

Gravitational Wave Astrophysics

Lecture 2

Filippo Santoliquido
GSSI and **INFN**, L'Aquila, Italy

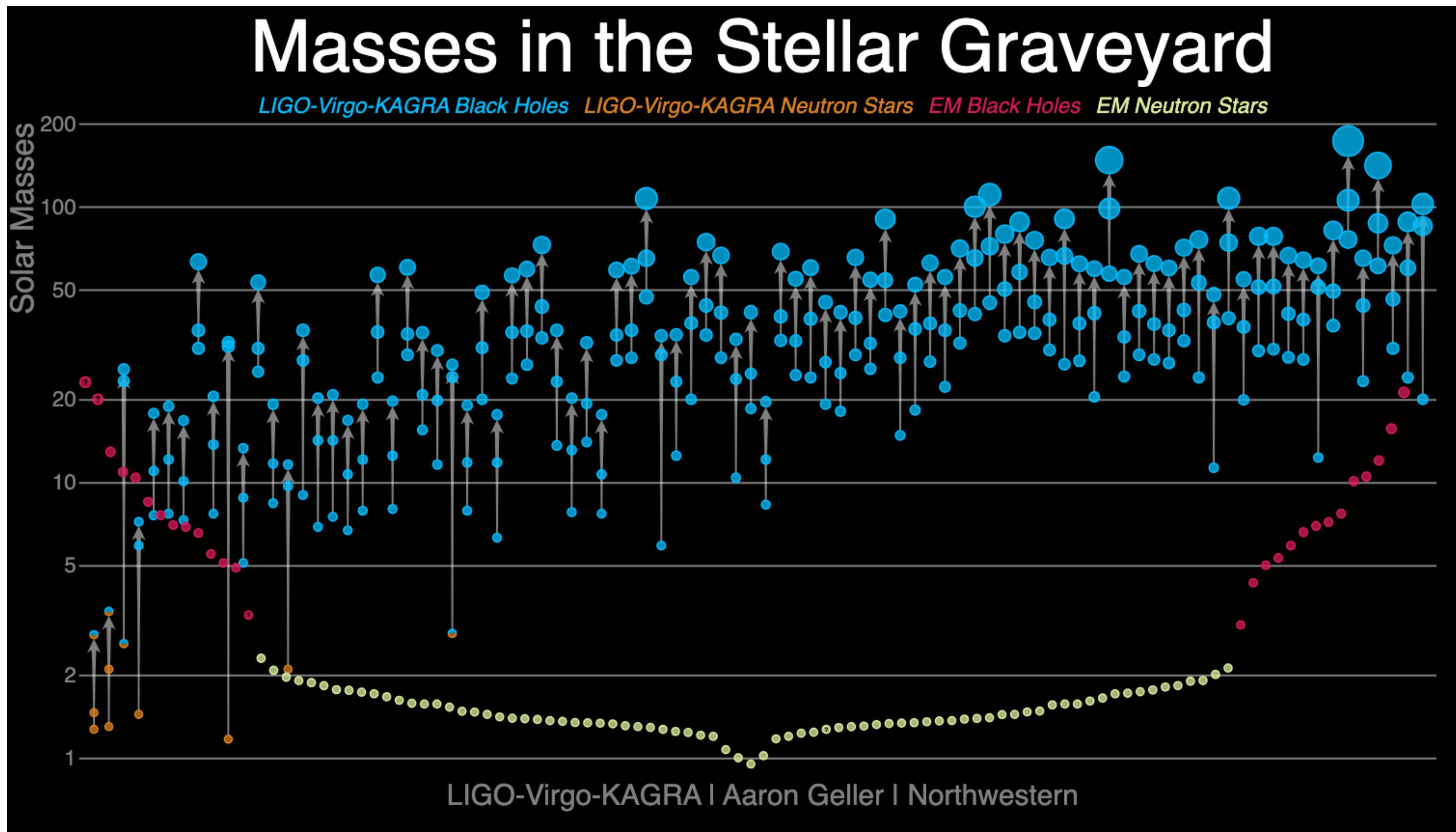
ICTP-SAIFR/IFT-UNESP, São Paulo, August 2024



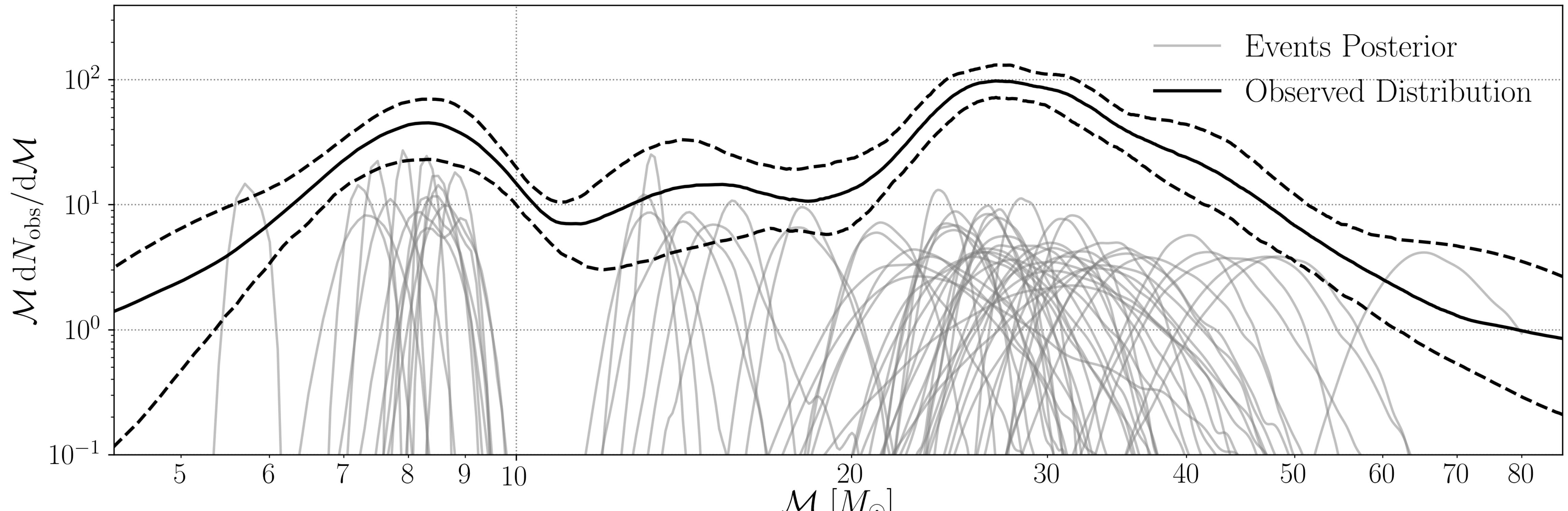
In this lecture, you'll learn

- Population analysis of GW events
- Astrophysics of compact objects:
 - Isolated formation channel
 - Dynamical formation channel

Population studies



Population studies



Credits: [The LIGO-Virgo-KAGRA collaboration 2021](#)

Parametric models

Hierarchical Bayesian Analysis

$$p(\theta | d) \propto \mathcal{L}(d | \theta)p(\theta)$$

$$\theta = \{m_1, m_2, d_L, \dots\}$$



$$p(\Lambda | \{d\}) \propto \mathcal{L}(\{d\} | \Lambda)p(\Lambda)$$

$$\{d\} = \{d_1, d_2, d_3, \dots\}$$

Parametric models

$$p(\Lambda \mid \{d\}) \propto \mathcal{L}(\{d\} \mid \Lambda) p(\Lambda)$$

Parametric models

$$p(\Lambda \mid \{d\}) \propto \mathcal{L}(\{d\} \mid \Lambda) p(\Lambda)$$

$$\mathcal{L}(\{d\} \mid \Lambda) \propto \prod_i^{N_{obs}} \frac{\int \mathcal{L}(d_i \mid \theta) \pi(\theta \mid \Lambda)}{\xi(\Lambda)}$$

Parametric models

$$p(\Lambda \mid \{d\}) \propto \mathcal{L}(\{d\} \mid \Lambda) p(\Lambda)$$

$$\mathcal{L}(\{d\} \mid \Lambda) \propto \prod_i^{N_{obs}} \frac{\int \mathcal{L}(d_i \mid \theta) \pi(\theta \mid \Lambda)}{\xi(\Lambda)}$$

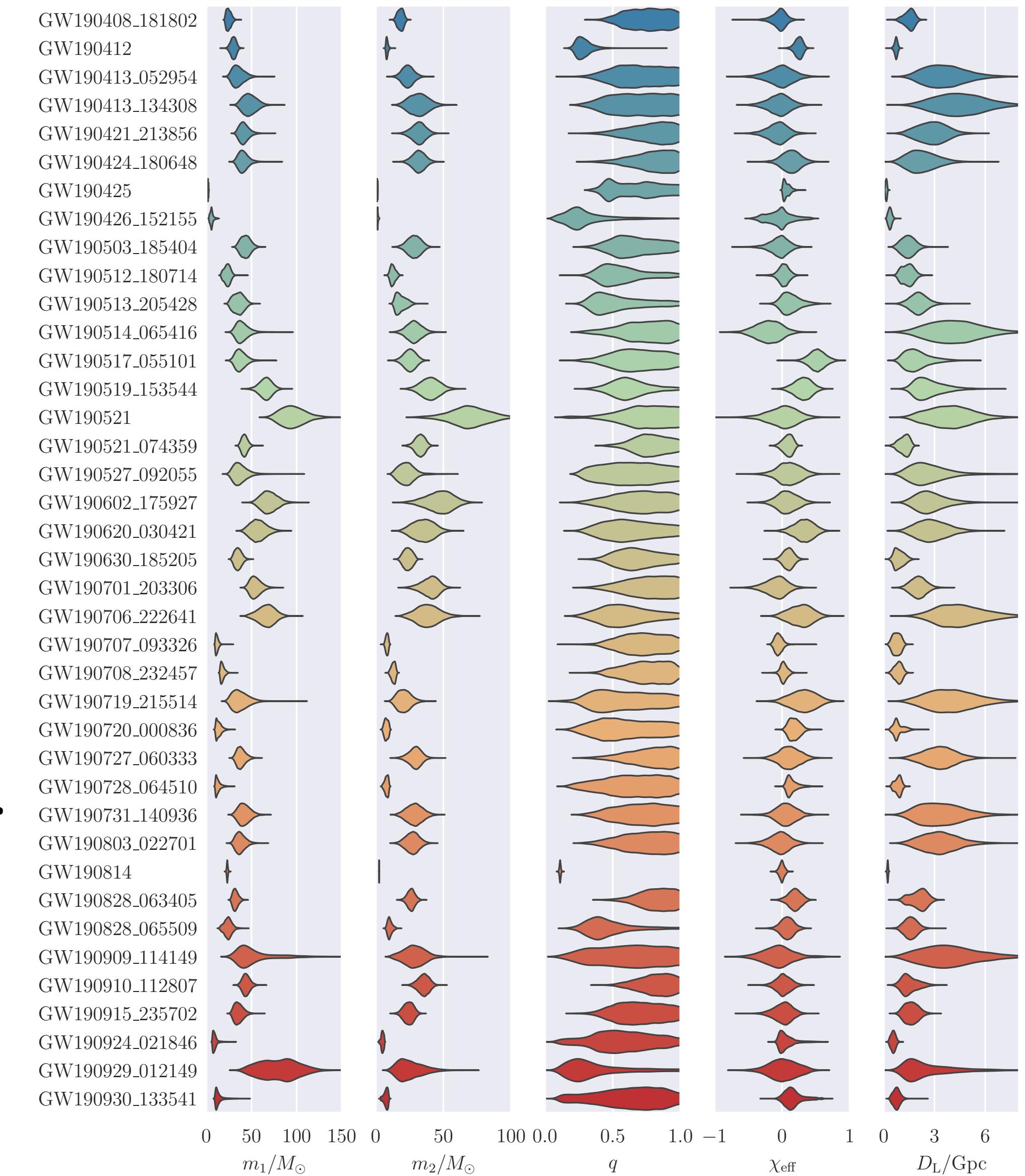
$$\mathcal{L}(\{d\} \mid \Lambda) \propto \prod_i^{N_{obs}} \frac{1}{\xi(\Lambda)} \left\langle \frac{\pi(\theta_j \mid \Lambda)}{\pi(\theta_j)} \right\rangle_{\theta_j \sim p(\theta \mid d_i)}$$

Parametric models

$$p(\Lambda \mid \{d\}) \propto \mathcal{L}(\{d\} \mid \Lambda) p(\Lambda)$$

$$\mathcal{L}(\{d\} \mid \Lambda) \propto \prod_i^{N_{obs}} \frac{\int \mathcal{L}(d_i \mid \theta) \pi(\theta \mid \Lambda)}{\xi(\Lambda)}$$

$$\mathcal{L}(\{d\} \mid \Lambda) \propto \prod_i^{N_{obs}} \frac{1}{\xi(\Lambda)} \left\langle \frac{\pi(\theta_j \mid \Lambda)}{\pi(\theta_j)} \right\rangle_{\theta_j \sim p(\theta \mid d_i)}$$

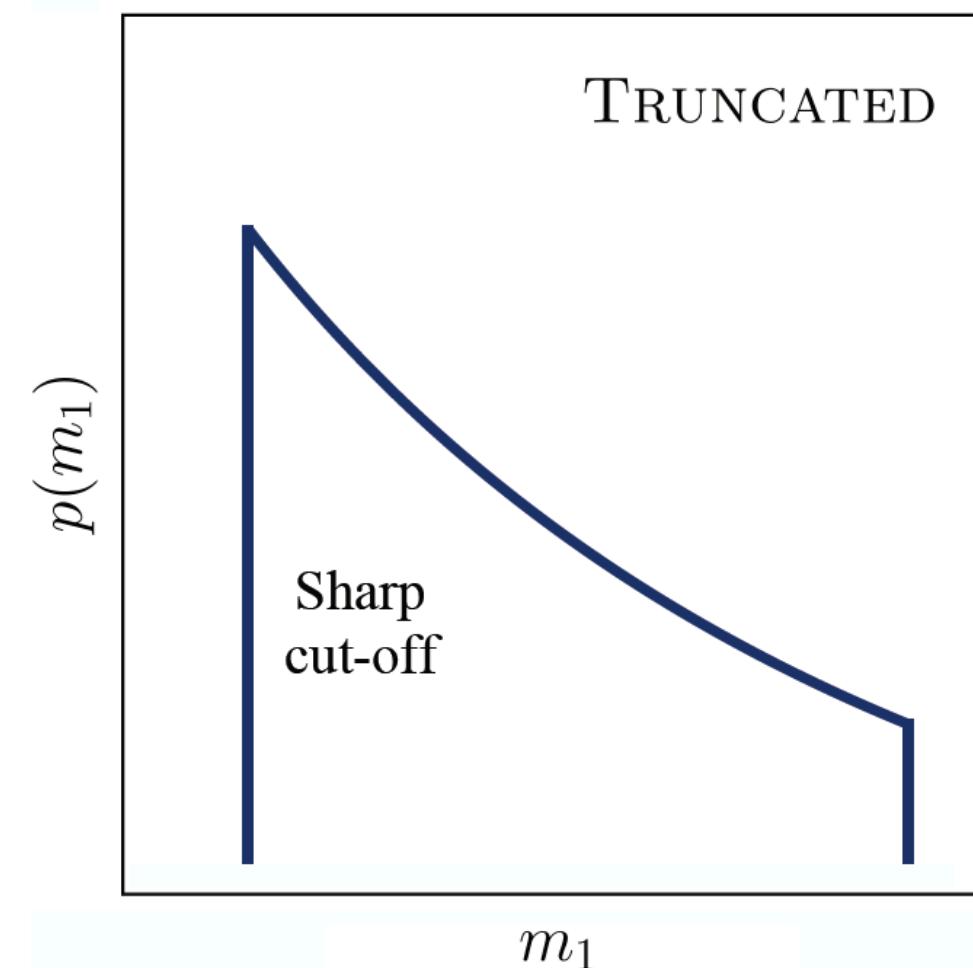


Parametric models

$$p(\Lambda | \{d\}) \propto \mathcal{L}(\{d\} | \Lambda) p(\Lambda)$$

Population models

$$\mathcal{L}(\{d\} | \Lambda) \propto \prod_i \frac{\int \mathcal{L}(d_i | \theta) \pi(\theta | \Lambda)}{\xi(\Lambda)}$$

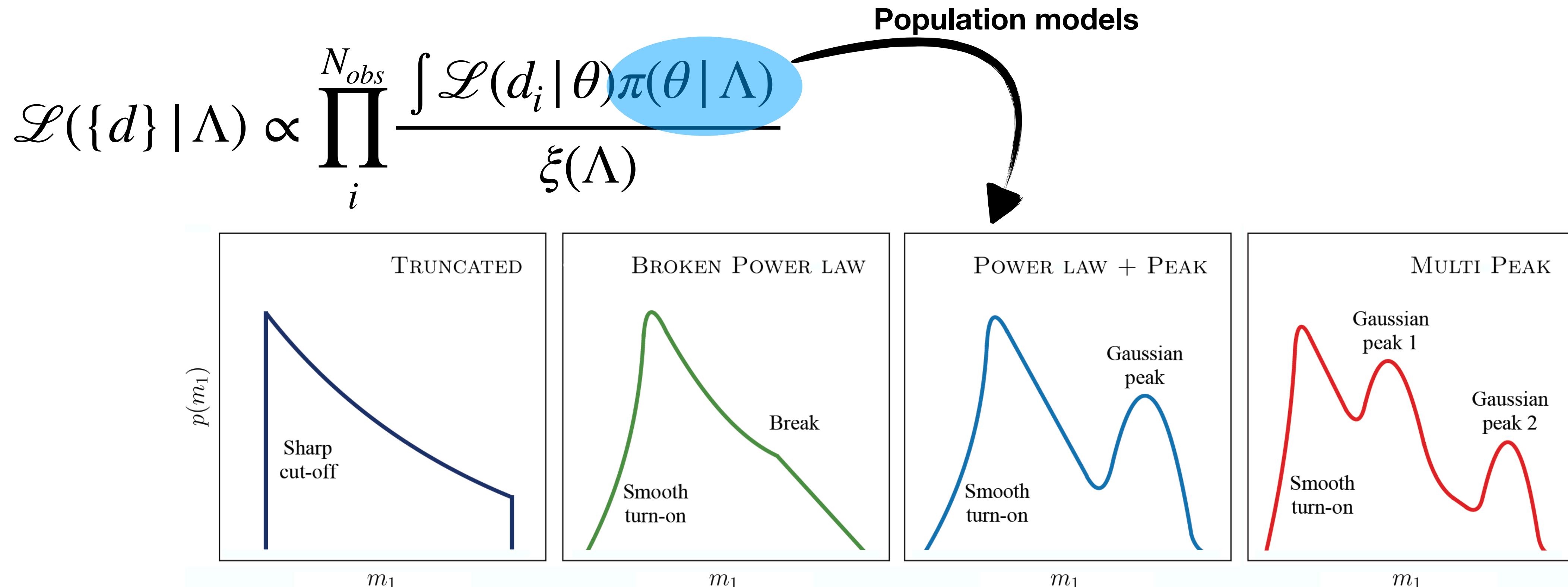


Why a power law?

$$\pi(m_1 | \alpha, m_{\min}, m_{\max}) \propto \begin{cases} m_1^{-\alpha} & m_{\min} < m_1 < m_{\max} \\ 0 & \text{otherwise,} \end{cases}$$

Parametric models

$$p(\Lambda | \{d\}) \propto \mathcal{L}(\{d\} | \Lambda) p(\Lambda)$$



Parametric models

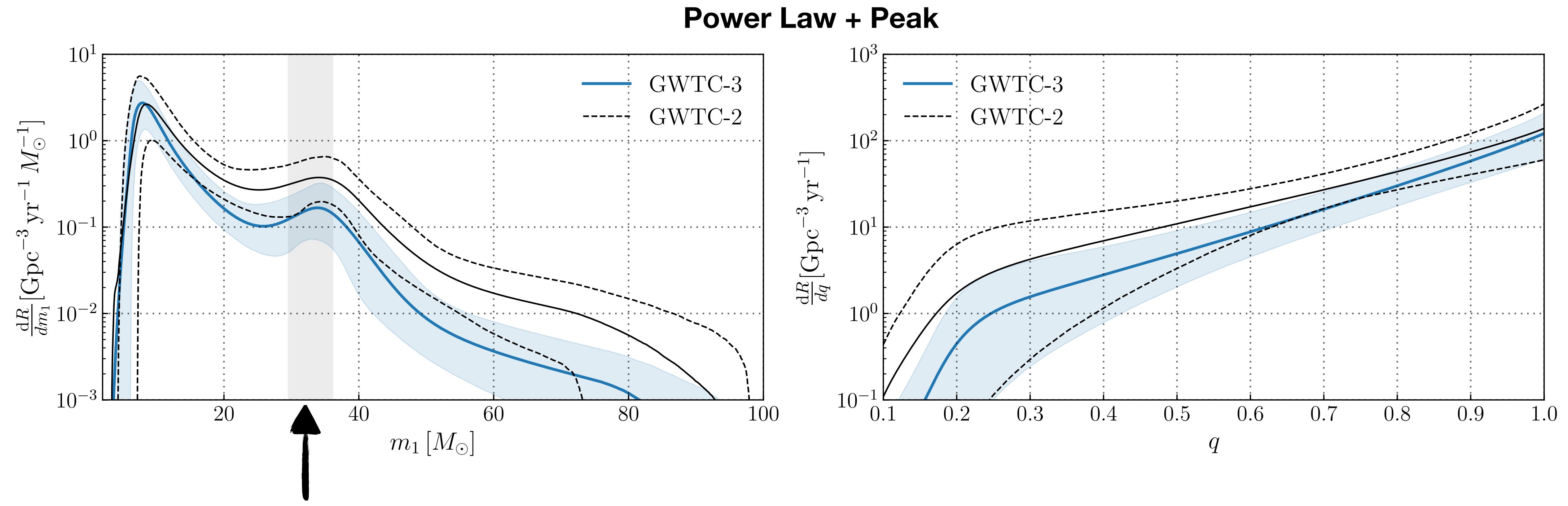
$$p(\Lambda | \{d\}) \propto \mathcal{L}(\{d\} | \Lambda) p(\Lambda)$$

$$\mathcal{L}(\{d\} | \Lambda) \propto \prod_i \frac{\int \mathcal{L}(d_i | \theta) \pi(\theta | \Lambda)}{\xi(\Lambda)}$$

Selection effects: fraction of merger that are detectable for a population with parameters Λ
We will define it in lecture #4

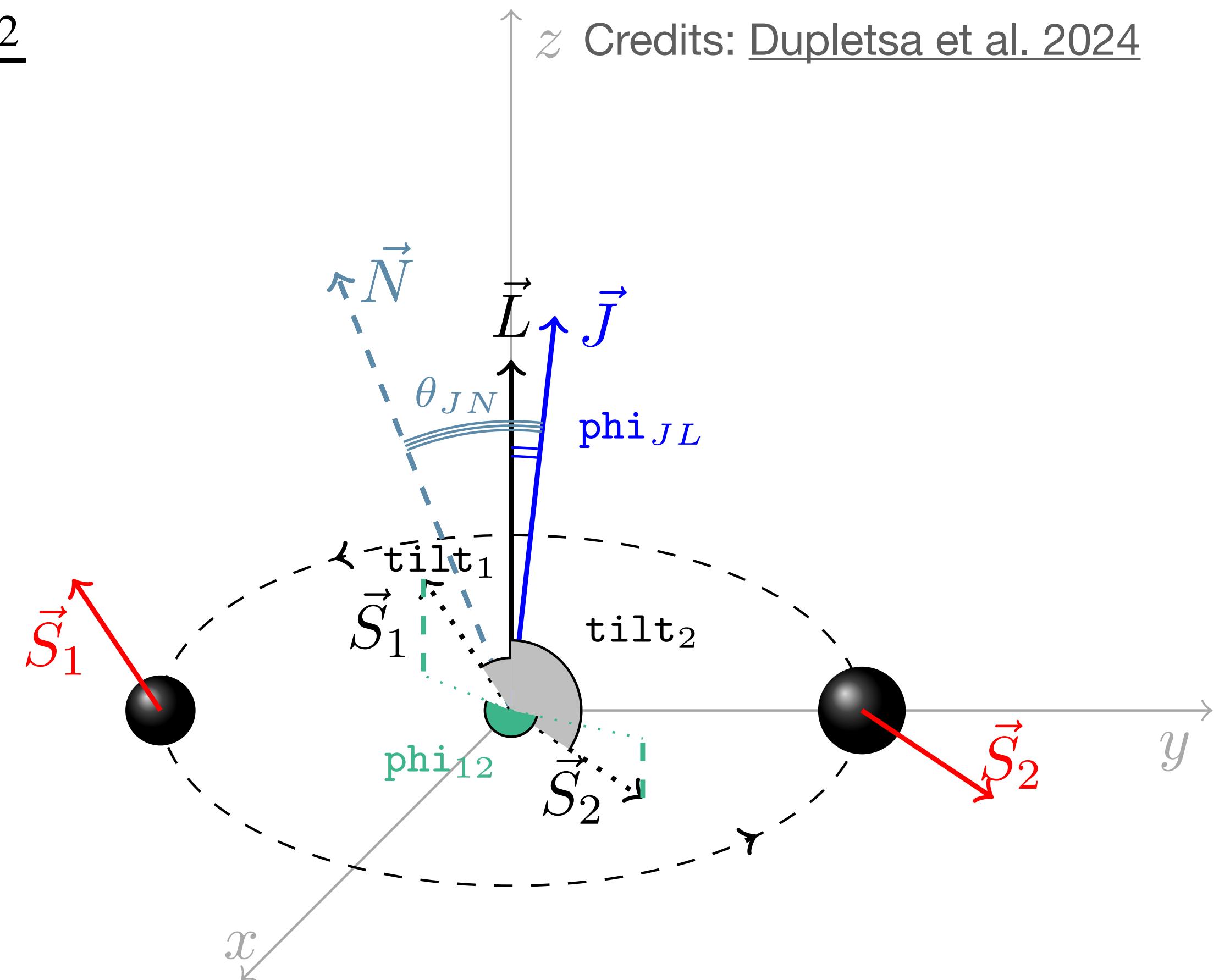
Masses

$$p(\Lambda | \{d\}) \propto \mathcal{L}(\{d\} | \Lambda) p(\Lambda)$$

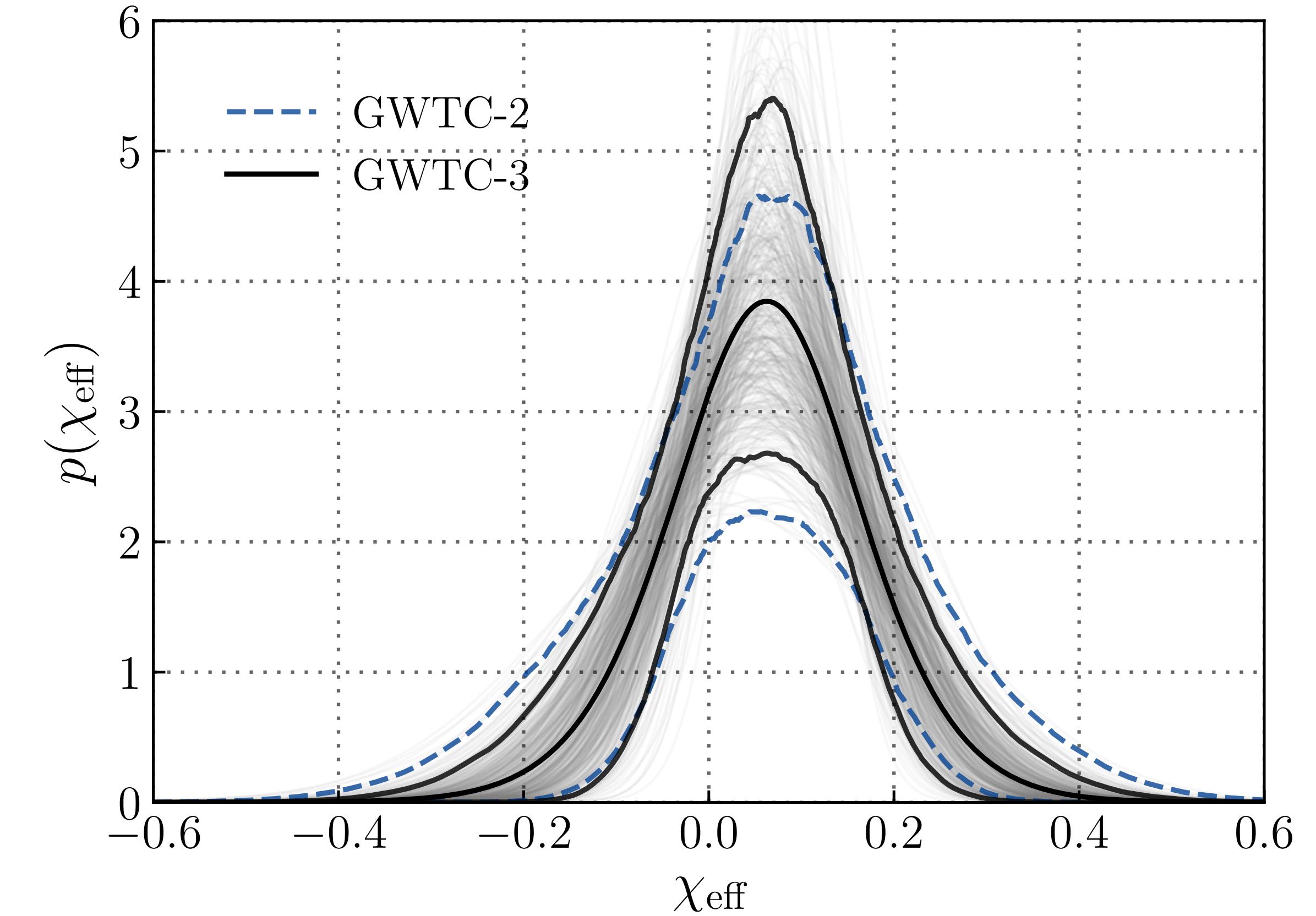
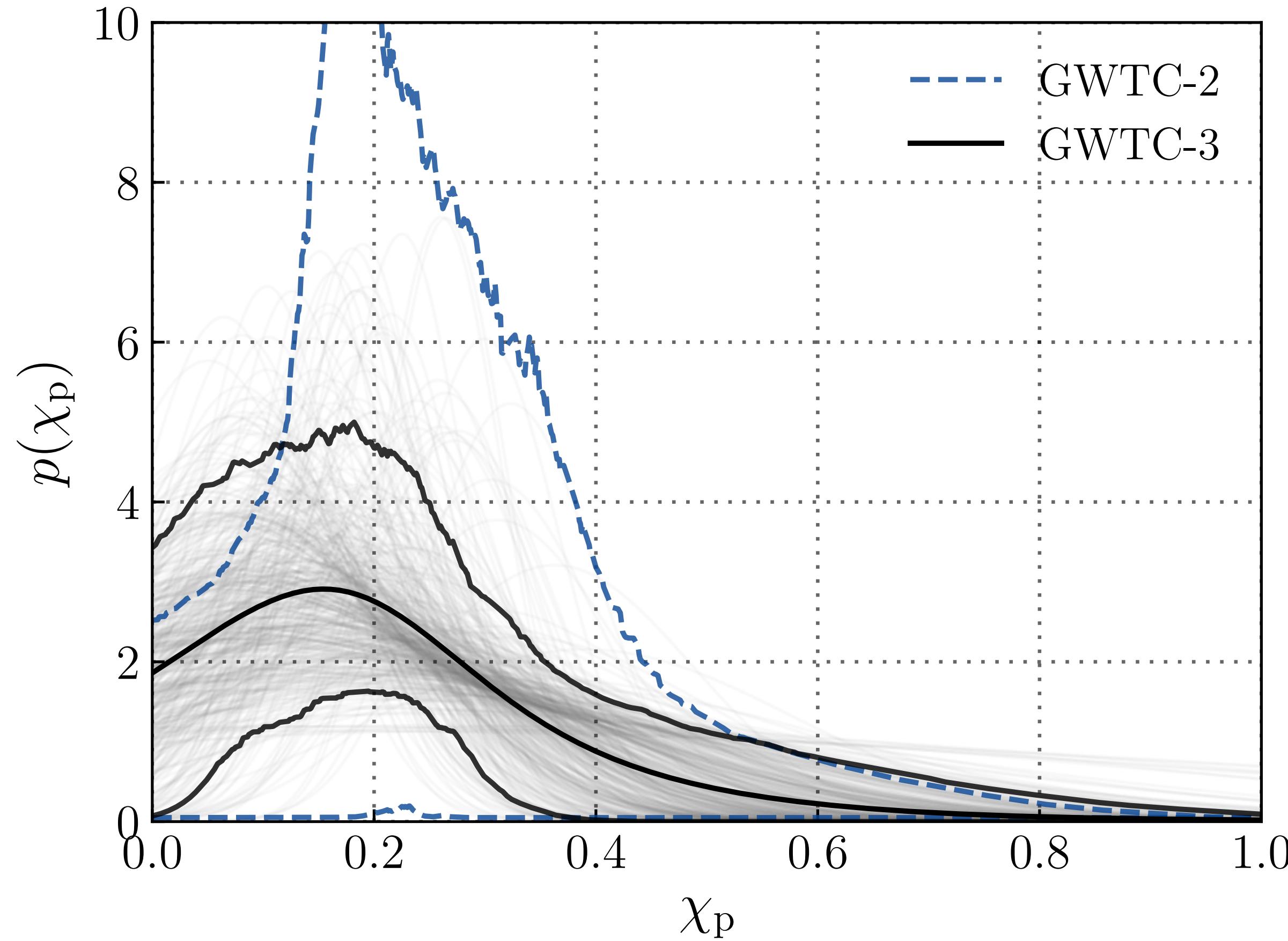


Spins

- $\chi_{\text{eff}} = (\mathbf{S}_1/m_1 + \mathbf{S}_2/m_2) \cdot \hat{\mathbf{L}}/M = \frac{\chi_1 \cos \theta_1 + q\chi_2 \cos \theta_2}{1 + q}$
- $\chi_p = \max \left[\chi_1 \sin \theta_1, \left(\frac{3 + 4q}{4 + 3q} \right) q\chi_2 \sin \theta_2 \right]$
- χ_{eff} and χ_p are approximately conserved quantities
- Dimension-less spin component $\chi_i = \mathbf{S}_i/m_i$ where \mathbf{S}_i is the individual spin
- $\hat{\mathbf{L}}$ is orbital angular momentum



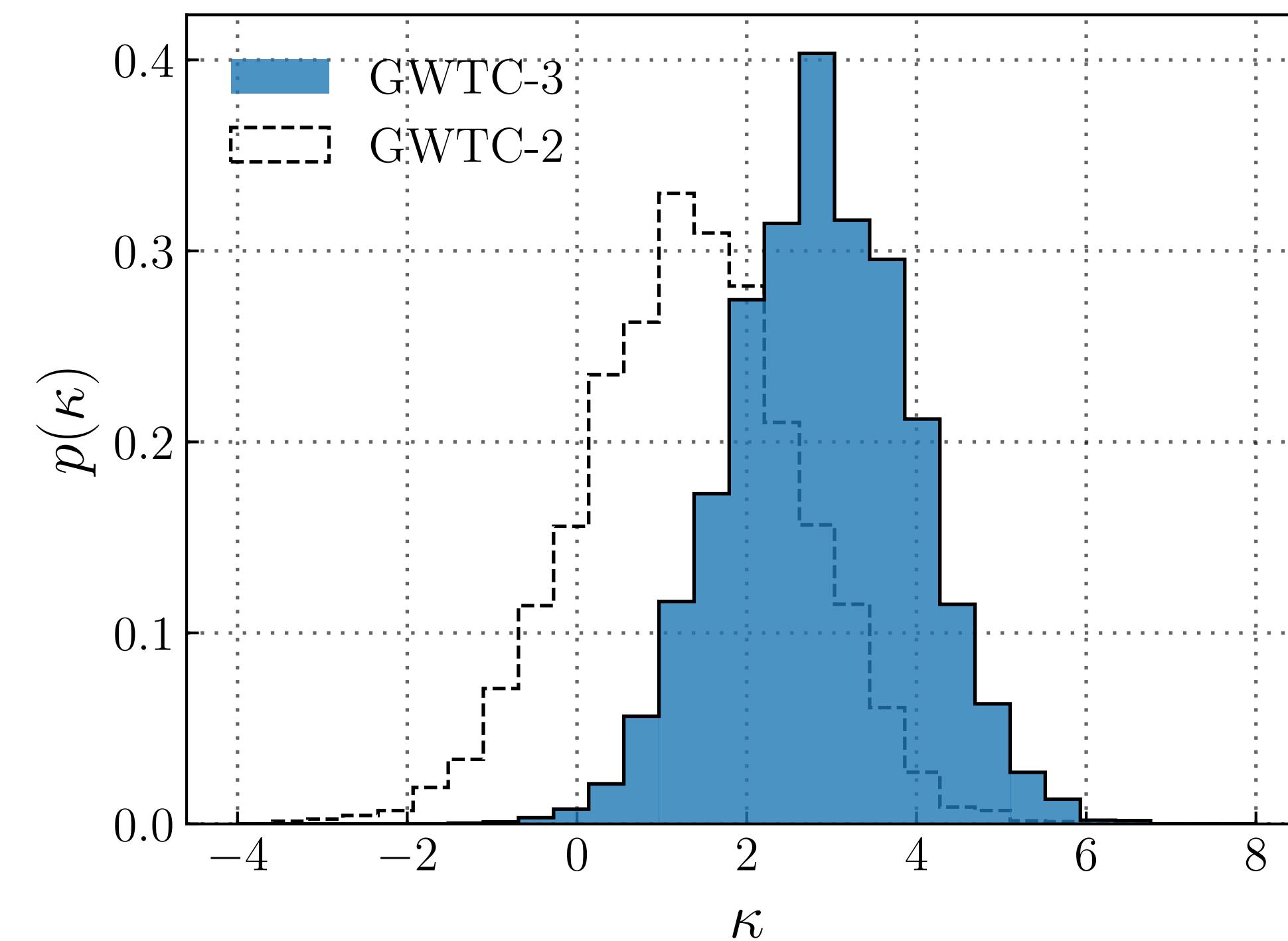
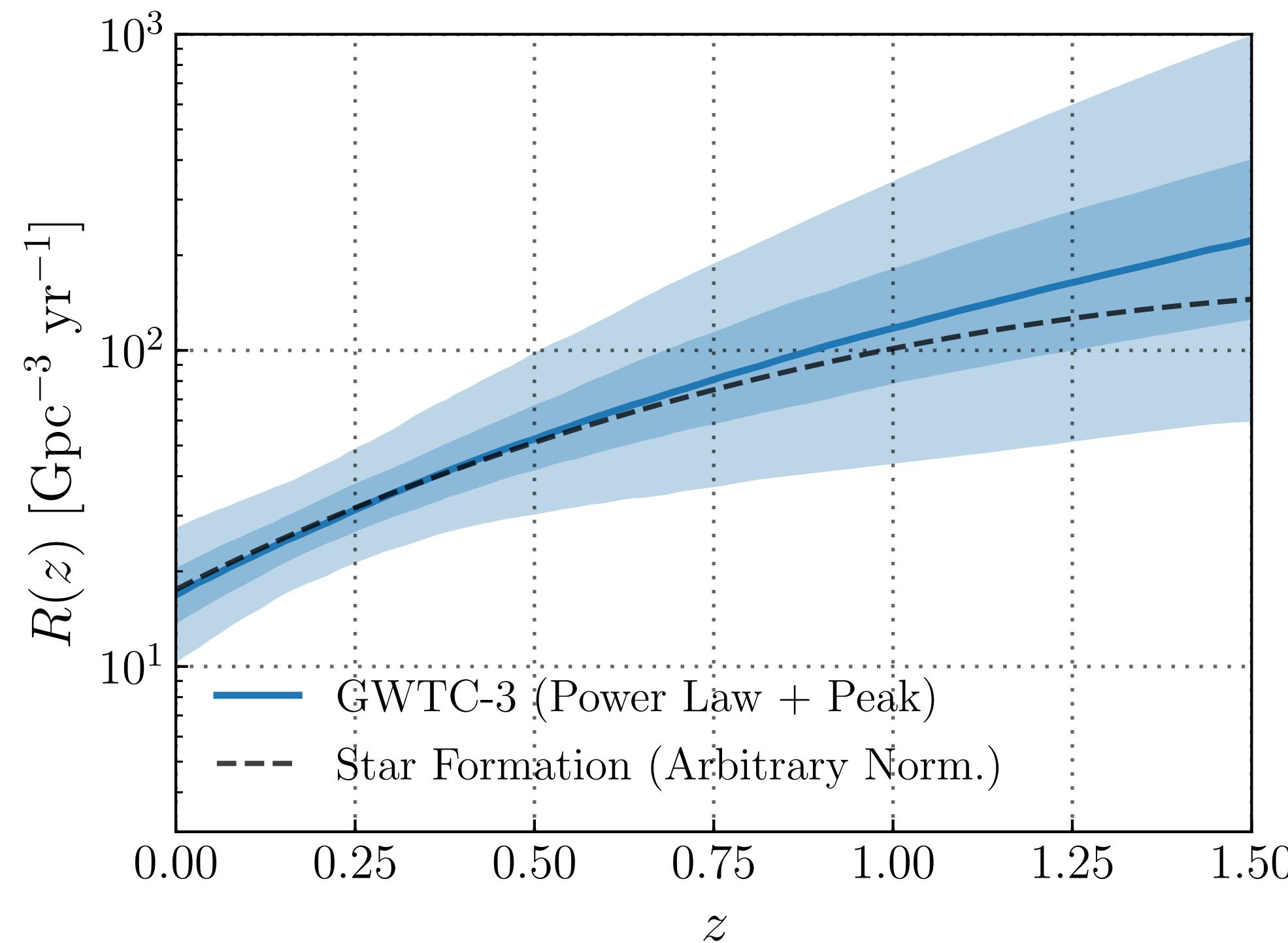
Spins



small but non-vanishing spins

Rates

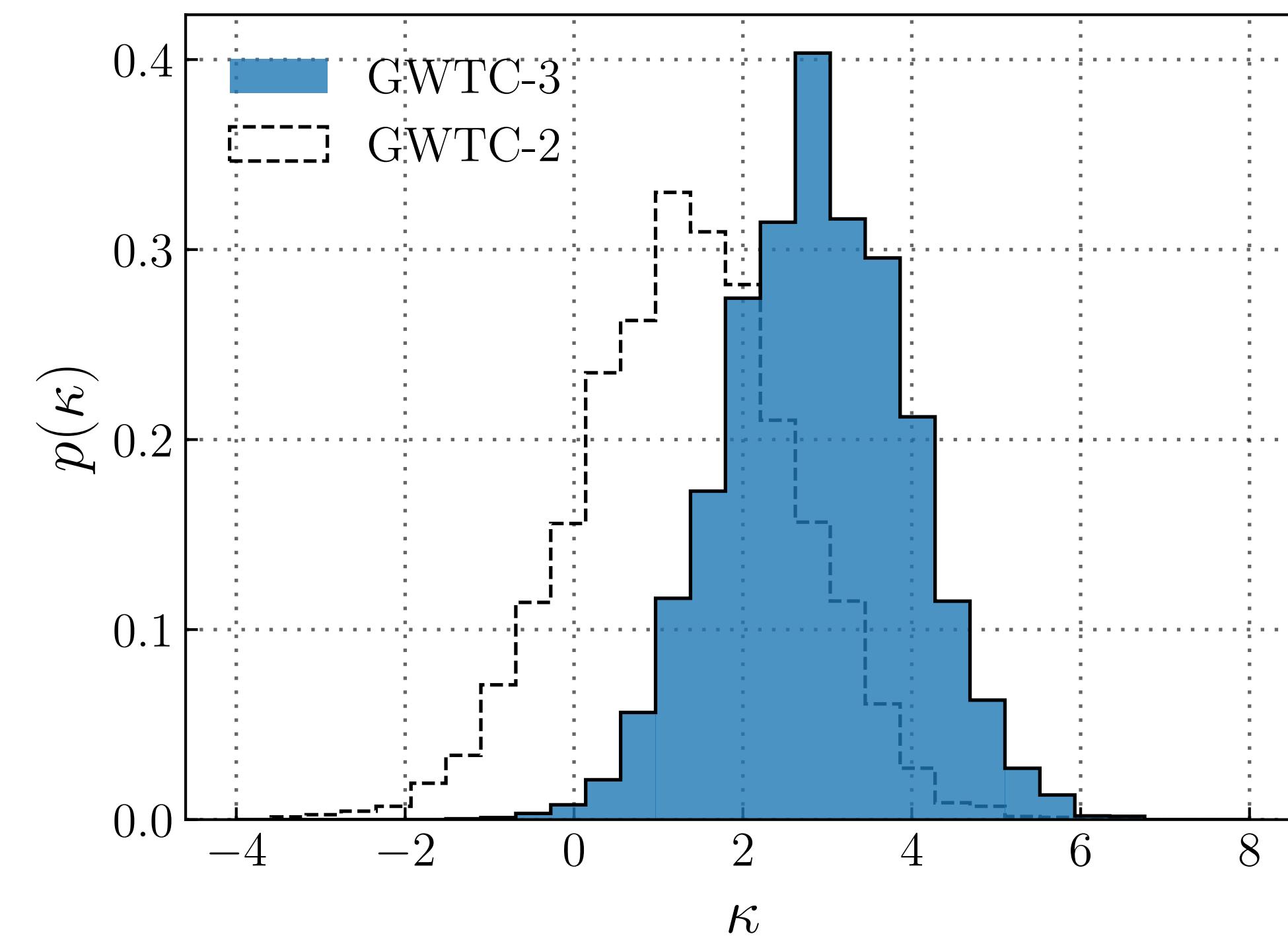
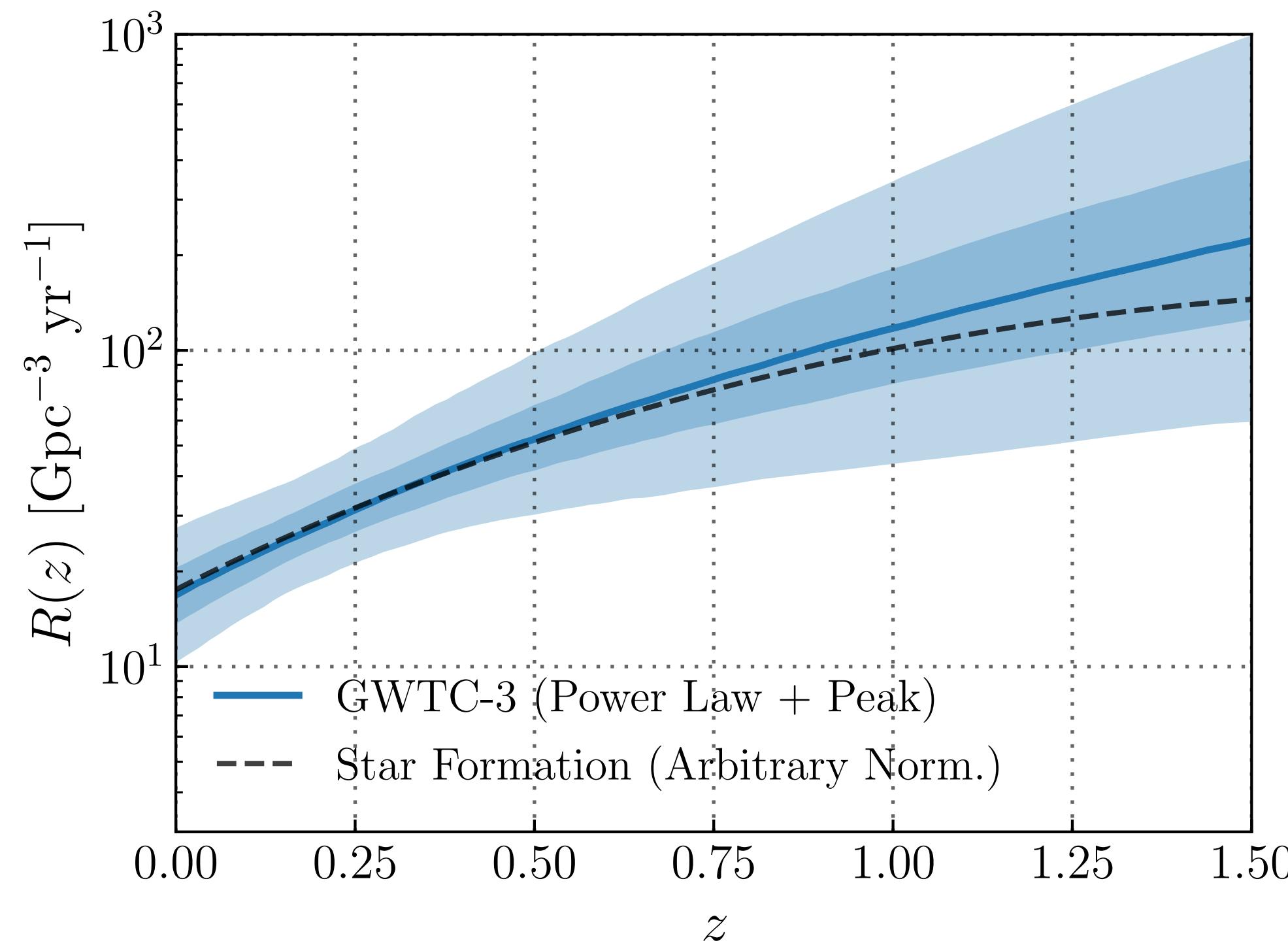
$$\mathcal{R} \propto (1 + z)^\kappa$$



Merger rate density is increasing with redshift

Rates

$$\mathcal{R} \propto (1+z)^\kappa$$



Today's hands-on section!

Astrophysics of compact objects: can our models predict observations?

NS vs BH

- Final stage of massive star evolution
- **NS mass** $\in [\sim 1, \sim 3] M_{\odot}$
 - Lower limit: **Chandrasekhar mass**: maximum mass for a star to be supported by electron degeneracy
 - Upper limit: **Tolman-Oppenheimer-Volkoff limit**: max mass above which no source pressure can counteract gravity
- **BH mass** $\gtrsim 3 M_{\odot}$

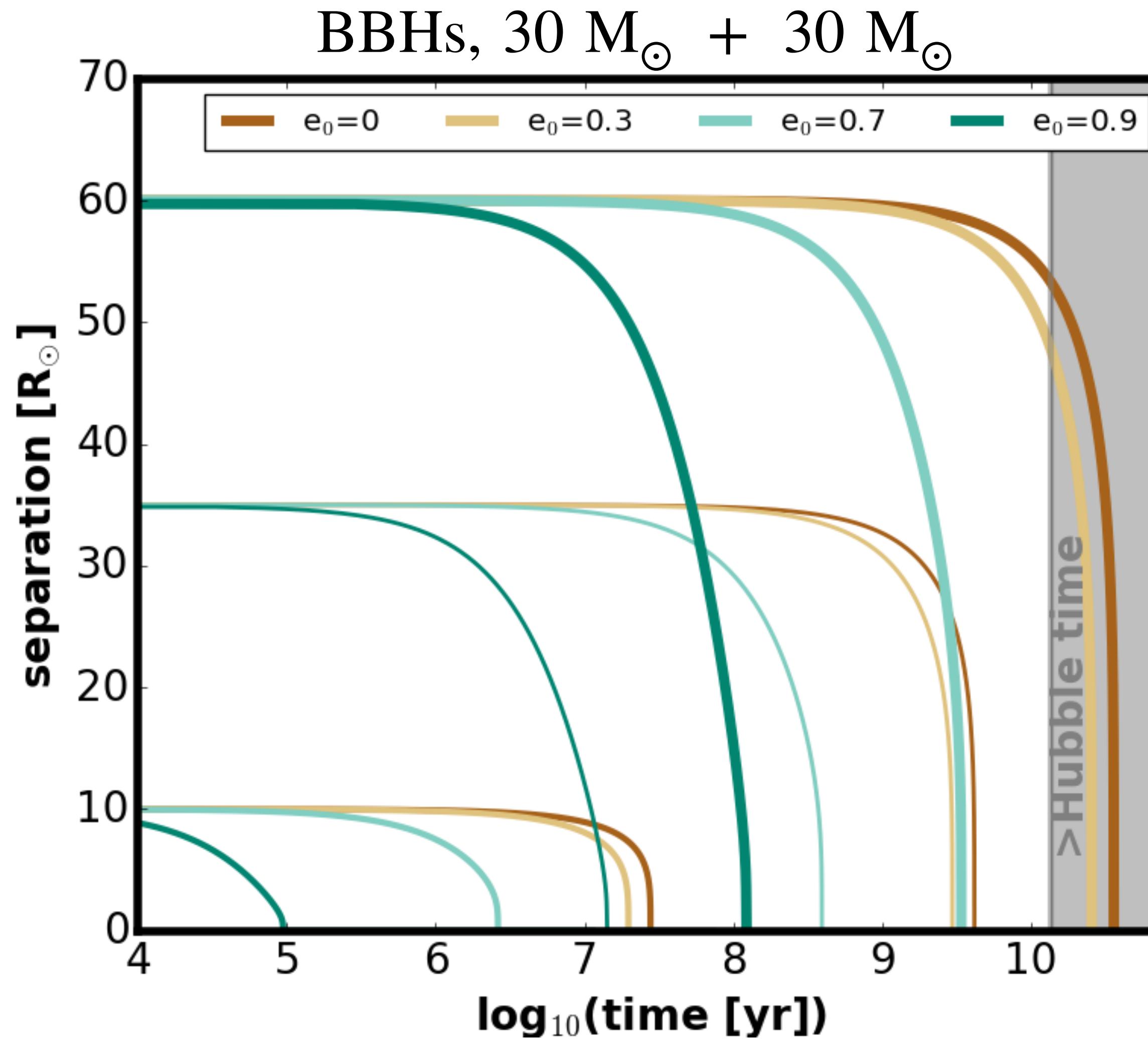
GW decay

- $\frac{dE_{\text{orb}}}{dt} = -\frac{Gm_1m_2}{2a^2}\frac{da}{dt}$, $\frac{dE_{\text{orb}}}{dt} \sim \frac{32}{5}\frac{G^4}{c^5}\frac{m_1m_2(m_1 + m_2)}{a^3}$

GW decay

- $\frac{dE_{\text{orb}}}{dt} = -\frac{Gm_1m_2}{2a^2} \frac{da}{dt}$, $\frac{dE_{\text{orb}}}{dt} \sim \frac{32}{5} \frac{G^4}{c^5} \frac{m_1m_2(m_1 + m_2)}{a^3}$
- Orbital decay: $\frac{da}{dt} = -\frac{64}{5} \frac{G^3 m_1 m_2 (m_1 + m_2)}{c^5 a^3 (1 - e^2)^{7/2}} \left(1 + \frac{73}{24} e^2 + \frac{37}{96} e^4 \right)$
- Circularisation: $\frac{de}{dt} = -\frac{304}{15} e \frac{G^3 m_1 m_2 (m_1 + m_2)}{c^5 a^4 (1 - e^2)^{5/2}} \left(1 + \frac{121}{304} e^2 \right)$

GW decay



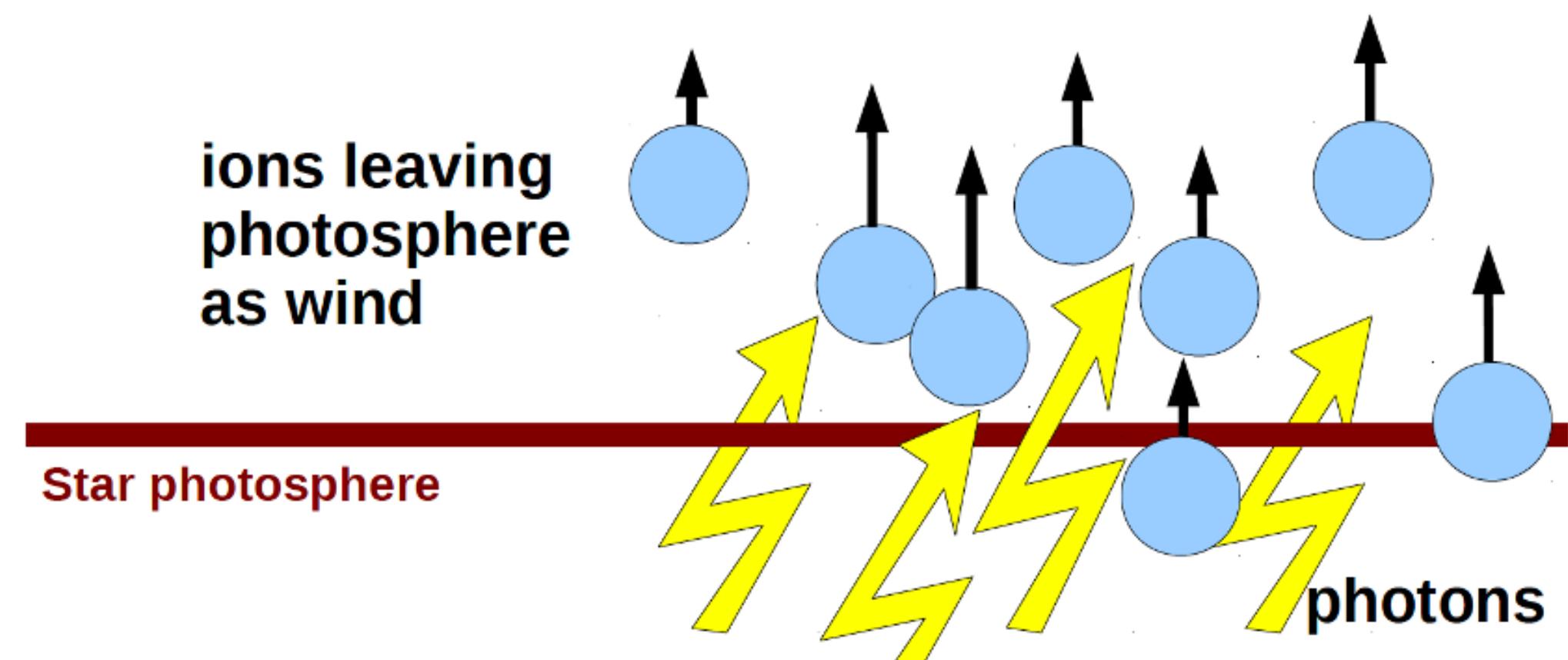
Credits: Martyna Chruślinska

GW decay

- Integrating $\frac{da}{dt}$ and assuming $\frac{de}{dt} = 0$:
$$t_{\text{GW}} = \frac{5}{256} \frac{c^5}{G^3} \frac{a^4}{m_1 m_2 (m_1 + m_2)} (1 - e^2)^{7/2}$$
- Example: $m_1 = m_2 = 1 \text{ M}_\odot$, $a = 1 \text{ AU} \rightarrow t_{\text{GW}} \sim 2 \times 10^{17} \text{ yr}$
- GW decay is significant only in very tight binaries \rightarrow
we need **astrophysical processes** to make the two compact objects merge

Single stellar evolution

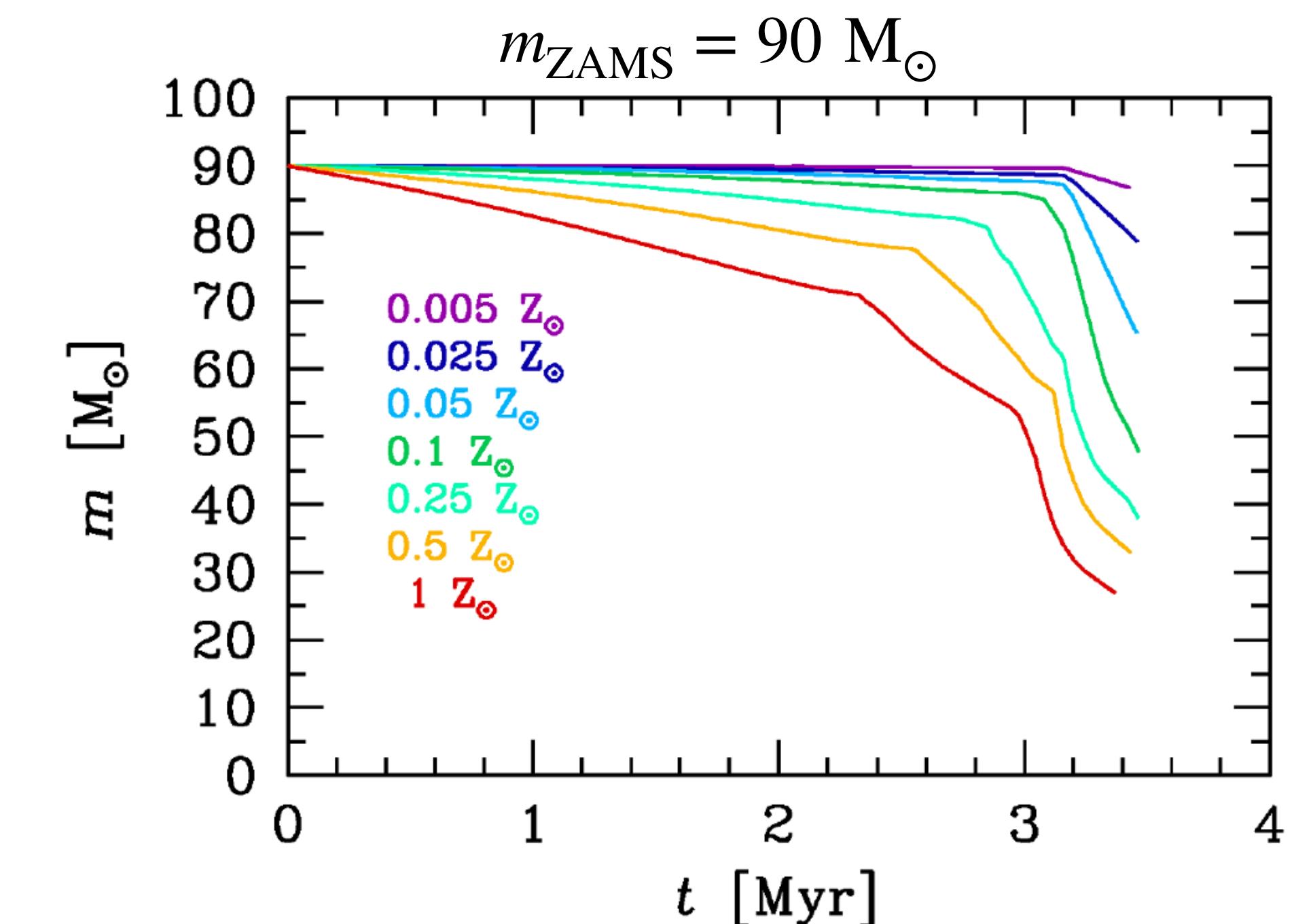
- **Metallicity:** fraction of every element heavier than hydrogen (X) and helium (Y)
➡ $Z = 1 - X - Y$
- **Sun:** $Z_{\odot} \sim 0.015 - 0.02$ ➡ metal rich star
- **Stellar winds:** photons in stellar atmosphere couples with ions ➡ transfer of linear momentum that can unbind ions



Credit: Michela Mapelli

Single stellar evolution

- Mass loss due to stellar winds depends on metallicity $\rightarrow \dot{M} \propto Z^\alpha$ with $\alpha \sim 0.5 - 0.9$
- Massive and metal rich star can lose $> 50\%$ of their mass due to stellar winds

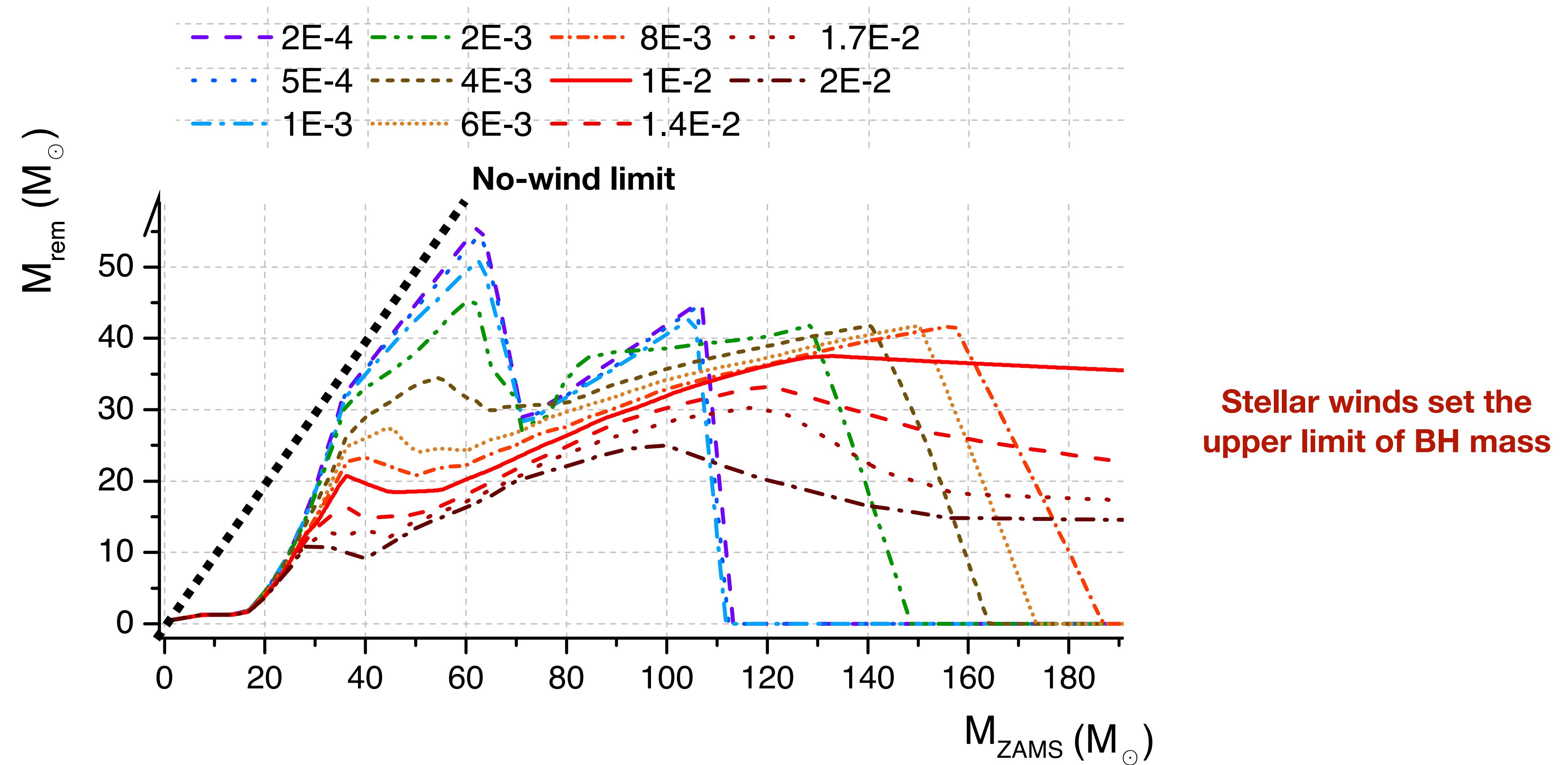


Credits: Michela Mapelli

Death of a star

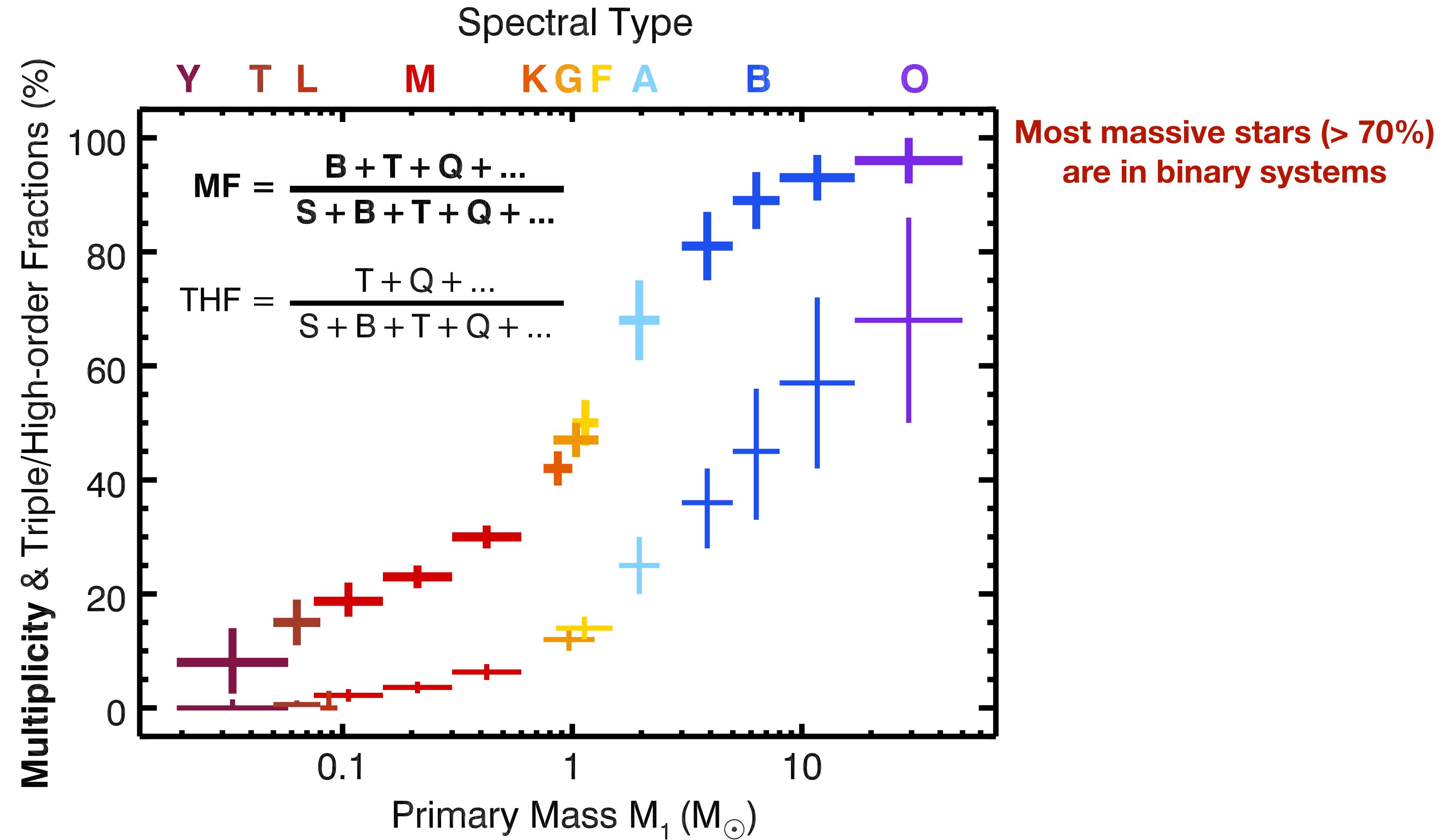
- When nuclear burning is over, the star is out of hydrostatic equilibrium and the **core collapses (CC)**
- $M_{\text{ZAMS}} \lesssim 8 M_{\odot} \rightarrow$ CC stops when electron degeneracy pressure balance gravity. Thermal pulses remove all the envelope, revealing the bare CO core \rightarrow **White Dwarf**
- $M_{\text{ZAMS}} \in [\sim 8, \sim 20] M_{\odot} \rightarrow$ CC stops when core reaches nuclear density and neutron degeneracy pressure balance gravity \rightarrow **Neutron Star**
- $M_{\text{ZAMS}} \gtrsim 20 M_{\odot} \rightarrow$ gravity is too strong and no source of pressure can stop CC \rightarrow **Black Hole**

Single stellar evolution



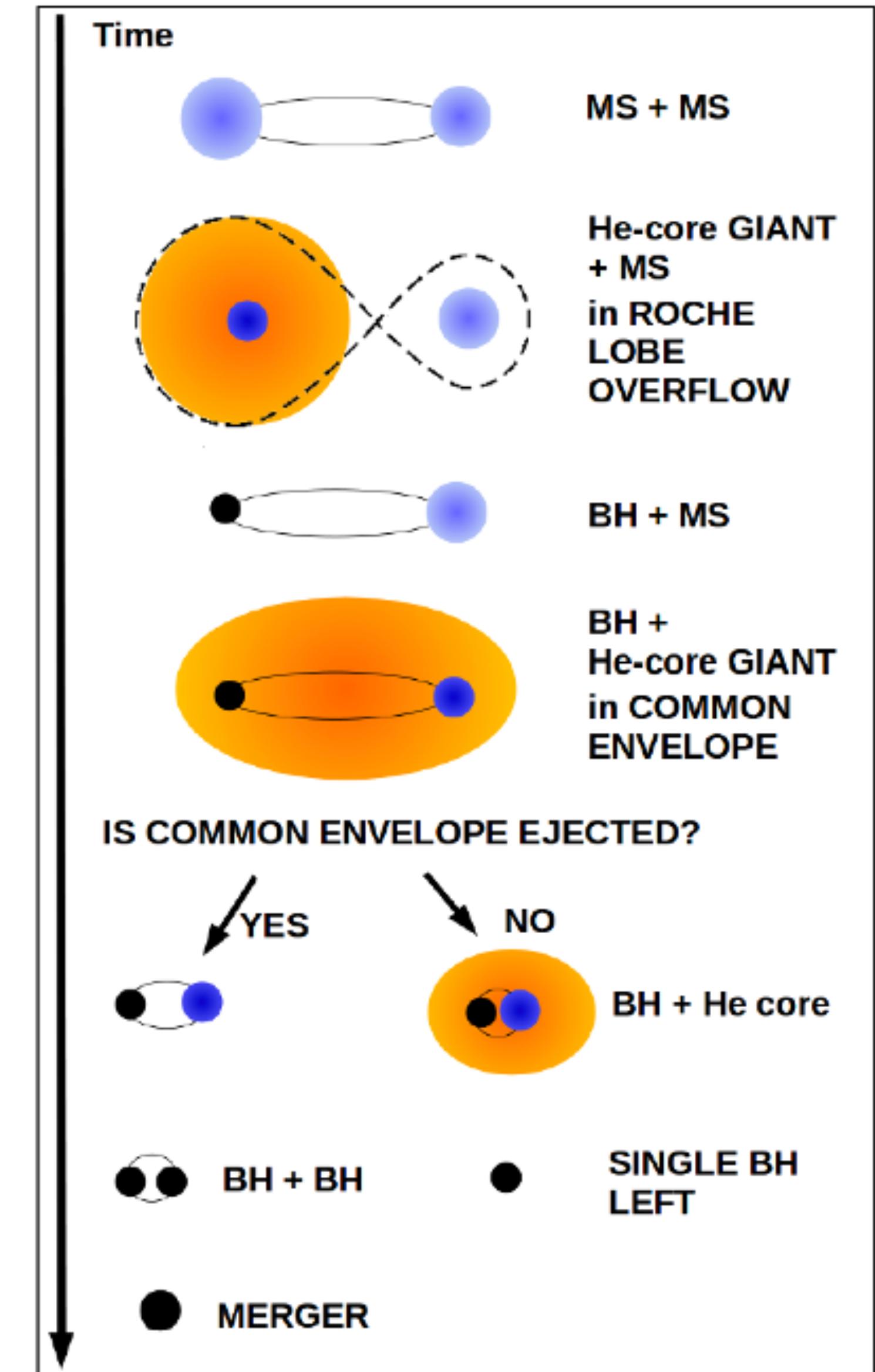
Refs: [Spera et al. 2015](#), [Spera and Mapelli 2017](#), [Mapelli 2018](#), [Spera et al. 2022](#)

Binary systems



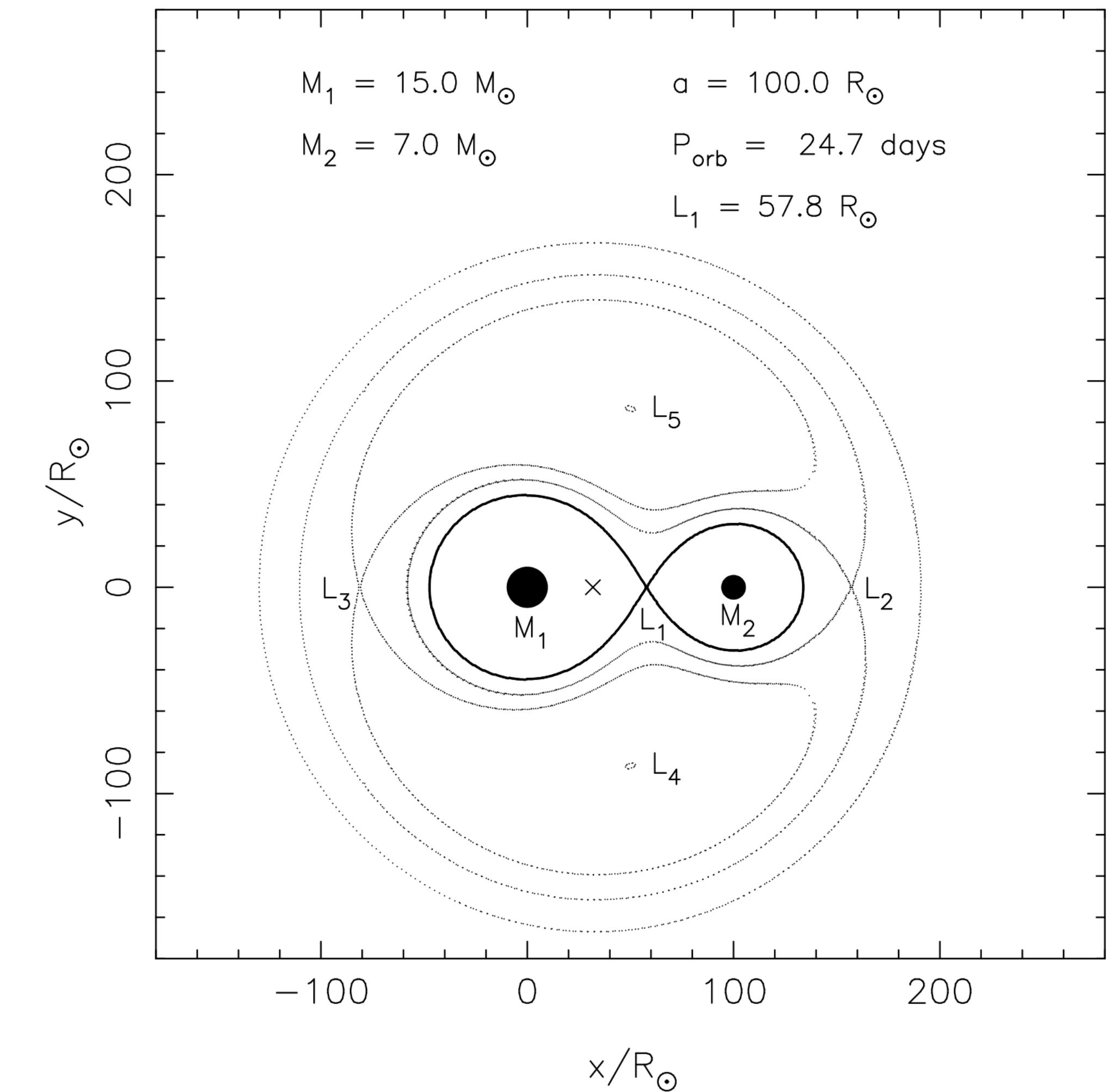
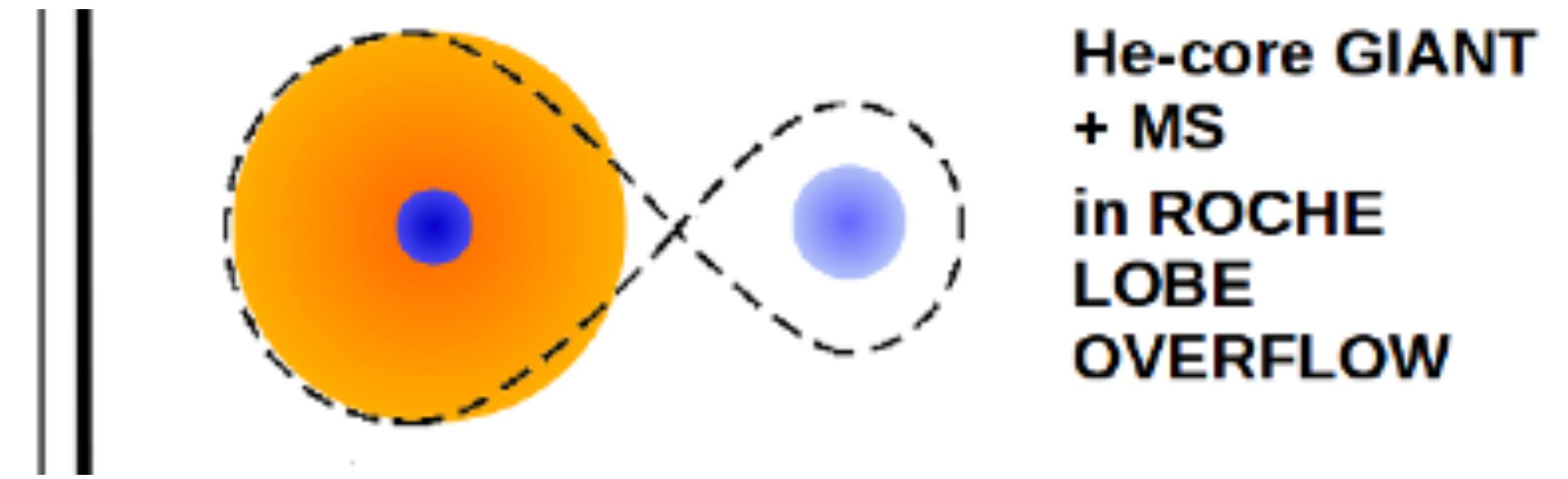
Isolated formation channel

- **Two stars** form from the same gas cloud and evolve into **two merging BHs**
- Binary evolution driven by two main processes:
 - **Mass transfer**
 - **Common Envelope**



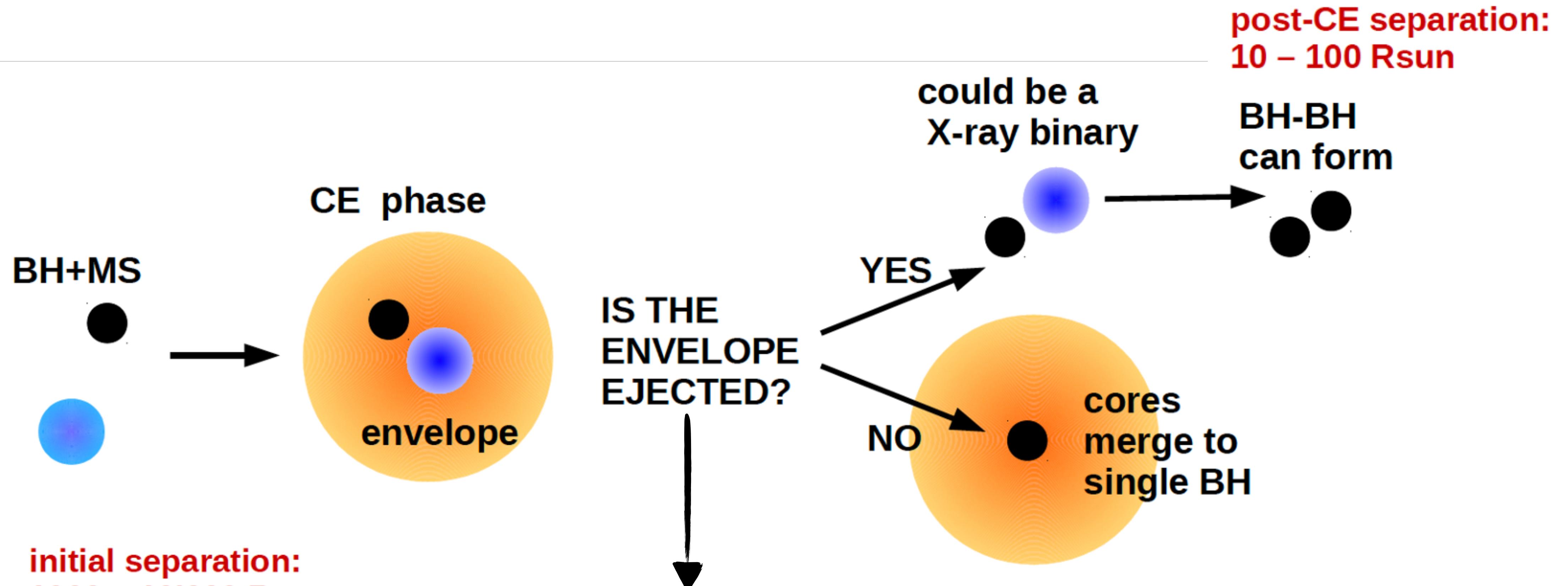
Mass transfer

- **Roche lobe:** $r_{\text{RL}} = a \frac{0.49q^{2/3}}{0.6q^{2/3} + \ln(1 + q^{1/3})}$
- **Mass transfer via Roche-lobe overflow:** mass can be transferred to the other object via L1
- **Orbit shrinks** in case of **stable** mass transfer



Common Envelope

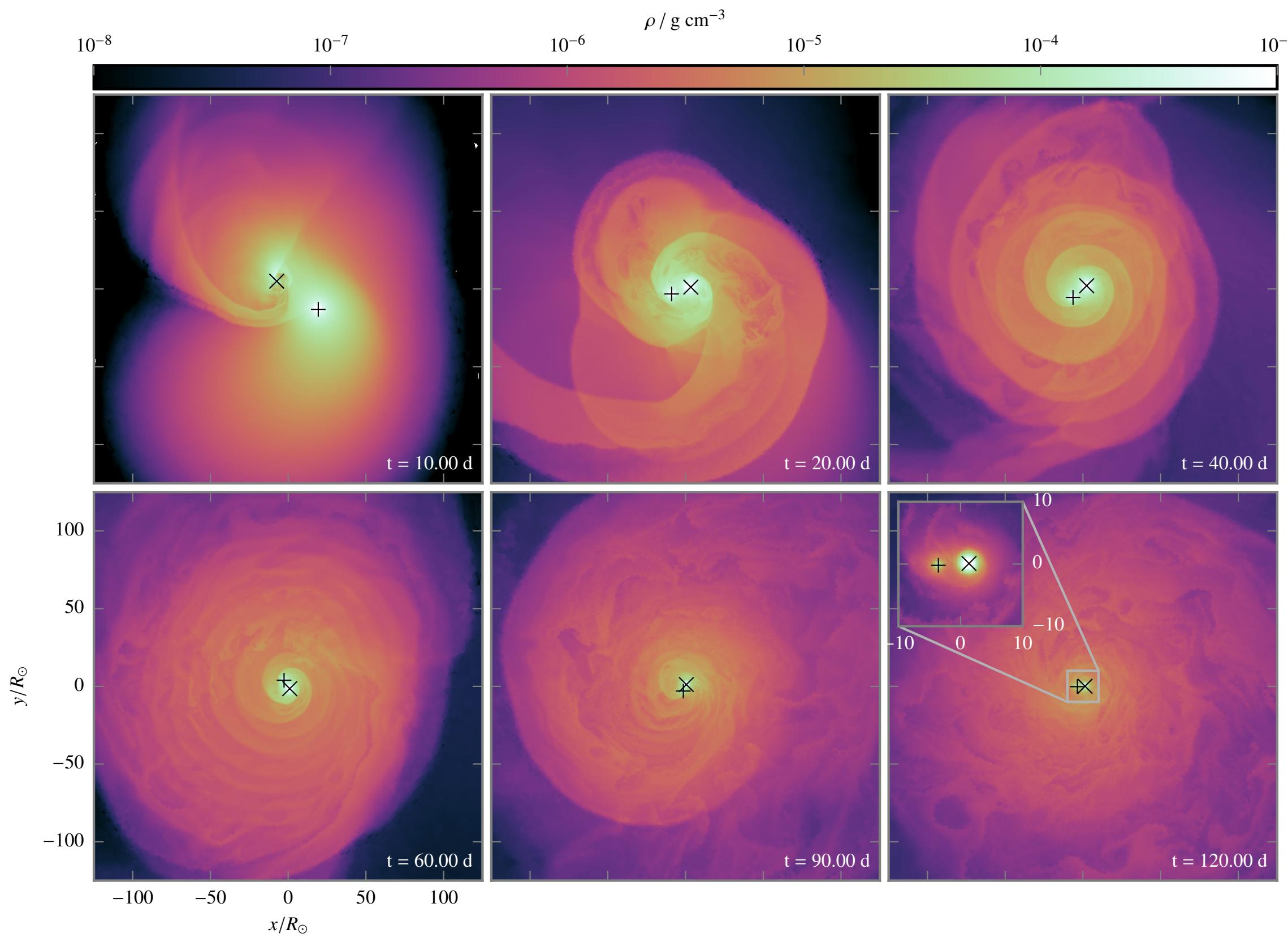
See [here](#) a short movie describing the CE phase



This is the most important question.
i.e., does the binary survive the CE phase?

Modelling the common envelope

Hydrodynamical simulations



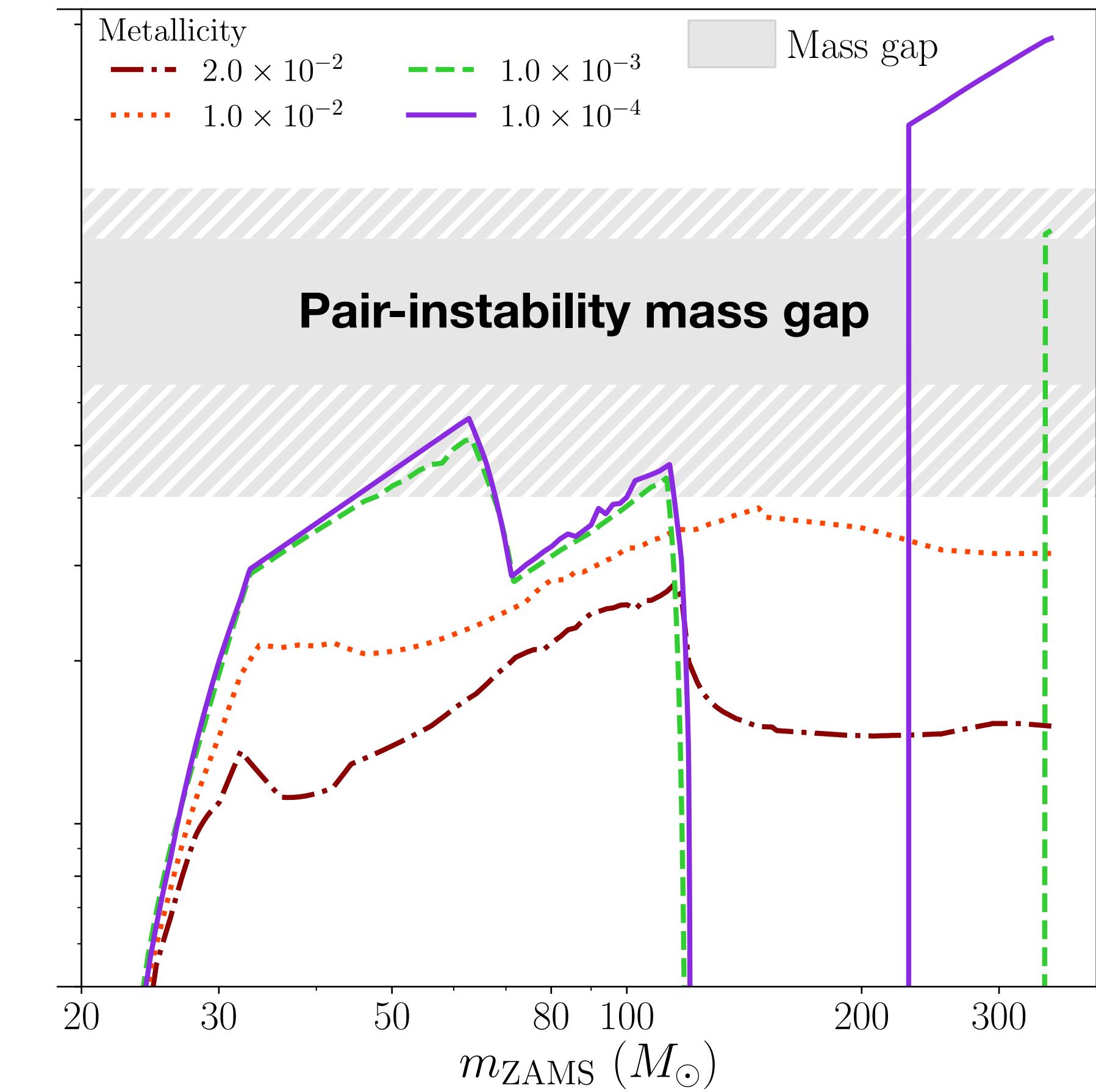
$\alpha\lambda$ -formalism

- $E_{\text{bind},\text{ini}} = -\frac{G}{\lambda} \left(\frac{M_1 M_{1,\text{env}}}{r_1} + \frac{M_2 M_{2,\text{env}}}{r_2} \right)$
- $E_{\text{orb},\text{ini}} = \frac{1}{2} \frac{GM_{c,1}M_{c,2}}{a_{\text{ini}}}$
- $E_{\text{bind},\text{ini}} = \Delta E_{\text{orb}} = \alpha(E_{\text{orb},\text{fin}} - E_{\text{orb},\text{ini}})$

Can we explain all GW observations with only the isolated formation channel ?

Pair-instability Super Nova

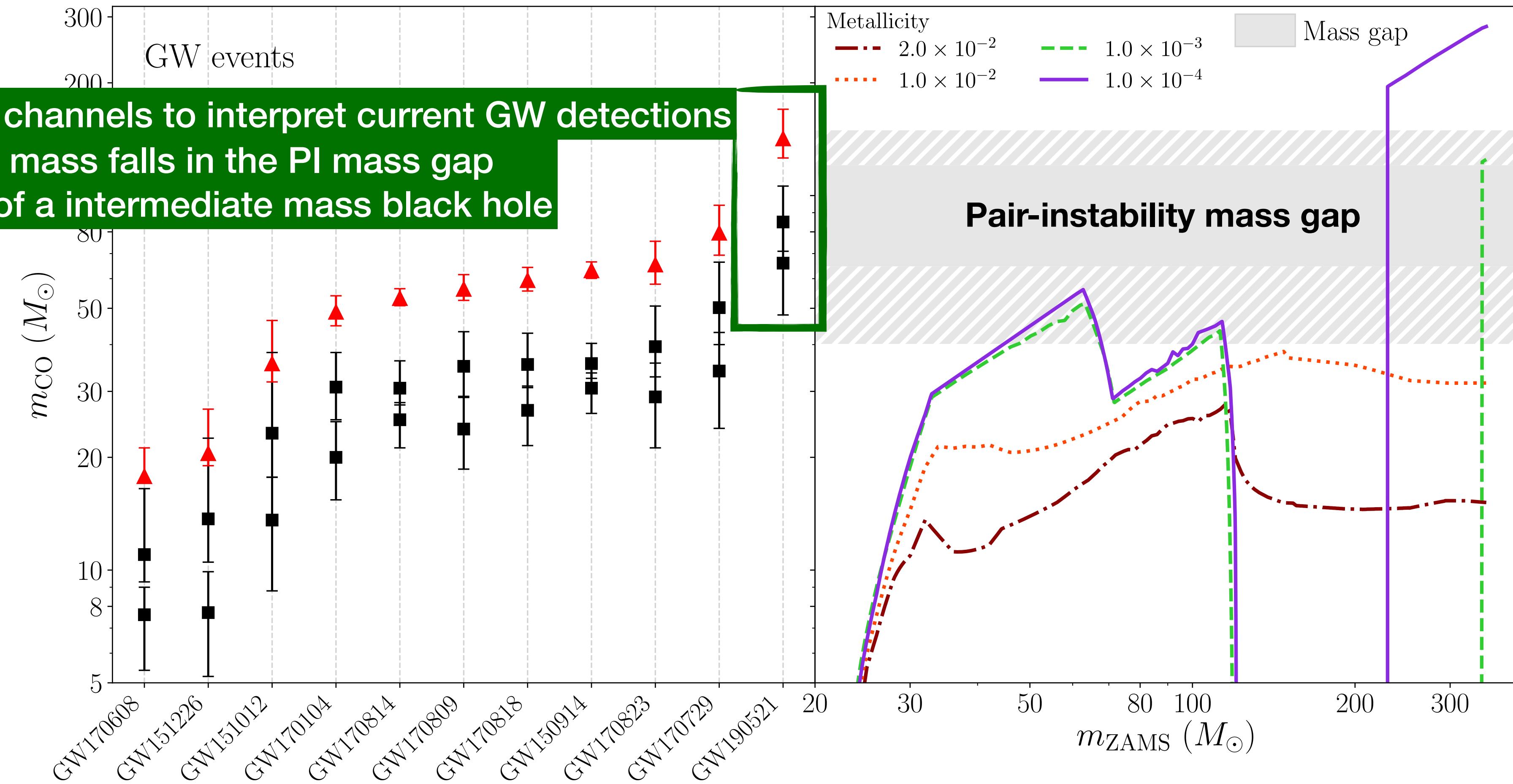
- Very massive stars ($M_{\text{He}} \gtrsim 64 M_{\odot}$)
- Central temperature $T > 7 \times 10^8 \text{ K}$
- Efficient production of gamma ray radiation in the core
- Gamma-ray photons scattering by atomic nucleus produce electron-positron **pairs**
- **Missing radiation pressure** produces dramatic **instability** and collapse, leaving no remnants ➔



Pair-instability Super Nova

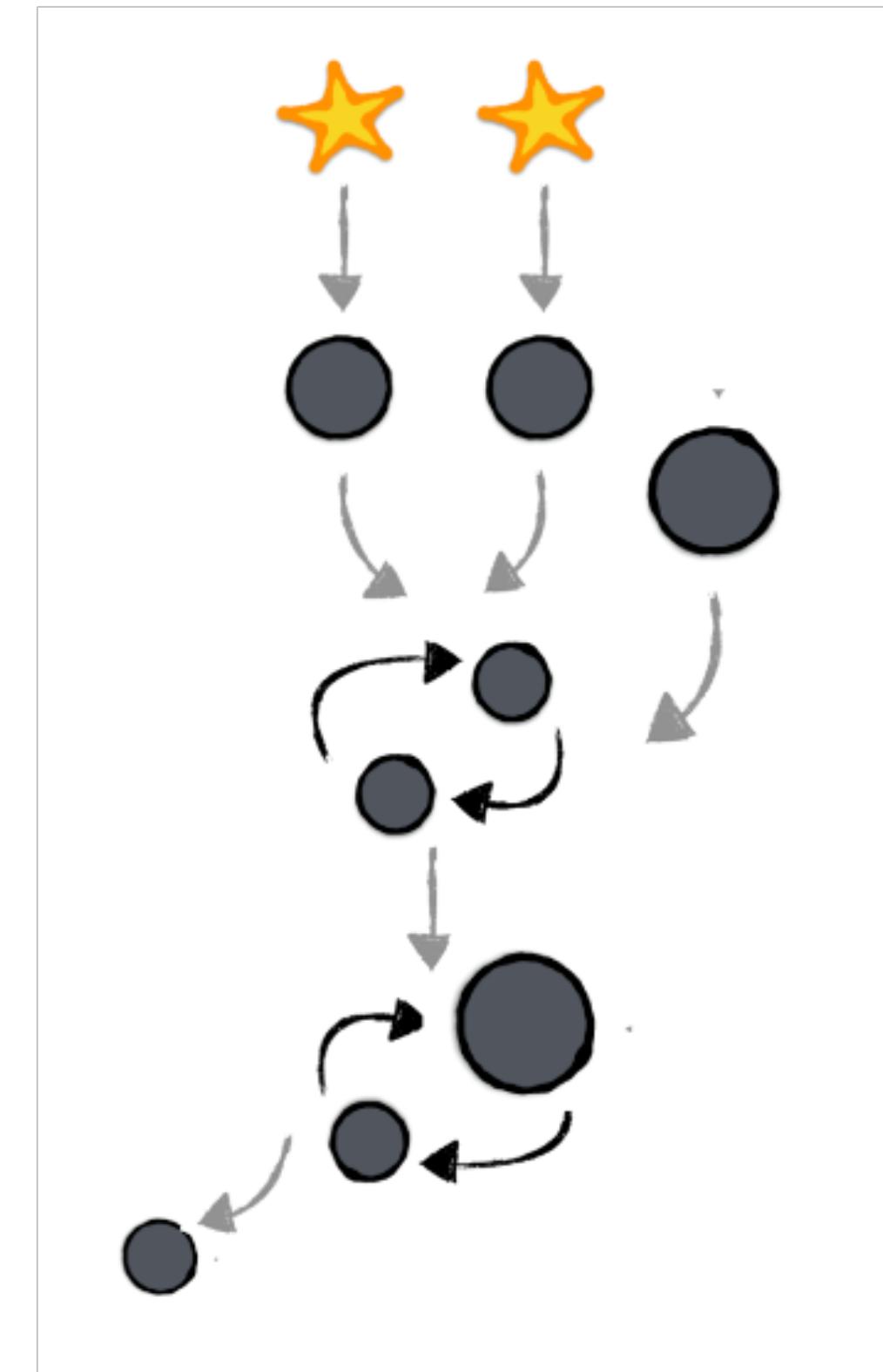
We need other formation channels to interpret current GW detections

1. Primary mass falls in the PI mass gap
2. Formation of a intermediate mass black hole



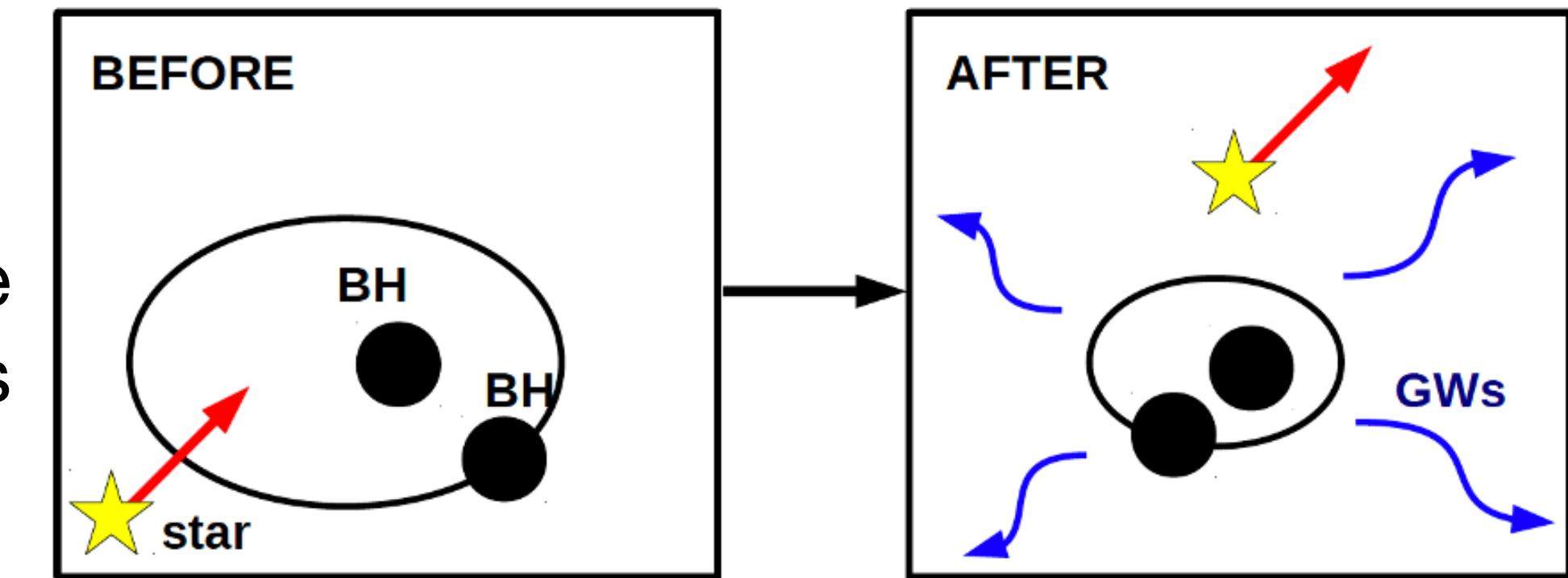
Dynamical formation channel

- Compact objects form and evolve with dynamical processes
- Dynamical processes have effect only with $\rho > 10^3 \text{ stars pc}^{-3}$ (i.e. **Globular Clusters, Nuclear Star Cluster, Young Star Clusters**)
- Star clusters are also an active environment of formation of massive stars

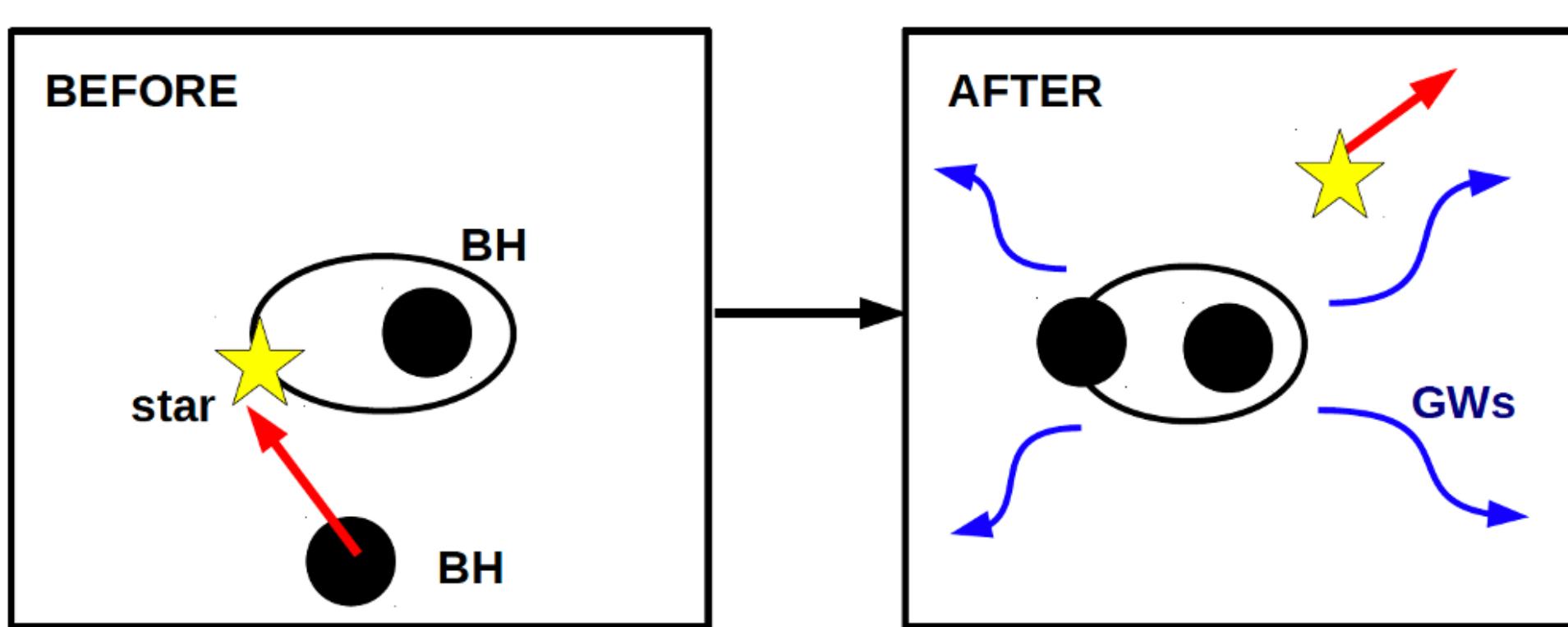


Dynamical formation channel: processes

Hardening: star gains kinetic energy from the binary system → binary system shrinks



Credits: Michela Mapelli

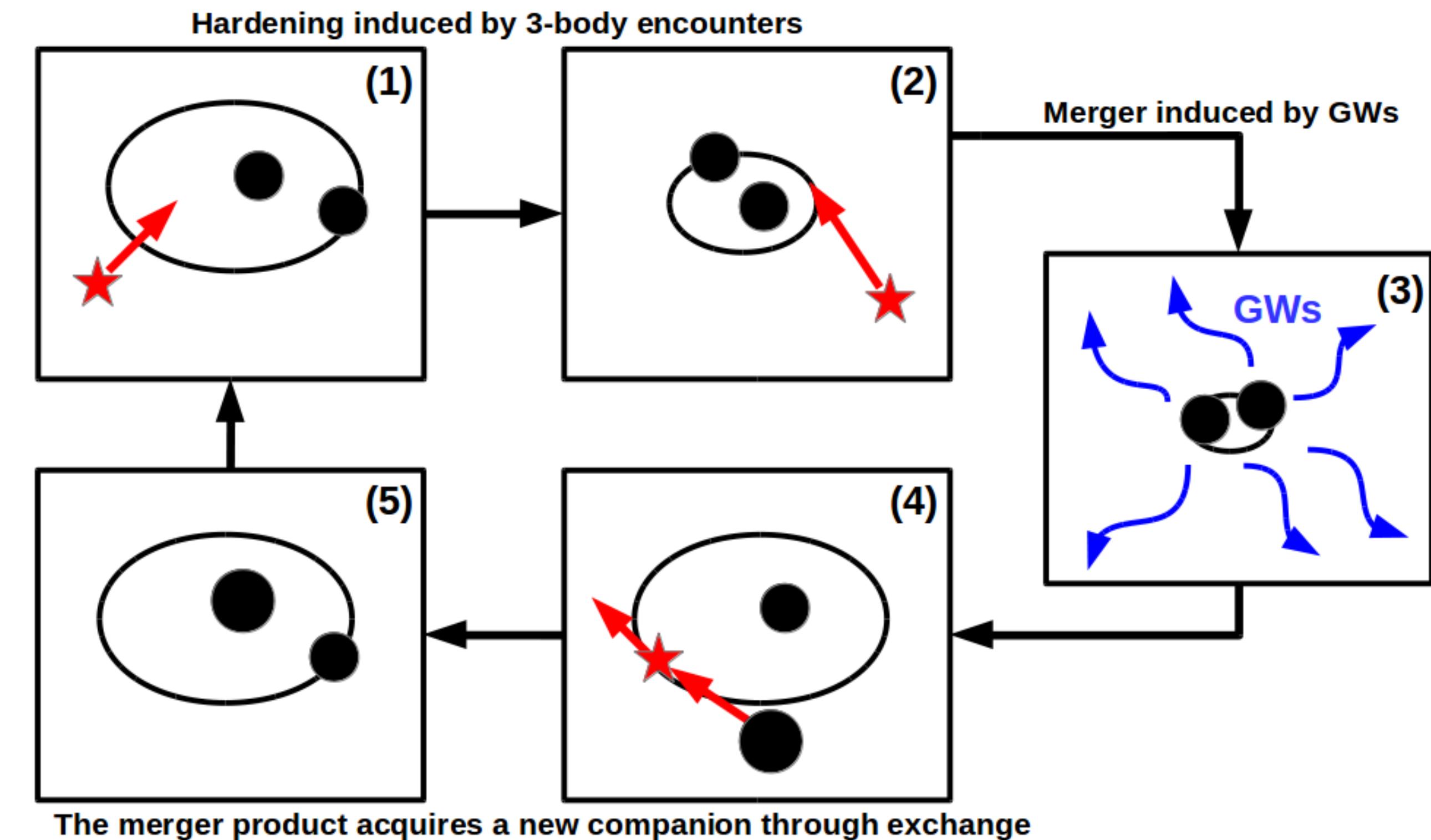


Credits: Michela Mapelli

Exchange: making single BH part of a binary systems → merger of massive BH with misaligned spins

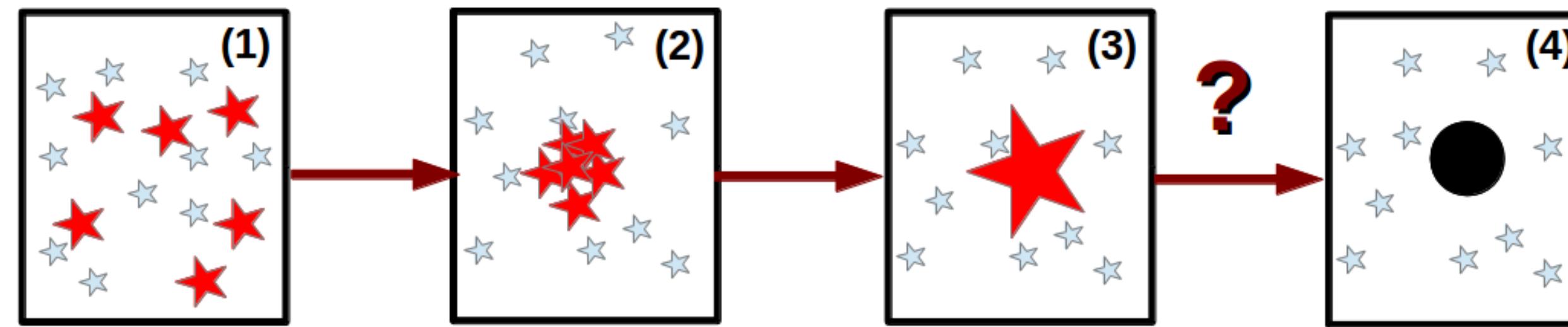
Dynamical formation channel: processes

Hierarchical mergers: Merger remnant can become part of a binary by exchange
➡ BH can grow in mass because of repeated mergers



Credits: Michela Mapelli

Runaway collisions

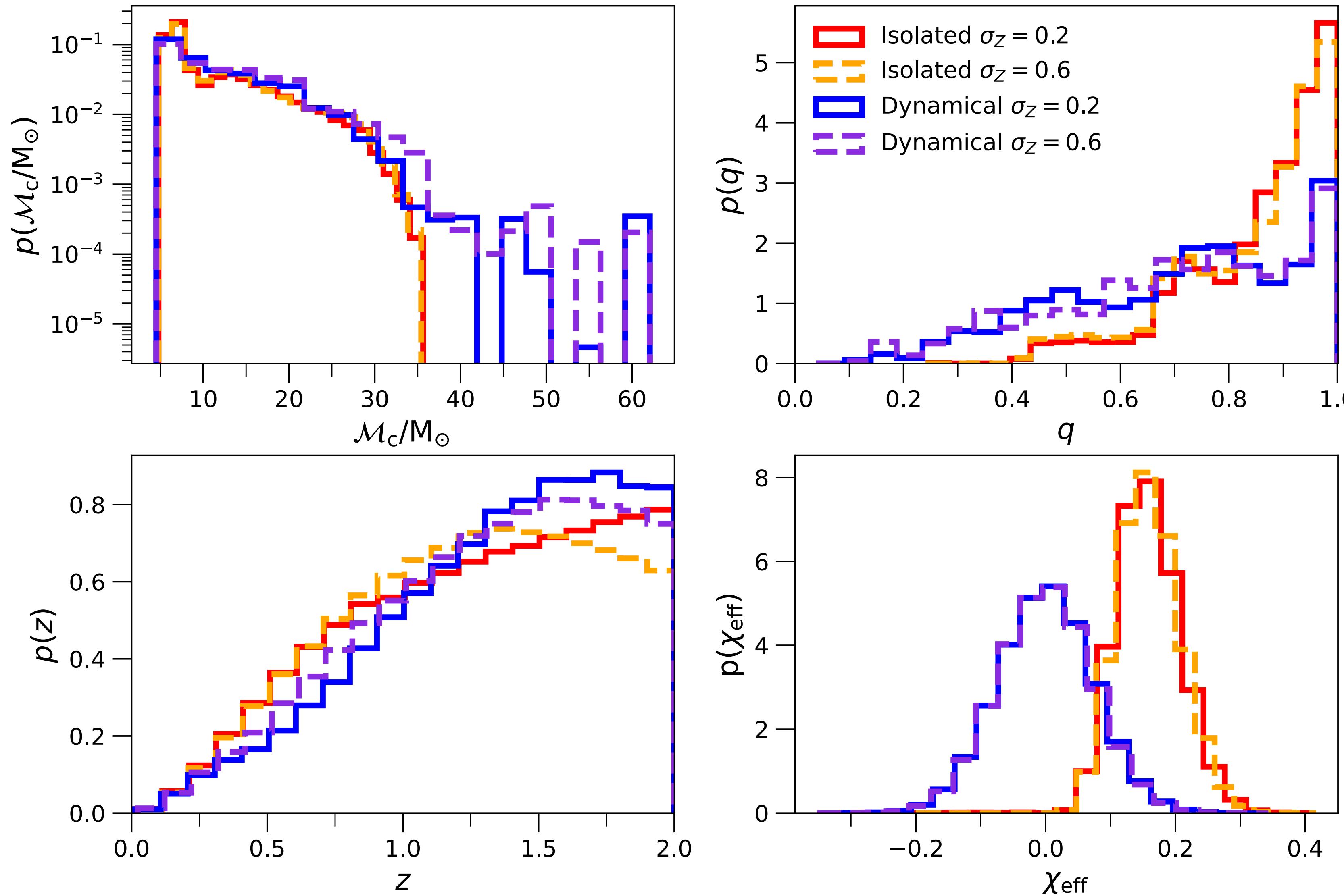


Credits: Michela Mapelli

- Mass segregation brings massive stars into the center
- Massive star collide, merge, and form super-massive stars capable of merging in the **pair-instability mass gap BH**

Is there a predominant formation channel?

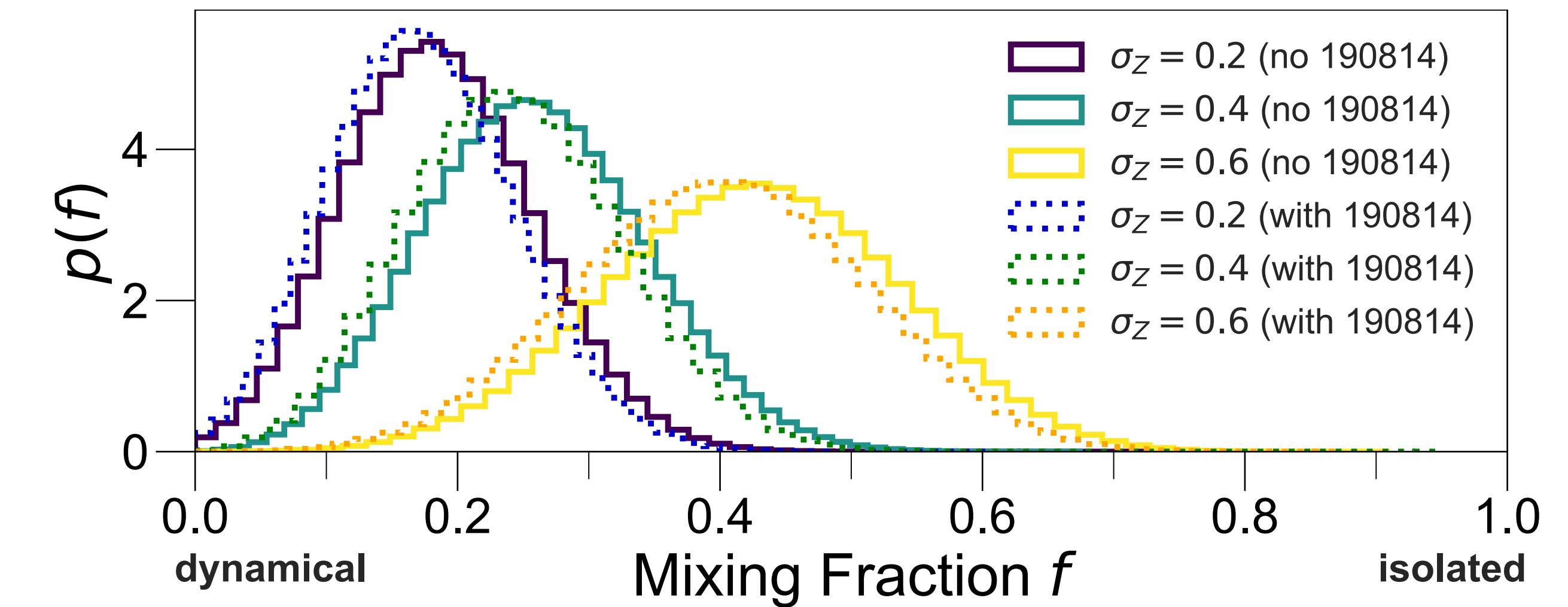
Signatures



Mixing fraction

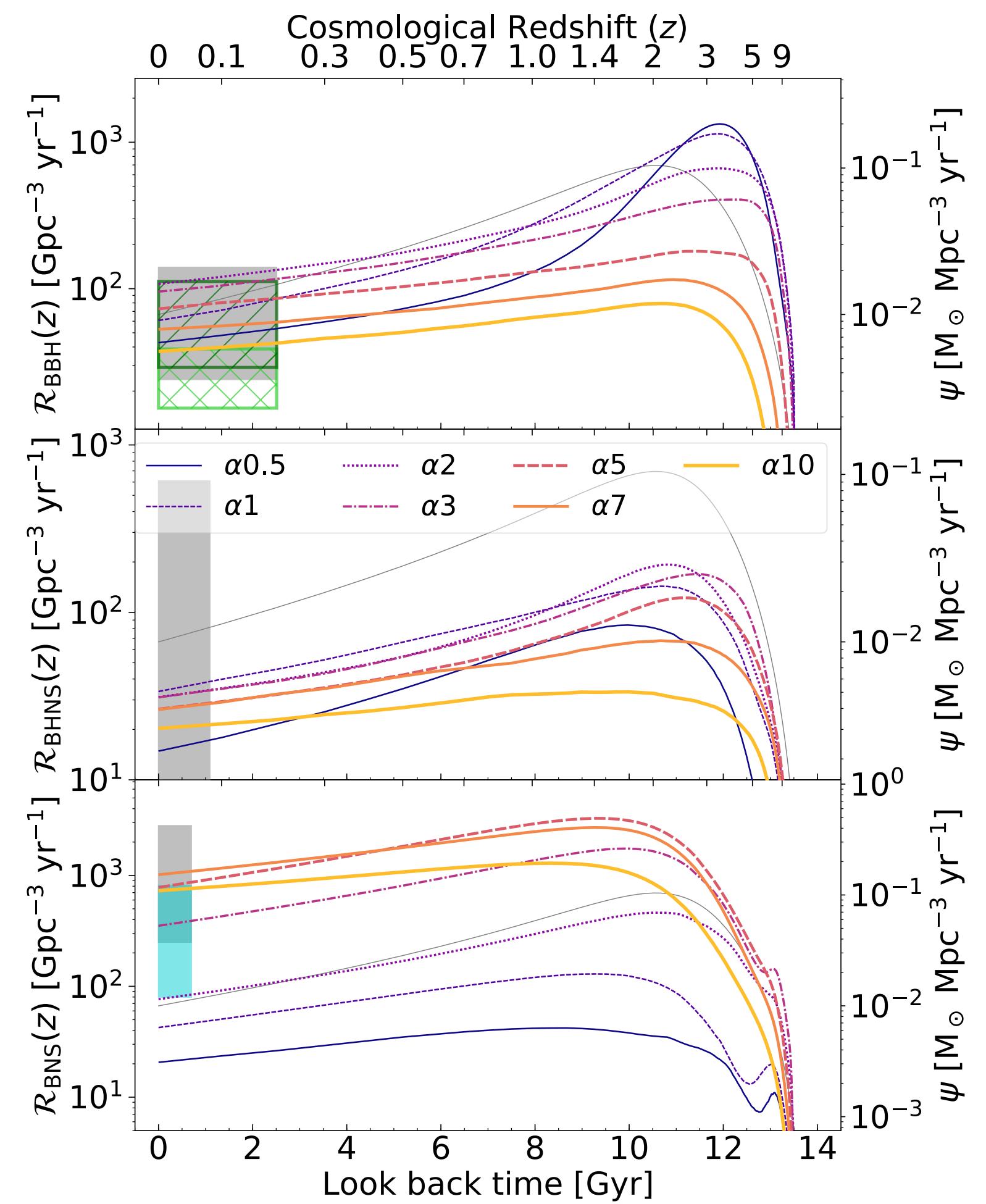
$$\mathcal{L}(\{d\} | \Lambda) \propto \prod_i^{N_{obs}} \frac{\int \mathcal{L}(d_i | \theta) \pi(\theta | \Lambda)}{\xi(\Lambda)}$$

$$\pi(\theta | \Lambda) = f p(\theta | \text{iso}, \sigma_z) + (1 - f)p(\theta | \text{dyn}, \sigma_z)$$



Today's hands on

- `cosmoRate`
- d_L (z , assuming a cosmology) is a waveform parameter
- A given population model must include a redshift distribution
- evaluate the merger rate density given a population of compact object mergers



What you did (not) learn today

Tomorrow

- Observed population properties
- Astrophysical processes driving the isolated and dynamical formation channels
- Multimessenger Astrophysics
- Properties of host galaxies of compact object mergers

Further reading:

- This lecture is based on lecture materials from Marica Branchesi, Jan Harms, Tito Dal Canton, Michela Mapelli, Giuliano Iorio, Gaston Escobar, and Eleonora Loffredo
- References:
 - **Non-parametric models:** [Mandel et al. 2017](#), [Li et al. 2021](#), [Rinaldi and Dal Pozzo 2022](#),
 - **Astrophysics of compact objects:** [Mapelli 2018](#), [Spera et al. 2022](#) (with dynamics), [Costa et al. 2023](#) (and references therein)
- **See you this afternoon!**