Mesh evaluation

The estimation of the spacing for the mesh along the y direction is performed through one of the utilities that can be found inside the folder `1-pre_processing/Stretching Mesh`. For a channel flow case, the default function of Incompact3d can be used, `stretching_parameter_channel.f90`.

For a temporal TBL case instead, a new function was developed, starting from the stretching subroutine that can be found inside the original solver of Incompact3d. This new function is a Python script, `mesh_evaluation_ttbl.py`. This function allows to estimate the mesh sizes and other relevant parameters by introducing the following inputs:

- Number of points: n_x,n_y,n_z.
- Domain dimensions: L_x,L_y,L_z.
- Stretching parameter: β.
- Kinematic viscosity: ν.
- Velocity of the wall: U_w.
- Time step: Δt.
- Tripping wire diameter: D.
- Skin friction coefficient at peak: $c_{f,max}$.
- BL thickness: δ.

And it produces the following outputs in `sim_settings.txt` file:

- Non-dimensional domain dimensions at IC and at peak c_f: L_x^+,L_y^+,L_z^+.
- CFL, Péclet, Numerical Fourier and stability parameter: Co, $\mathcal{D}$, $P\acute{e}$, S.
- Mesh size in y direction at the first element near the wall at IC, at peak c_f and at $Re_\tau = 500$: Δy_1^+.
- Mesh sizes in x and z directions at IC, at peak c_f and at $Re_\tau = 500$: Δx^+, Δz^+.
- Mesh size in y direction at the BL edge at $Re_\tau = 500$: Δy_δ^+.
- Number of mesh nodes `npvis` in the viscous sublayer at c_f peak.
- Number of mesh nodes `npsl` in the initial shear layer.
- Approximate initial shear layer momentum thickness θ_{sl}.
- Initial shear layer thickness δ_{99}^+.
- Shear velocity u_τ at IC, at peak c_f and at $Re_\tau = 500$.
- Total number of mesh points, n_{tot}.

In the file `mesh_y.txt` instead:

- Dimensional mesh spacings in y direction: Δy.
- Aspect Ratios (ARs) in the xy plane of grid elements (ratio width/height of a single element).
- Growth Rates (GRs) in the xy plane of grid elements (ratio of heights of adjacent elements).

It is worth noticing that in a temporal BL simulation, the peak c_f value constraints the height of the first cell at the wall Δy_1, as in standard CFD simulations. However, the decrease of c_f along the simulation (and thus the increase in viscous length δ_ν) imposes a constraint for the domain dimensions L_x, L_y, L_z since they appear progressively "smaller" (their non-dimensional counterparts decreases). Too low values of L_x^+,L_y^+,L_z^+ must be avoided in order to do not enforce a too strong periodicity in the turbulent structures at the wall.

For numerical stability, it is recommended to maintain $Co < 1$, $D < 0.5$, $P\acute{e} < 2$ and $S < 1$.

Pre-processing quantities

Calculated quantities for a temporal TBL in `mesh_evaluation_ttbl.py`.

Shear velocity at IC

$$\frac{\partial U}{\partial y} = -\frac{U_w}{4\theta_{sl}} \cdot \frac{1}{\cosh^2\left(\frac{D}{2\theta_{sl}}\right)}$$

$$u_\tau = \sqrt{\nu \left| \frac{\partial U}{\partial y} \right|}$$

Shear velocity at peak friction coefficient

$$u_\tau = U_w \sqrt{\frac{c_f}{2}}$$

Initial velocity profile

$$U_0^+(y) = \frac{U_w^+}{2} + \frac{U_w^+}{2} \tanh\left[\frac{D}{2\theta_{sl}}\left(1 - \frac{y}{D}\right)\right]$$

Initial shear layer momentum thickness

$$\theta_{sl} \approx \frac{54\nu}{U_w}$$

Initial Courant-Friedrichs-Lewy number (< 1)

$$Co = \frac{U_w \Delta t}{\Delta x}$$

Initial Numerical Fourier number (< 0.5)

$$\mathcal{D} = \frac{\nu \Delta t}{\Delta x^2}$$

Initial Péclet number (< 2)

$$Pé = \frac{U_w \Delta x}{\nu}$$

Initial stability parameter (< 1)

$$S = \frac{U_w^2 \Delta t}{2\nu}$$