high_order_integrals

high_order_integrals is a small Python program that is able to calculate 6th order accurate thickness parameters $(\delta_{99}, \delta^*, \theta)$ for a temporal turbulent boundary layer (TTBL). It also calculates the related Reynolds numbers $(Re_\tau, Re_{\delta^*}, Re_\theta)$, the streamwise shear velocity $u_{\tau,x}$ and the streamwise friction coefficient $c_{f,x}$.
It employs mean statistics calculated with `post_incompact3d`, in particular mean velocity profile and mean streamwise velocity gradient. `high_order_integrals` is able to automatically open each `mean_stats` and `vort_stats` files, based on inputs given through `post.prm`. Time-units are read from the `.xdmf` snapshots' headers at the folder corresponding to the flow realization 1.

High order interpolation

The mean velocity profile is reconstructed with the SciPy function `InterpolatedUnivariateSpline`, with a spline of order 5 that passes through all data points. Integrals are evaluated with the `integral` command of the same function. In this manner, the integrals calculated are 6th order accurate.

List of calculated quantities

- BL thickness δ_{99} (O(6))

$$\delta_{99} \equiv y : \langle u(y) \rangle = 0.01 U_w$$

- Displacement thickness (O(6))

$$\delta^* = \int_0^\infty \frac{\langle u \rangle}{U_w} dy$$

- Momentum thickness (O(6))

$$\theta = \int_0^\infty \frac{\langle u \rangle}{U_w} \left(1 - \frac{\langle u \rangle}{U_w}\right) dy$$

- Friction Reynolds number (O(6))

$$Re_\tau = \frac{u_\tau \delta_{99}}{\nu} = \delta_{99}^+$$

- Reynolds number based on displacement thickness (O(6))

$$Re_{\delta^*} = \frac{U_w \delta^*}{\nu}$$

- Reynolds number based on momentum thickness (O(6))

$$Re_\theta = \frac{U_w \theta}{\nu}$$

- Streamwise shear velocity (O(6))

$$u_{\tau,x} = \sqrt{\nu \left| \frac{\partial U}{\partial y} \right|}$$

- Streamwise friction coefficient (O(6))

$$c_{f,x} = 2\left(\frac{u_{\tau,x}}{U_w}\right)^2$$