PROBLEM SET 1

You are advised, but not obliged, to work on this problem set in groups of up to three people. Groups can change for different problem sets. You are free to use any software you are familiar with. Each group has to hand in one solution sheet (available for each problem set on Canvas) together with the unique calculation file (Matlab/R/Julia or whatever software you used). Answers in the solution sheet should be typed (or written legibly). The deadline for this problem set is October 18 at 23:59 CEST. Late submissions will not be accepted by the system.

In this problem set, we will apply some key concepts of the portfolio theory, using the constituents of the S&P500 stock index. The reason is that S&P500 is one of the most traded indices that is often considered as one of the main performance indicators of the US stock market and, sometimes, of the whole US economy. Also, S&P500 is widely used as a benchmark for measuring the performance of different stock portfolios.

For the tasks below, you would need the following data files:

- *data_ps1.xls* - contains information on S&P500 stocks. Sheet *"components"* includes the weight of each stock in the index as of 09/06/2019. Sheet *"prices"* includes series of historical daily closing prices[1] of the index constitutes for a period 01/01/2019 – 30/08/2019. All the quotes are expressed in US dollars;
- risk-free rate R_f: take the average of the 3-month Treasury bill rate (daily) between 01/01/2019 – 30/08/2019. It can be obtained from here. N/A values should be ignored.

  Hint: pay attention that T-bill rates are reported in annualized values in percents. Please note: in the tasks below, take care that all your returns (i.e. stock returns as well as risk-free rate) are consistent in terms of periodicity (daily vs annualized) and are either all raw or all in percentage terms.

PART I. (25 points)

1. (0 points, warm-up) Have a look at how many constituents the S&P500 has. Google out why there are more than 500 stocks.

[1] These are close prices adjusted for splits.

For the rest of this task, we will use all stocks except for $AMCR$, $CTVA$, DOW, FOX and $FOXA$, for which there is no data at the beginning of the sample. Please keep only the rest 500 stocks for the tasks (2)-(6).

2. (5 points) Using the price data for the 500 stocks, calculate their arithmetic returns. Form an equally-weighted portfolio from these stocks and calculate the average daily and annualized returns μ, daily and annualized standard deviation σ, daily and annualized Sharpe ratio SR.

3. (5 points) Now, calculate the daily/weekly/monthly and annualized μ, σ, and SR, but using the actual weights of constitutes in the S&P500 (sheet *"components"*). Briefly comment on the difference between three measures for the two portfolios.

4. (5 points) Assume that you can borrow at the risk-free rate R_f (that has been downloaded). Calculate μ, σ, and SR for the portfolio with weights from (3) given a Leverage ratio v of 1, 3 and 5. What happens to μ, σ, and SR as you increase leverage?

Hint: See equations (2.1) to (2.3) in the lecture notes for calculating the effect of leverage on μ and σ of the portfolio returns.

5. (3 points) If one invested 1$ in the S&P500 on 01/01/2019, what would be the value of this investment on 30/08/2019 (assuming zero transaction costs)? What is the cumulative return of this investment (in %)?

6. (7 points) The optimal portfolio weight of the risky asset depends on the ratio of the average returns to the variance of total return — see (5.3) from the lecture notes. For instance, if the average return is 6%, the variance is 0.03 and risk aversion coefficient is, say, 2, then this investor should invest all his money in stocks *(v=1)*. Does this advice scale with horizon? Would you advise that an investor with the investment horizon of 20 years allocate more wealth to stocks than an investor with a 1-year horizon, because he/she has time to wait out the turbulent time and because stocks are "safer in the long run"? Explain.

PART II. (25 points)

In this task, we will construct the mean-variance frontier for two stocks, which comprise the largest share of the S&P500: MSFT (Microsoft Corp.) and AAPL (Apple Inc.). Use annualized returns for this task. Varying portfolio weights of MSFT from 0 to 1 (0% to 100%), plot (on X-axis is the standard deviation and on Y-axis is the expected return) the mean-variance frontier for these stocks, if the correlation between them is equal to:

(a) their actual correlation,

(b) -1,
(c) 0,
(d) 1.

Hint: Compute the new mean-variance frontier each time with the new covariance matrix. Compare your results with the Figure 3.3 from the lecture notes.

PART III. (50 points)

Let us now increase our investment universe to the 30 S&P500 stocks with the largest weighting. Recalculate the weights inside the new 30-stock index proportionally to their actual weights in the S&P500 (so that weights of 30 stocks add up to 1). Use the the risk-free rate R_f from the downloaded file. Take care that all your returns (i.e. stock returns as well as risk-free rate) are consistent in terms of periodicity (daily vs annualized) and are either all raw or all in percentage terms.

1. (22 points) Is this subsample of 30 stocks the tangency portfolio? Find the weights of risky assets in the 30-stock tangency portfolio. Compare the weights of the largest top10 actual index constitutes with their weights in the tangency portfolio.

Hint: Make sure your weights into risky assets sum up to one (zero investment into the R_f asset).

2. (22 points) Now let us check if the largest 30 stocks of the S&P500 form the minimum-variance portfolio (MVP)? Assuming that short sales are allowed, compute the weights in the 30-stock MVP. Compare the actual weights of the largest top10 index constitutes with the weights in the MVP portfolio.

Hint: Note that negative portfolio weights are possible, but that they have to add to 1. The function pfvar.m from the Matlab intro session might be of a help.

3. (6 points) Why is the variance of the MVP equal to the covariance of the MVP with the tangency portfolio? Report the value and provide your argumentation.

4* - extra 10 points[2]: Is the whole S&P500 the tangency portfolio? Calculate the weights of all 500 constitutes inside the tangency portfolio. Comment on the difference between the top10 constitutes between this tangency portfolio and 30-stock tangency portfolio from (1).

[2]you may skip this task without losing any points; however, you can get some for completing it (i.e. you can achieve max. 110% grade)