Theoretical Problem set - Industrial Organization 2022

Bertrand/Cournot

1. Consider two firms that each have the demand functions

$$
\begin{aligned}
q_1 &= \frac{1}{2}\left[\alpha - \beta\left(1+\frac{\gamma}{2}\right)p_1 + \frac{\beta\gamma}{2}p_2\right] \\
q_2 &= \frac{1}{2}\left[\alpha - \beta\left(1+\frac{\gamma}{2}\right)p_2 + \frac{\beta\gamma}{2}p_1\right]
\end{aligned}
$$

where p_i is price of firm i, q_i is quantity produced for firm i, α and β are parameters and γ measures the degree of product differentiation (these demand functions are associated with Shubik and Levitan - originally in "Market structure and behavior", Harvard University Press (1980).
a) Find the Bertrand (Nash in prices) equilibrium for a duopoly facing the above demand functions and constant marginal costs c.
b) The corresponding inverse demand functions are

$$
\begin{aligned}
p_1 &= \frac{\alpha}{\beta} - \frac{1}{\beta}\frac{2+\gamma}{1+\gamma}\left(q_1 + \frac{\gamma}{2+\gamma}q_2\right) \\
p_2 &= \frac{\alpha}{\beta} - \frac{1}{\beta}\frac{2+\gamma}{1+\gamma}\left(q_2 + \frac{\gamma}{2+\gamma}q_1\right)
\end{aligned}
$$

Find the Cournot (Nash in quantities) equilibrium for a duopoly facing the above inverse demand functions and constant marginal costs c.
c) Graph the reaction functions under a set of parameter values that you determine yourself for the solutions from a) and b). What are equilibrium prices, quantities and profits under your parameterization? Discuss any differences between a) and b).
d) Repeat the exercize in c) but for a higher γ,discuss the difference.

Linear city

Let there be two firms, 1 and 2 with zero costs. Tastes are assumed to be uniformly distributed on the unit interval. Firms are located at a and $(1-b)$ in this interval. Consumers buy one of the products (at most) and experience a disutility cost that is a function of the distance between their favorite variety in product space and the products offered. The utility of a consumer with taste $x \in [0,1]$ is

$$U(\theta) = \begin{cases} s - t(x-a)^2 - p_1 \text{ if buy from firm 1} \\ s - t(1-b-x)^2 - p_2 \text{ if buy from firm 2} \\ 0 \text{ otherwise} \end{cases}$$

where s is the reservation price, t times the squared distance gives the disutility cost and $p_1$and p_2 are the prices charged by the firms.
a) Taking locations as exogenous find the equilibrium profits assuming that firms compete Nash in prices. Note that the first step in the analysis is to find demand for each of the firms' products.
b) Use a value of t of your own choosing. What are profits if $a = 0.2$ and $b = 0.2$?
c) Use the same value of t as in b). What are profits if $a = 0.4$ and $b = 0.1$? What are profits if $a = 0$ and $b = 0$? Discuss the differences.
d) Use the same values of a and b as in c) but choose a lower t. What are profits now?
e) Introduce vertical differentiation by choosing $s_1 > s_2$. Calculate profits for same values as in c). Discuss the differences.