Mesh evaluation

The estimation of the spacing for the mesh along the y direction is performed through one of the utilities that can be found inside the folder `1-pre_processing/Stretching Mesh`. For a channel flow case, the default function of Incompact3d can be used, `stretching_parameter_channel.f90`.

For a temporal TBL case instead, a new function was developed, starting from the stretching subroutine that can be found inside the original solver of Incompact3d. This new function is a Python script, `mesh_evaluation_ttbl.py`. This function allows to estimate the mesh size by introducing the following inputs:

- Number of points: n_x,n_y,n_z.
- Domain dimensions: L_x,L_y,L_z.
- Stretching parameter: β.
- Skin friction coefficient: c_f.
- Kinematic viscosity: ν.
- Velocity of the wall: U_w.
- Time step: Δt.
- Tripping wire diameter: D.

And it produces the following outputs:

- Non-dimensional domain dimensions at IC and at peak c_f: L_x^+,L_y^+,L_z^+.
- CFL, Péclet, Numerical Fourier and stability parameter: Co, $\mathcal{D}$, $P\acute{e}$, S.
- Mesh size at the first element near the wall at peak c_f: Δy_1^+.
- Mesh size at the last element away from the wall at peak c_f: Δy_n^+.
- Mesh spacings in x and z directions at peak c_f: Δx^+, Δz^+.
- Aspect ratios (ARs) of grid elements at bottom and top walls in x and z directions: AR_{x_1}, AR_{x_n}, AR_{z_1}, AR_{z_n}.
- Number of mesh nodes `npvis` in the viscous sublayer at c_f peak.
- Number of mesh nodes `npsl` in the initial shear layer.
- Approximate initial shear layer momentum thickness θ_{sl}.
- Initial shear layer thickness δ_{99}^+.

It is worth noticing that in a temporal BL simulation, the peak c_f value constraints the height of the first cell at the wall Δy_1, as in standard CFD simulations. However, the decrease of c_f along the simulation (and thus the increase in viscous length δ_ν) imposes a constraint for the domain dimensions L_x, L_y, L_z since they appear progressively "smaller" (their non-dimensional counterparts decreases). Too low values of L_x^+,L_y^+,L_z^+ must be avoided in order to do not enforce a too strong periodicity in the turbulent structures at the wall.

For numerical stability, it is recommended to maintain $Co < 1$, $D < 0.5$, $P\acute{e} < 2$ and $S < 1$.

Pre-processing quantities

Calculated quantities for a temporal TBL in `mesh_evaluation_ttbl.py`.

Shear velocity at IC

$$\frac{\partial U}{\partial y} = -\frac{U_w}{4\theta_{sl}} \cdot \frac{1}{\cosh^2\left(\frac{D}{2\theta_{sl}}\right)}$$

$$u_\tau = \sqrt{\nu \left| \frac{\partial U}{\partial y} \right|}$$

Shear velocity at peak friction coefficient

$$u_\tau = U_w \sqrt{\frac{c_f}{2}}$$

Initial velocity profile

$$U_0^+(y) = \frac{U_w^+}{2} + \frac{U_w^+}{2} \tanh\left[\frac{D}{2\theta_{sl}}\left(1 - \frac{y}{D}\right)\right]$$

Initial shear layer momentum thickness

$$\theta_{sl} \approx \frac{54\nu}{U_w}$$

Initial Courant-Friedrichs-Lewy number

$$Co = \frac{U_w \Delta t}{\Delta x}$$

Initial Numerical Fourier number

$$\mathcal{D} = \frac{\nu \Delta t}{\Delta x^2}$$

Initial Péclet number

$$P\acute{e} = \frac{U_w \Delta x}{\nu}$$

Initial stability parameter

$$S = \frac{U_w^2 \Delta t}{2\nu}$$