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Chiral Matter

from quarks to quantum materials

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Contents

1. Chirality in gauge theories
2. Chiral magnetic effect (CME) and anomaly-induced transport
3. CME in heavy ion collisions
4. CME in condensed matter
5. Chirality, quantum entanglement and the parton model

Chirality: the definition

Greek word: χειρ (cheir) - hand



Lord Kelvin (1893):

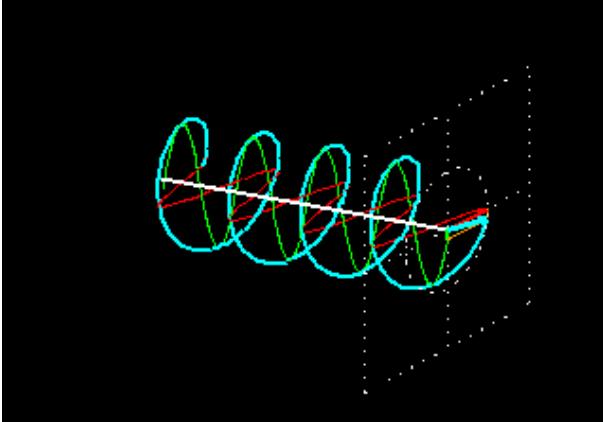
“I call any geometrical figure, or groups of points, chiral, and say it has chirality, if its image in a plane mirror, ideally realized, cannot be brought to coincide with itself.”

Light and electromagnetism

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \quad \nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$



Maxwell theory
is left-right
symmetric

THE
LONDON, EDINBURGH AND DUBLIN
PHILOSOPHICAL MAGAZINE
AND
JOURNAL OF SCIENCE.
[FOURTH SERIES.]

MARCH 1861.

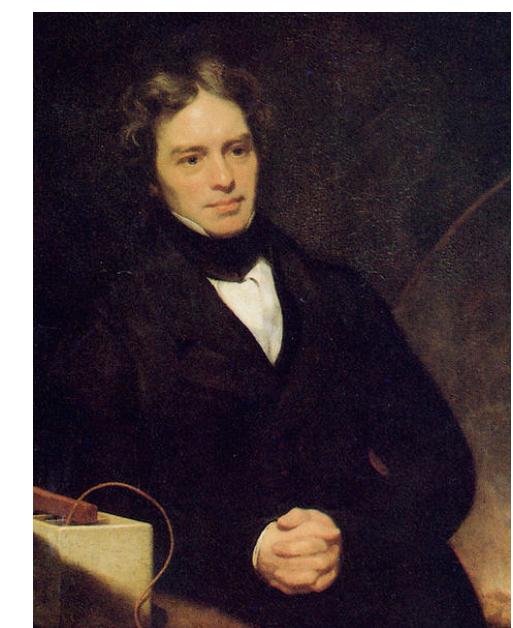
XXV. *On Physical Lines of Force.* By J. C. MAXWELL, Professor of Natural Philosophy in King's College, London*. PART I.—*The Theory of Molecular Vortices applied to Magnetic Phenomena.*

IN all phenomena involving attractions or repulsions, or any forces depending on the relative position of bodies, we have to determine the *magnitude* and *direction* of the force which would act on a given body, if placed in a given position.

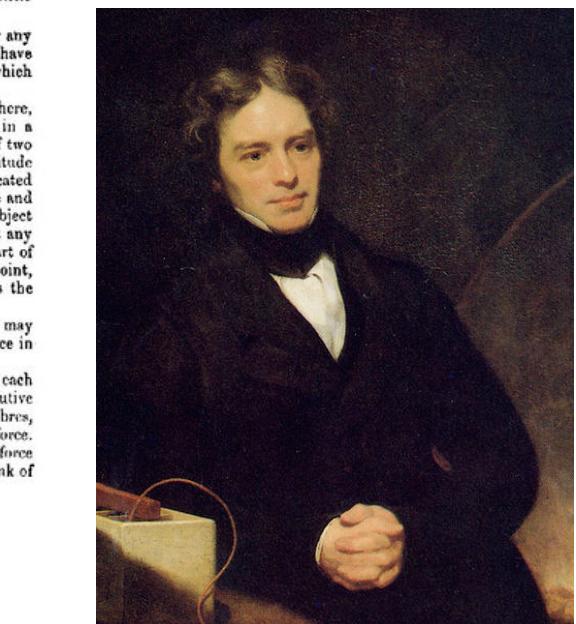
In the case of a body acted on by the gravitation of a sphere, this force is inversely as the square of the distance, and in a straight line to the centre of the sphere. In the case of two attracting spheres, or of a body not spherical, the magnitude and direction of the force vary according to more complicated laws. In electric and magnetic phenomena, the magnitude and direction of the resultant force at any point is the main subject of investigation. Suppose that the direction of the force at any point is known, then, if we draw a line so that in every part of its course it coincides in direction with the force at that point, this line may be called a *line of force*, since it indicates the direction of the force in every part of its course.

By drawing a sufficient number of lines of force, we may indicate the direction of the force in every part of the space in which it acts.

Thus if we strew iron filings on paper near a magnet, each filing will be magnetized by induction, and the consecutive filings will unite by their opposite poles, so as to form fibres, and these fibres will indicate the direction of the lines of force. The beautiful illustration of the presence of magnetic force afforded by this experiment, naturally tends to make us think of

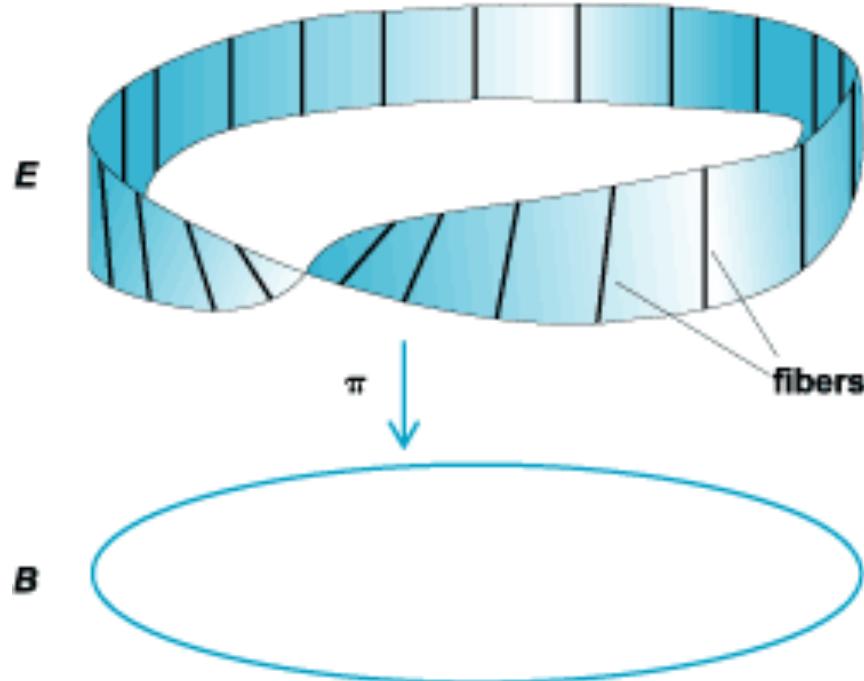


James C. Maxwell, 1831-1879



Michael Faraday, 1791-1867

Gauge fields and topology

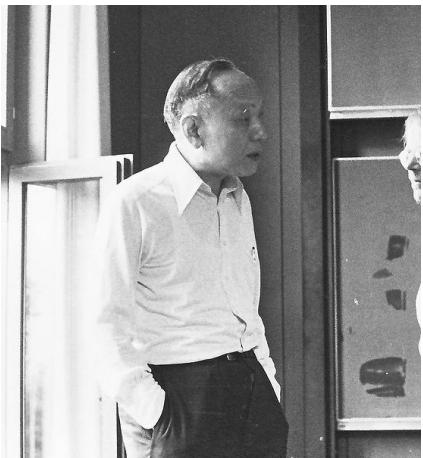


NB: Maxwell
electrodynamics
as a curvature
of a line bundle

Möbius strip, the simplest nontrivial example of a fiber bundle

Gauge theories “live” in a fiber bundle space that possesses non-trivial topology (knots, links, twists,...)

Chern-Simons forms



6. Applications to 3-manifolds

In this section M will denote a compact, oriented, Riemannian 3-manifold, and $F(M) \xrightarrow{\pi} M$ will denote its $SO(3)$ oriented frame bundle equipped with the Riemannian connection θ and curvature tensor Ω . For A, B skew symmetric matrices, the specific formula for P_1 shows $P_1(A \otimes B) = -(1/8\pi^2) \operatorname{tr} AB$. Calculating from (3.5) shows

$$3.1) \quad TP_1(\theta) = \frac{1}{4\pi^2} \{ \theta_{12} \wedge \theta_{13} \wedge \theta_{23} + \theta_{12} \wedge \Omega_{12} + \theta_{13} \wedge \Omega_{13} + \theta_{23} \wedge \Omega_{23} \}.$$

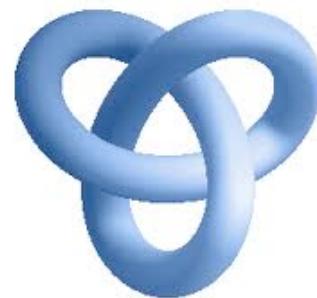
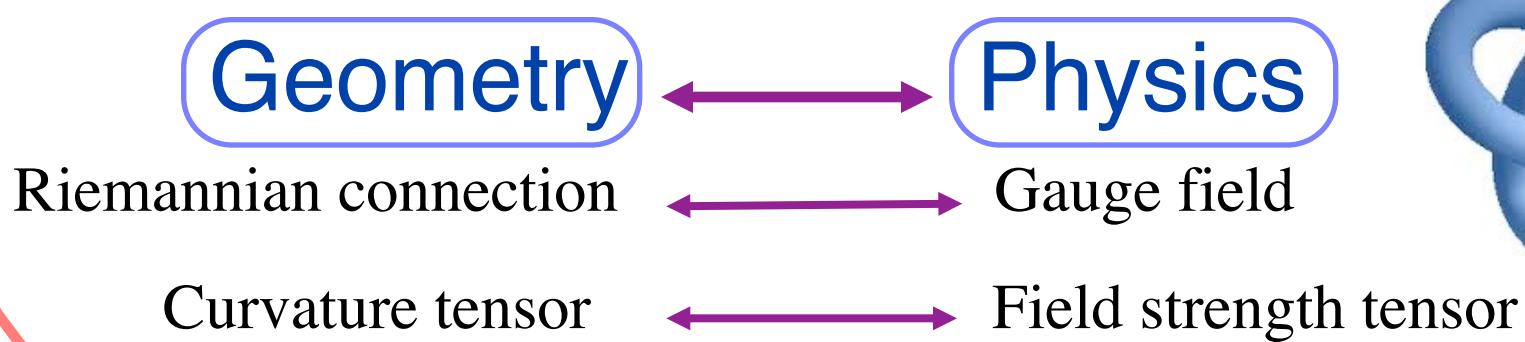
What does it mean for a gauge theory?

Chern-Simons theory

CHARACTERISTIC FORMS

$$(6.1) \quad TP_1(\theta) = \frac{1}{4\pi^2} \{ \theta_{12} \wedge \theta_{13} \wedge \theta_{23} + \theta_{12} \wedge \Omega_{12} + \theta_{13} \wedge \Omega_{13} + \theta_{23} \wedge \Omega_{23} \} .$$

What does it mean for electromagnetism?



$$S_{CS} = \frac{k}{8\pi} \int_M d^3x \, \epsilon^{ijk} \left(A_i F_{jk} + \frac{2}{3} A_i [A_j, A_k] \right)$$

“magnetic helicity”

Chern-Simons form and circularly polarized light

How to describe the helicity of the circularly polarized light?

Magnetic helicity itself does not obey electric-magnetic symmetry of Maxwell equations in vacuum:

$$\mathbf{E} \rightarrow \cos \theta \ \mathbf{E} + \sin \theta \ \mathbf{B}$$

$$\mathbf{B} \rightarrow \cos \theta \ \mathbf{B} - \sin \theta \ \mathbf{E}$$

Heaviside, 1892
Larmor, 1897

We can however enforce this symmetry by introducing, in addition to the magnetic helicity, the dual pseudovector gauge potential \mathbf{C} . In Coulomb gauge \mathbf{C} is defined by:

$$\nabla \mathbf{A} = \nabla \mathbf{C} = 0$$

$$\mathbf{E} = -\nabla \times \mathbf{C} = -\dot{\mathbf{A}}$$

$$\mathbf{B} = \nabla \times \mathbf{A} = -\dot{\mathbf{C}}$$

Bateman, 1915

Optical helicity of the circularly polarized light

Electric-magnetic transformation

$$\mathbf{E} \rightarrow \cos \theta \ \mathbf{E} + \sin \theta \ \mathbf{B}$$

$$\mathbf{B} \rightarrow \cos \theta \ \mathbf{B} - \sin \theta \ \mathbf{E}$$

is induced by

$$\mathbf{A} \rightarrow \cos \theta \ \mathbf{A} + \sin \theta \ \mathbf{C}$$

$$\mathbf{C} \rightarrow \cos \theta \ \mathbf{C} - \sin \theta \ \mathbf{A}$$

We can now define the **optical helicity** by adding CS terms for A and C:

$$H \equiv \frac{1}{2} \int d^3x (\mathbf{A} \cdot (\nabla \times \mathbf{A}) + \mathbf{C} \cdot (\nabla \times \mathbf{C})) = \frac{1}{2} \int d^3x (\mathbf{A} \cdot \mathbf{B} - \mathbf{C} \cdot \mathbf{E})$$

Optical helicity of the circularly polarized light

The optical helicity

$$H = \frac{1}{2} \int d^3x (\mathbf{A} \cdot \mathbf{B} - \mathbf{C} \cdot \mathbf{E})$$

is invariant under electric-magnetic symmetry

$$\mathbf{A} \rightarrow \cos \theta \ \mathbf{A} + \sin \theta \ \mathbf{C}$$

$$\mathbf{C} \rightarrow \cos \theta \ \mathbf{C} - \sin \theta \ \mathbf{A}$$

It is a T-even, P-odd quantity that is conserved *in the absence of interactions with chiral (P-odd) matter*:

$$\frac{dH}{dt} = 0$$

Chern-Simons theory

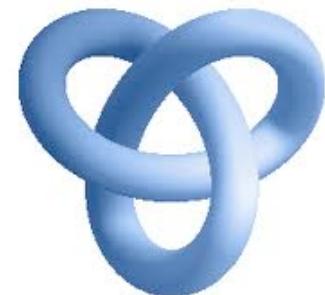
CHARACTERISTIC FORMS

$$(6.1) \quad TP_1(\theta) = \frac{1}{4\pi^2} \{ \theta_{12} \wedge \theta_{13} \wedge \theta_{23} + \theta_{12} \wedge \Omega_{12} + \theta_{13} \wedge \Omega_{13} + \theta_{23} \wedge \Omega_{23} \} .$$

What does it mean for electromagnetism?

Geometry

Physics



Riemannian connection

Gauge field

Curvature tensor

Field strength tensor

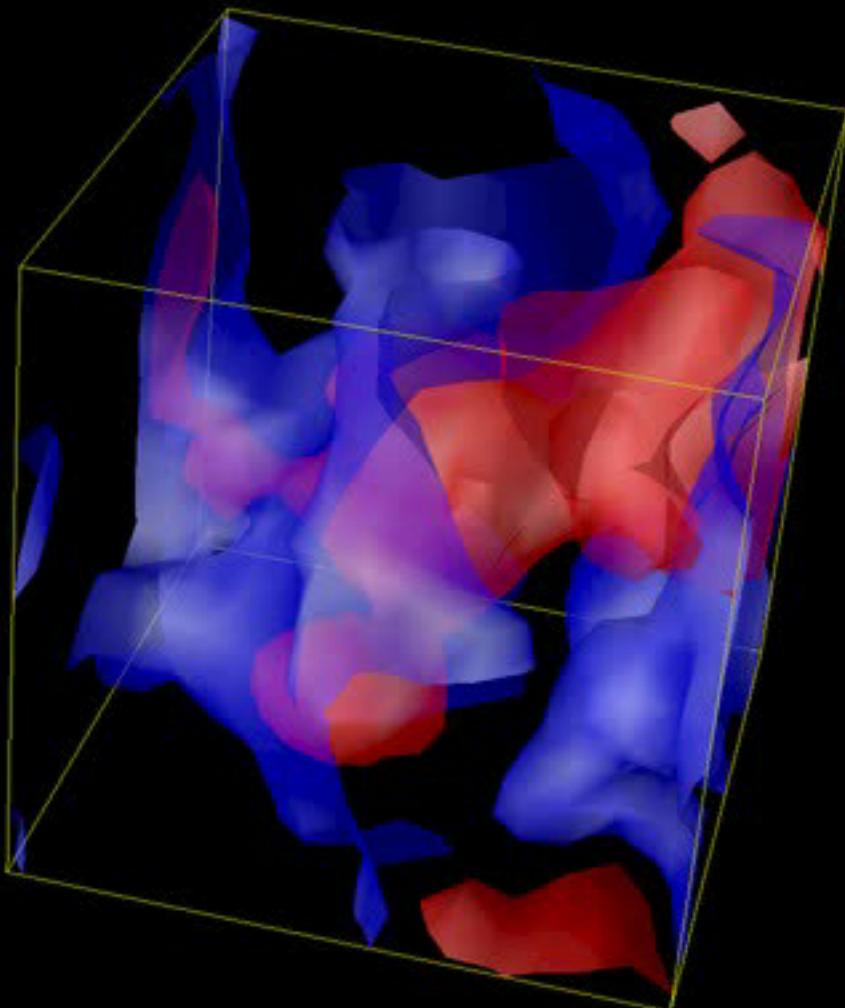
$$S_{CS} = \frac{k}{8\pi} \int_M d^3x \ \epsilon^{ijk} \left(A_i F_{jk} + \frac{2}{3} A_i [A_j, A_k] \right)$$

“magnetic helicity”

Non-Abelian helicity

“Topological foam” in QCD vacuum, (3+1) Dimensions

ITEP Lattice Group



Chirality in electrodynamics, (3+1)D: Maxwell-Chern-Simons theory

$$\mathcal{L}_{\text{MCS}} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - A_\mu J^\mu + \frac{c}{4} P_\mu J_{CS}^\mu$$

$$J_{CS}^\mu = \epsilon^{\mu\nu\rho\sigma} A_\nu F_{\rho\sigma} \quad P_\mu = \partial_\mu \theta = (\dot{\theta}, \vec{P})$$

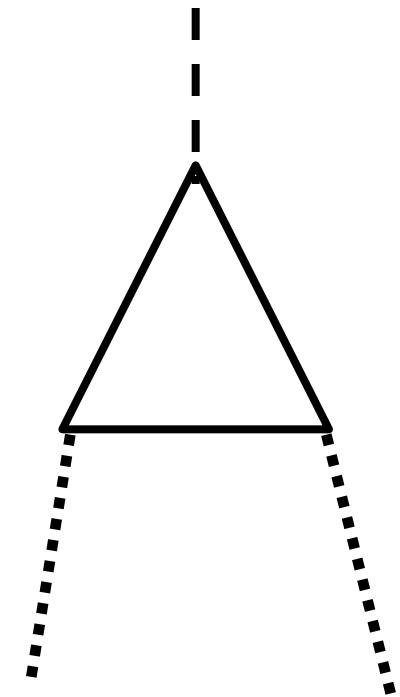
$$\vec{\nabla} \times \vec{B} - \frac{\partial \vec{E}}{\partial t} = \vec{J} + \underline{c \left(\dot{\theta} \vec{B} - \vec{P} \times \vec{E} \right)},$$

$$\vec{\nabla} \cdot \vec{E} = \rho + \underline{c \vec{P} \cdot \vec{B}},$$

$$\vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0,$$

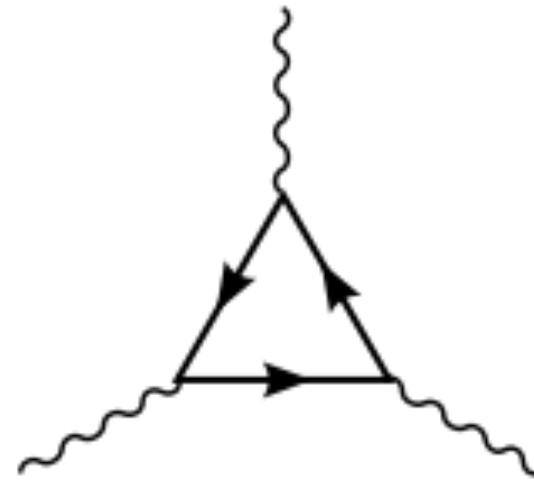
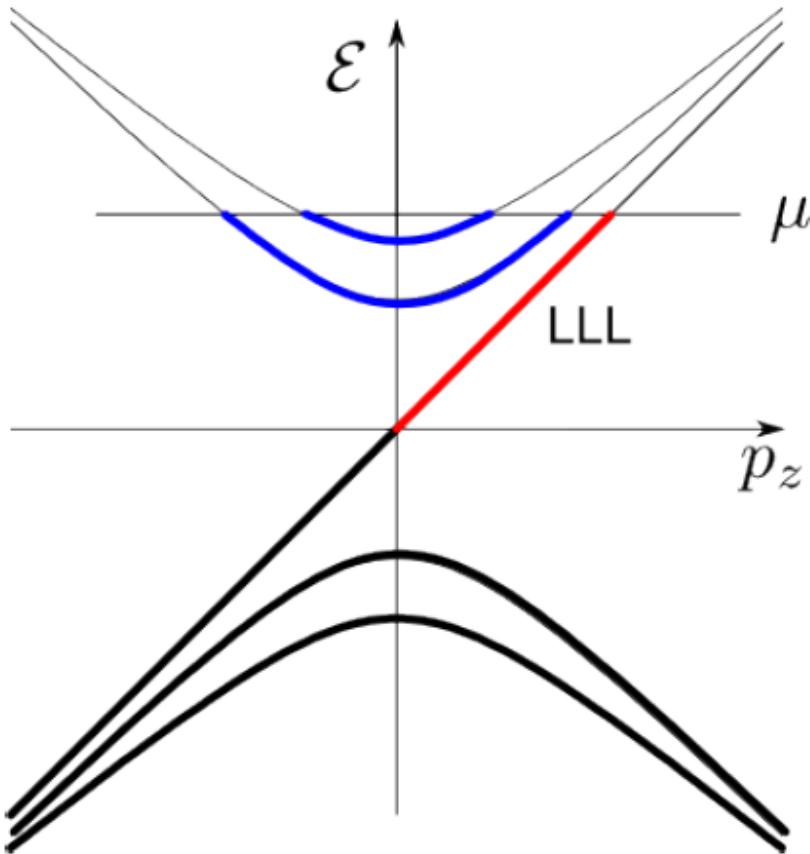
$$\vec{\nabla} \cdot \vec{B} = 0,$$

Chiral current



Photons

Chiral anomaly



In classical background fields (E and B), chiral anomaly induces a collective motion in the Dirac sea

Problem:

Derive the action describing a source of chirality described by $\theta(x,t)$ for non-linear electrodynamics, e.g. for Born-Infeld action:

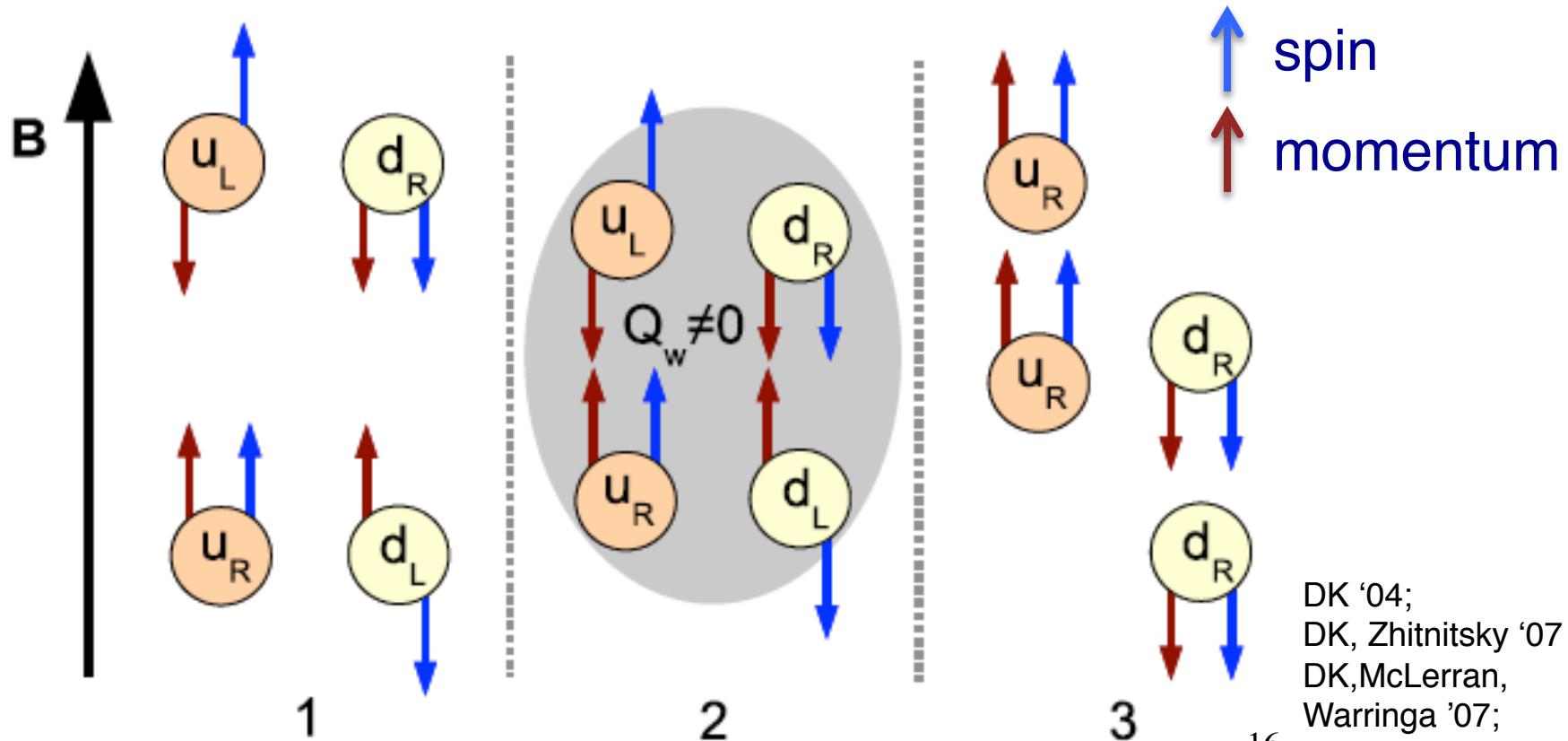
$$\mathcal{L} = -b^2 \sqrt{1 - \frac{E^2 - B^2}{b^2} - \frac{(\mathbf{E} \cdot \mathbf{B})^2}{b^4}} + b^2$$

Note: the BI action obeys the electric-magnetic symmetry

Applications include the generation of second harmonic in chiral nanotubes, see F.Qin, “Superconductivity in a chiral nanotube” Nature Comm. 2017; WS₂

Chirality in 3D: the Chiral Magnetic Effect

chirality + magnetic field = current



Early work on currents in magnetic field due to P violation

(see DK, Prog.Part.Nucl.Phys. 75 (2014) 133
for a complete (?) list of references)

A.Vilenkin (1980) “Equilibrium parity-violating current in a magnetic field”;
(1980) “Cancellation of equilibrium parity-violating currents”

G. Eliashberg (1983) JETP 38, 188
L. Levitov, Yu.Nazarov, G. Eliashberg (1985) JETP 88, 229

M. Joyce and M. Shaposhnikov (1997) PRL 79, 1193;
M. Giovannini and M. Shaposhnikov (1998) PRL 80, 22

A. Alekseev, V. Cheianov, J. Frohlich (1998) PRL 81, 3503

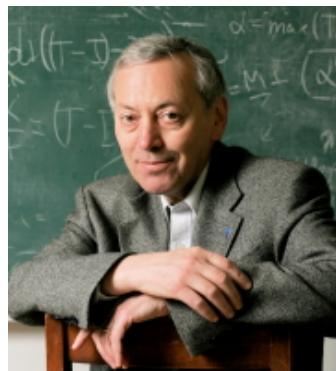
Equilibrium parity-violating current in a magnetic field

Alexander Vilenkin

Physics Department, Tufts University, Medford, Massachusetts 02155

(Received 1 August 1980)

It is argued that if the Hamiltonian of a system of charged fermions does not conserve parity, then an equilibrium electric current parallel to \vec{B} can develop in such a system in an external magnetic field \vec{B} . The equilibrium current is calculated (i) for noninteracting left-handed massless fermions and (ii) for a system of massive particles with a Fermi-type parity-violating interaction. In the first case a nonzero current is found, while in the second case the current vanishes in the lowest order of perturbation theory. The physical reason for the cancellation of the current in the second case is not clear and one cannot rule out the possibility that a nonzero current appears in other models.



But: no current in equilibrium



C.N. Yang

Cancellation of equilibrium parity-violating currents

Alexander Vilenkin

Physics Department, Tufts University, Medford, Massachusetts 02155

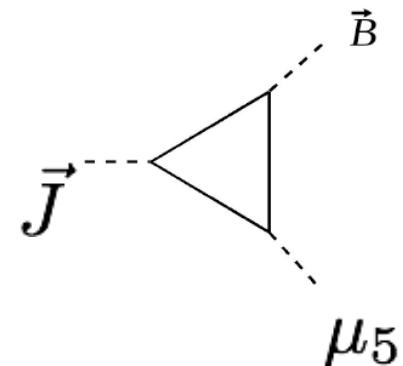
Chiral Magnetic Effect

DK'04; K.Fukushima, DK, H.Warringa, PRD'08;
Review and list of refs: DK, arXiv:1312.3348

Chiral chemical potential is formally equivalent to a background chiral gauge field: $\mu_5 = A_5^0$

In this background, and in the presence of B , vector e.m. current is generated:

$$\partial_\mu J^\mu = \frac{e^2}{16\pi^2} \left(F_L^{\mu\nu} \tilde{F}_{L,\mu\nu} - F_R^{\mu\nu} \tilde{F}_{R,\mu\nu} \right)$$



Compute the current through

$$J^\mu = \frac{\partial \log Z[A_\mu, A_\mu^5]}{\partial A_\mu(x)}$$

The result:

$$\vec{J} = \frac{e^2}{2\pi^2} \mu_5 \vec{B}$$

Coefficient is fixed by the axial anomaly, no corrections

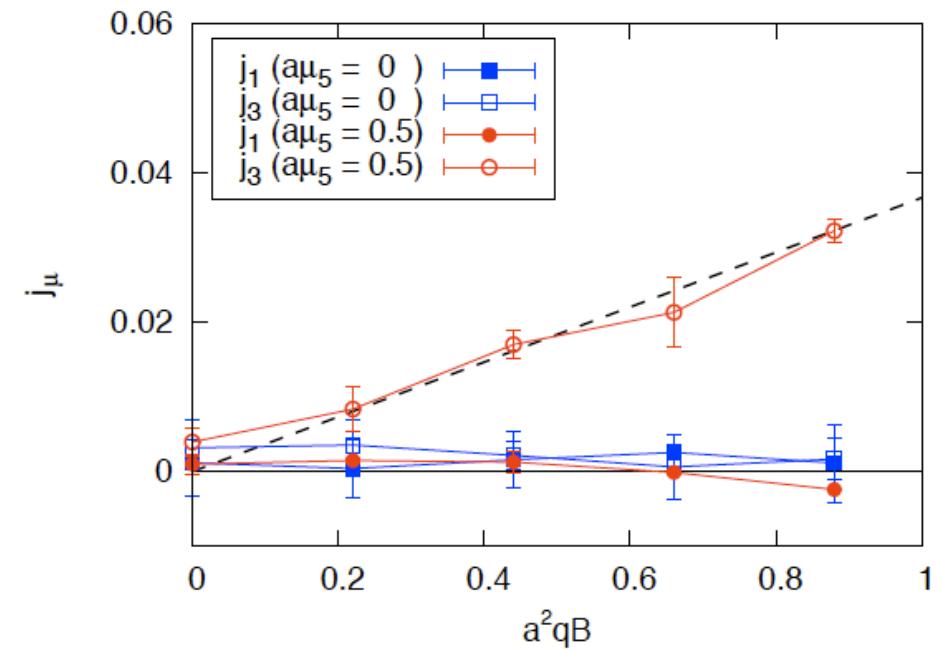
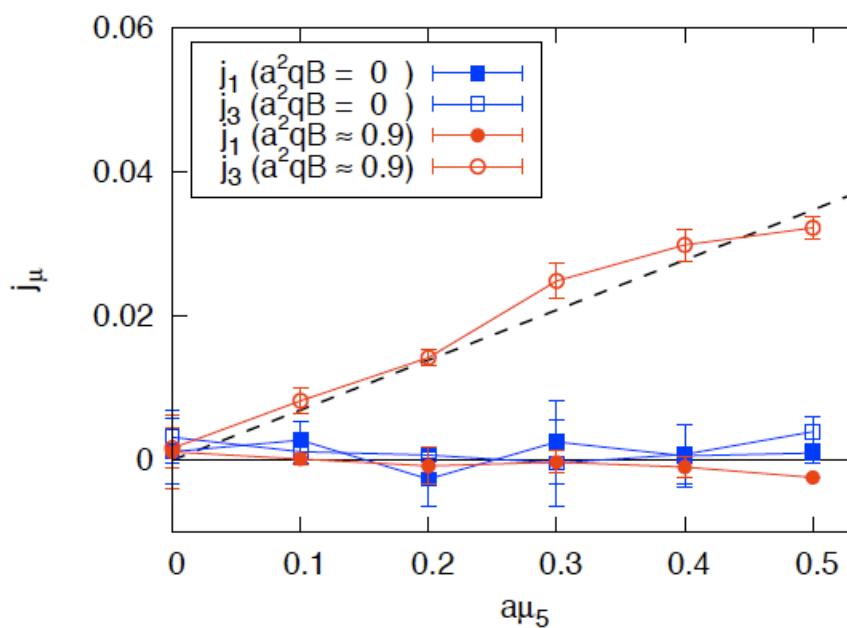
Chiral magnetic effect in lattice QCD with chiral chemical potential

Arata Yamamoto

Department of Physics, The University of Tokyo, Tokyo 113-0033, Japan

(Dated: May 3, 2011)

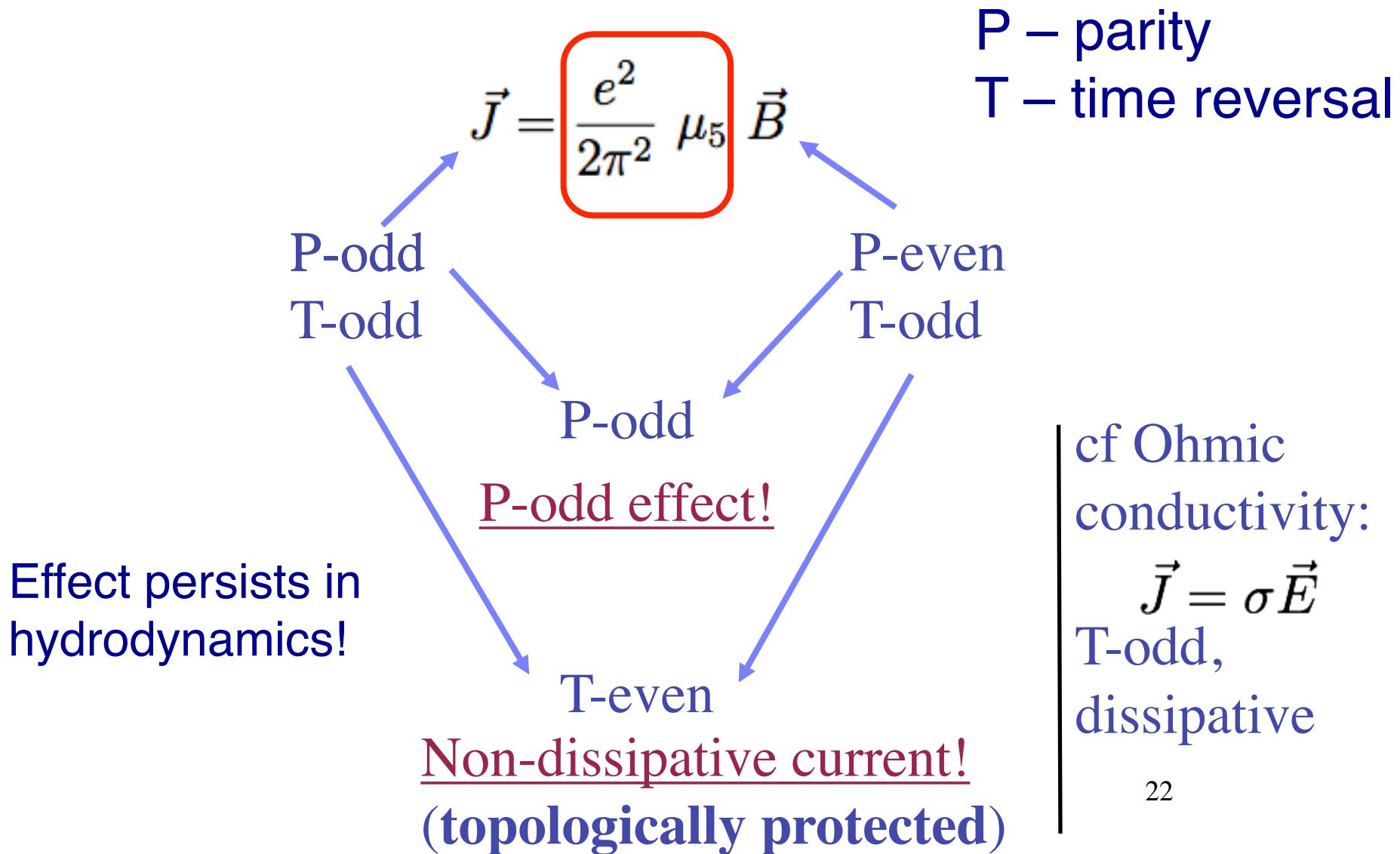
We perform a first lattice QCD simulation including two-flavor dynamical fermion with chiral chemical potential. Because the chiral chemical potential gives rise to no sign problem, we can exactly analyze a chirally asymmetric QCD matter by the Monte Carlo simulation. By applying an external magnetic field to this system, we obtain a finite induced current along the magnetic field, which corresponds to the chiral magnetic effect. The obtained induced current is proportional to the magnetic field and to the chiral chemical potential, which is consistent with an analytical prediction.



Systematics of anomalous conductivities

	Magnetic field	Vorticity
Vector current	$\frac{\mu_A}{2\pi^2}$	$\frac{\mu\mu_A}{2\pi^2}$
Axial current	$\frac{\mu}{2\pi^2}$	$\frac{\mu^2 + \mu_A^2}{4\pi^2} + \frac{T^2}{12}$

Chiral magnetic conductivity: discrete symmetries



CME as a new type of superconductivity

London theory of superconductors, '35:

$$\vec{J} = -\lambda^{-2} \vec{A} \quad \nabla \cdot \vec{A} = 0$$



Fritz and Heinz London

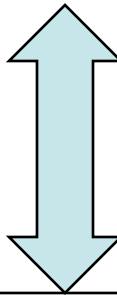
$$\vec{E} = -\dot{\vec{A}}$$

$$\vec{E} = \lambda^2 \dot{\vec{J}}$$

CME:

$$\vec{J} \sim \mu_5 \vec{B}$$

for $\vec{E} \parallel \vec{B}$



$$\vec{E} \sim B^{-2} \dot{\vec{J}}$$

assume that chirality
is conserved:

$$\mu_5 \sim \vec{E} \vec{B} t$$

superconducting
current, tunable
by magnetic field!

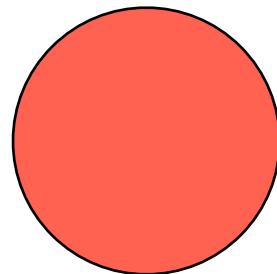
Hydrodynamics and symmetries

- Hydrodynamics: an effective low-energy TOE. States that the response of the fluid to slowly varying perturbations is completely determined by conservation laws (energy, momentum, charge, ...)
- Conservation laws are a consequence of symmetries of the underlying theory
- What happens to hydrodynamics when these symmetries are broken by quantum effects (anomalies of QCD and QED)?

No entropy production from P-odd anomalous terms

DK and H.-U. Yee, 1105.6360; PRD

Entropy grows



$$\partial_\mu s^\mu \geq 0$$

Allows to compute analytically 13 out of 18
anomalous transport coefficients in 2nd order
relativistic hydrodynamics

Mirror reflection:
entropy decreases ?

$$\partial_\mu s^\mu \leq 0$$

Decrease is ruled
out by 2nd law of
thermodynamics



$$\partial_\mu s^\mu = 0$$
 25

Conformally invariant Chiral magnetohydrodynamics

$$T_{\alpha\beta\dots}^{\mu\nu\dots}(x) \rightarrow e^{w\phi(x)} T_{\alpha\beta\dots}^{\mu\nu\dots}(x)$$

$w = [\text{mass dimension}] + [\#\text{ of upper indices}] - [\#\text{ of lower indices}]$

$$\mathcal{D}_\mu f = \nabla_\mu f + w \mathcal{W}_\mu$$

$$\mathcal{W}_\mu = u^\nu \nabla_\nu u_\mu - \frac{(\nabla_\nu u^\nu)}{3} u_\mu$$

$$\begin{aligned} & \sigma^{\mu\nu} \mathcal{D}_\nu \bar{\mu} , \quad \omega^{\mu\nu} \mathcal{D}_\nu \bar{\mu} , \quad \Delta^{\mu\nu} \mathcal{D}^\alpha \sigma_{\nu\alpha} , \quad \Delta^{\mu\nu} \mathcal{D}^\alpha \omega_{\nu\alpha} , \quad \sigma^{\mu\nu} \omega_\nu , \\ & \sigma^{\mu\nu} E_\nu , \quad \sigma^{\mu\nu} B_\nu , \quad \omega^{\mu\nu} E_\nu , \quad \omega^{\mu\nu} B_\nu , \quad u^\nu \mathcal{D}_\nu E^\mu , \\ & \epsilon^{\mu\nu\alpha\beta} u_\nu E_\alpha \mathcal{D}_\beta \bar{\mu} , \quad \epsilon^{\mu\nu\alpha\beta} u_\nu B_\alpha \mathcal{D}_\beta \bar{\mu} , \quad \epsilon^{\mu\nu\alpha\beta} u_\nu E_\alpha B_\beta , \quad \epsilon^{\mu\nu\alpha\beta} u_\nu \mathcal{D}_\alpha E_\beta , \quad \epsilon^{\mu\nu\alpha\beta} u_\nu \mathcal{D}_\alpha B_\beta . \end{aligned} \tag{2.60}$$

$$\omega^\mu \equiv \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} u_\nu \omega_{\alpha\beta} = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} u_\nu \nabla_\alpha u_\beta \quad E^\mu = F^{\mu\nu} u_\nu \quad , \quad B^\mu = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} u_\nu F_{\alpha\beta}$$

$$\sigma_{\mu\nu} = \frac{1}{2} (\mathcal{D}_\mu u_\nu + \mathcal{D}_\nu u_\mu) \quad , \quad \omega_{\mu\nu} = \frac{1}{2} (\mathcal{D}_\mu u_\nu - \mathcal{D}_\nu u_\mu) \quad , \quad \mathcal{D}_\mu u_\nu = \sigma_{\mu\nu} + \omega_{\mu\nu}$$

Problem:

Derive the equations of Chiral Magnetohydrodynamics for non-conformal fluids assuming that the breaking of conformal invariance is due to the scale anomaly,

$$\partial_\mu s^\mu = \theta_\mu^\mu$$

$$s^\mu = x_\nu \theta^{\mu\nu}$$

In some SUSY theories with electric-magnetic duality, scale and chiral anomaly are related, so this should reflect on hydrodynamics.

Why is this relevant?

Scale invariance

Scale transformations (dilatations)
are defined by

$$x \rightarrow e^\lambda x$$

the corresponding
dilatational current is

$$s^\mu = x_\nu \theta^{\mu\nu}$$



Hermann Weyl
(1885-1955)

It is conserved
(a theory is scale-invariant)
if the energy-momentum is
traceless:

$$\partial_\mu s^\mu = \theta_\mu^\mu$$

Scale invariance in QCD

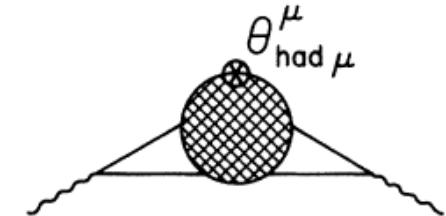
The trace of the energy-momentum tensor in QCD
(computed in classical field theory) is

$$\Theta_\alpha^\alpha = \sum_{l=u,d,s} m_l \bar{q}_l q_l + \sum_{h=c,b,t} m_h \bar{q}_h q_h$$

Two problems:

1. Potentially large contribution from heavy quarks to the masses of light hadrons
2. If we forget about heavy quarks, all hadron masses must be equal to zero in the chiral limit

Scale anomaly in QCD



The quantum effects (loop diagrams) modify the expression for the trace of the energy-momentum tensor:

$$\Theta_\alpha^\alpha = \frac{\beta(g)}{2g} G^{\alpha\beta a} G_{\alpha\beta}^a + \sum_{l=u,d,s} m_l (1 + \gamma_{m_l}) \bar{q}_l q_l + \sum_{h=c,b,t} m_h (1 + \gamma_{m_h}) \bar{Q}_h Q_h,$$

Running coupling \rightarrow dimensional transmutation \rightarrow mass scale

Gross, Wilczek;
Politzer

$$\beta(g) = -b \frac{g^3}{16\pi^2} + \dots, \quad b = 9 - \frac{2}{3} n_h,$$

Ellis, Chanowitz;
Crewther;
Collins, Duncan,
Joglekar; ...

At small momentum transfer, heavy quarks decouple:

$$\sum_h m_h \bar{Q}_h Q_h \rightarrow -\frac{2}{3} n_h \frac{g^2}{32\pi^2} G^{\alpha\beta a} G_{\alpha\beta}^a + \dots$$

SVZ '78

so only light quarks enter the final expression

$$\Theta_\alpha^\alpha = \frac{\tilde{\beta}(g)}{2g} G^{\alpha\beta a} G_{\alpha\beta}^a + \sum_{l=u,d,s} m_l \bar{q}_l q_l,$$

The proton mass

At zero momentum transfer, the matrix elements of the energy-momentum tensor are

$$\langle P | \theta^{\mu\nu} | P \rangle = 2P^\mu P^\nu$$

so that the trace of the energy-momentum tensor defines the masses of hadrons:

$$\langle P | \theta_\mu^\mu | P \rangle = 2M^2$$

$$\Theta_\alpha^\alpha = \frac{\tilde{\beta}(g)}{2g} G^{\alpha\beta a} G_{\alpha\beta}^a + \sum_{l=u,d,s} m_l \bar{q}_l q_l,$$

In the chiral limit, the entire mass is from gluons!

The proton mass

At finite quark mass, contribution from “sigma-terms”

$$\Sigma_{\pi N} = \hat{m} \langle p | \bar{u}u + \bar{d}d | p \rangle$$

can be extracted from pion-nucleon scattering or
measured on the lattice

e.g. Y.-B.Yang et al
arXiv:1511.09089

Sometimes interpreted as either

1. Contribution from quark masses
or
1. Contribution from chiral symmetry breaking

But the interpretation is more subtle

The proton mass

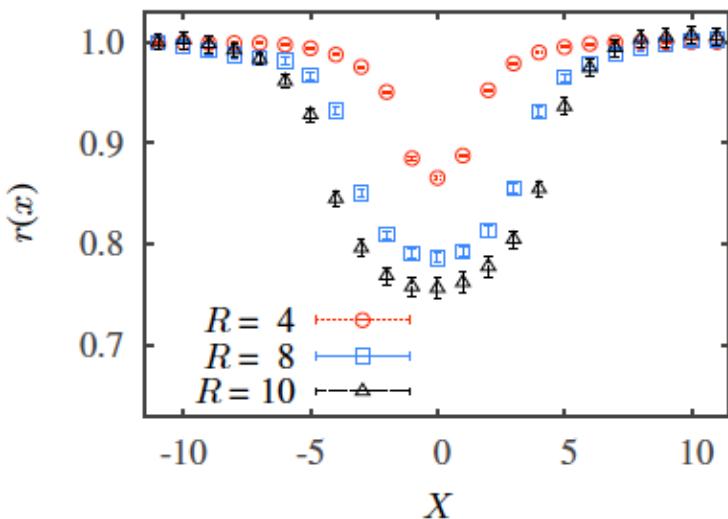
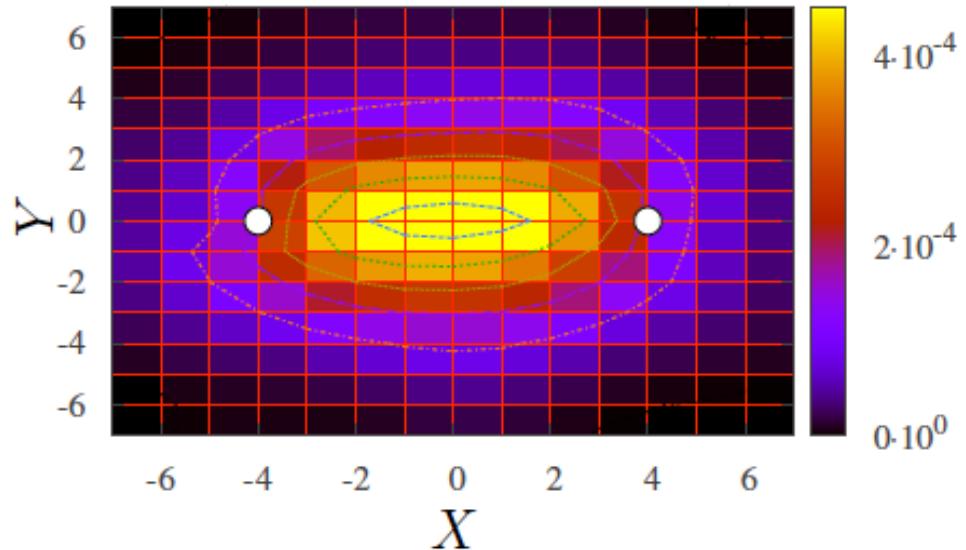
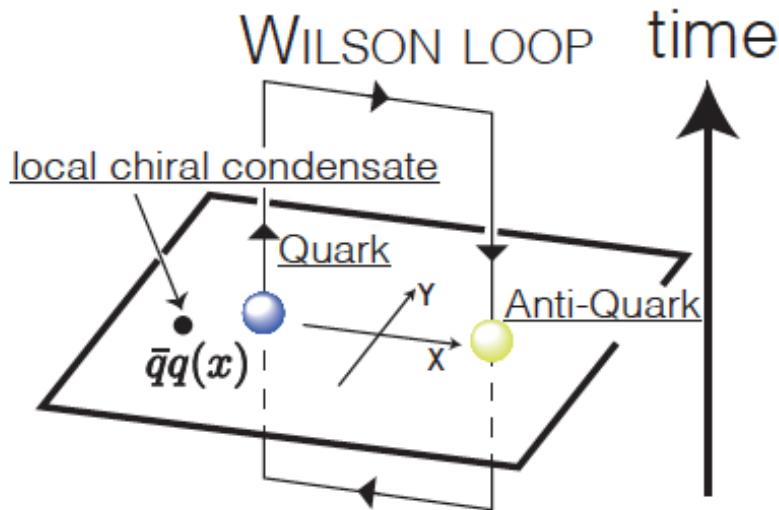
The matrix elements over a hadron state have to be understood as the **difference** of the value of the measured quantity in the hadron and in the vacuum, e.g.

$$\langle P | \bar{q}q | P \rangle = \langle P | \int d^3x \ \bar{q}(x)q(x) | P \rangle - \langle 0 | \bar{q}q | 0 \rangle V_P$$

This difference results from the partial **restoration** of spontaneously broken chiral symmetry inside the hadron

e.g., Donoghue, Nappi '86

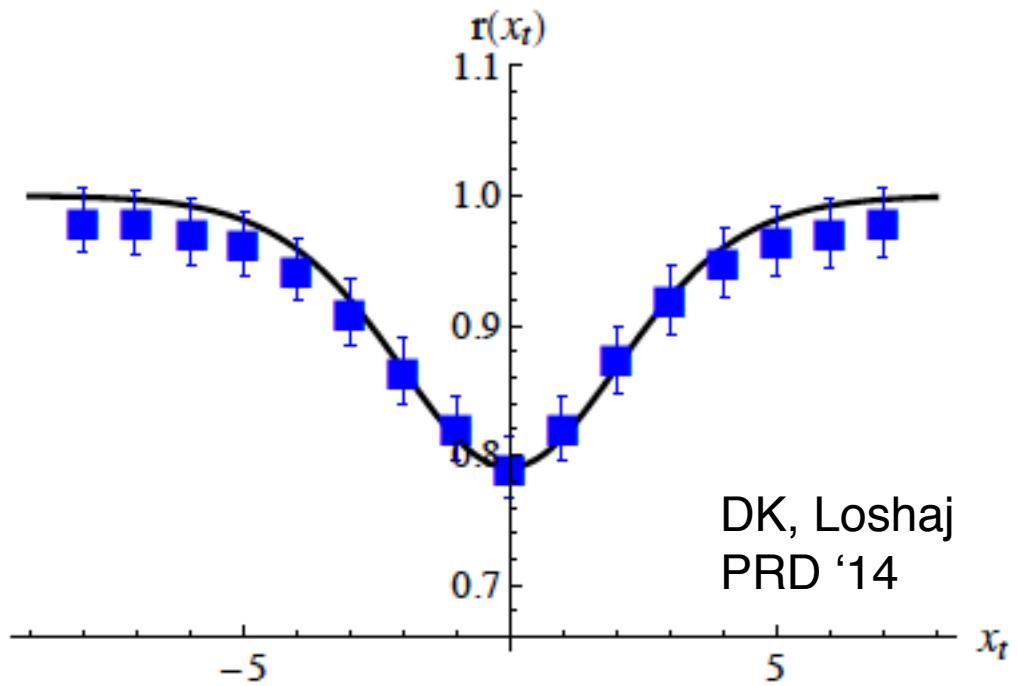
Partial restoration of chiral symmetry inside the nucleon



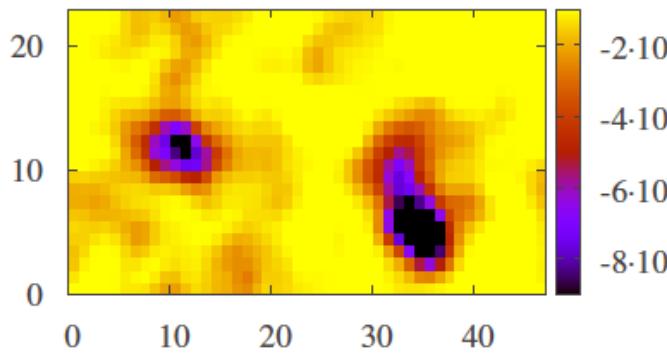
Significant suppression of
the local quark condensate
by the confining flux tube!

Iritani, Cossu, Hashimoto
arXiv:1502.04845 PRD

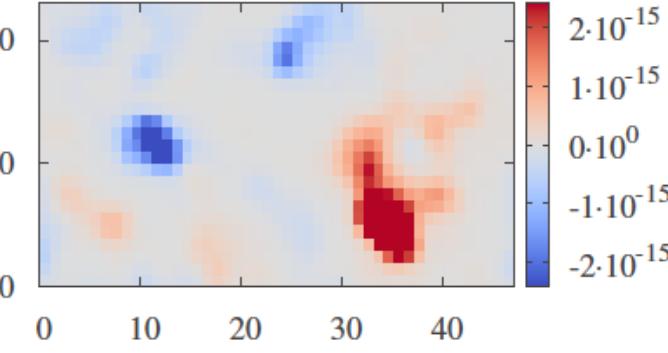
Partial restoration of chiral symmetry inside the nucleon



(a) local chiral condensate



(c) topological charge density



A possible mechanism of chiral condensate suppression involves the chiral anomaly –

so the entire mass of the proton might originate from anomalies – scale and chiral !

The proton mass as a result of the vacuum polarization induced by the presence of the proton

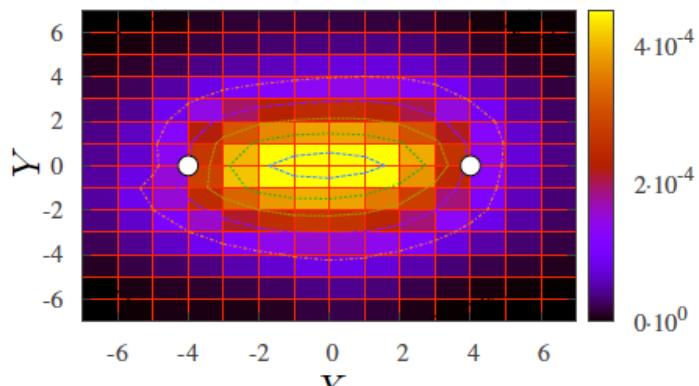
$$\Theta_{\alpha}^{\alpha} = \frac{\tilde{\beta}(g)}{2g} G^{\alpha\beta a} G_{\alpha\beta}^a + \sum_{l=u,d,s} m_l \bar{q}_l q_l,$$

Polarization of the gluon field;

~ 90% of the proton's mass ?

Polarization of the quark condensate;

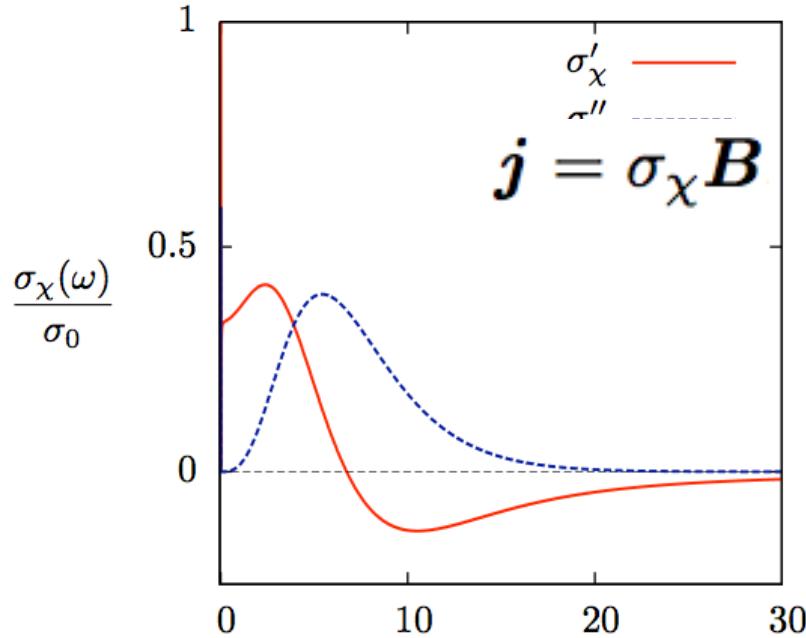
numerically, ~ 80 MeV using



Y.-B.Yang et al
arXiv:1511.09089

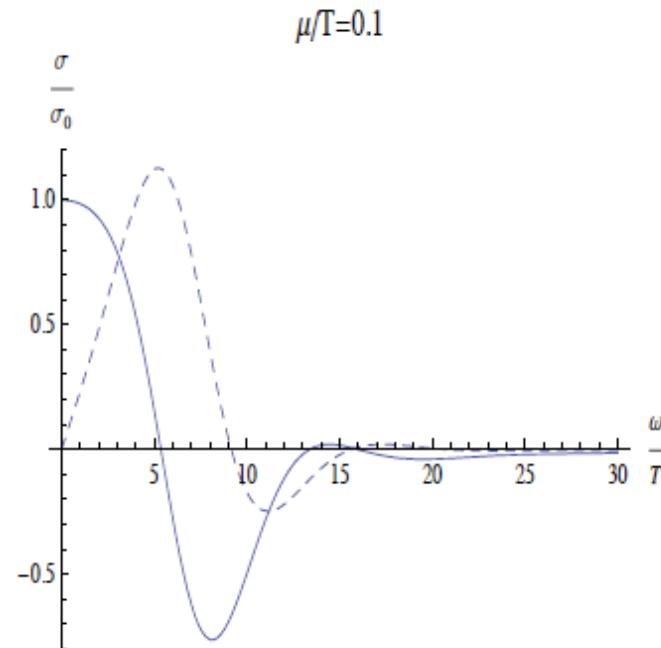
Dynamical chiral magnetic effect

Weak coupling



D.K., H. Warringa
Phys Rev D80 (2009) 034028

Strong coupling



H.-U. Yee, arXiv:0908.4189,
JHEP 0911:085, 2009;

DK,
M.Stephanov,
H.-U.Yee.
1612.01674
PRD'17

A.Rebhan, A.Schmitt, S.Stricker JHEP 0905, 084 (2009), G.Lifshytz, M.Lippert, arXiv:0904.4772; A. Gorsky, P. Kopnin, A. Zayakin, arXiv:1003.2293, A.Gynther, K. Landsteiner, F. Pena Benitez, JHEP 1102 (2011) 110; V. Rubakov, arXiv:1005.1888, C. Hoyos, T. Nishioka, A. O'Bannon, JHEP1110 (2011) 084; ...

CME persists at strong coupling - hydrodynamical formulation?

The CME in relativistic hydrodynamics: The Chiral Magnetic Wave

$$\begin{pmatrix} \vec{j}_V \\ \vec{j}_A \end{pmatrix} = \frac{N_c e \vec{B}}{2\pi^2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \mu_V \\ \mu_A \end{pmatrix}$$

Propagating chiral wave: (if chiral symmetry
is restored)

$$\left(\partial_0 \mp \frac{N_c e B \alpha}{2\pi^2} \partial_1 - D_L \partial_1^2 \right) j_{L,R}^0 = 0$$

Gapless collective mode is the carrier of CME current in MHD:

$$\omega = \mp v_\chi k - iD_L k^2 + \dots$$

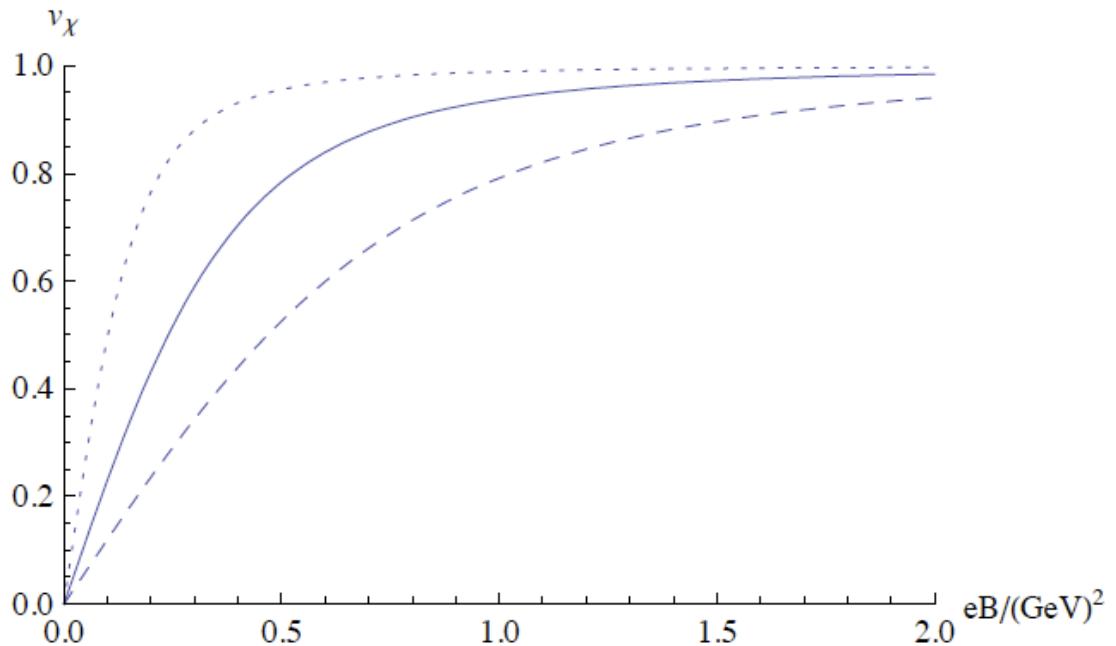
DK, H.-U. Yee,
arXiv:1012.6026 [hep-th];
PRD



The Chiral Magnetic Wave: oscillations of electric and chiral charges coupled by the chiral anomaly



In strong magnetic field, CMW propagates with the speed of light!



Chiral Magnetic Wave in real time!

Anomalous transport in real time

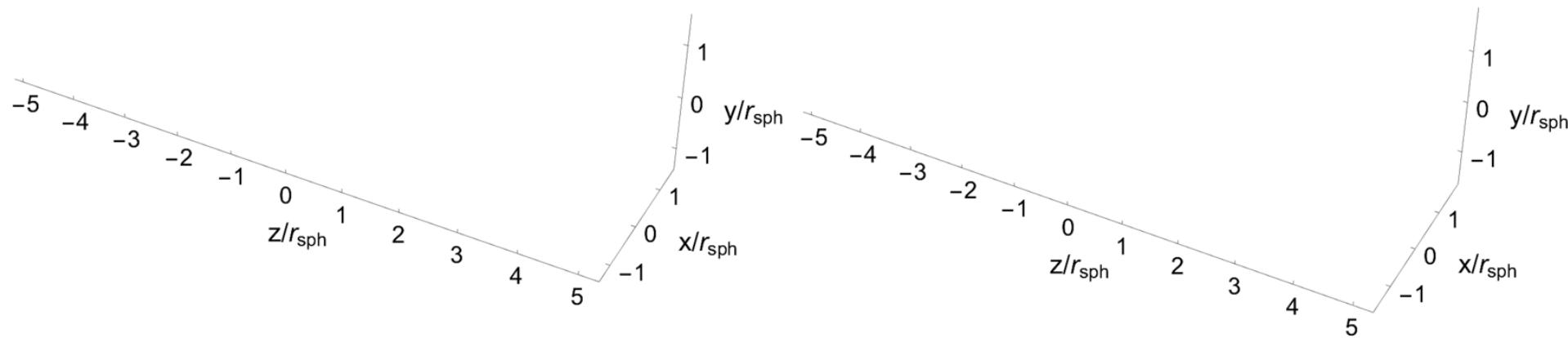
j_a^0 :axial charge

j_v^0 :vector charge

B

$t/t_{\text{sph}}=0$

$t/t_{\text{sph}}=0$

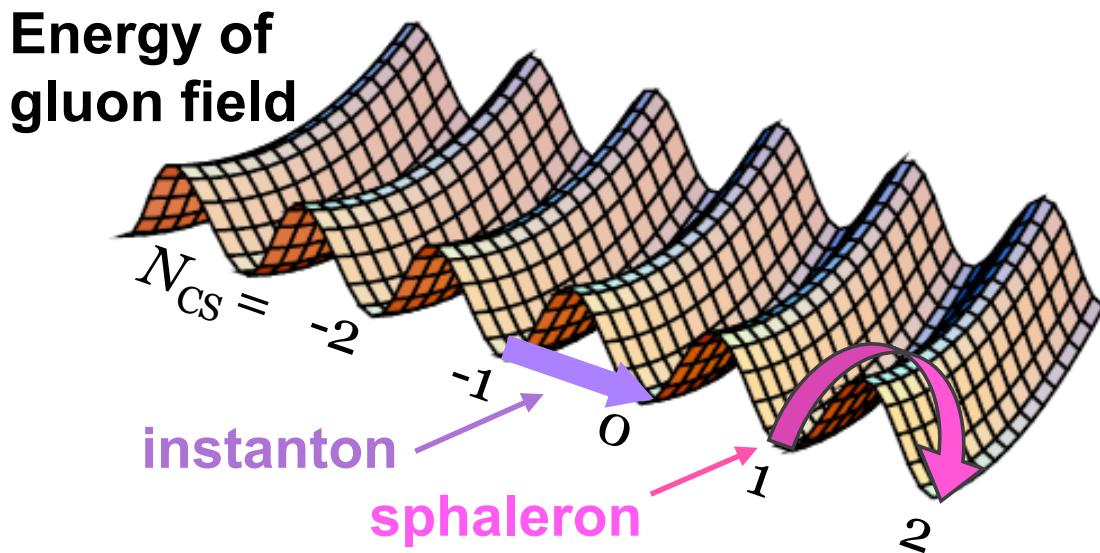


Static U(1) magnetic field in z -dir

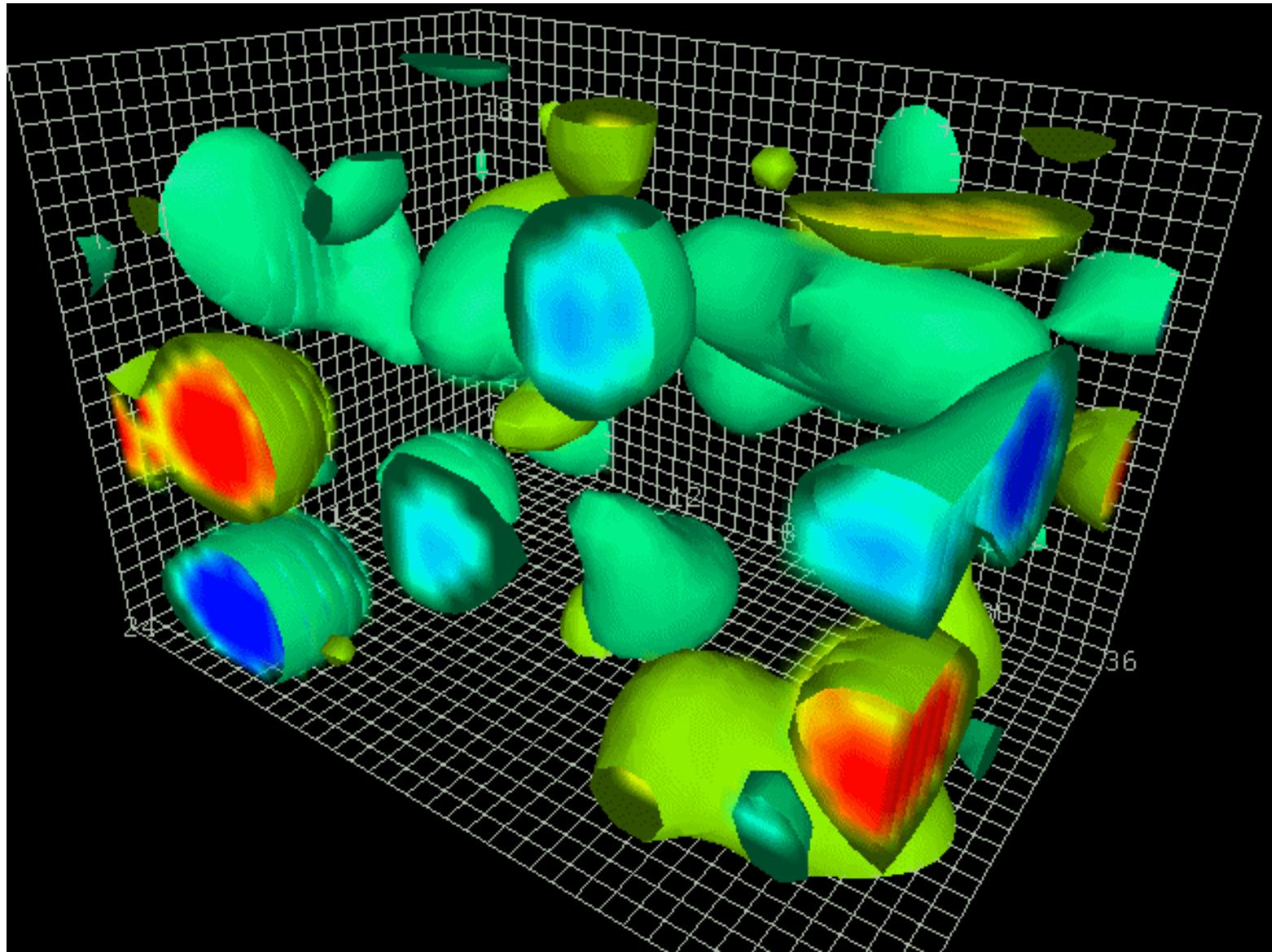
Topology and QCD vacuum

The instanton solutions in Minkowski space-time describe the tunneling events between the topological sectors of the vacuum marked by different integer values of

$$N_{CS} \equiv \int d^3x K_o$$
$$K_\mu = \frac{1}{16\pi^2} \epsilon_{\mu\alpha\beta\gamma} \left(A_\alpha^a \partial_\beta A_\gamma^a + \frac{1}{3} f^{abc} A_\alpha^a A_\beta^b A_\gamma^c \right)$$



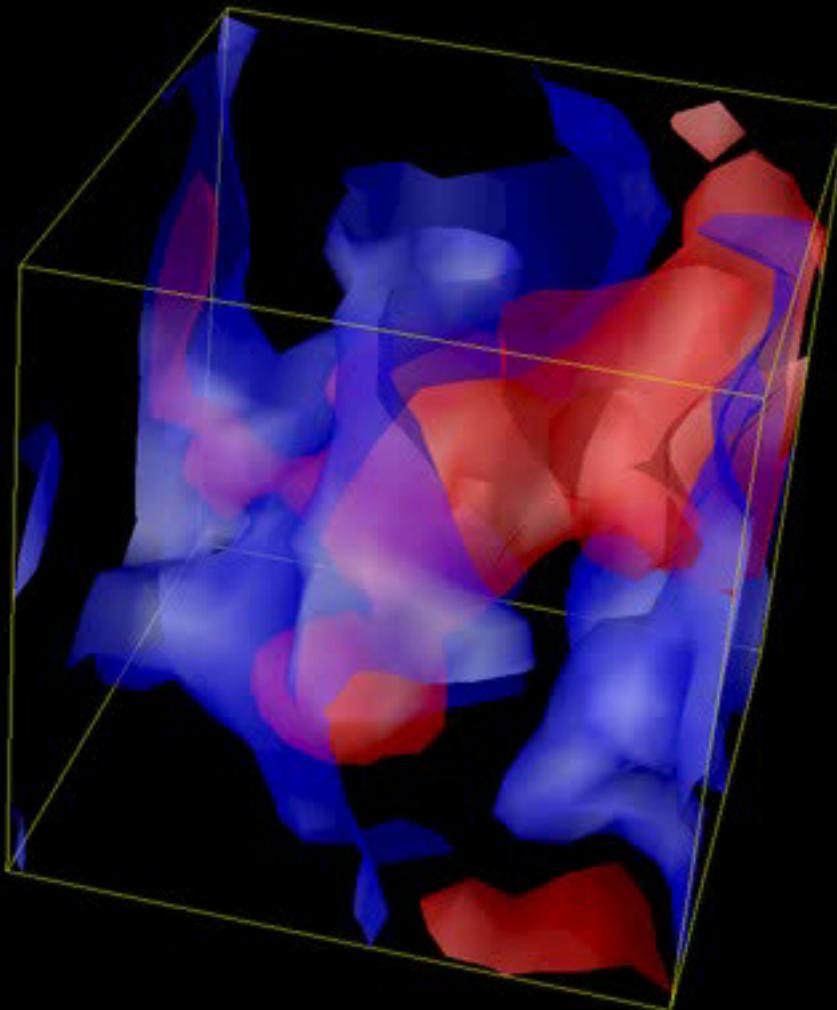
Topological number fluctuations in QCD vacuum ("cooled" configurations)



D. Leinweber

Topological number fluctuations in QCD vacuum

ITEP Lattice Group

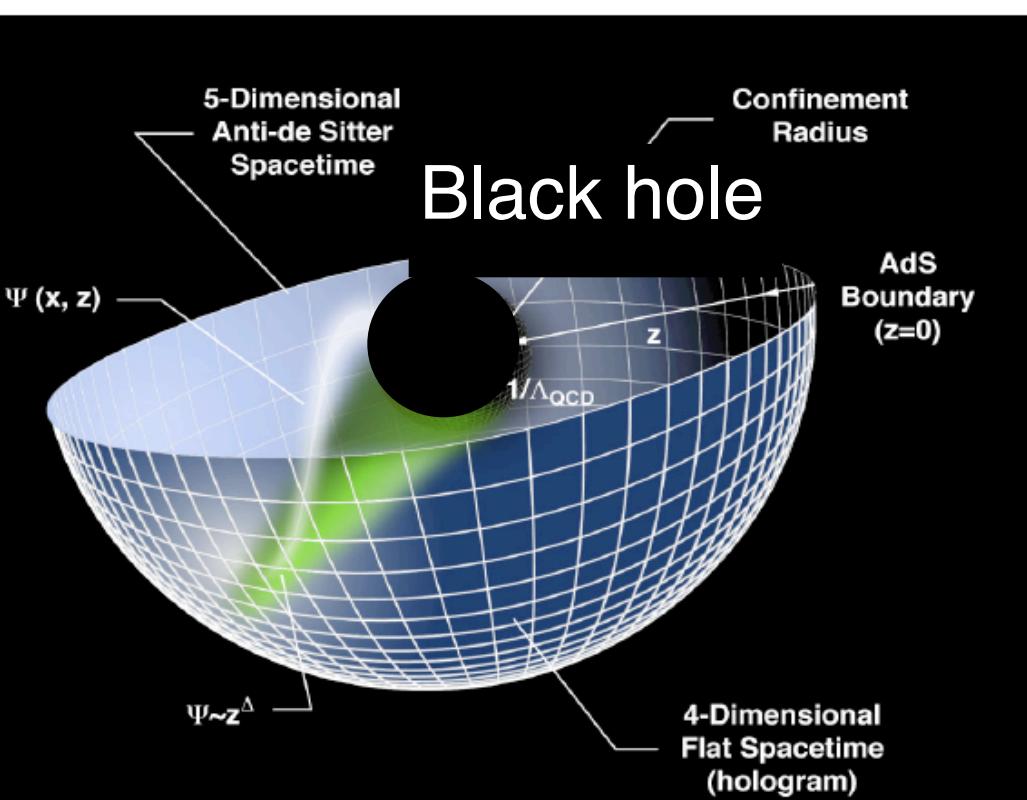


Topological number diffusion at strong coupling

Chern-Simons number
diffusion rate
at strong coupling

$$\Gamma = \frac{(g_{\text{YM}}^2 N)^2}{256\pi^3} T^4$$

D.Son,
A.Starinets
hep-th/
020505



NB: This calculation is analogous to the calculation of shear viscosity that led to the “perfect liquid”

The Chern-Simons diffusion rate in an external magnetic field

strongly coupled N=4 SYM plasma in an external U(1)_R magnetic field through holography

G. Basar, DK, Phys Rev D,
arXiv:1202.2161

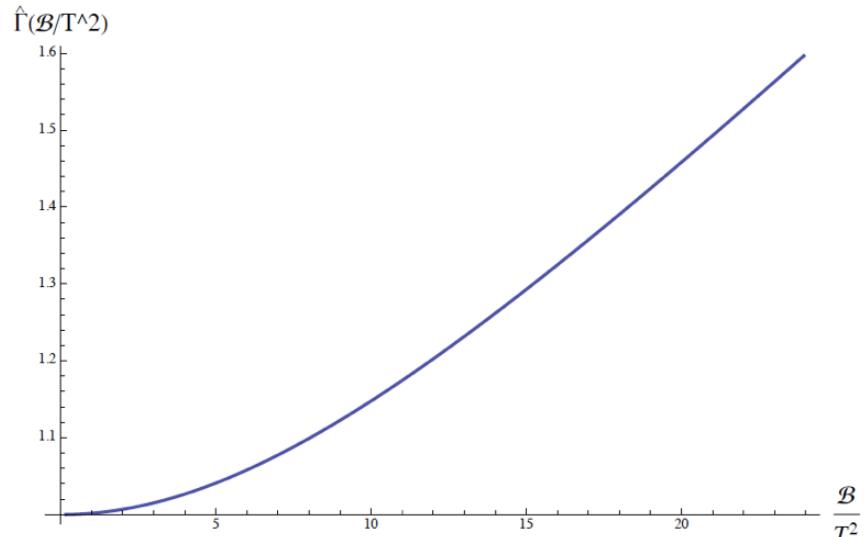
weak field:

$$\Gamma_{CS} = \frac{(g^2 N)^2}{256\pi^3} T^4 \left(1 + \frac{1}{6\pi^4} \frac{\mathcal{B}^2}{T^4} + \mathcal{O}\left(\frac{\mathcal{B}^4}{T^8}\right) \right)$$

strong field increases the rate:

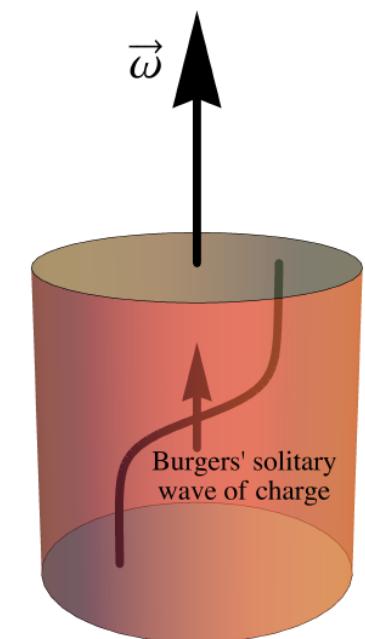
$$\Gamma(\mathcal{B}, T) = \frac{(g^2 N)^2}{384\sqrt{3}\pi^5} \mathcal{B} T^2$$

dimensional reduction⁴⁵



Anomalous transport induced by vorticity

Consider a “hot” system (QGP, DSM) with $\frac{\mu}{T} \ll 1$



The chemical potential is then proportional to charge density:

$$\mu \approx \chi^{-1}\rho + \mathcal{O}(\rho^3)$$

the CME current is

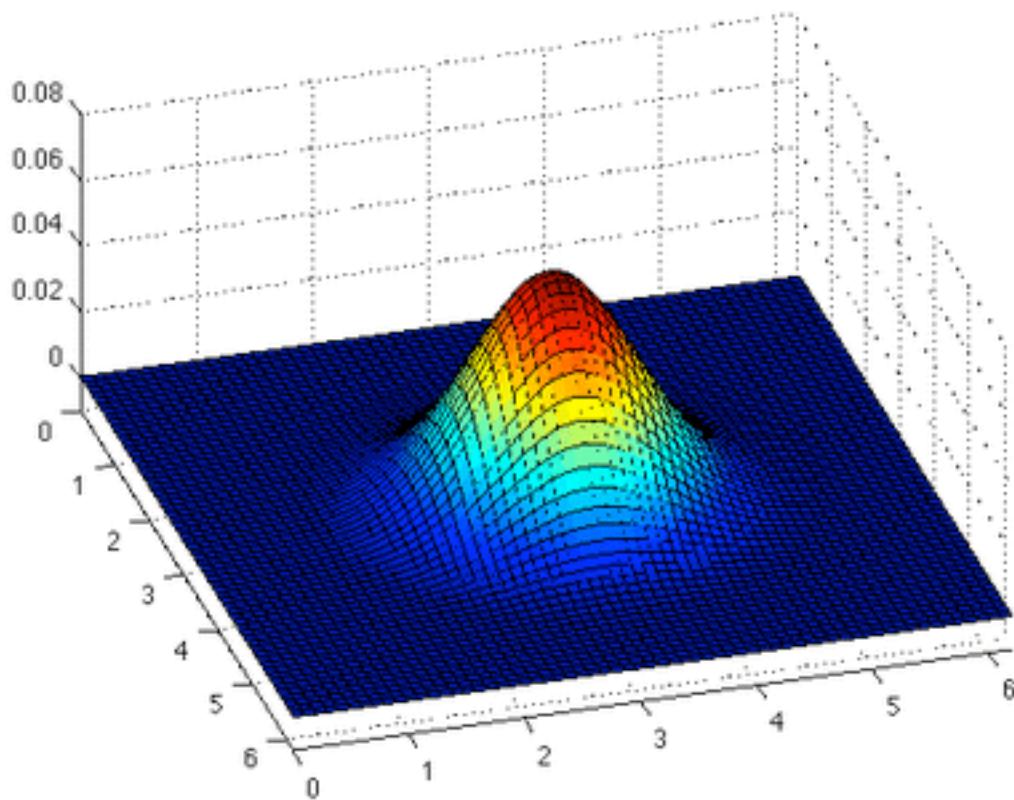
$$J^3 = \frac{ke}{4\pi^2} \left(\chi^{-2}\rho^2 + \frac{\pi^2}{3}T^2 \right) \omega - D\partial_3\rho + \mathcal{O}(\partial^2, \rho^3)$$

and the charge conservation $\partial_t\rho + \partial_3 J^3 = 0$ leads to

$$\partial_t\rho + C\rho\partial_x\rho - D\partial_x^2\rho = 0 \quad C = \frac{ke\omega}{2\pi^2\chi^2} \quad x \equiv x^3$$

The Burgers' equation

$$\partial_t \rho + C \rho \partial_x \rho - D \partial_x^2 \rho = 0$$

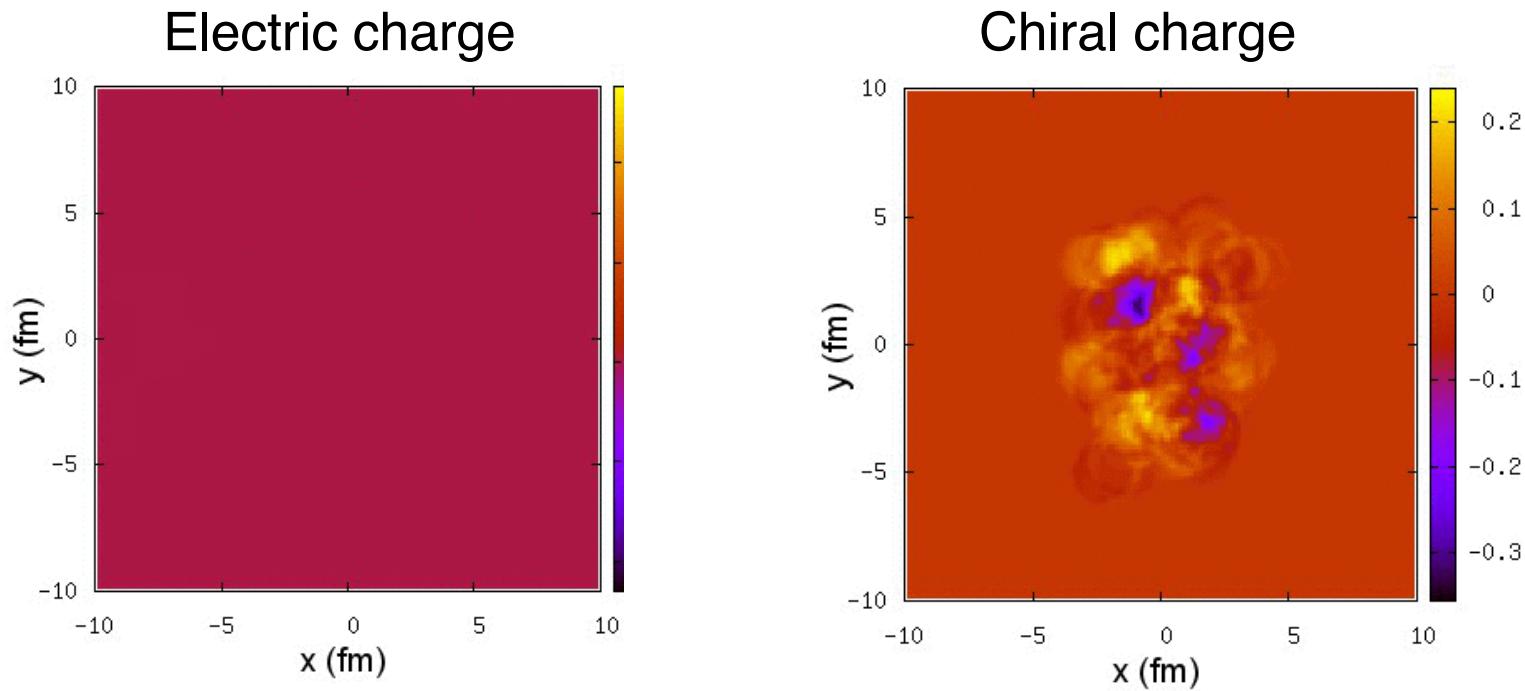


Exactly soluble by
Cole-Hopf
transformation -

initial value problem,
integrable dynamics

describes shock
waves, solitons, ...

CMHD



Y.Hirono, T.Hirano, DK, (Stony Brook – Tokyo), arxiv:1412.0311
(3+1) ideal CMHD (Chiral MagnetoHydroDynamics)

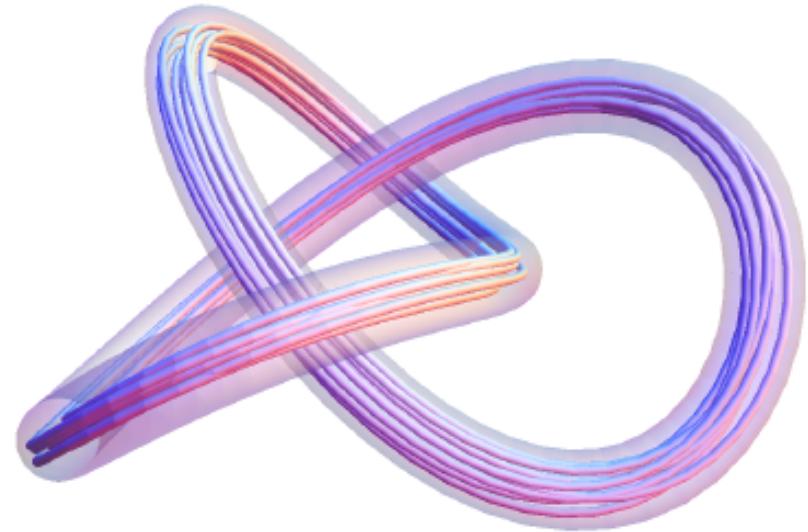
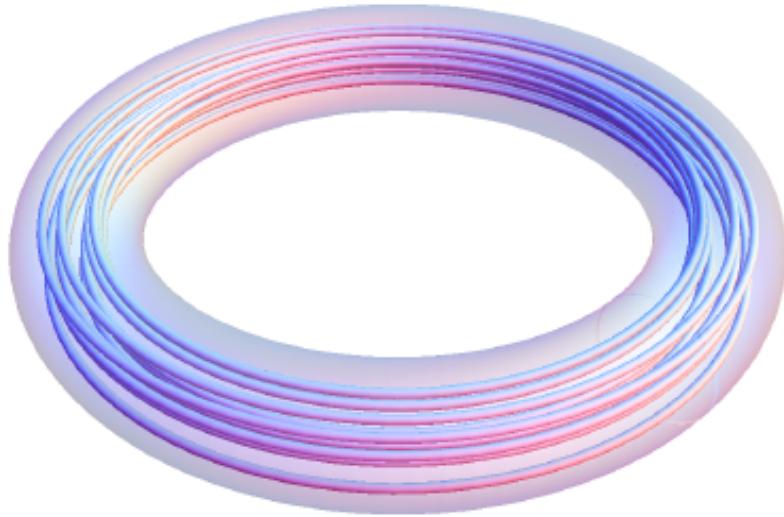
BEST Theory Collaboration (DOE)

Quantized CME from knot reconnections

Magnetic helicity is the measure
of “knottedness” of magnetic flux
- Chern-Simons 3-form

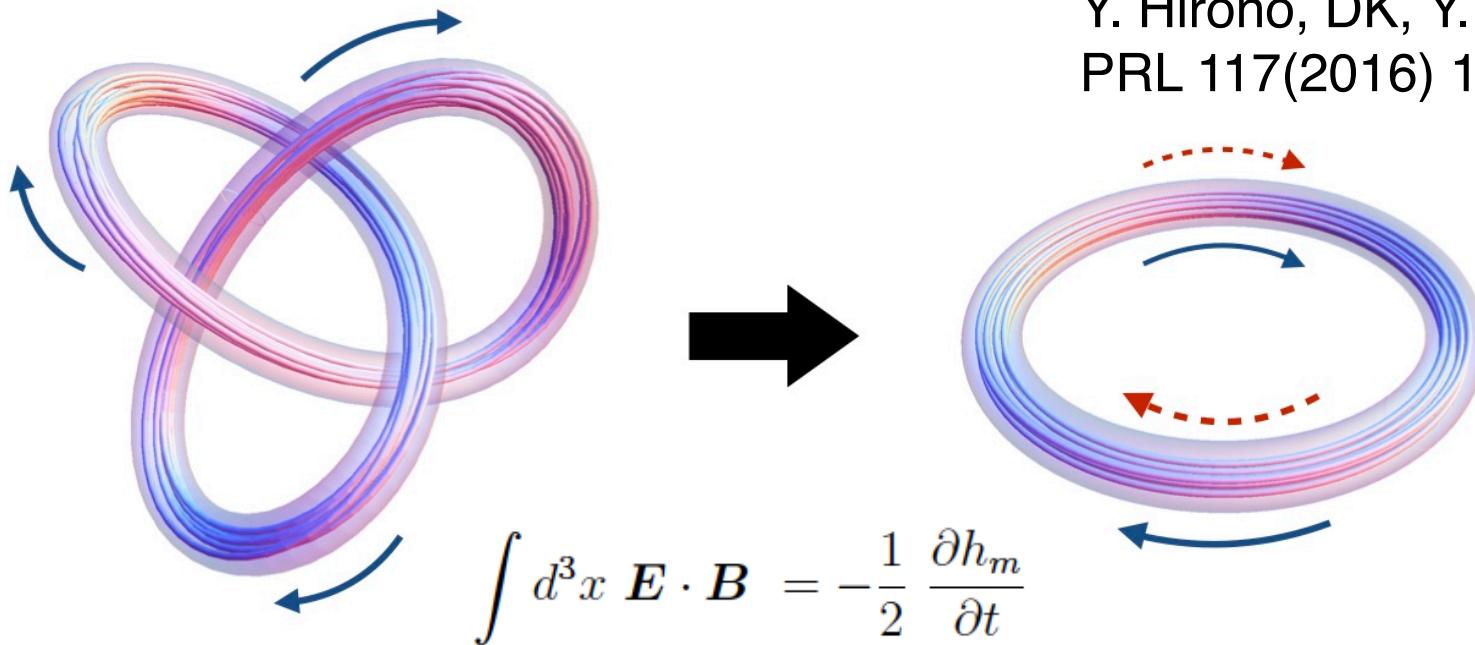
$$h_m \equiv \int d^3x \ A \cdot B$$

Y. Hirono, DK, Y. Yin,
PRL 117(2016) 172301



Consider a tube (unknot) of magnetic flux, with chiral fermions localized on it.

To turn it into a (chiral) knot, we need a magnetic reconnection.
What happens to the fermions during the reconnection?

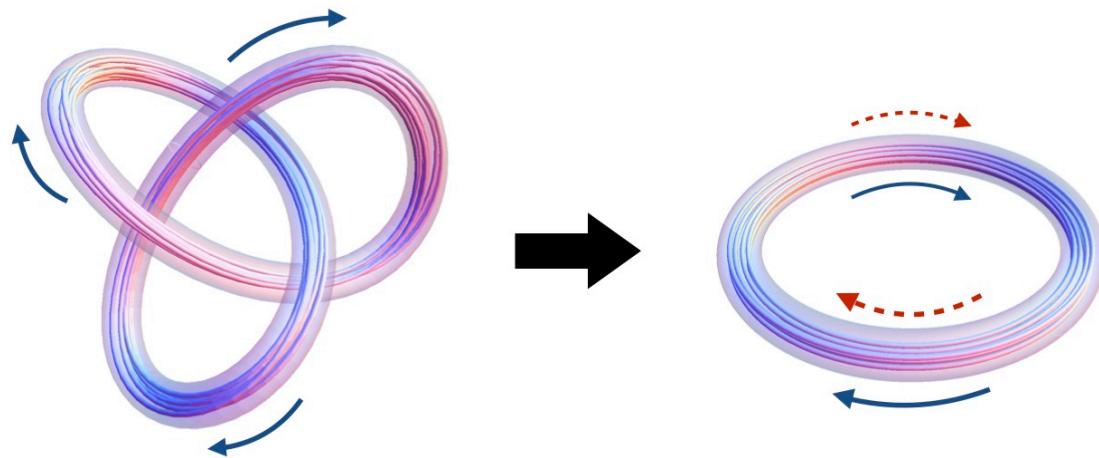


Changing magnetic flux through the area spanned by the tube will generate the electric field (Faraday's induction):

$$\frac{d}{dt} \Phi_B = - \oint_C \mathbf{E} \cdot d\mathbf{x}$$

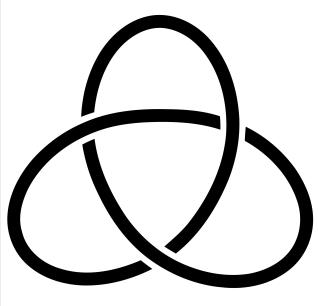
The electric field will generate electric current of fermions (chiral anomaly in 1+1 D):

$$\Delta J = \Delta J_R + \Delta J_L = \frac{q^3 \Phi^2}{2\pi^2 L}$$



Helicity change per magnetic reconnection is $\Delta\mathcal{H} = 2\Phi^2$.

Multiple magnetic reconnections leading to non-chiral knots do not induce net current (need to break left-right symmetry).



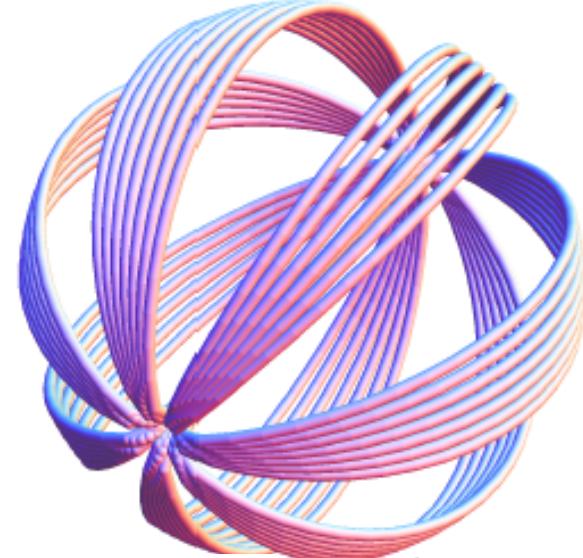
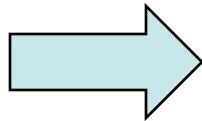
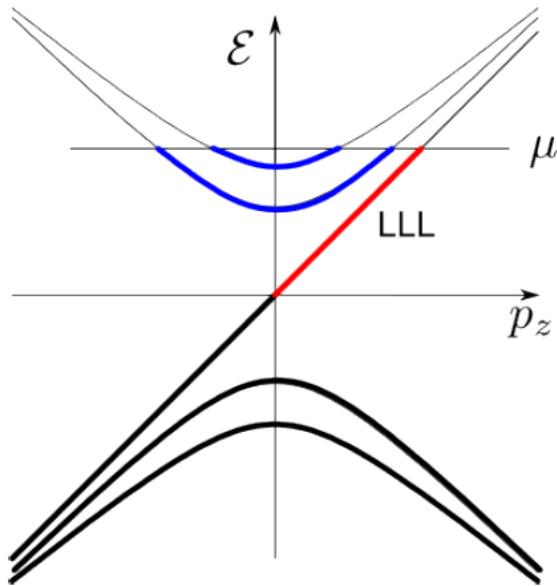
For N_+ positive and N_- negative crossings on a planar knot diagram, the total magnetic helicity is:

$$\mathcal{H} = 2(N_+ - N_-)\Phi^2$$

The total current induced by reconnections to a chiral knot:

$$J = \frac{q^3 \mathcal{H}}{4\pi^2 L}$$

Chirality transfer from fermions to magnetic helicity



Chandrasekhar-Kendall states
(ApJ, 1957)

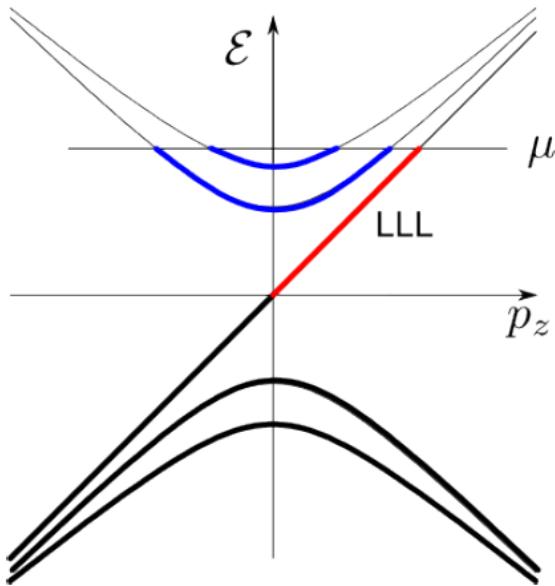
$$h_m \equiv \int d^3x \mathbf{A} \cdot \mathbf{B}$$

$$\partial_\mu j_A^\mu = C_A \mathbf{E} \cdot \mathbf{B}$$

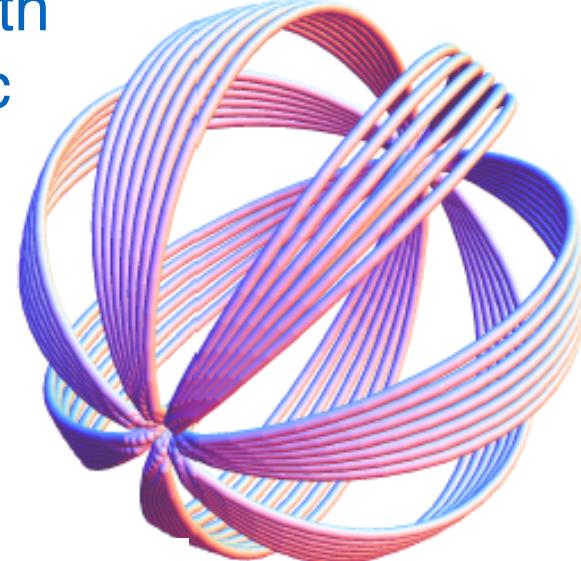
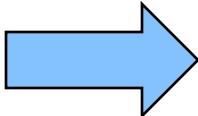
$$h_0 \equiv h_m + h_F = \text{const}$$

$$\int d^3x \mathbf{E} \cdot \mathbf{B} = -\frac{1}{2} \frac{\partial h_m}{\partial t}$$

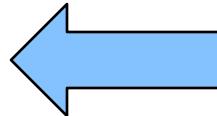
Inverse cascade of magnetic helicity



Instability at $k < C_A \mu_A$ leads
to the growth
of magnetic
helicity



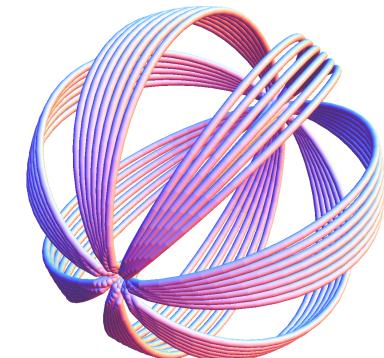
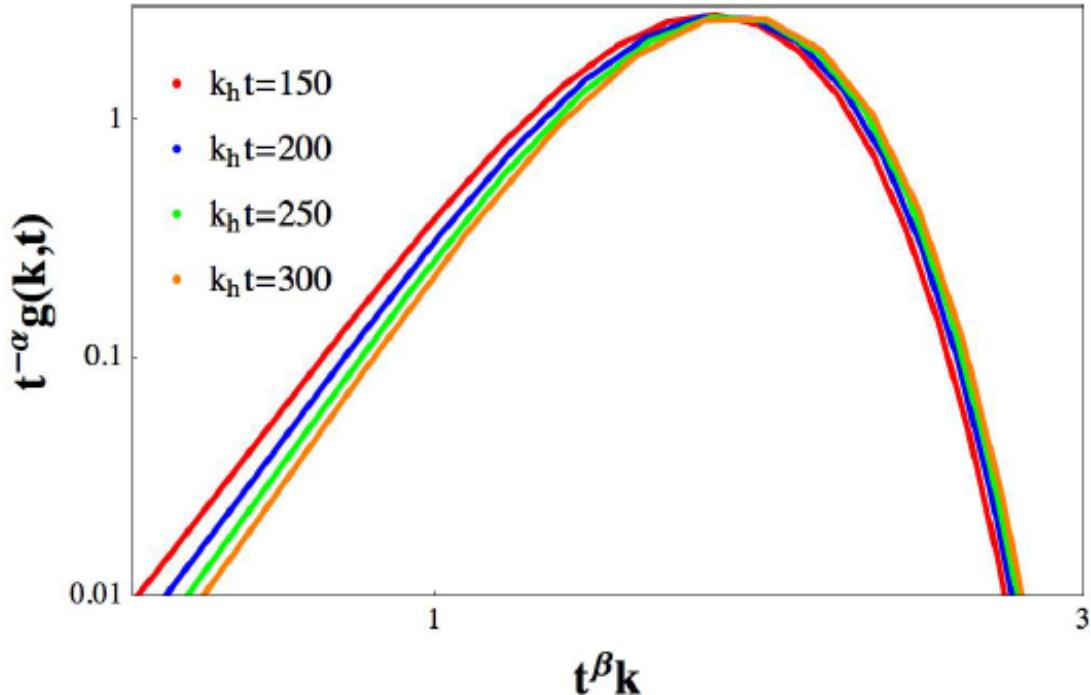
Increase of
magnetic
helicity reduces μ_A



Inverse cascade:

- M.Joyce and M.Shaposhnikov, PRL 79 (1997) 1193;
R.Jackiw and S.Pi, PRD 61 (2000) 105015;
A.Boyarsky, J.Frohlich, O.Ruchayskiy, PRL 108 (2012) 031301;
PRD 92 (2015) 043004;
H.Tashiro, T.Vachaspati, A.Vilenkin, PRD 86 (2012) 105033

Self-similar cascade of magnetic helicity driven by CME

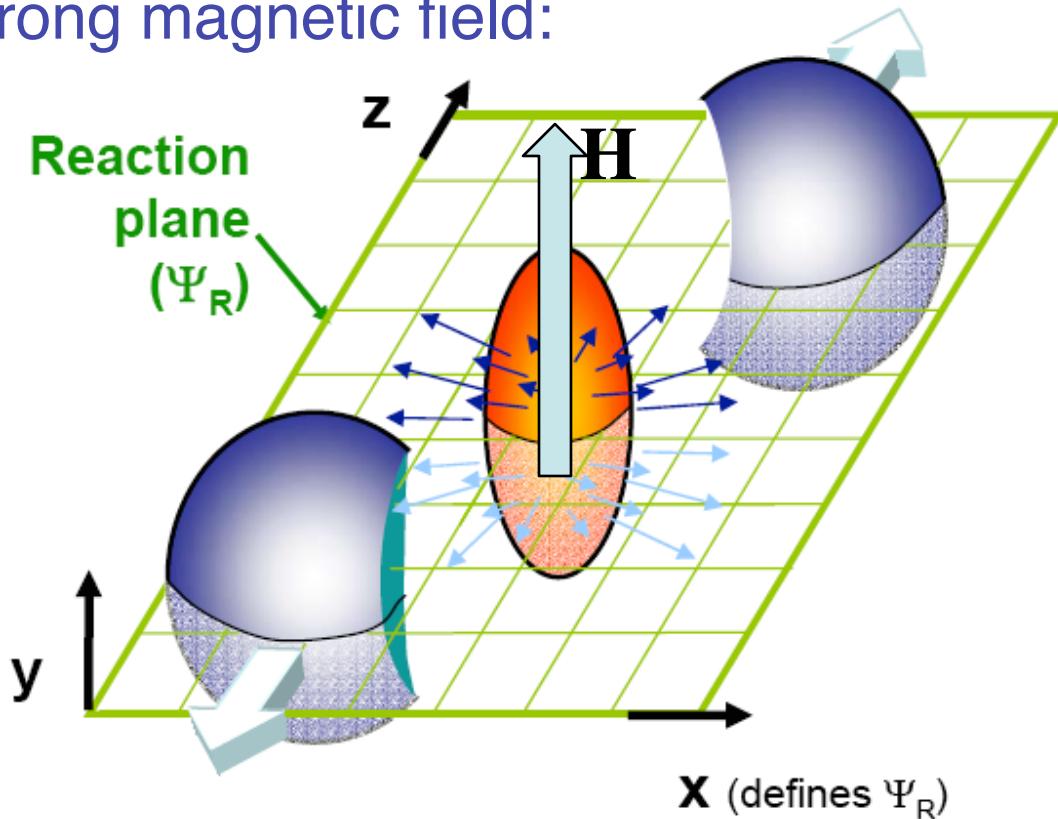


$$g(k, t) \sim t^\alpha \tilde{g}(t^\beta k) \quad \alpha = 1, \quad \beta = 1/2$$

Y. Hirono, DK, Y. Yin, Phys.Rev.D92 (2015) 125031;
N. Yamamoto, Phys.Rev.D93 (2016) 125016

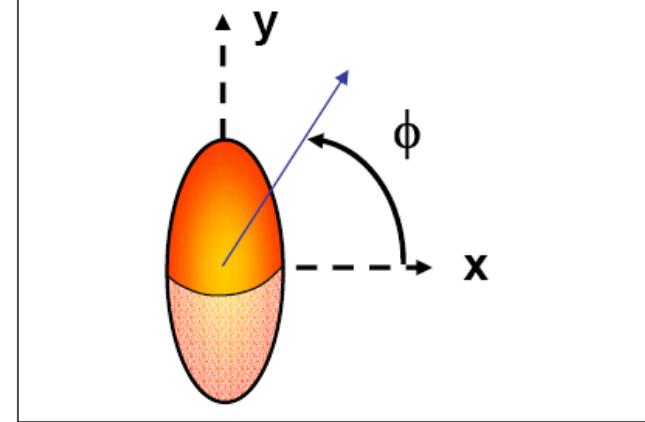
Is there a way to observe CME in nuclear collisions at RHIC?

Relativistic ions create
a strong magnetic field:

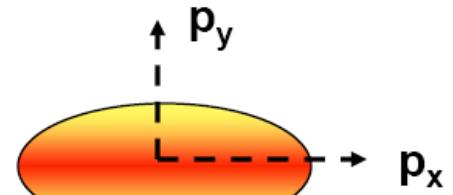


DK, McLerran, Warringa '07

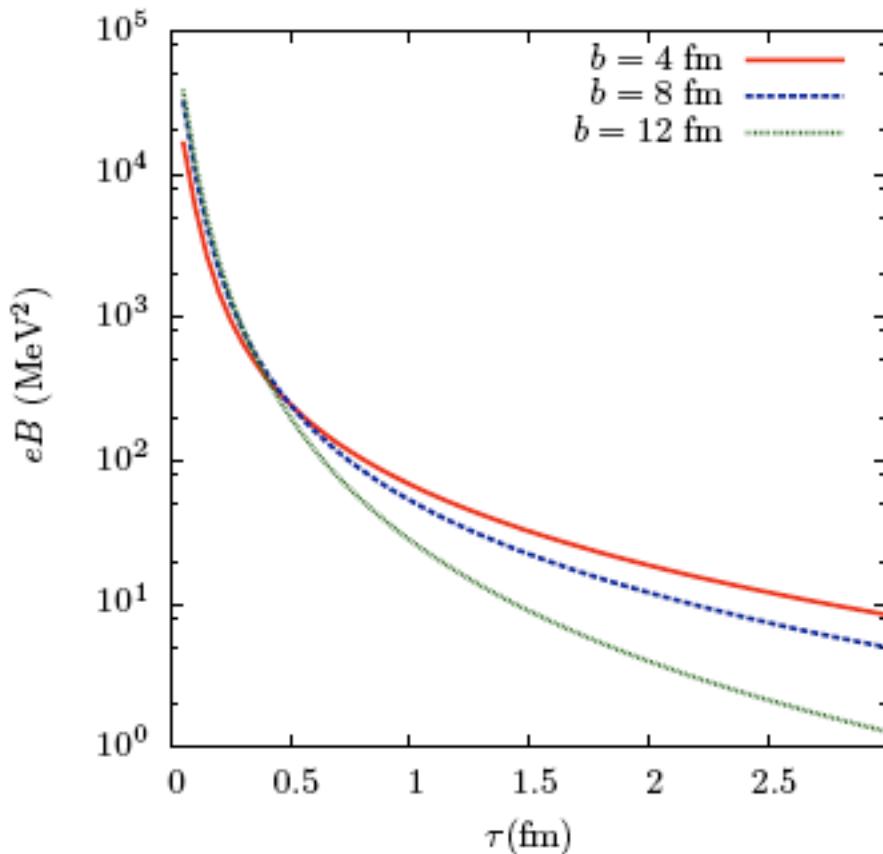
Initial spatial anisotropy



Final momentum anisotropy



Heavy ion collisions as a source of the strongest magnetic fields available in the Laboratory



DK, McLerran, Warringa,
Nucl Phys A803(2008)227

Fig. A.2. Magnetic field at the center of a gold-gold collision, for different impact parameters. Here the center of mass energy is 200 GeV per nucleon pair ($Y_0 = 5.4$).

Comparison of magnetic fields



The Earth's magnetic field 0.6 Gauss



A common, hand-held magnet 100 Gauss
The strongest steady magnetic fields achieved so far in the laboratory 4.5×10^5 Gauss

The strongest man-made fields ever achieved, if only briefly 10^7 Gauss

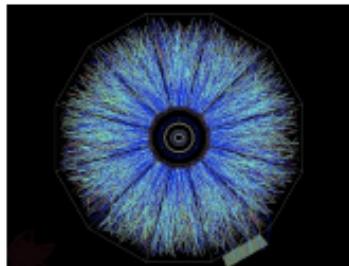


Typical surface, polar magnetic fields of radio pulsars 10^{13} Gauss

Surface field of Magnetars 10^{15} Gauss

<http://solomon.as.utexas.edu/~duncan/magnetar.html>

Heavy ion collisions: the strongest magnetic field ever achieved in the laboratory



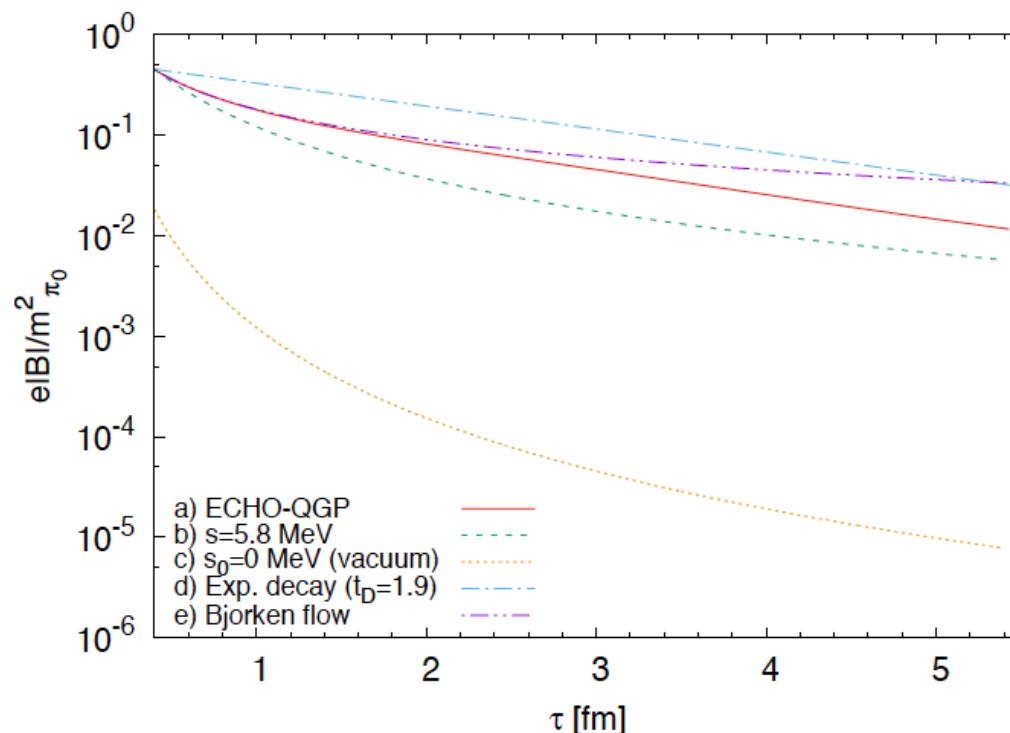
Off central Gold-Gold Collisions at 100 GeV per nucleon
 $eB(\tau=0.2 \text{ fm}) = 10^3 \sim 10^4 \text{ MeV}^2 \sim 10^{17} \text{ Gauss}$

Evolution of magnetic field in full ideal MHD

Numerical magneto-hydrodynamics for relativistic nuclear collisions

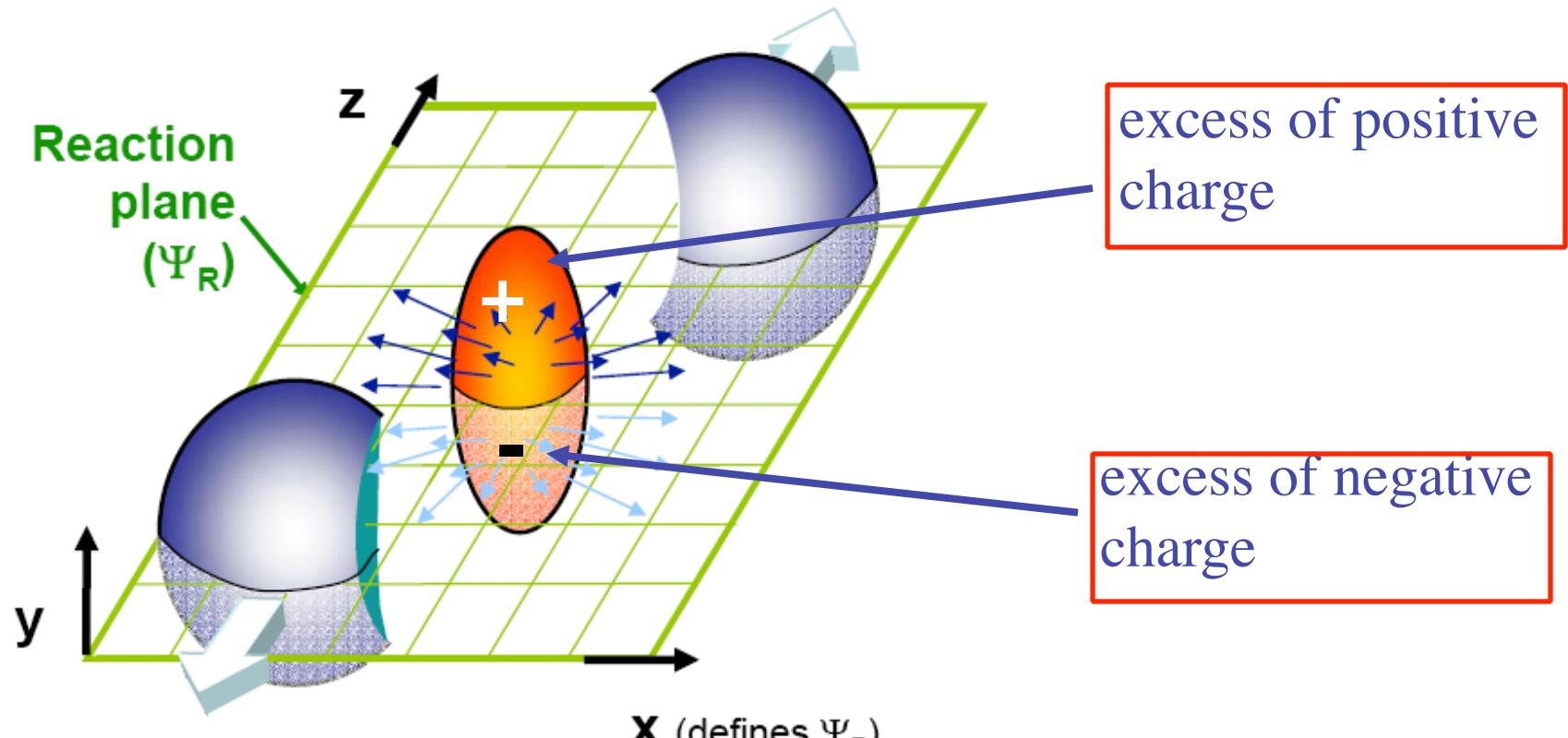
Gabriele Inghirami,^{1, 2, 3, 4,*} Luca Del Zanna,^{5, 6, 7} Andrea Beraudo,⁸
Mohsen Haddadi Moghaddam,^{9, 8} Francesco Becattini,^{5, 6} and Marcus Bleicher^{1, 2, 3, 4}

arxiv:1609.03042



Charge asymmetry w.r.t. reaction plane as a signature of chirality imbalance

Electric dipole moment due to chiral imbalance

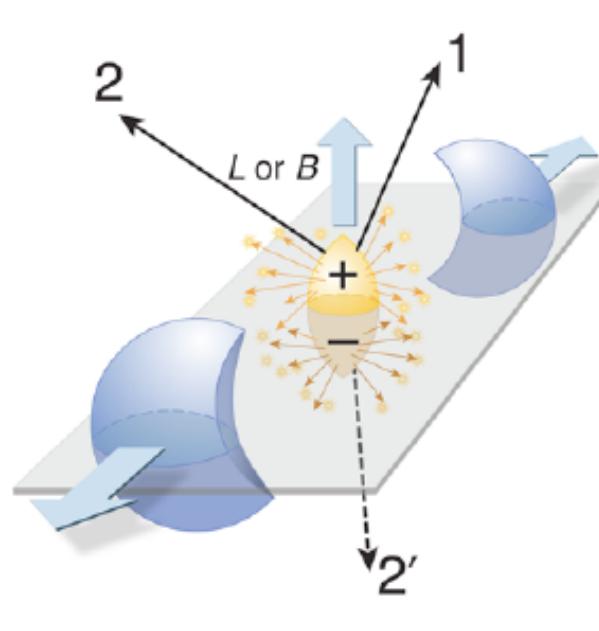


$$\Theta \rightarrow \Theta(x,t)$$



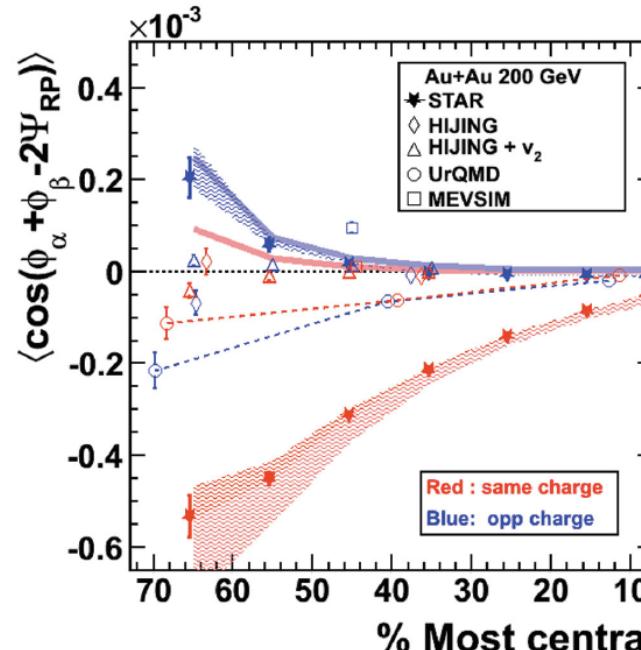
Azimuthal Charged-Particle Correlations and Possible Local Strong Parity Violation

(STAR Collaboration)



S.Voloshin '04

$$\begin{aligned} \gamma &\equiv \langle \cos(\phi_\alpha + \phi_\beta - 2\Psi_{RP}) \rangle = \langle \cos \Delta\phi_\alpha \cos \Delta\phi_\beta \rangle - \langle \sin \Delta\phi_\alpha \sin \Delta\phi_\beta \rangle \\ &= [\langle v_{1,\alpha} v_{1,\beta} \rangle + B_{IN}] - [\langle a_\alpha a_\beta \rangle + B_{OUT}] \approx -\langle a_\alpha a_\beta \rangle + [B_{IN} - B_{OUT}], \end{aligned}$$



**NB: P-even quantity (strength of P-odd fluctuations)
 – subject to large background contributions**

Observation of charge-dependent azimuthal correlations in pPb collisions and its implication for the search for the chiral magnetic effect

arxiv:1610.00263

The CMS Collaboration*

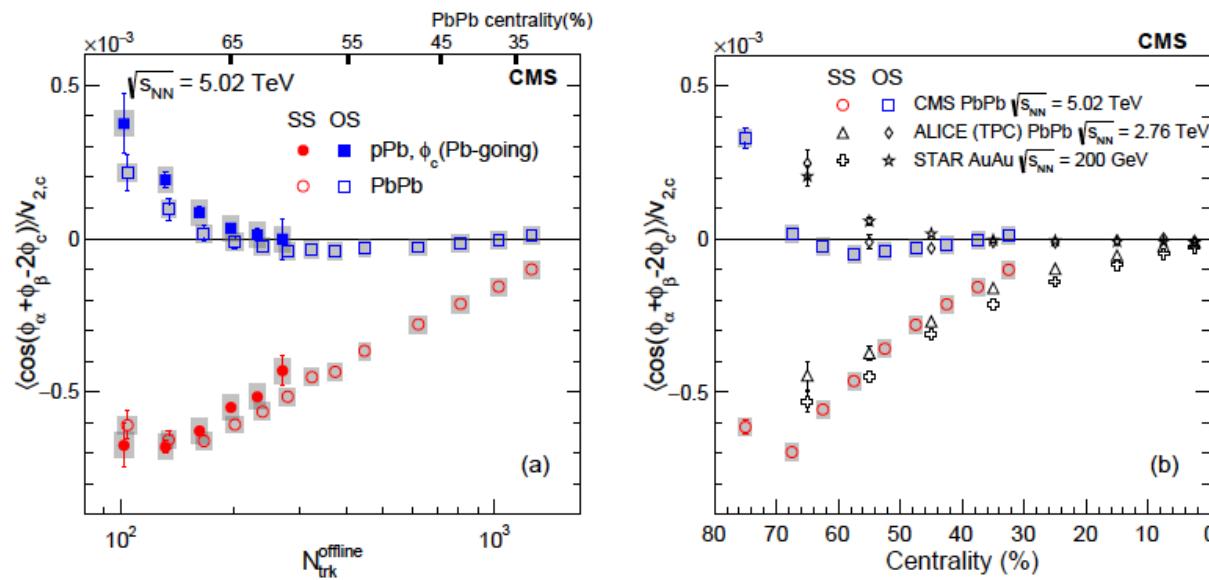
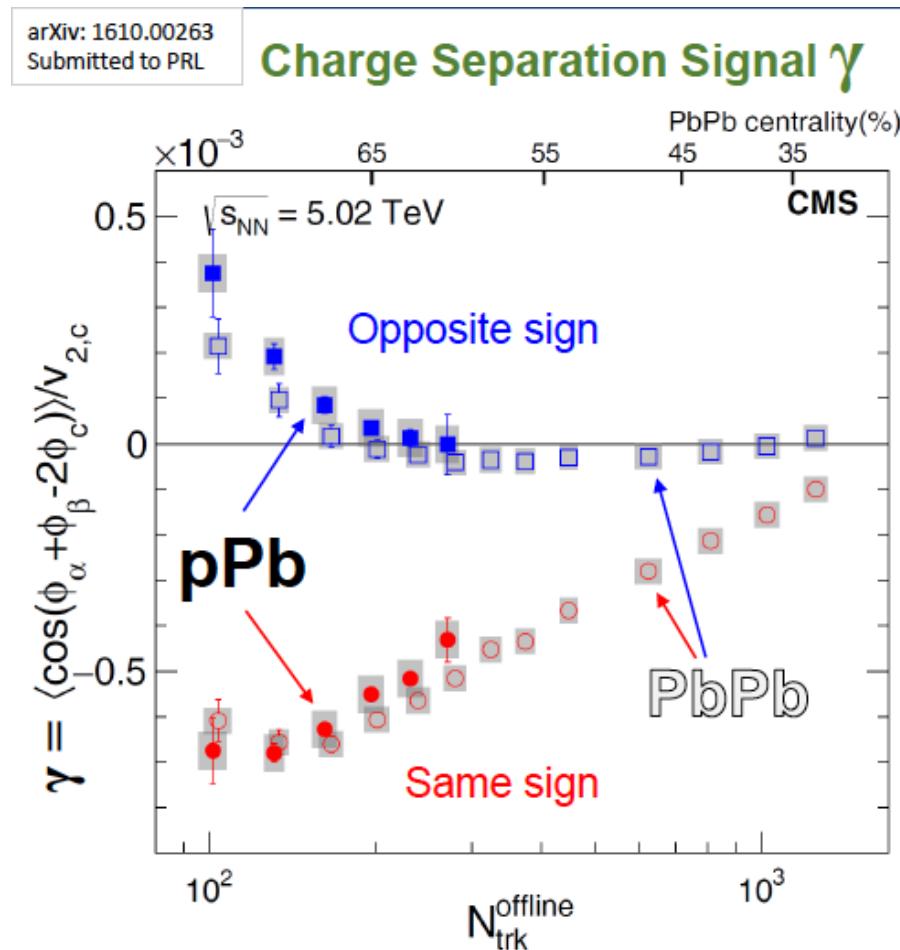


Figure 2: In (a), the same sign (SS) and opposite sign (OS) three-particle correlator averaged over $|\eta_\alpha - \eta_\beta| < 1.6$ as a function of $N_{\text{trk}}^{\text{offline}}$ in pPb and PbPb collisions at $\sqrt{s_{\text{NN}}} = 5.02 \text{ TeV}$ are shown. In (b), the same correlation as a function of centrality is presented in PbPb collisions at $\sqrt{s_{\text{NN}}} = 5.02 \text{ TeV}$ from CMS, at $\sqrt{s_{\text{NN}}} = 2.76 \text{ TeV}$ from ALICE, and in AuAu collisions at $\sqrt{s_{\text{NN}}} = 0.2 \text{ TeV}$ from STAR. Statistical and systematic uncertainties are indicated by the error bars and shaded regions, respectively.

Background
everywhere?
(dAu at RHIC!)

Magnetic field
in pA?

CMS: Surprising scaling of pA and AA results at different energies?

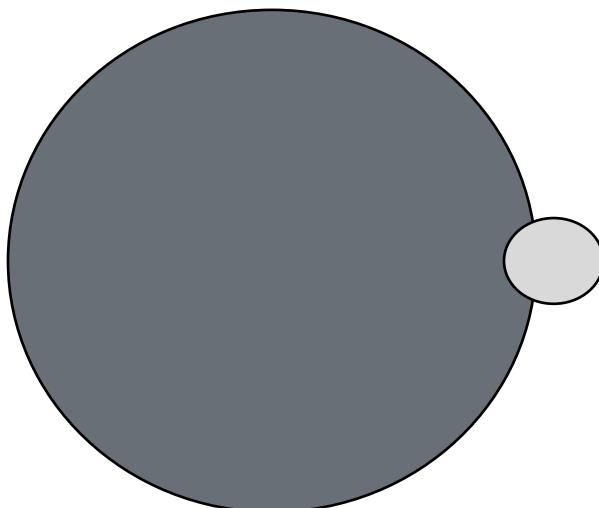


But: different dependence on rapidity difference between α and β

PbPb and pPb with the same event multiplicity are similar...!
Challenge to CME interpretation!

Some comments:

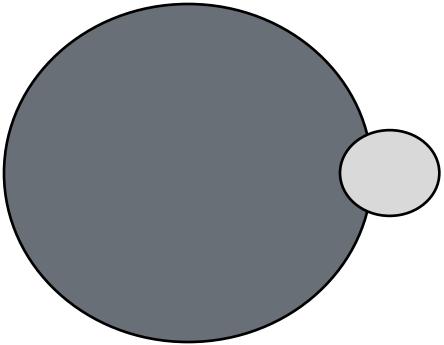
1. The scaling is a challenge to both CME and background interpretations, since background scales as v_2/N , and v_2 is different ($\sim 30\%$?) in pA and AA at the same multiplicity. Even more challenging for RHIC vs LHC comparison.
2. In pA, one expects much weakened, but non-zero correlations between magnetic field B and reaction plane due to the gradient of nuclear density. For a black disk:



This configuration yields
 B orthogonal to the reaction plane;

Its contribution is suppressed by

$$(R_N/R_A)^2$$



This configuration yields
B orthogonal to the reaction plane (RP);
Its contribution is suppressed by
 $(R_N/R_A)^2$

The proton is always much smaller than the nucleus... or is it?

The proton size grows with energy:

$$R_p^2(s) = R_p^2(s_0) + \alpha'_P \ln(s/s_0)$$

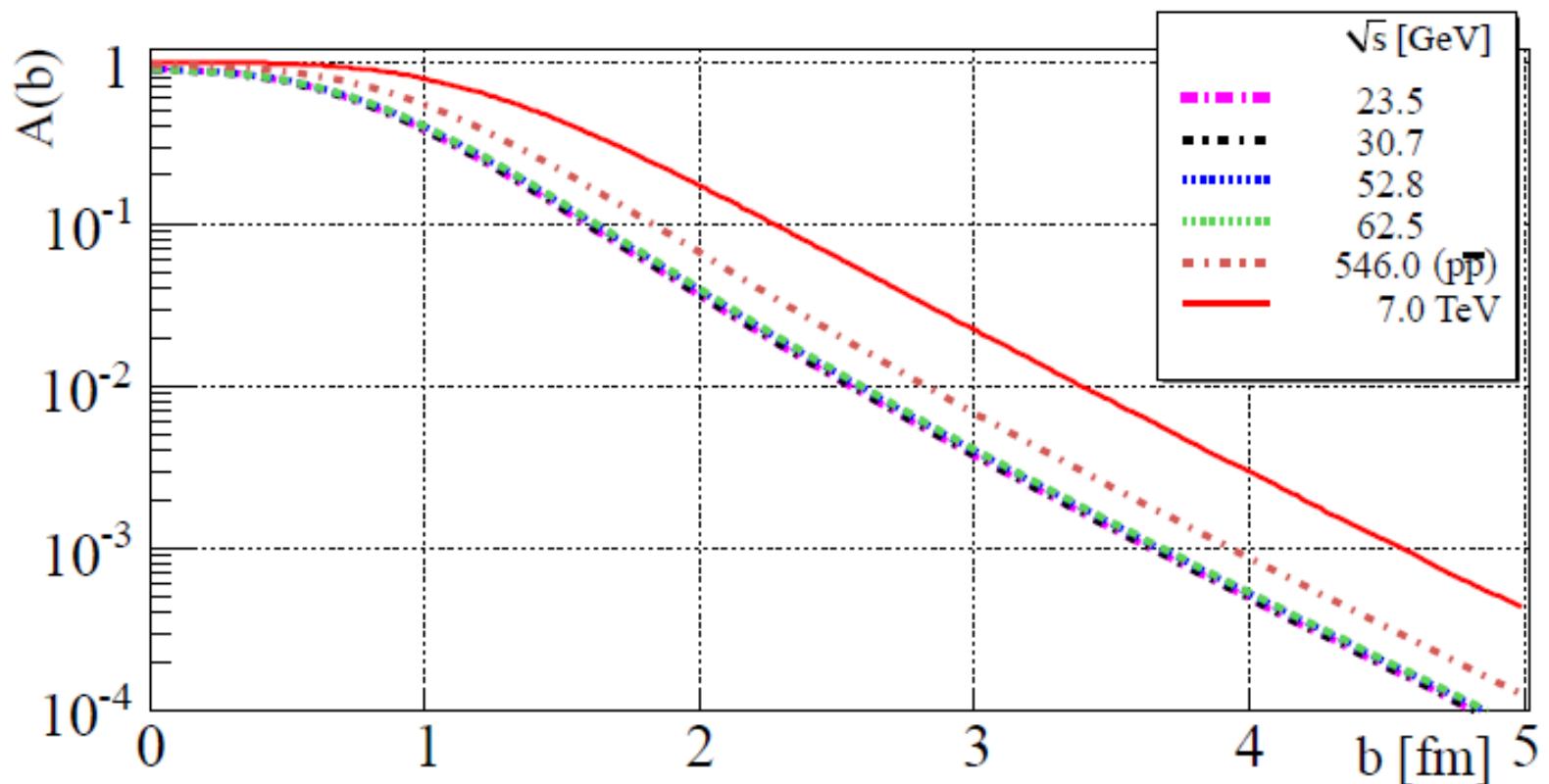
Gribov diffusion; Shrinkage of diffraction peak. Even at LHC,
still a relatively modest size growth [TOTEM: non-linear dependence]

But: the second term is due to the number of parton splittings –
in high multiplicity N events, can expect larger than average
size of the proton,

$$R_p^2(s; N) = \bar{R}_p^2(s) \frac{N}{\bar{N}}$$

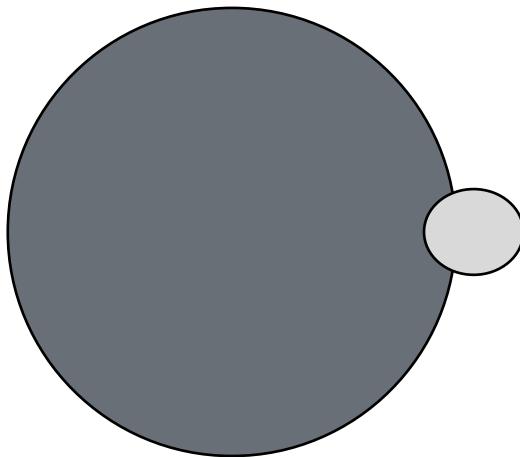
Can this effect give a sizeable correlation between B^{64} and RP?

The growth of the proton size at high energies

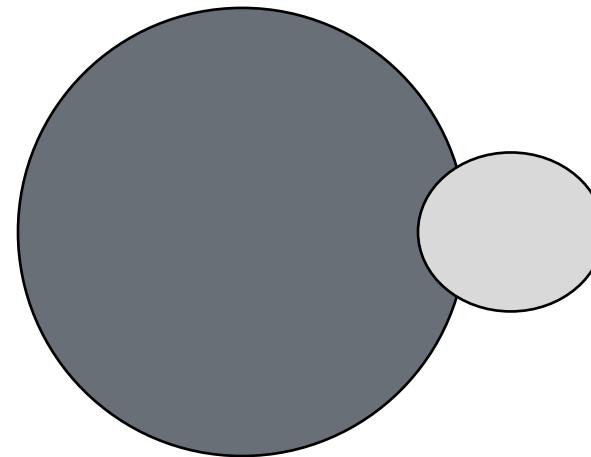


TOTEM Collaboration

Average
Multiplicity:



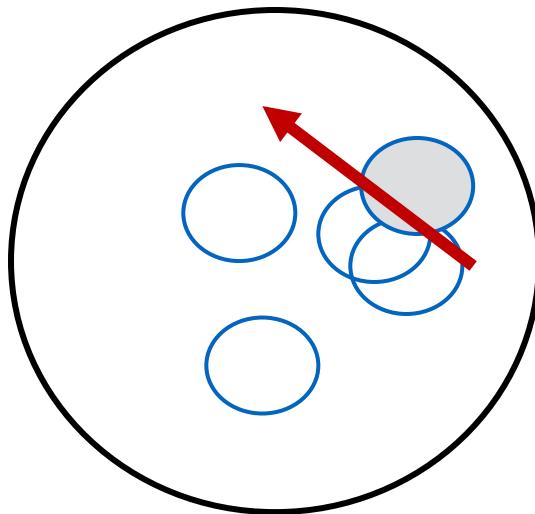
High
Multiplicity:



$$R_p^2(s; N) = \bar{R}_p^2(s) \frac{N}{\bar{N}}$$

Can this effect give a sizeable correlation between B and RP?

3. Even in pA collisions, **vorticity** has to be correlated with the reconstructed reaction plane:



Perhaps, the **Chiral Vortical Effect (CVE)**?

Can this be studied in **high multiplicity pp collisions?**
(small B, high vorticity, can check scaling expected for background vs CVE)

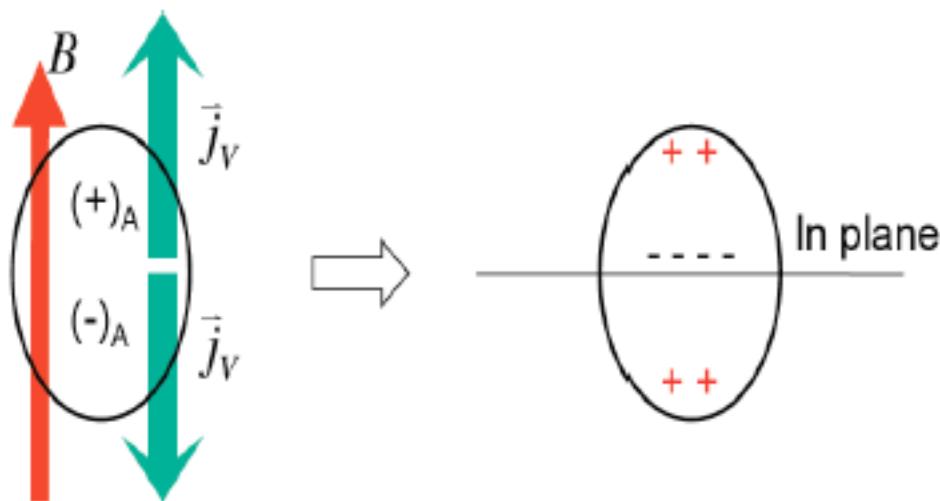
Is there a different observable with a controlled initial state?

The Chiral Magnetic Wave: controlling the initial state

Finite baryon density + CMW = electric quadrupole moment of QGP

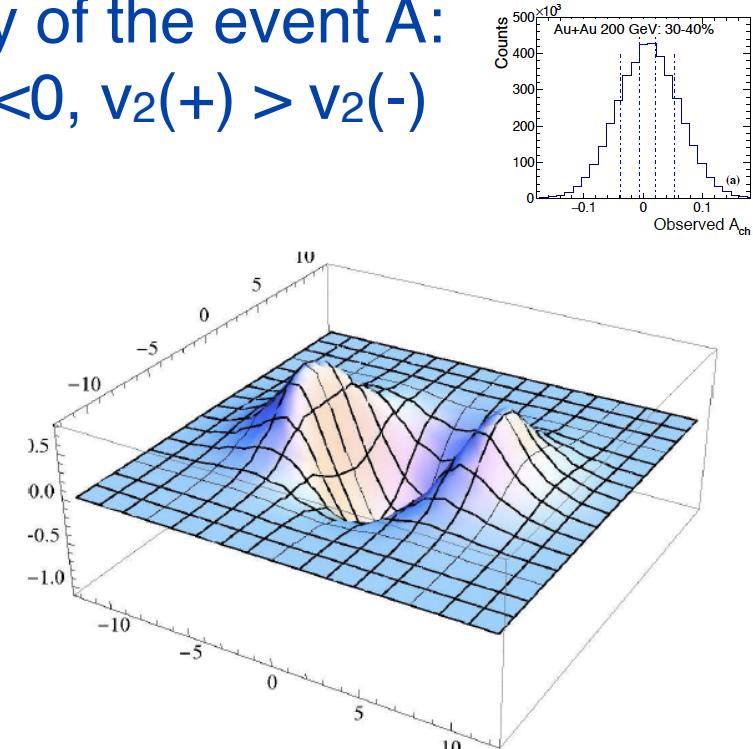
Signature - difference of elliptic flows of positive and negative pions determined by total charge asymmetry of the event A:

$$\text{at } A>0, v_2(-) > v_2(+); \quad \text{at } A<0, v_2(+) > v_2(-)$$



$$v_2^- - v_2^+ = C + 2\left(\frac{q_e}{\bar{\rho}_e}\right)A_{\pm}$$

$$A_{\pm} = (\bar{N}_+ - \bar{N}_-)/(\bar{N}_+ + \bar{N}_-)$$

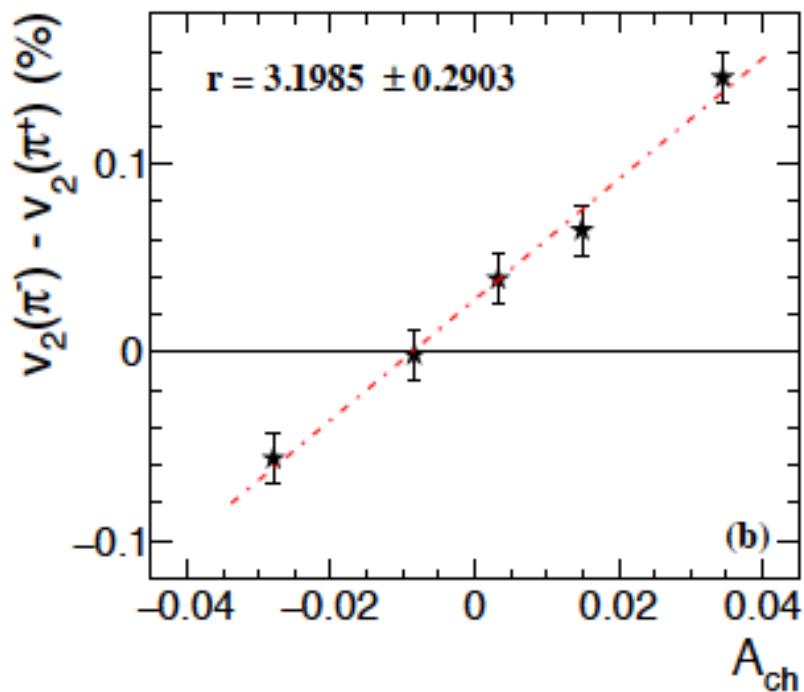
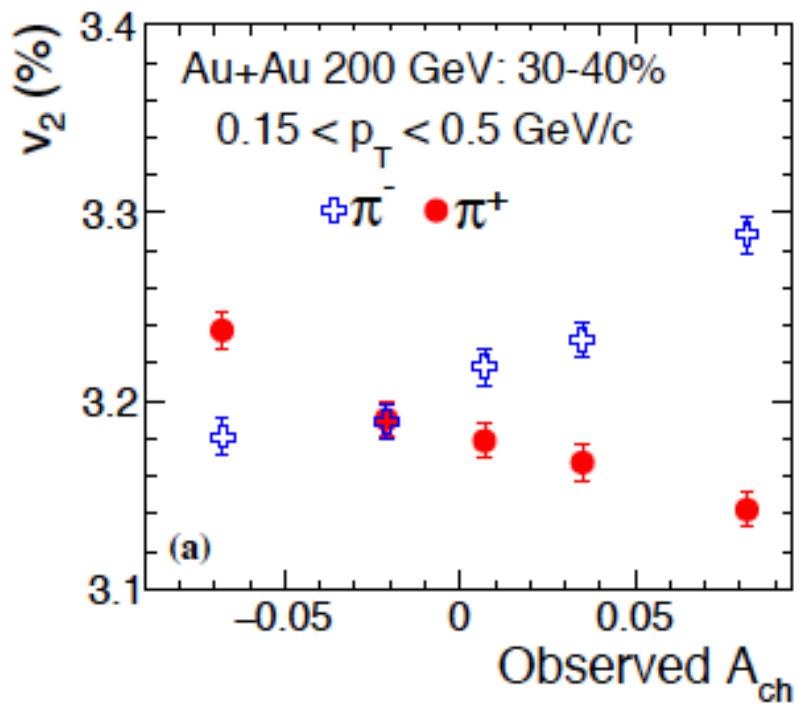


Y.Burnier, DK, J.Liao, H.Yee,
PRL 2011

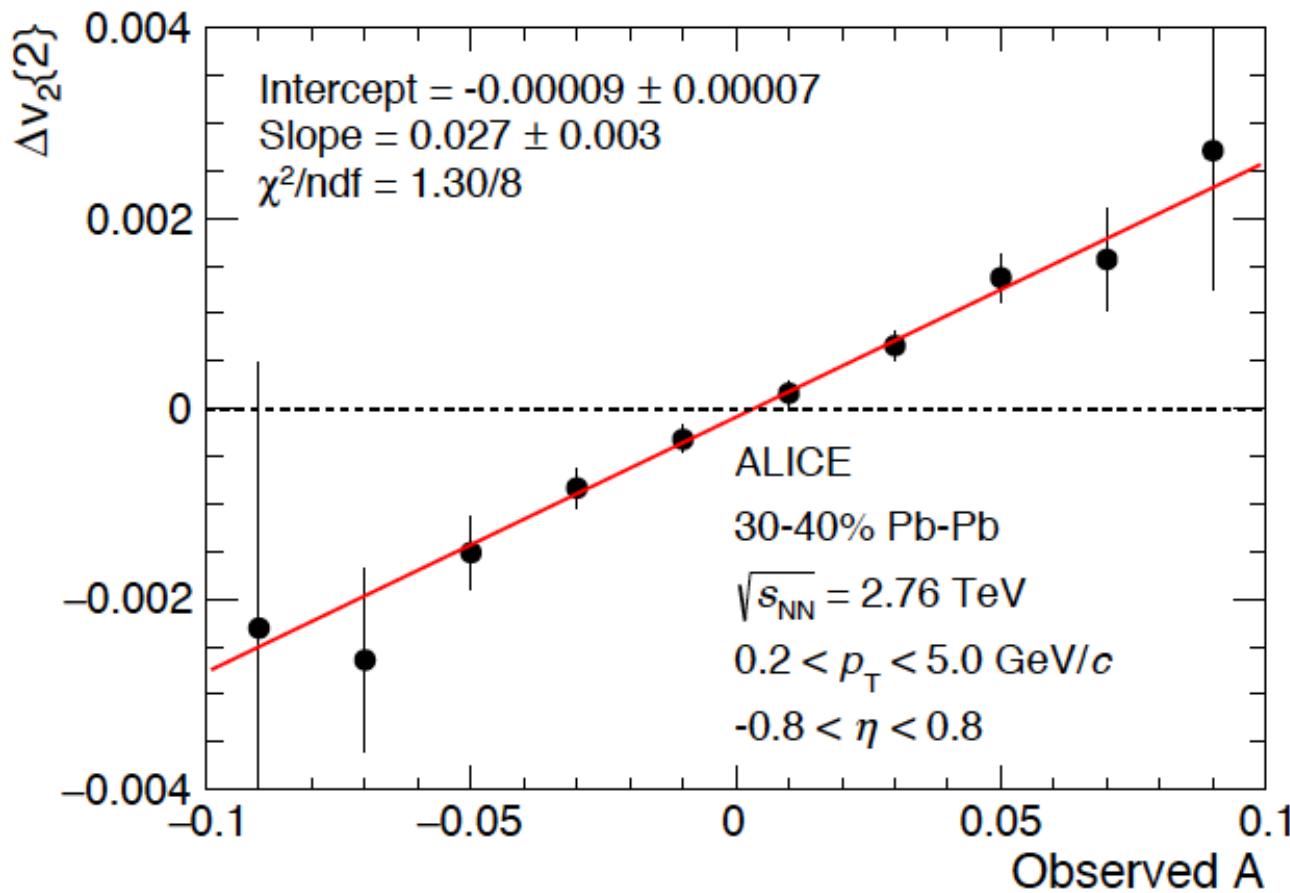
Observation of charge asymmetry dependence of pion elliptic flow and the possible chiral magnetic wave in heavy-ion collisions

(STAR Collaboration)

arXiv:1504.02175



ALICE Coll. at the LHC



ALICE Coll, Phys. Rev. C93 (2016) 044903

Chiral Magnetic Effect Task Force Report

Vladimir Skokov (co-chair),^{1,*} Paul Sorensen (co-chair),^{2,†} Volker Koch,³

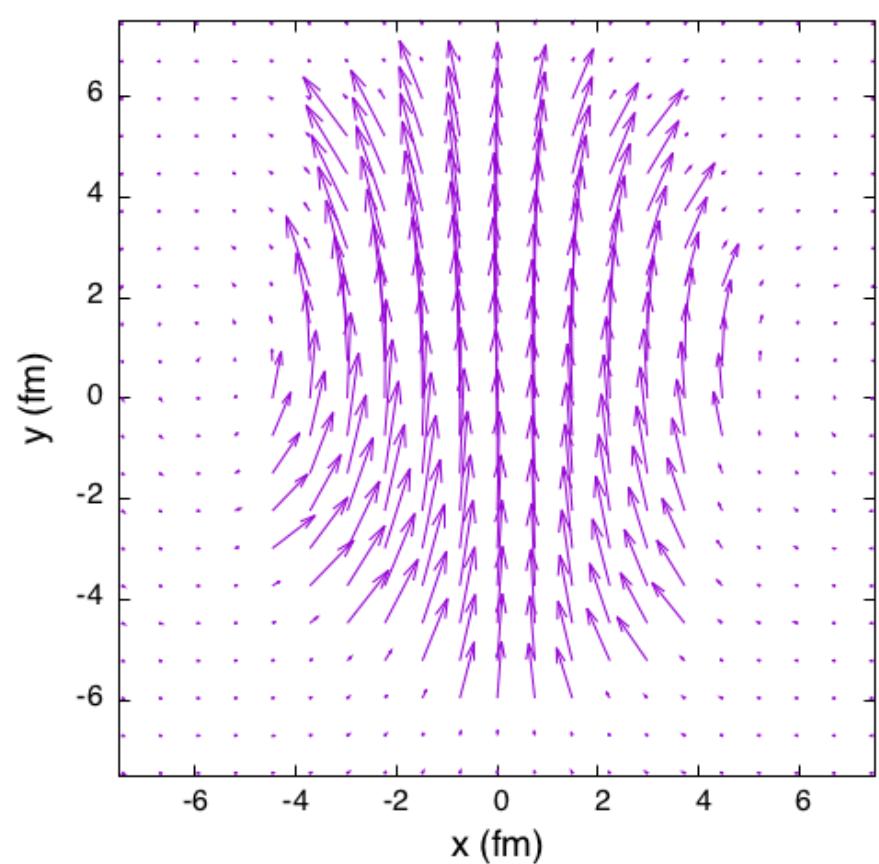
Soeren Schlichting,² Jim Thomas,³ Sergei Voloshin,⁴ Gang Wang,⁵ and Ho-Ung Yee^{6,1}

arxiv:1608.00982

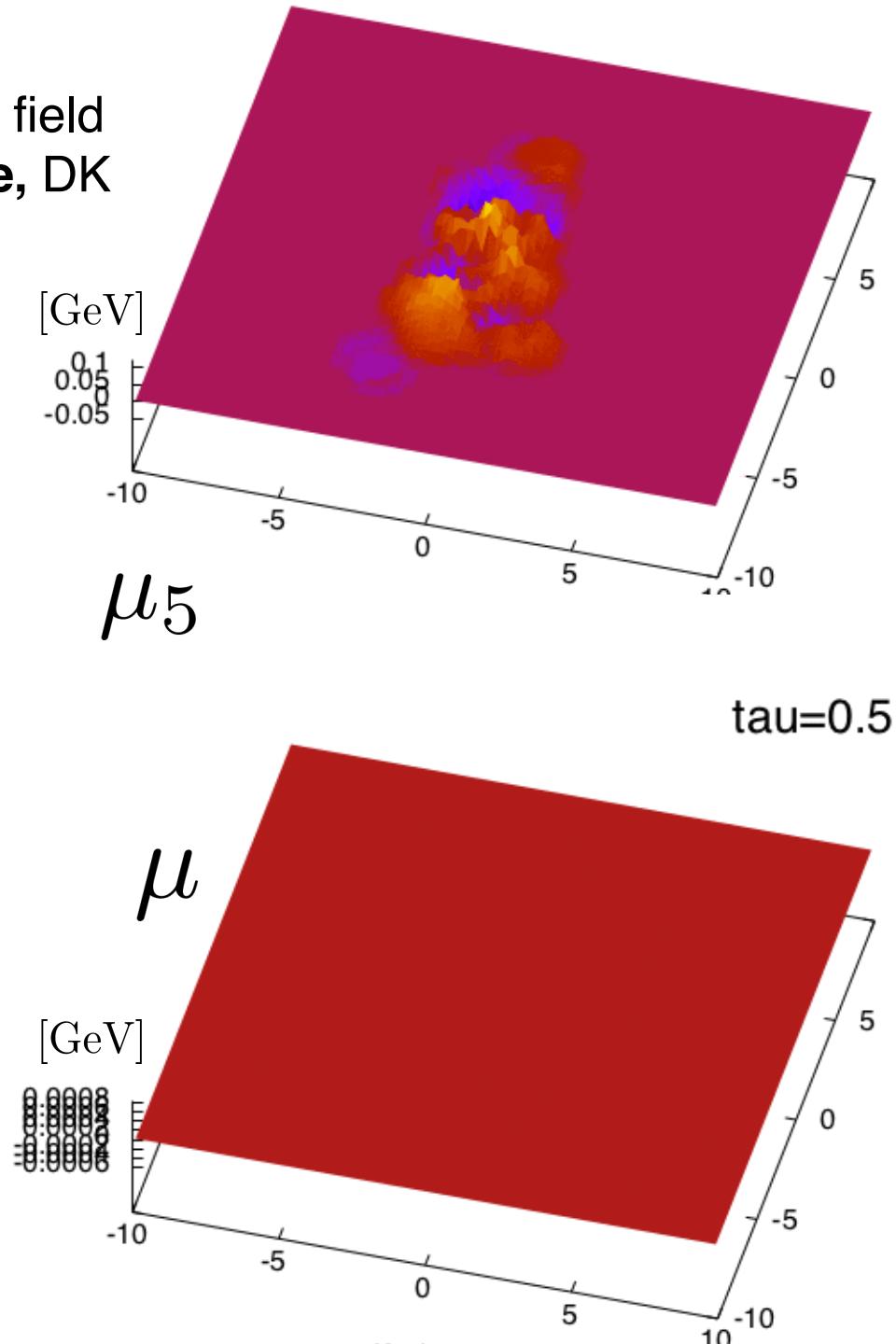
The unique identification of the chiral magnetic effect in heavy-ion collisions would represent one of the highlights of the RHIC physics program and would provide a lasting legacy for the field. The current plan for completing the RHIC mission envisions a second phase of RHIC. We have specifically investigated the case for colliding nuclear isobars (nuclei with the same mass but different charge) and find the case compelling. We recommend that a program of nuclear isobar collisions to isolate the chiral magnetic effect from background sources be placed as a high priority item in the strategy for completing the RHIC mission.

Approved dedicated 2018 CME run at RHIC with
Zr (Z=40), Ru (Z=44) isobars

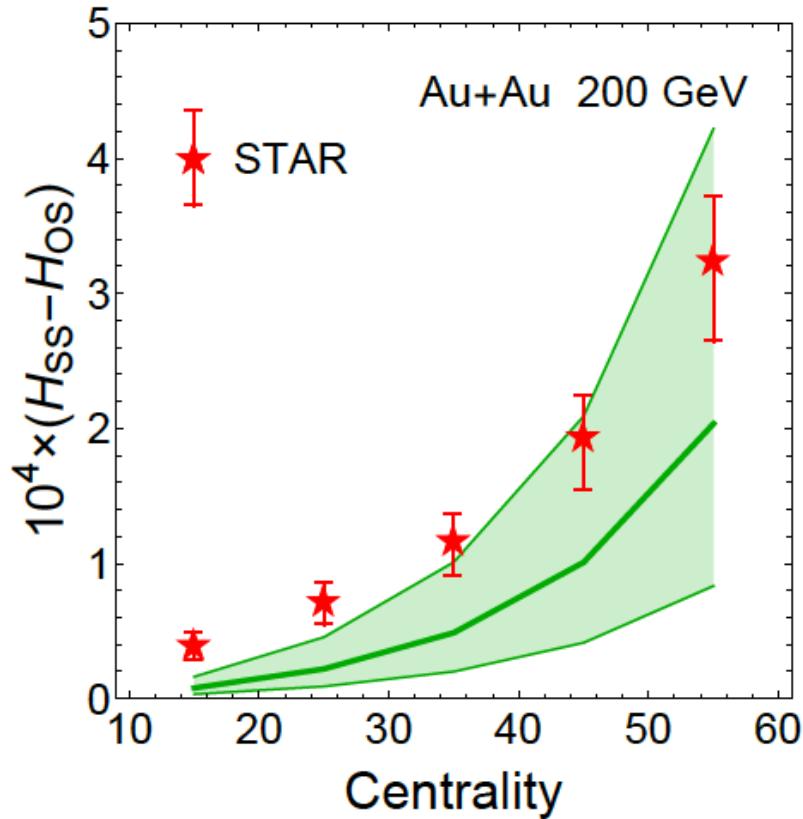
CMHD with dynamical MHD magnetic field
from ECHO-QGP: **Y. Hirono, M. Mace, DK**



B field evolution
in transverse plane



Anomalous viscous hydrodynamics



Anomalous currents
as perturbations on
top of “conventional” (2+1)D
VISHNU viscous hydrodynamics;
background magnetic field.

Y.Jiang, S.Shi, Y.Yin, J.Liao,
Arxiv:1611.04586

FIG. 3: (color online) The azimuthal correlation observable ($H_{ss} - H_{Os}$) for various centrality, computed from AVFD simulations and compared with STAR data [20], with the green band spanning the range of key parameter from $Q_s^2 = 1\text{GeV}^2$ (bottom edge) to $Q_s^2 = 1.5\text{GeV}^2$ (top edge).

Broader implications: Dirac & Weyl semimetals



SOVIET PHYSICS JETP

VOLUME 32, NUMBER 4

APRIL, 1971

*POSSIBLE EXISTENCE OF SUBSTANCES INTERMEDIATE BETWEEN METALS AND
DIELECTRICS*

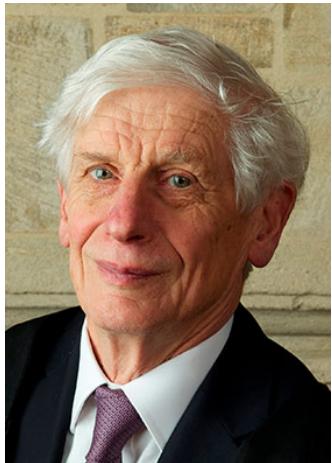
A. A. ABRIKOSOV and S. D. BENESLAVSKII

L. D. Landau Institute of Theoretical Physics

Submitted April 13, 1970

Zh. Eksp. Teor. Fiz. 59, 1280–1298 (October, 1970)

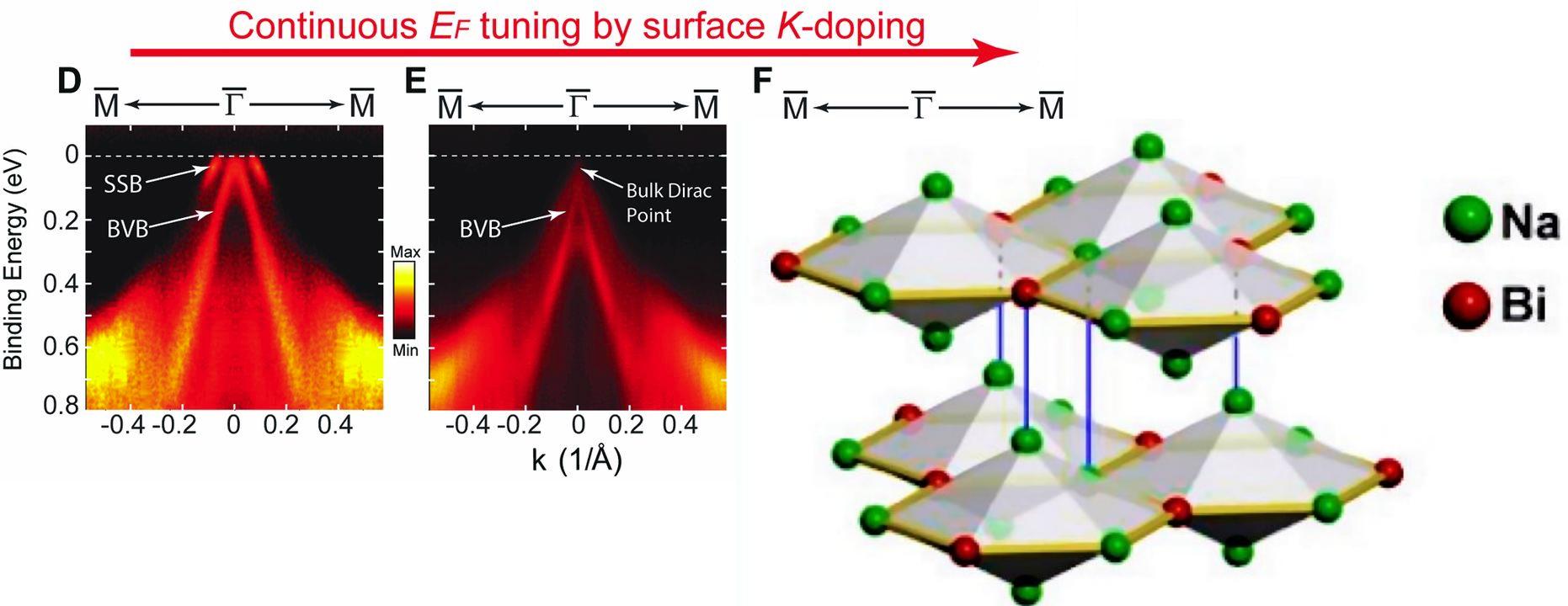
The question of the possible existence of substances having an electron spectrum without any energy gap and, at the same time, not possessing a Fermi surface is investigated. First of all the question of the possibility of contact of the conduction band and the valence band at a single point is investigated within the framework of the one-electron problem. It is shown that the symmetry conditions for the crystal admit of such a possibility. A complete investigation is carried out for points in reciprocal lattice space with a little group which is equivalent to a point group, and an example of a more complicated little group is considered. It is shown that in the neighborhood of the point of contact the spectrum may be linear as well as quadratic.



Scientific Background on the Nobel Prize in Physics 2016

TOPOLOGICAL PHASE TRANSITIONS AND TOPOLOGICAL PHASES OF MATTER

The discovery of Dirac semimetals – 3D chiral materials



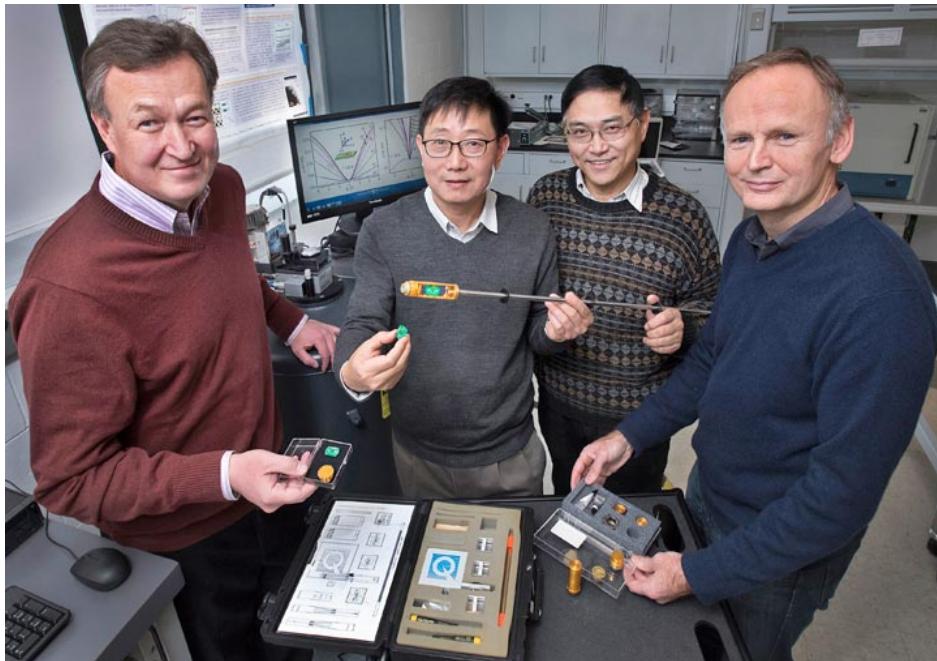
Z.K.Liu et al., Science 343 p.864 (Feb 21, 2014)

CME in condensed matter:

Observation of the chiral magnetic effect in ZrTe₅

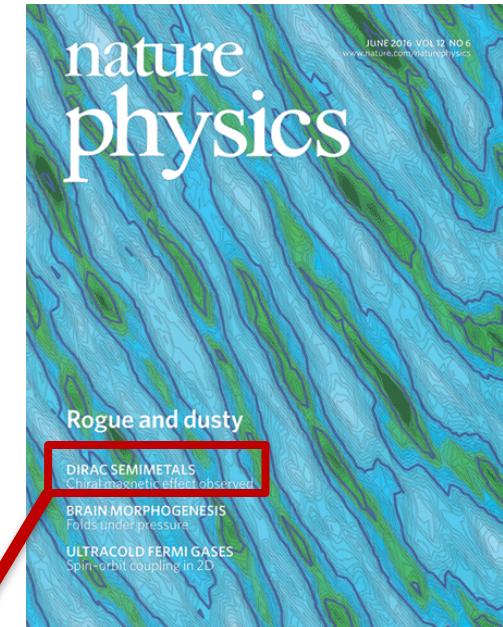
Qiang Li,¹ Dmitri E. Kharzeev,^{2,3} Cheng Zhang,¹ Yuan Huang,⁴ I. Pletikosić,^{1,5} A. V. Fedorov,⁶ R. D. Zhong,¹ J. A. Schneeloch,¹ G. D. Gu,¹ and T. Valla¹

BNL - Stony Brook - Princeton - Berkeley



Nature Phys.
12 (2016) 550

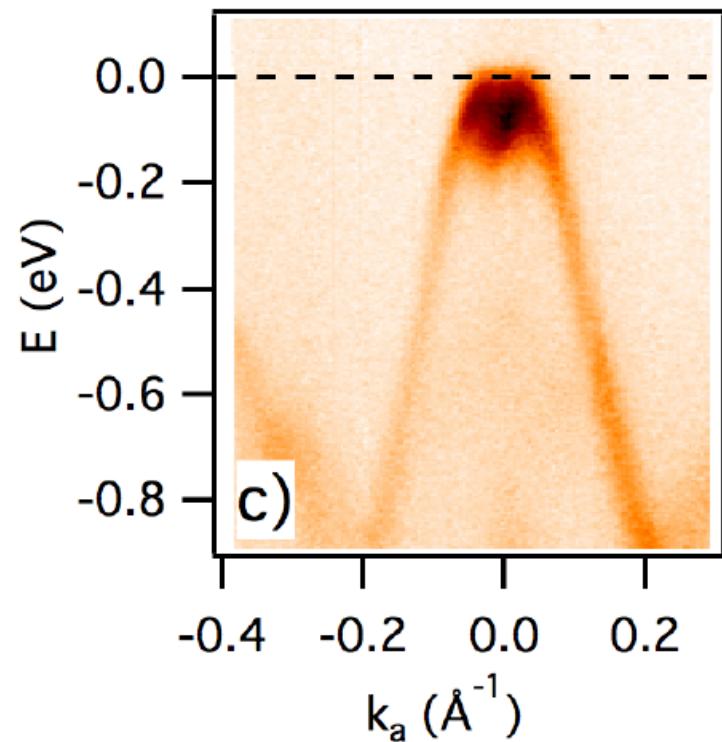
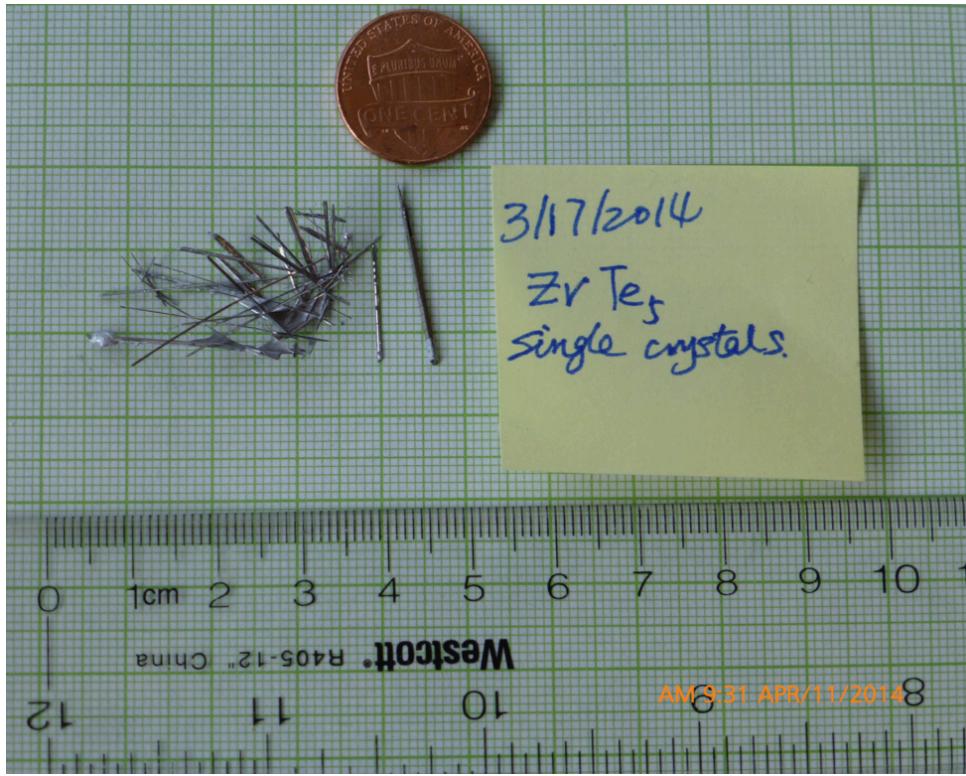
DIRAC SEMIMETALS
Chiral magnetic effect observed



arXiv:1412.6543 [cond-mat.str-el]

Observation of the chiral magnetic effect in ZrTe₅

Qiang Li,¹ Dmitri E. Kharzeev,^{2,3} Cheng Zhang,¹ Yuan Huang,⁴ I. Pletikosić,^{1,5} A. V. Fedorov,⁶ R. D. Zhong,¹ J. A. Schneeloch,¹ G. D. Gu,¹ and T. Valla¹



arXiv:1412.6543 (December 2014); Nature Physics **12**, 550 (2016)

Observation of the chiral magnetic effect in ZrTe₅

Qiang Li,¹ Dmitri E. Kharzeev,^{2,3} Cheng Zhang,¹ Yuan Huang,⁴ I. Pletikosić,^{1,5} A. V. Fedorov,⁶ R. D. Zhong,¹ J. A. Schneeloch,¹ G. D. Gu,¹ and T. Valla¹

Put the crystal in parallel E, B fields – the anomaly generates chiral charge:

$$\frac{d\rho_5}{dt} = \frac{e^2}{4\pi^2\hbar^2c} \vec{E} \cdot \vec{B} - \frac{\rho_5}{\tau_V}.$$

and thus the chiral chemical potential:

$$\mu_5 = \frac{3}{4} \frac{v^3}{\pi^2} \frac{e^2}{\hbar^2c} \frac{\vec{E} \cdot \vec{B}}{T^2 + \frac{\mu^2}{\pi^2}} \tau_V.$$

Observation of the chiral magnetic effect in ZrTe₅

Qiang Li,¹ Dmitri E. Kharzeev,^{2,3} Cheng Zhang,¹ Yuan Huang,⁴ I. Pletikosić,^{1,5} A. V. Fedorov,⁶ R. D. Zhong,¹ J. A. Schneeloch,¹ G. D. Gu,¹ and T. Valla¹

so that there is a chiral magnetic current:

$$\vec{J}_{\text{CME}} = \frac{e^2}{2\pi^2} \mu_5 \vec{B}.$$

resulting in the quadratic dependence of CME conductivity on B:

$$J_{\text{CME}}^i = \frac{e^2}{\pi\hbar} \frac{3}{8} \frac{e^2}{\hbar c} \frac{v^3}{\pi^3} \frac{\tau_V}{T^2 + \frac{\mu^2}{\pi^2}} B^i B^k E^k \equiv \sigma_{\text{CME}}^{ik} E^k.$$

adding the Ohmic one – negative magnetoresistance

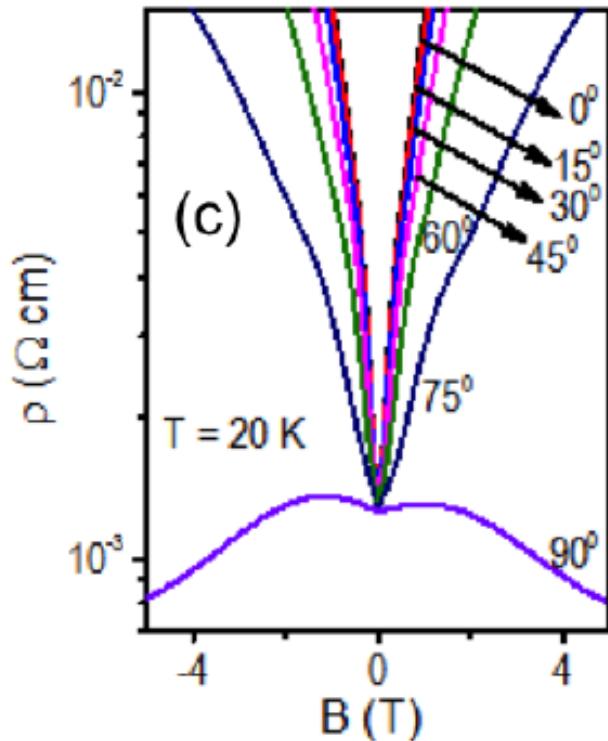
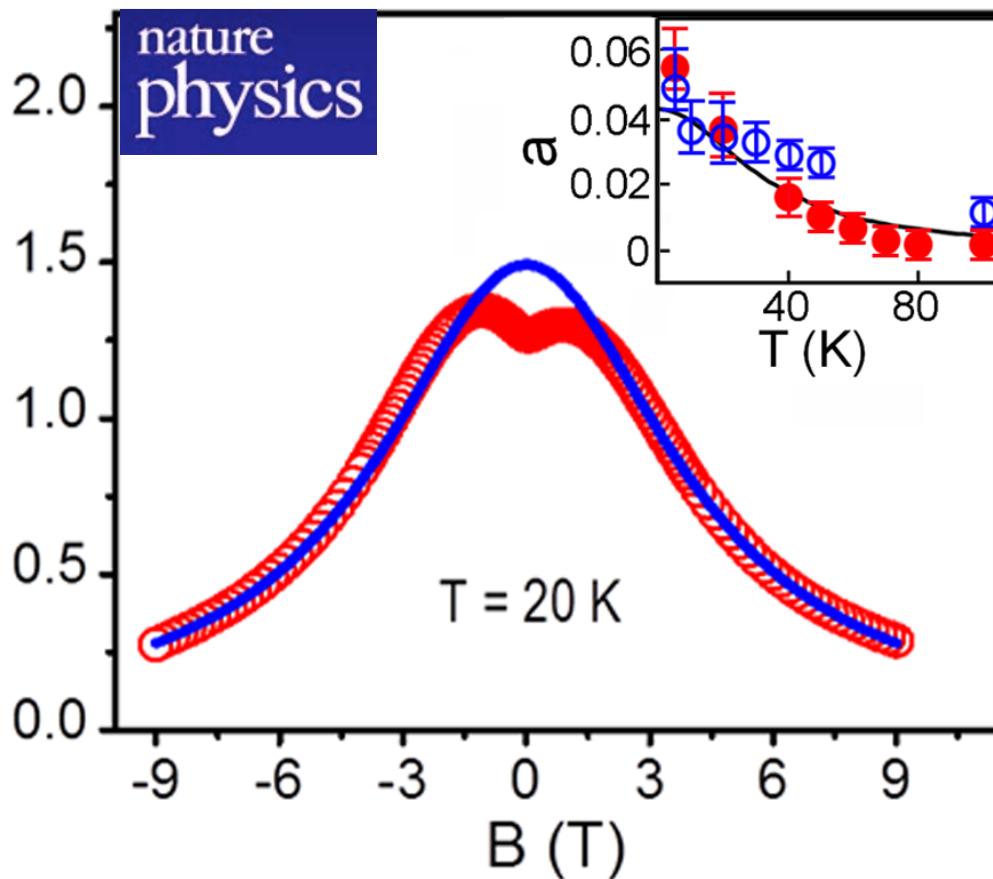
Son, Spivak, 2013
80

Chiral Magnetic Effect Generates Quantum Current

Separating left- and right-handed particles in a semi-metallic material produces anomalously high conductivity

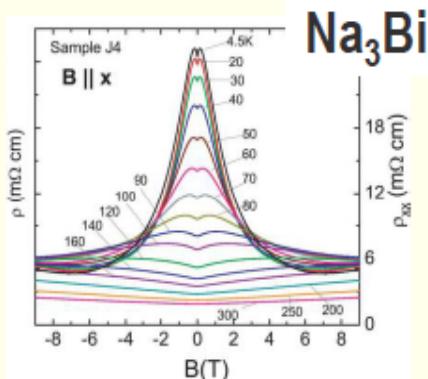
February 8, 2016

Nature Physics 12, 550 (2016)



Chiral magnetic effect in Dirac/Weyl semimetals

Dirac semimetals:



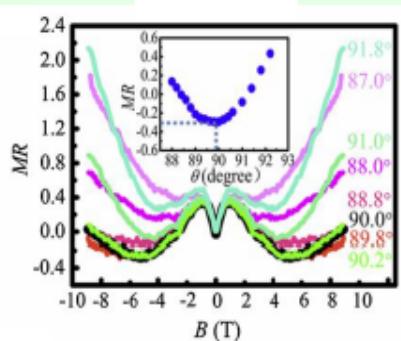
ZrTe₅ - Q. Li, D. Kharzeev, et al (BNL and Stony Brook Univ.)
arXiv:[1412.6543](https://arxiv.org/abs/1412.6543); doi:10.1038/NPHYS3648

Na₃Bi - J. Xiong, N. P. Ong et al (Princeton Univ.)
arXiv:[1503.08179](https://arxiv.org/abs/1503.08179); Science 350:413,2015

Cd₃As₂- C. Li et al (Peking Univ. China)
arXiv:[1504.07398](https://arxiv.org/abs/1504.07398); Nature Commun. 6, 10137 (2015).

Weyl semimetals

TaAs



TaAs - X. Huang et al (IOP, China)
arXiv:[1503.01304](https://arxiv.org/abs/1503.01304); Phys. Rev. X 5, 031023

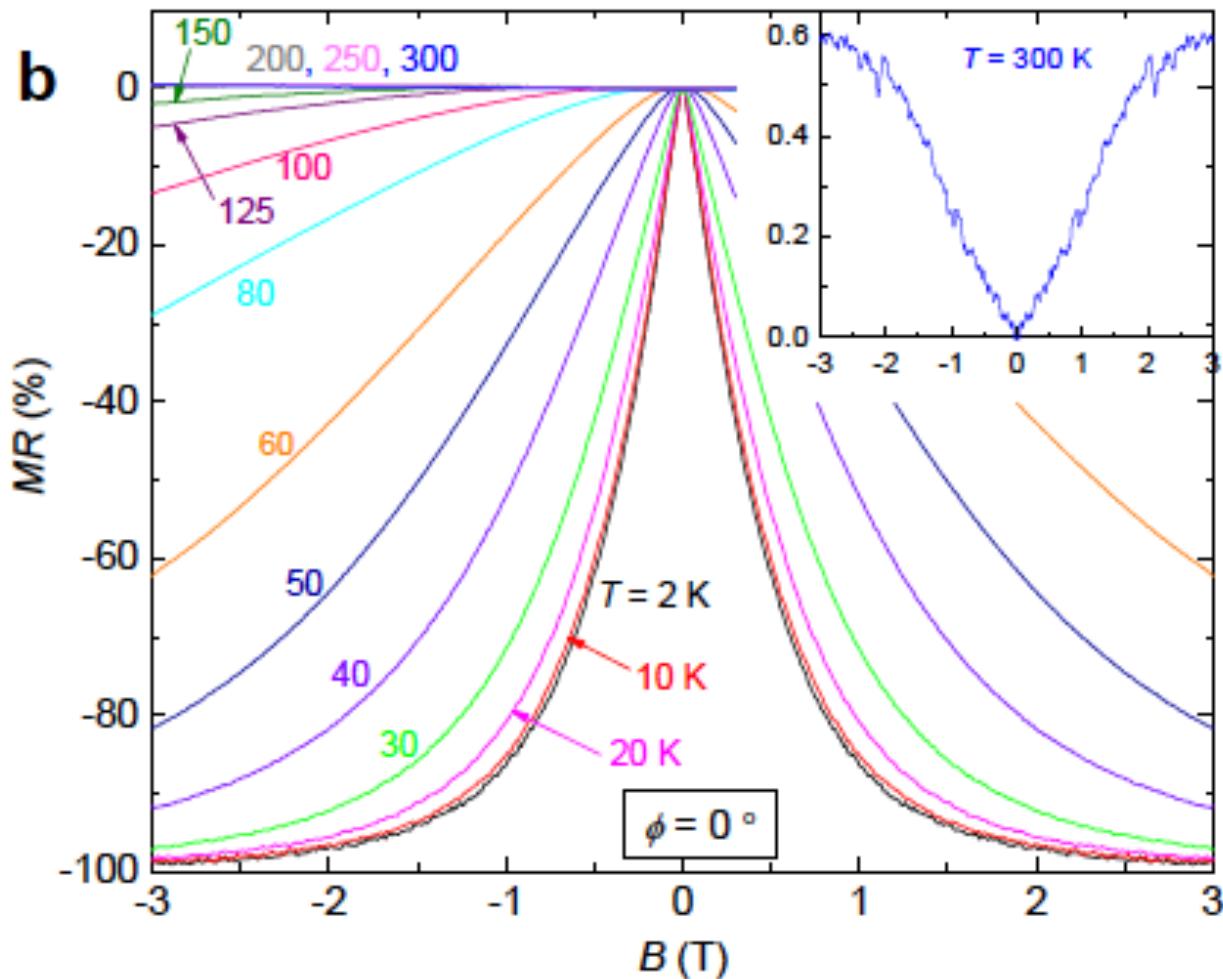
NbAs - X. Yang et al (Zhejiang Univ. China)
arXiv:[1506.02283](https://arxiv.org/abs/1506.02283)

NbP - Z. Wang et al (Zhejiang Univ. China)
arXiv:[1504.07398](https://arxiv.org/abs/1504.07398)

TaP - Shekhar, C. Felser, B. Yang et al (MPI-Dresden)
arXiv:[1506.06577](https://arxiv.org/abs/1506.06577)

Bi_{1-x}Sb_x at x ≈ 0.03 - Kim, et al. "Dirac versus Weyl Fermions in Topological Insulators: Adler-Bell-Jackiw Anomaly in Transport Phenomena. Phys. Rev. Lett., 111, 246603 (2013).

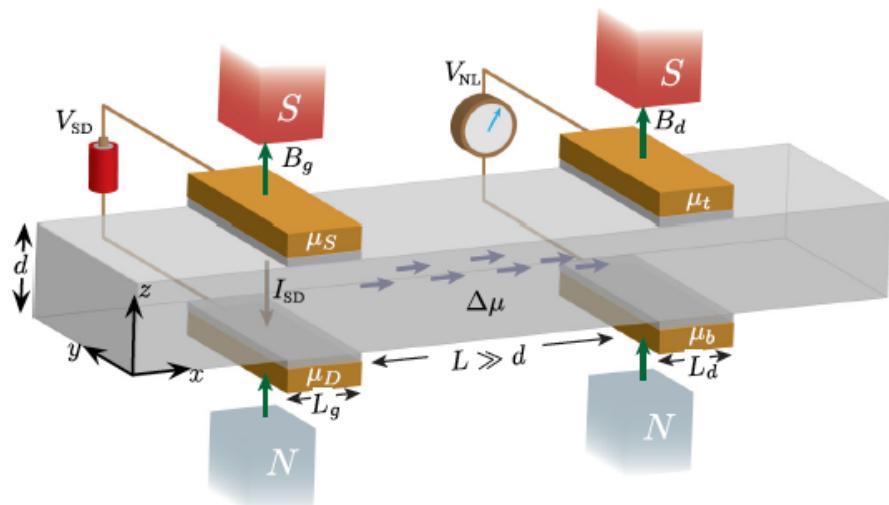
Negative MR in TaAs₂



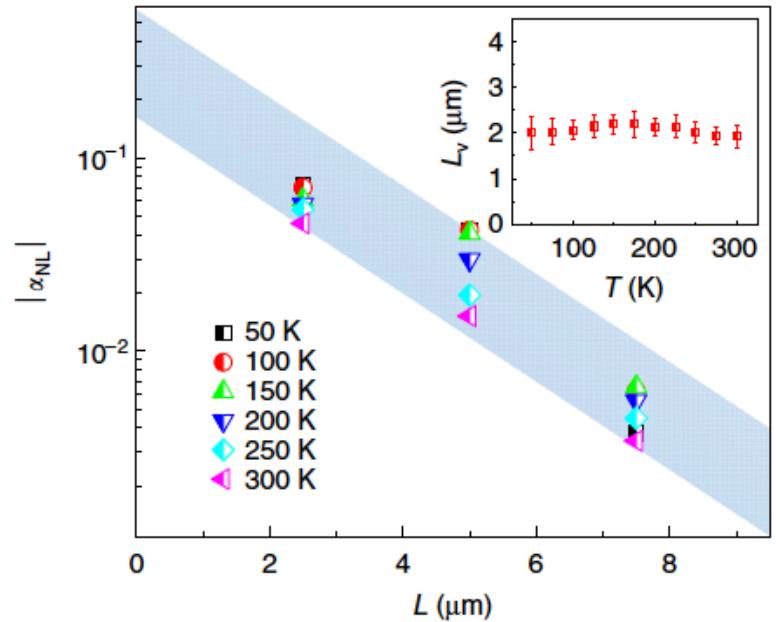
Towards the room temperature CME

Nonlocal chiral transport

C.Zhang et al, Nature Comm.'17
DOI: 10.1038/ncomms13741



S.Parameswaran et al,
PRX4, 031035 (2014)



$$|V_{NL}(x)| \propto V_{SD} e^{-\frac{L}{L_v}}$$

CME as a new type of superconductivity

London theory of superconductors, '35:

$$\vec{J} = -\lambda^{-2} \vec{A} \quad \nabla \cdot \vec{A} = 0$$



Fritz and Heinz London

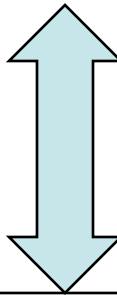
$$\vec{E} = -\dot{\vec{A}}$$

$$\vec{E} = \lambda^2 \dot{\vec{J}}$$

CME:

$$\vec{J} \sim \mu_5 \vec{B}$$

for $\vec{E} \parallel \vec{B}$



$$\vec{E} \sim B^{-2} \dot{\vec{J}}$$

assume that chirality
is conserved:

$$\mu_5 \sim \vec{E} \vec{B} t$$

superconducting
current, tunable
by magnetic field!

CME in Dirac metals

G.Monteiro, A.Abanov, DK,
arXiv:1507.05077; PhysRevB

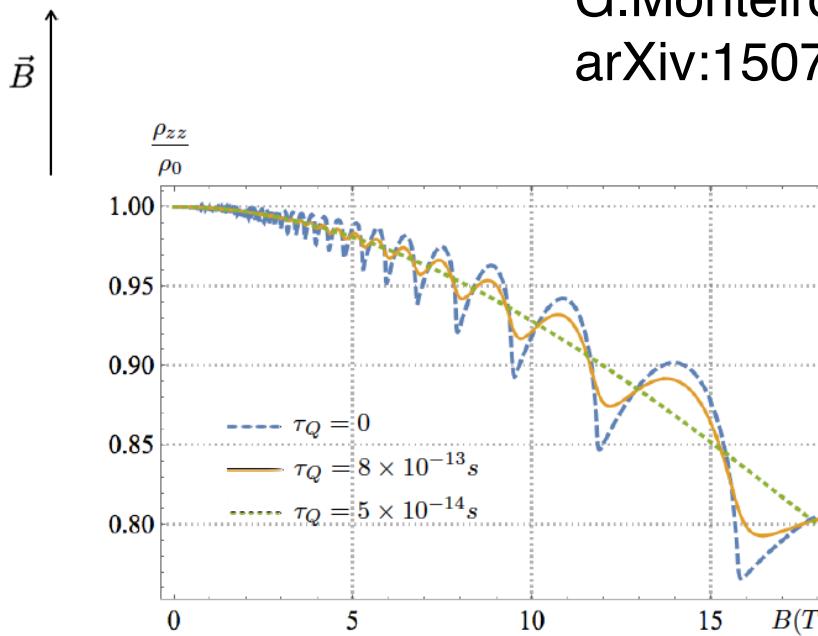
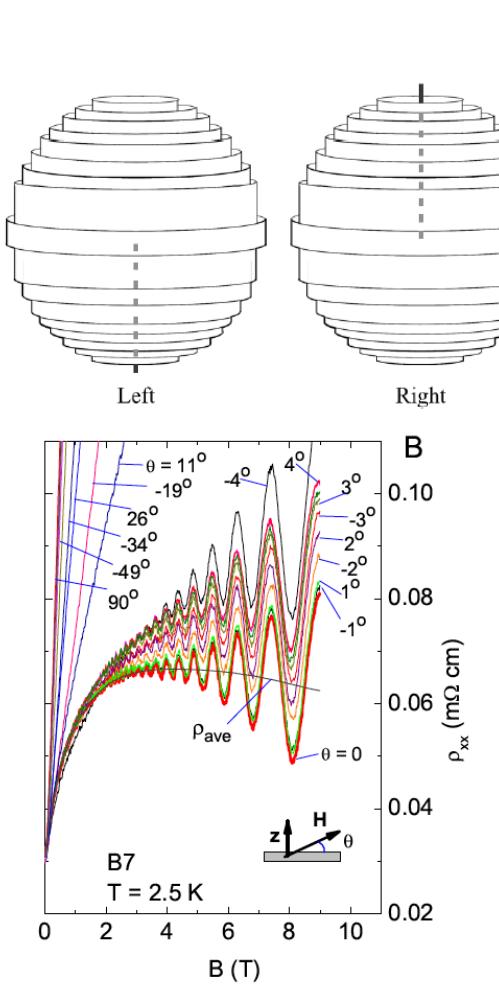


FIG. 2: Longitudinal magnetoresistance as a function of the magnetic field. We used numerical values consistent with [17]: $k_F = 3.8 \times 10^8 \text{ m}^{-1}$, $v_F = 9.3 \times 10^5 \text{ m/s}$, $\tau = 8 \times 10^{-13} \text{ s}$, $T = 2.5 \text{ K}$. The plots are made for three values of $\tau_Q/\tau = 0, 1, 16$ and for $\tau_v = 10\tau$.

**What is
the origin of
positive MR
in weak field?**

Quantum oscillations in CME conductivity

$$\chi = \frac{\partial \rho_5}{\partial \mu_5}$$

S.Kaushik, DK,
arXiv:1703.05865

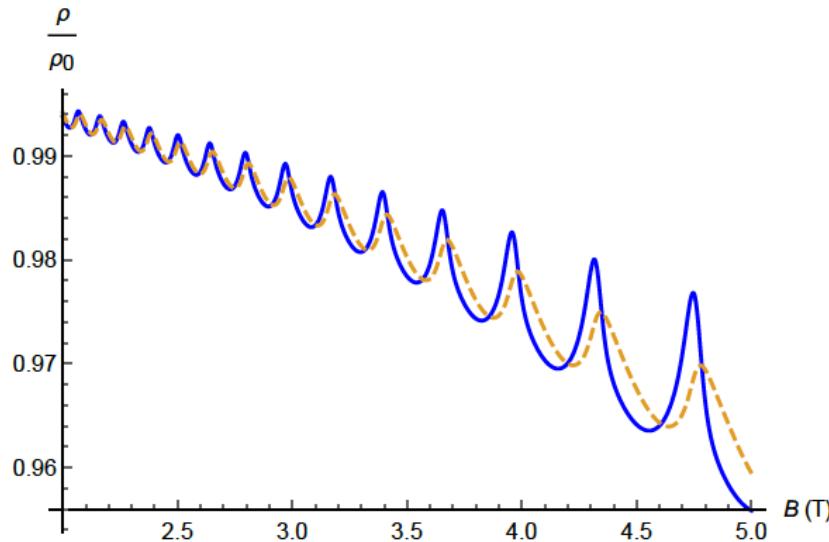


FIG. 2: ρ_{zz}/ρ_0 vs B for $\mu = 150$ meV, $v = c/600$, $T = 1.74$ K, $\Gamma = 0.3$ meV, and $\tau_V/\tau = 20$. The solid line represents the full prediction taking account of the quantum CME oscillations, see (32); the dashed line represents only the SdH oscillations given by (31). The quantum CME oscillations become larger than the SdH oscillations at $B \simeq 3$ T.

$$\begin{aligned} \chi \approx & \frac{\mu^2}{\pi^2 v^3} + \frac{T^2}{3v^3} + \frac{\mu E_L}{2\pi^2 v^3} \sum_{l=1}^{\infty} \frac{1}{\sqrt{l}} \frac{\left(l \frac{4\pi^2 \mu T}{E_L^2}\right)}{\sinh\left(l \frac{4\pi^2 \mu T}{E_L^2}\right)} \\ & \times \exp(-4\pi l \Gamma \mu / E_L^2) [\cos(2\pi l \mu^2 / E_L^2) + \sin(2\pi l \mu^2 / E_L^2)] \end{aligned}$$

Chiral photonics

Faraday rotation due to surface states in $(\text{Bi}_{1-x}\text{Sb}_x)_2\text{Te}_3$ topological insulator

Y. M. Shao^{1,*}, K. W. Post,¹, J. S. Wu¹, S. Dai¹, A. J. Frenzel¹, A. R. Richardella², J. S. Lee²,

N. Samarth², M. M. Fogler¹, A. V. Balatsky^{3,4}, D. E. Kharzeev^{5,6} and D. N. Basov¹

¹*Physics Department, University of California-San Diego, La Jolla, California 92093, USA*

²*Department of Physics, The Pennsylvania State University, University Park, Pennsylvania 16802, USA*

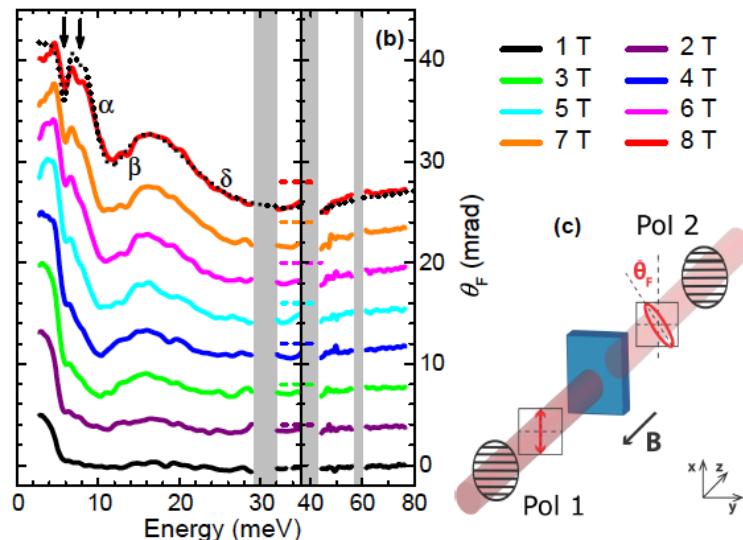
³*Nordita, KTH Royal Institute of Technology and Stockholm University,*

Roslagstullsbacken 23, SE-106 91 Stockholm, Sweden

⁴*Institute for Materials Science, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA*

⁵*Department of Physics and Astronomy, Stony Brook University, Stony Brook, New York 11794-3800, USA*

⁶*Department of Physics, Brookhaven National Laboratory, Upton, New York 11973, USA*



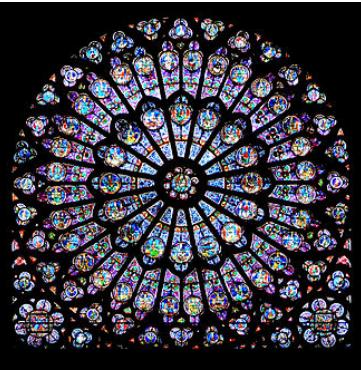
NANO
LETTERS

Nano Letters, 2017

Response of surface states
grows linearly in B!

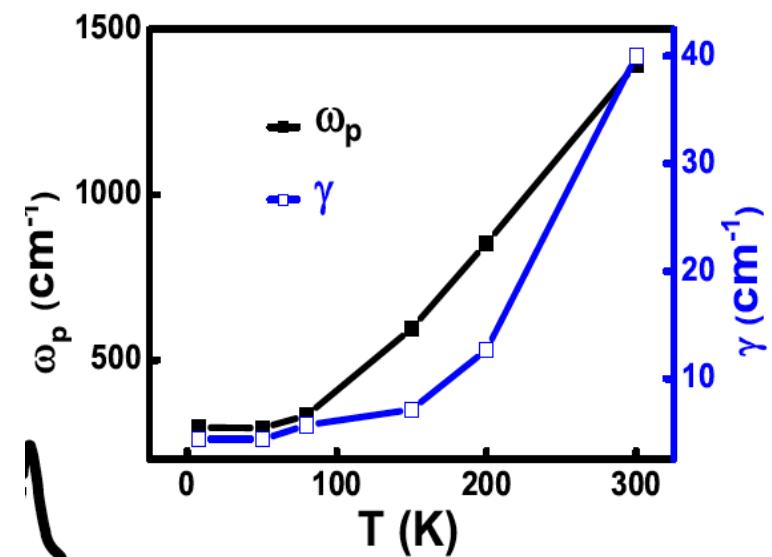
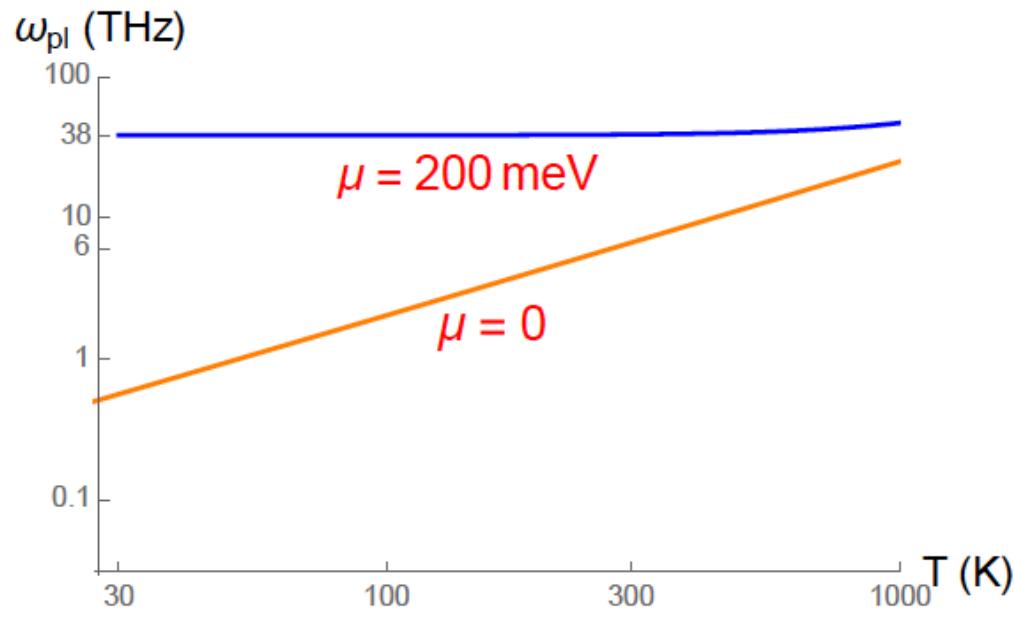
$$\vec{\nabla} \cdot \vec{E} = \rho + c \vec{P} \cdot \vec{B}$$

Rotation of light polarization on
axion domain walls in the Universe?



Chiral photonics

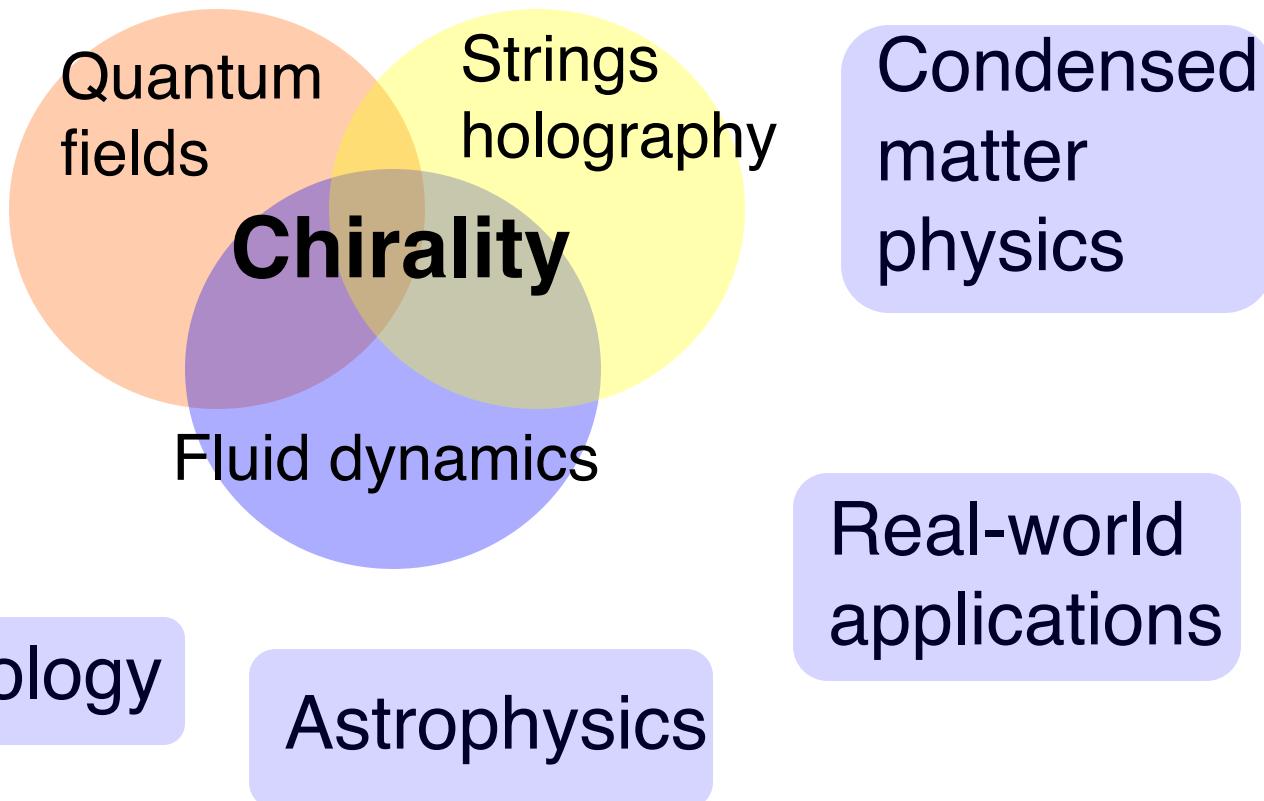
Plasmons (collective excitations) in
Dirac semimetals have THz frequency range



DK, R. Pisarski, H.-U. Yee,
PRL(2015); arXiv:1412.6106

R. Chen et al, “Optical spectroscopy
of 3D Dirac semimetal ZrTe_5 ”
arXiv:1505.00307

Summary



Reviews:

DK, K. Landsteiner, A. Schmitt, H.U.Yee (Eds),
“Strongly interacting matter in magnetic fields”,
Springer, 2013; arxiv:1211.6245

DK, “The chiral magnetic effect and anomaly-induced transport”,
Prog.Part.Nucl.Phys. 75 (2014) 133; arxiv: 1312.3348

DK, “Topology, magnetic field and strongly interacting matter”,
arxiv: 1501.01336; Ann. Rev. Nucl. Part. Science (2015)

DK, J.Liao, S.Voloshin, G.Wang, “Chiral magnetic and vortical effects
in high-energy nuclear collisions: A status report” Prog. Part. Nucl.
Phys. 88 (2016) 1