardless. I decide to focus on the case where all consumers are served as it is the most interesting case and the algebra does not get unwieldy. The monopoly case can be solved for analytically but is not very interesting by itself.

To find the demand facing each firm, we must find the indifferent consumer under the assumption that all interior consumers purchase, and then think about cases where the consumer's utilities are positive, i.e the edge consumers.

> Throughout, I will assume $a < (1-b)$. This is WLOG as we can just switch the two.

To find the indifferent consumer, equate utility from buying from either firm. This corresponds to a point in between a and $(1-b)$. Note that I let s_1 and s_2 be part of the solution by not assuming they are identical since this will help in part e. I also denote differences by a prime, so $p' = p_1 - p_2$ for notational convenience.

$$s_1 - t(x-a)^2 - p_1 = s_2 - t(1-b-x)^2 - p_2.$$

Move x terms to RHS, expand squares and note that squared x-terms cancel out to find:

$$\frac{s' - p'}{t} = a^2 - (1-b)^2 - 2x(a + -(1-b)).$$

Isolate x for the location of the indifferent consumer (which depends on relative prices). Also make use of the conjugate rule when simplifying:

$$x^* = \frac{s' - p'}{2t(1-b-a)} + \frac{1-b+a}{2}.$$

Notice how the second term denotes the midpoint between a and $1-b$, and that the first term tilts the solution towards 0 if $p_2 < p_1$, or $s_1 < s_2$.

We now have demand for firm 1, under the assumption that there exists an indifferent consumer:

$$q_1(p_1, p_2) = x^*(p_1, p_2) - \underline{x}(p_1)$$

Since we also assumed everyone buys, we know that $\underline{x} = 0$.

And demand facing firm 2, assuming there is an indifferent consumer, is consequently:

$$q_2(p_1, p_2) = \overline{x}(p_2) - x^*(p_1, p_2)$$

With $\overline{x} = 1$.

Both firms will profit maximize, taking each others' location and price as given so we therefore look for a Nash equilibrium and use the first order conditions of the two profit equations:

- $\Pi_1(p_1, p_2) = p_1 q_1(p_1, p_2)$, with FOC:
  - $x^*(p_1, p_2) - \underline{x}(p_1) - p_1 \left[\frac{1}{2t(1-b-a)}\right] = 0$
- $\Pi_2(p_1, p_2) = p_2 q_2(p_1, p_2)$ with FOC:
  - $\overline{x}(p_2) - x^*(p_1, p_2) - p_2 \left[\frac{1}{2t(1-b-a)}\right] = 0$
  - Note, the two first order conditions are only valid for interior solutions where there is an indifferent consumer and $\underline{x} = 0, \overline{x} = 1$.

The system has two equations and two unknowns p_1, p_2 that we can solve for:

I find that $p_1 = 2t(1-b-a) - p_2$. Plugging into demand for firm 2 gives:

- $3p_2 = (4-2m)t(1-b-a) - s'$ with $m = \frac{1-b+a}{2}$.
- $3p_1 = (4+2m)t(1-b-a) + s'$

CASE 2: Two monopolists

Next, we can consider the possibility where there is no consumer that is indifferent between firm 1 and firm 2, so x^* becomes irrelevant. Here, both firms have local monopolies and does not take the other firm's price into account. Solving for this is straightforward and I think is ruled out by the sentence "firms compete Nash in prices", if the firms have monopolies they don't compete so I don't write out the solution to that.

CASE 3: Firms compete, and not everyone buys
I think these cases are complicated judging from the profit equations. Firms will have stronger incentives to lower their price as they not just shift the indifferent consumer away, they also gain consumers not covered by the market. We'd therefore expect to see lower prices, but I won't solve for this case.

Profits given parameter values

b) Symmetrical scenario

Let $t = 1$, then when $a = 0.2, b = 0.2$:

- $^*p_1 = p_2 = 0.6$
- $x^* = 1/2$
- $q_1 = q_2 = 1/2$
- $\Pi_1 = \Pi_2 = 0.3$

c) Repositioning the firms

Let $t = 1$, then when $a = 0.4, b = 0.1$

- $^*p_1 = 0.55, p_2 = 0.45$
- $q_1 = 0.55, q_2 = 0.45$
- $\Pi_1 = 0.3025,$
- $\Pi_2 = 0.205.$

In this scenario, firm 1 is relatively close to the middle. It therefore serves a large share fo consumers without competition, and competes for customers from an advantageous point. Firm 1 ends up selling beyond the mid-point due to its better location.

If we now put the firms in the two corners, I find:

- $^*p_1 = 1, p_2 = 1$
- $q_1 = 0.5, q_2 = 0.5$
- $\Pi_1 = 0.5, \Pi_2 = 0.5$

I.e. prices, quantities, and profits become higher because competition is lower under these positionings, assuming the market continues to be served. Note that both firms prefer this scenario over being given interior solutions if that means the other firm is also interior, but would prefer to be interior themselves, holding the rival fixed at the corner.

d) A lower t

I now let $t = 1/2$ and find for $a = 0.4, b = 0.1$. This has the meaning that customers find it less costly to travel and should experience higher utility overall, all else equal.

The resulting profits are:

- $\Pi_1 = 0.15125$,
- $\Pi_2 = 0.10125$.

Profits become lower, whereas quantities are the same, so prices went down. Why did firms lower prices if consumers experience higher utility? The answer is increasing competition for customers between the firms, who are now more likely to change firm given a price reduction.

For the corner case, I find:

- $\Pi_1 = 0.25$,
- $\Pi_2 = 0.25$.

Again, profits are lower than in c due to price reductions stemming from increased competition due to lower travel costs. The overall effect here is larger because all consumers are interior, so there are no customers that will never swap firms given a change in price.

e) Introduce vertical differentiation

For the final scenario, I let $s_1 = 1, s_2 = 0$, $t = 1$ as in c.
We now have in the asymmetric case that:

- $\Pi_1 = 0.78\ldots$
- $\Pi_1 = 0.0136\ldots$

A higher degree of vertical differentiation allows firm 1 to gain large profits, whereas firm 2 earn almost zero profits. This is natural as firm 1 has a good positioning towards the center, and a higher valued product.

And in the corner case:

- $\Pi_1 = 0.888\ldots$
- $\Pi_1 = 0.222\ldots$

Industry profits increase under this scenario, and consumption is dominated by firm 1 who has a favourable position and a higher quality product. Industry profits can increase thanks to the industry generating more welfare to consumers, which it is able to monetize. Ie. not all gets loss in competition over consumers despite all consumers being contested.

Firm 2 does relatively well in this scenario thanks to the quadratic costs, low quality producers really benefit extra from not competing with high quality producers.