

Few words about Andrea

## **40 years of CFT**

Jacopo Viti (INFN, Florence)

**GGI, February 2024**

# 1. Entanglement in CFTs

- Struggling with Andrea in 2007 : “**Entanglement entropies in two-dimensional CFTs**”

- Entropies for free fermions at finite temperature
- **Full solution:** Azeynagi, Nishioka, Takayanagi [PRD 2007]
- **Excited states:** Alcaraz, Berganca, Sierra [PRL 2011]

UNIVERSITÀ DEGLI STUDI DI FIRENZE  
Facoltà di Scienze Matematiche Fisiche e Naturali



*Tesi di Laurea Specialistica in Fisica*

ENTROPIA DI ENTANGLEMENT IN  
TEORIE INVARIANTI CONFORMI  
BIDIMENSIONALI

Candidato: **Jacopo Viti**.....  
Relatore: **Dott. Andrea Cappelli**.....

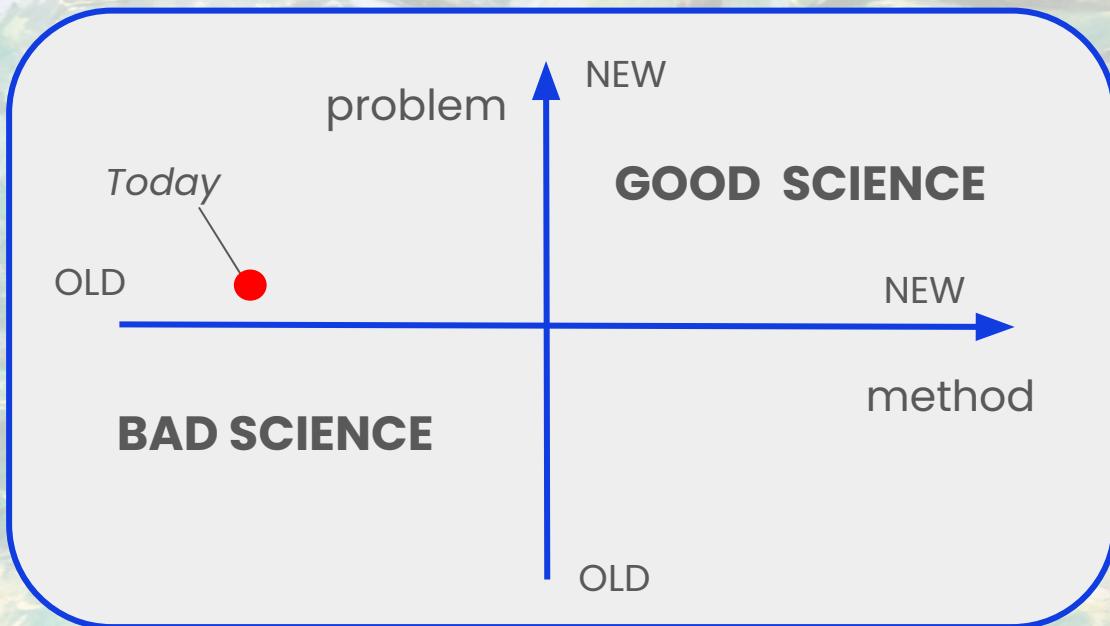
Anno Accademico 2006/07

## 2.Cappelli last theorem (in his own words)

“It is likely that if you write a paper with Andrea, it will be your **last good paper**”

### 3. Cappelli taxonomy for research

- **Classification** of modular invariant CFT p.f. [Cappelli, Itkykson, Zuber NPB 1986]



**Cappelli complementarity principle (in his own words):**

*"If you want to have a good approximation of reality, negate everything Andrea told you."*

# Tracy-Widom distribution in the six-vertex model with DWBC

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# 1. The six vertex model

## Basics

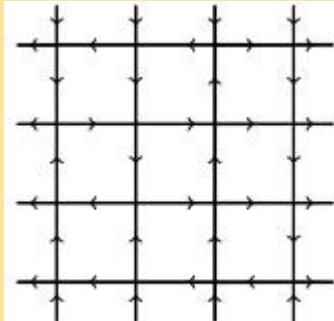
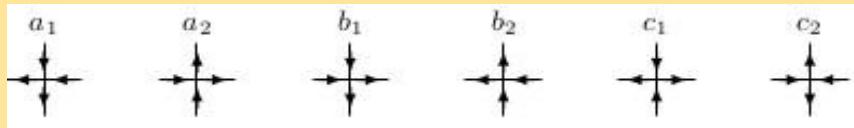
- Residual entropy of **ice** [Pauling '35]

## Vertex weights

$$W(a_i) = a, W(b_i) = b, W(c_i) = c$$

- Exact **free energy** on a square lattice with pbc [Lieb '67]
- Different kind of bcs (free and fixed)

## Six possible vertex (v) configurations



$$Z = \sum_{\text{conf}} \prod_v W(v)$$

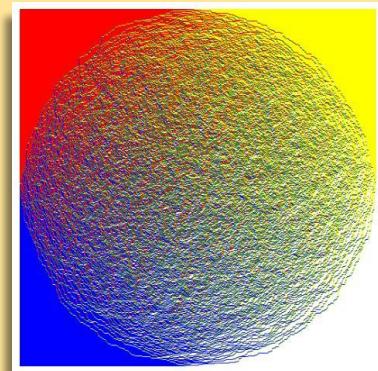
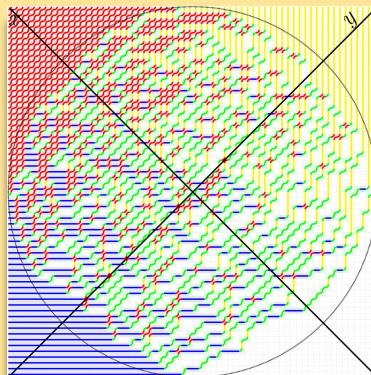
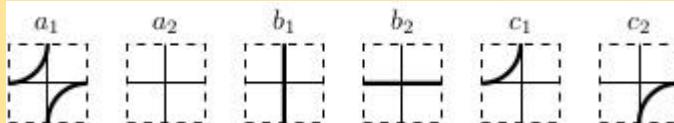
# 2. Domain wall boundary conditions

## Domain wall bcs [Korepin '82]

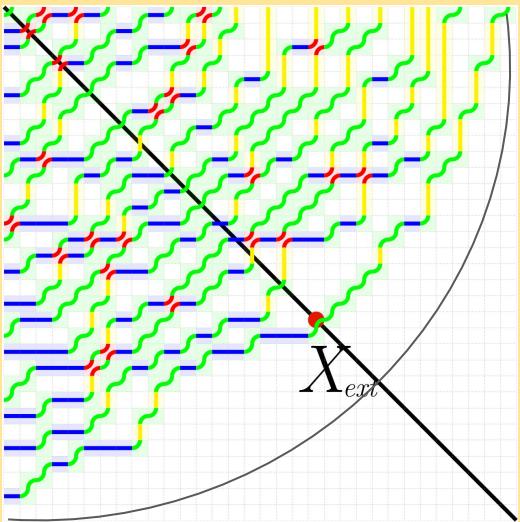
- Paths come in from the top and go out on the left
- Vertices fluctuate within **deterministic curves** [Jokush, Propp and Shor '98]-[Colomo and Pronko 2010] for  $N \rightarrow \infty$
- Define for later purposes

$$\Delta = \frac{a^2 + b^2 - c^2}{2ab}$$

## Non-intersecting path interpretation



# 3. Boundary fluctuations



## Theorem [Johansson 2005]

Consider the first occurrence of a vertex different from the one on the frozen corner.

For  $\Delta = 0$  and  $N$  large enough :

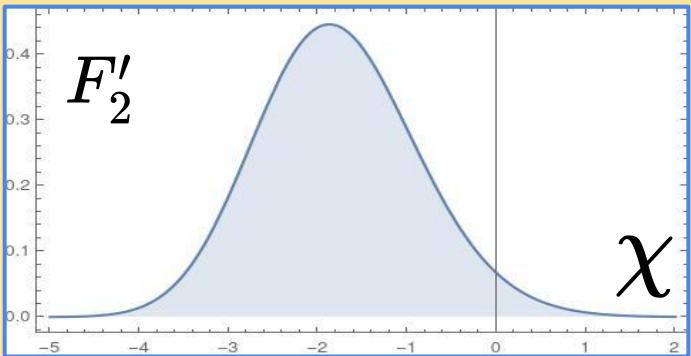
$$X_{ext} = \Lambda N + \Gamma N^{1/3} \chi; \quad \text{Prob}(\chi) = F_2'(\chi)$$

- **Purpose:** Test numerically universality of the **Tracy-Widom** distribution at  $\Delta \neq 0$

# 4. Tracy-Widom distribution

- Fluctuations of the **largest eigenvalue** of an  $N \times N$  Hermitian random matrix

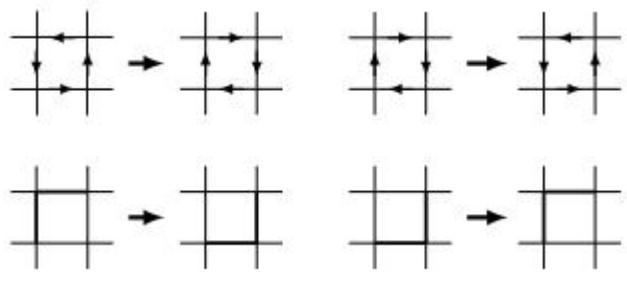
$$\text{Prob} \left( aN^{\frac{1}{6}} (\lambda_{\max} - \sqrt{2N}) < \chi \right) = F_2(\chi)$$



- Vicious Random walkers, random permutations, KPZ equation  
(**Universality**, see [Deift 2006],  
also in Quanta magazine 2014)

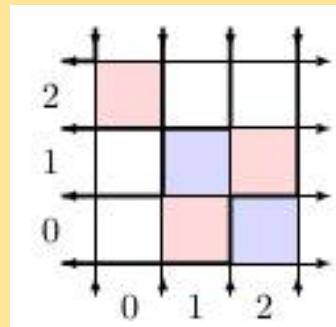
# 5. Monte Carlo algorithm

- **Local** Glauber dynamics



- Flip a vertex with probability

$$P(v) = \frac{\prod_{v' \in \text{plaquette}(v)} W(v')}{R}$$



- **Algorithm** proposed by [Allison, Reshetikhin 2006]
- Rejections for  $\Delta < -1$
- Density profiles, several bcs. [Lyberg, Korepin and V. 2016, 2018].
- **GPU** implementation [Keating, Sridhar 2018]

# 6. Direct analysis

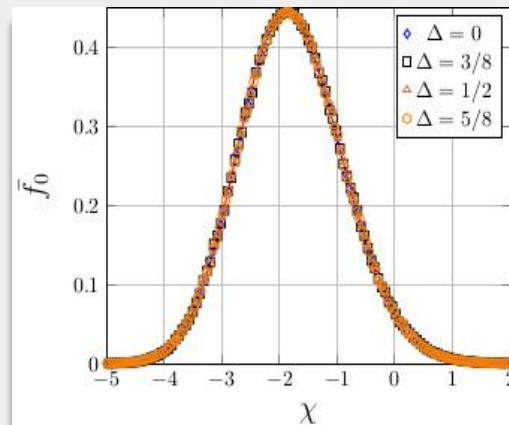
[with I. Lyberg V. Korepin, 2023]

- At finite N construct the histogram:

$$f_N \left( \frac{X_{ext} - \Lambda_N}{\Gamma_N} \right)$$

- Fit to the TW distribution and determine:

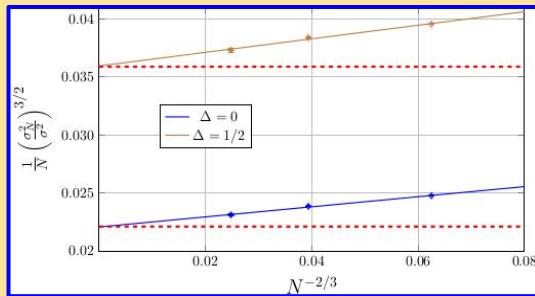
$$\Lambda_N, \Gamma_N$$



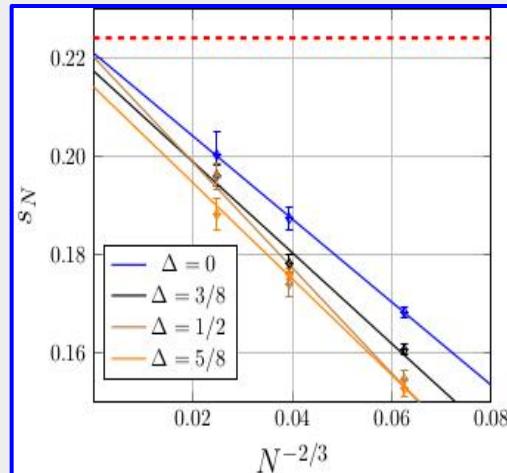
The fitting procedure does not distinguish TW from a Gaussian

# 7. Central moments

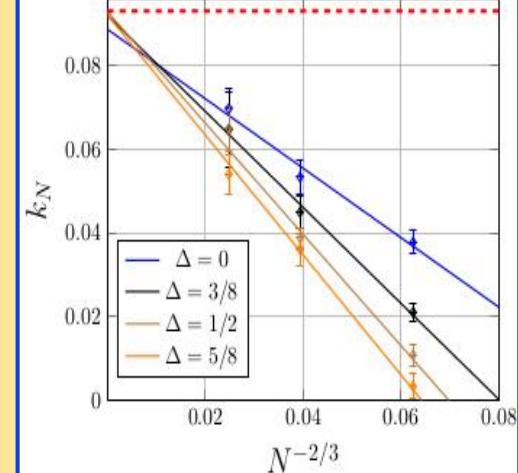
[with I. Lyberg, V. Korepin 2023.]



- From the **variance** we can extract  $\Gamma^3$
- See: [Allegra, Dubail, Stephan, V; 2016]; [Spohn, Praehofer; 2023]



- **Skewness** (third moment)



- **Excess kurtosis** (fourth moment)

# 8. Conclusions, recap

- In the **six vertex model** with DWbc the case  $\Delta = 0$  maps to free fermions. Obtaining results away from this point is hard.
- Provided **numerical evidence** of the existence of Tracy–Widom scaling for fluctuations of the arctic curves for  $\Delta \neq 0$  (Universality)
- **Analytical approaches** (?)