

## Quick Incompact3d guide and notes

This is a quick guide on to how to use **Incompact3d** and to know some details of its functioning. The present guide and the `modified` version of the code (baseline `v4.0`) are unofficial. Please refer to the official website of the code: <https://www.incompact3d.com> and the major references [1],[2],[3],[4].

### Compiling

Compilation is first prepared with `cmake`, that generates the `make` file.

It is suggested to create a *build* directory in the same folder of the solver:

```
mkdir build
```

In the *build* directory run

```
cmake ../
```

After that, the *makefile* will be created. If a specific version of the Fortran compiler is needed, the option to be added is the following (e.g. `gfortran` compiler):

```
cmake -DCMAKE_Fortran_COMPILER=gfortran ../
```

and equivalently for the Intel compiler `ifort`:

```
cmake -DCMAKE_Fortran_COMPILER=ifort ../
```

It is now possible to compile the binary file in the build directory with the following command:

```
make -j n
```

where `n` is the number of processors that will be used to compile the program.

### Basic functioning

In this section, the main features of the code that can be useful to run simulations are reported.

1. The code is designed to solve non-dimensional Navier-Stokes equations, since the only input parameter is  $Re$ . However, consistency of other parameters (e.g. inlet velocity and domain dimensions) allow to have dimensional simulations. The  $Re$  specified in the input file is used to compute the kinematic viscosity as

$$\nu = \frac{1}{Re}$$

so we are assuming unitary reference length and velocity scales. Reference scales depend then on the specific flow case.

#### Channel flow

For a channel flow, the default condition of simulation is to enforce a constant flow rate (CFR), while constant pressure gradient (CPG) can be enabled if necessary. Both conditions are rescaled with the centerline Reynolds number the corresponding laminar Poiseuille flow,  $Re_0 = \frac{U_0 h}{\nu}$ . A relation is available in order to estimate the

related friction Reynolds number  $Re_\tau$ :

$$Re_\tau \approx 0.116 Re_0^{0.88}$$

With CPG option enabled, the same relation is used to impose the same non-dimensionalization, since  $Re_\tau$  must be specified in the input file instead of  $Re_0$ :

$$Re_0 \approx \left( \frac{Re_\tau}{0.116} \right)^{1/0.88}$$

In CFR conditions, being  $U_0$  constant and unitary, this means that the bulk velocity  $U_B$  must be kept to the value of  $2/3$ . This can be demonstrated being the laminar profile parabolic.

Finally, in order to estimate the related bulk Reynolds number  $Re_B = \frac{2U_B h}{\nu}$ , the following relation can be employed (Pope, "Turbulent Flows"):

$$Re_\tau \approx 0.09 Re_B^{0.88} \Rightarrow Re_B \approx \left( \frac{Re_\tau}{0.09} \right)^{1/0.88}$$

2. Pressure field is made non-dimensional with the reference specific kinetic energy content  $q_{ref} = \frac{1}{2} \rho U_{ref}^2$ , where  $U_{ref}$  is the reference velocity of the case. The code performs operations on the pressure field with a factor  $\Delta t$ , but it is rescaled before saving the snapshots with `rescale_pressure` subroutine.
3. To evaluate the stretching parameter  $\beta$  for the mesh elements in  $y$  direction, some useful scripts can be found in the folder `1-pre_processing/stretching_mesh`. The default Fortran program of Incompact3d is `stretching_parameter_channel.f90` and it is used to setup channel flow simulations. For temporal TBL simulations, the Python script `mesh_evaluation_ttbl.py` was developed. This script can handle also pre-processing of channel flow cases. Low  $\beta$  values correspond to a strong stretching of the mesh, while high  $\beta$  values correspond to almost uniform mesh.
4. Variables are saved on  $y_p$  points, that are the faces of the grid elements, while  $y_{pi}$  points are the centers of the grid elements ( $i$  : internal).
5. Boundary values of velocity are specified through the  $b_{ijk}$  variables, where  $i$  is the wall-normal direction of the boundary,  $j$  is the direction of the specific velocity component and  $k$  specifies if we are considering the bottom or the top walls (e.g.  $b_{yx1}$  refers to the  $x$  velocity component, specified at the bottom boundary with normal direction  $y$ ).
6. Velocity boundary conditions are specified in the input file `input.i3d` with the following variables: `nclx1`, `nclxn`, `ncly1`, `nclyn`, `nclz1`, `nclzn`, that specify the normal direction to the boundary and if we are considering the first or the last element along the specific direction. Values that can be adopted are: 0, for *periodic BC*, 1 for *free-slip BC* and 2 for *Dirichlet BC* (so imposed velocity value). Boundary conditions can be different along the same direction, so different combinations can be enforced. Due to the nature of the code, pressure boundary conditions are not needed.
7. Boundary conditions for scalar fields have the same functioning of the velocity BCs. They are called: `nclxS1`, `nclxSn`, `nclyS1`, `nclySn`, `nclzS1`, `nclzSn`.
8. The hyperviscous option for the second order derivative is a way to increase the numerical dissipation of the standard 6th order accurate scheme (that is sub-dissipative). In this manner, it is possible to increase the modified wavenumber at high wavenumbers, thus increasing the *dissipation error*  $E_{diss}$ , similarly to what is observed in high-order upwind schemes. This approach allows to prevent *wiggles* and thus to improve stability, even at high cell Reynolds number (or numerical Péclet)  $Pé \approx 200$ . The two parameters available can be used also to control the numerical dissipation in the context of ILES simulations.
9. Implicit time integration is available for the diffusive terms in  $y$  direction. From preliminary analyses, it appears that it allows to drop the restriction due to the stability parameter  $S < 1$  of fully explicit time integration schemes (Thompson et al. (1985)). The stability parameter  $S$  can be calculated in the three directions

$$S_i = \frac{u_i^2 \Delta t}{2\nu}$$

considering  $i = x, y, z$ . With implicit diffusion for  $y$  direction terms, the hyperviscous operator for the second order derivative can be used only with Dirichlet conditions on both boundaries. It is also important to notice that this approach is experimental, but channel flow simulations have been validated with data from literature, in both cases of fixed and oscillating walls.

10. Avoid to cancel the latest `restart.info` file, in order to being able to correctly calculate the current time unit (since the current time unit before restart is read from `restart.info` file itself, that is located in the `data/restart_info` folder).

11. The total shear velocity is calculated as:

$$u_\tau = \sqrt{\nu \frac{\partial U_{\parallel, w}}{\partial y}}$$

where  $U_{\parallel} = \sqrt{U^2 + W^2}$  is the parallel velocity to the wall. Similarly, the  $x$  and  $z$  components are defined as:

$$u_{\tau, x} = \sqrt{\nu \left| \frac{\partial U_w}{\partial y} \right|} \text{ and } u_{\tau, z} = \sqrt{\nu \left| \frac{\partial W_w}{\partial y} \right|}.$$

12. The (total) friction coefficient  $c_f$  is calculated as:

$$c_f = 2 \left( \frac{u_\tau}{U_{ref}} \right)^2$$

where  $U_{ref}$  is the reference velocity according to the specific flow case ( $U_w$  for a TTBL and  $U_B$  for a channel). The same approach is followed to calculate the streamwise and the spanwise friction coefficients,  $c_{f, x}$  and  $c_{f, z}$ .

13. The restart procedure of the code does not compromise the order of accuracy of the time-integration, since old time steps used for Adams–Bashforth schemes are stored in the `checkpoint` files.

## Visualization

Visualization of the results can be performed by opening the `.xdmf` snapshot files of the folder `data` with a visualization/post-processing software (e.g. Paraview). Moreover, it is also possible to visualize 2D instantaneous  $z$ -normal planes of the scalar field (if present) and  $x$ -normal planes of streamwise vorticity. This can be done by opening `.xdmf` files that can be found inside the `data/planes` folder. This can be useful for large cases, being a single plane much less memory-demanding. The frequency of saving of the planes can be set with the parameter `ioutput_plane` in the input file.

## Power input for TTBLs

The power input  $P_{in}$  for a TTBL with fixed walls is calculated in the following manner (assuming  $\rho = 1$ ):

$$P_{in, fw} = \tau_{w, x} U_w = \mu \frac{\partial U}{\partial y} U_w = u_{\tau, x}^2 U_w$$

where  $u_{\tau, x}$  is the streamwise shear velocity. For the oscillating walls case, the additional power input required to move the wall in  $z$  direction is considered:

$$P_{in, ow} = \tau_{w, x} U_w + \tau_{w, z} W_w = \mu \frac{\partial U}{\partial y} U_w + \mu \frac{\partial W}{\partial y} W_w = u_{\tau, x}^2 U_w + u_{\tau, z}^2 W_w$$

## Spanwise wall oscillations

Spanwise wall oscillations are calculated in the following manner:

$$w_w(t) = A \sin \left( \frac{2\pi}{T} t + \phi_{in} \pi \right)$$

where  $A$  is the amplitude,  $T$  is the period and  $\phi_{in}$  is the initial phase of oscillation, given as fraction of  $\pi$ . Wall oscillations are enabled by setting `iswitch_wo = 1`. Parameters for wall oscillations are read from the input file. If feedback control is not enabled,  $A$  and  $T$  are in external units, while in case of closed-loop control (`ifedback_control = 1`), the parameters are considered as rescaled in wall-units with the run-time streamwise shear velocity  $u_{\tau,x}$ .

## Use of external FFT libraries

It is possible to use different FFT libraries with respect to the `generic` implementation (default one) of Incompact3d. In the folder `5-external`, the FFT library `FFTW v3.3.10` is present. This library must be compiled before linking to Incompact3d. The following commands must be run inside the `FFTW` folder:

```
sh configure --prefix=/scratch2/fmoroni/Incompact3d/5-external/fftw-3.3.10 --
exec-prefix=/scratch2/fmoroni/Incompact3d/5-external/fftw-3.3.10 --enable-
shared --enable-threads

make

make install
```

`--enable-shared` allows to create the `.so` file (shared library/object) required by Incompact3d, while `--enable-threads` allows for multi-threading. Once the `FFTW` installation is completed, it is possible to specify the FFT implementation in the `cmake` command in the setup of Incompact3d:

```
cmake -DFFT_Choice=fftw3 -DCMAKE_Fortran_COMPILER=gfortran ../
```

FFT options available are: `generic`, `fftw3` and `mkl`.

## References

1. [High-order compact schemes for incompressible flows, A simple and efficient method with quasi-spectral accuracy - Laizet & Lamballais - 2009](#)
2. [Incompact3d - A powerful tool to tackle turbulence problems with up to  \$O\(10^5\)\$  computational cores - Laizet & Li - 2011](#)
3. [Xcompact3D - An open-source framework for solving turbulence problems on a Cartesian mesh - Bartholomew et al. - 2020](#)
4. [Straightforward high-order numerical dissipation via the viscous term for DNS and LES - Lamballais et al. - 2011](#)
5. [Turbulent Flows - Pope](#)
6. [The cell Reynolds number myth - Thompson et al. - 1985](#)