

Gravitational Wave Astrophysics

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In these lectures, you'll learn

1. Detectors, waveforms, parameter estimation
2. Population analysis and astrophysics of compact objects
3. Multimessenger astrophysics and host galaxies
4. Population III stars, Einstein Telescope and the future of GWs

You'll learn how to use

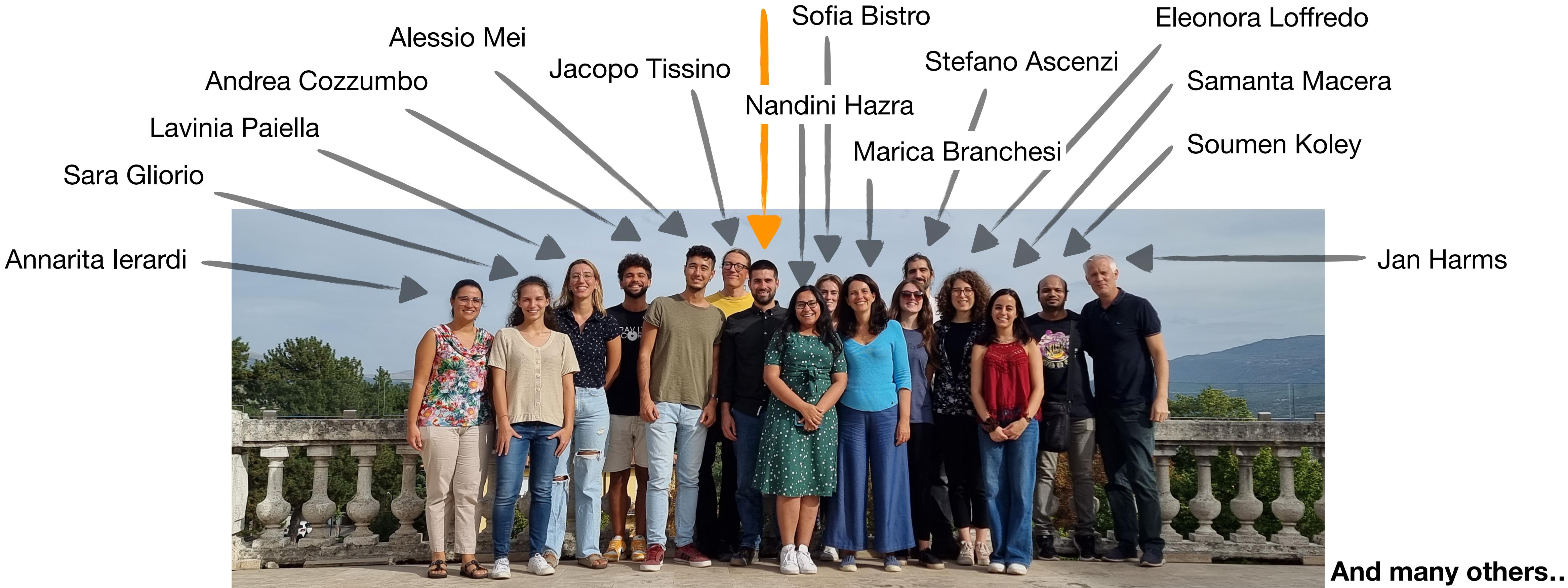
1. GWFish
2. cosmoRate



In this lecture, you'll learn

- GW milestones
- Detection process
- Parameter estimation of single events

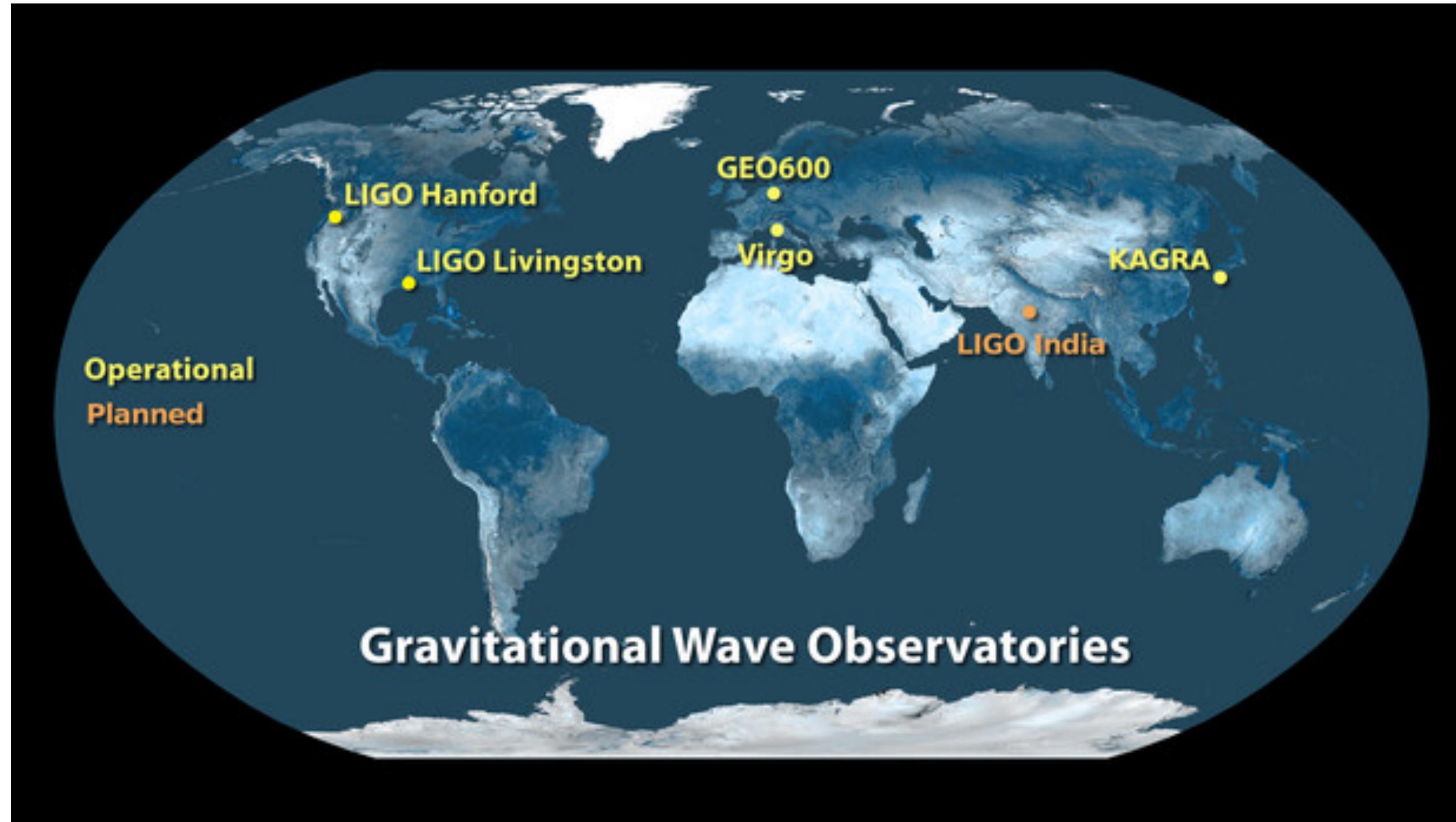
I introduce myself



Contact me at filippo.santoliquido@gssi.it or visit <https://filippo-santoliquido.github.io/>

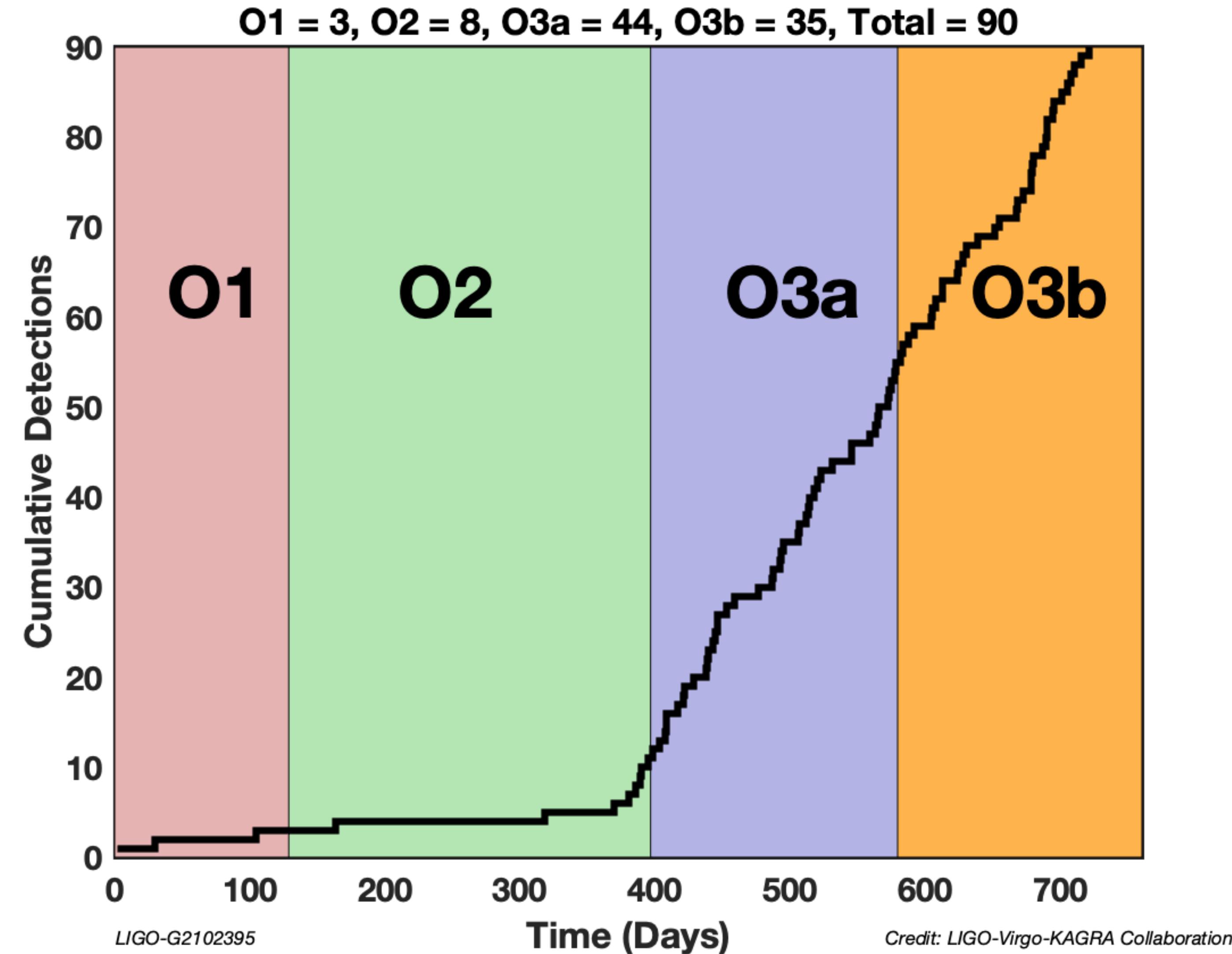
Ask questions, please

Detectors

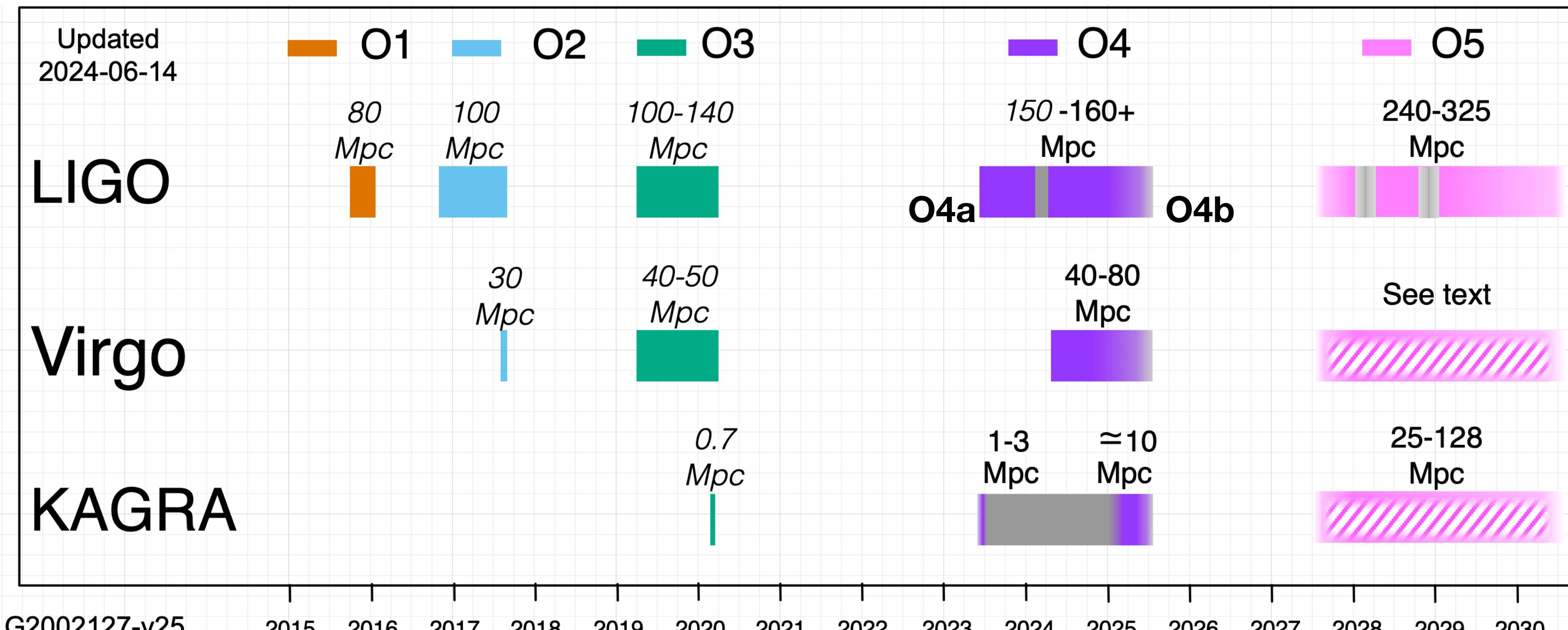


Credits: LIGO-Virgo-KAGRA collaboration

Observing runs

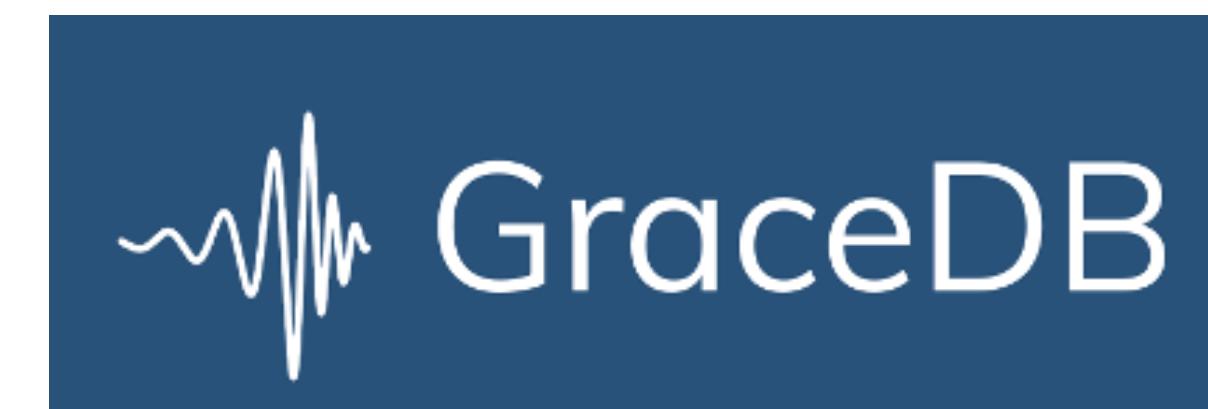


Ongoing observing run



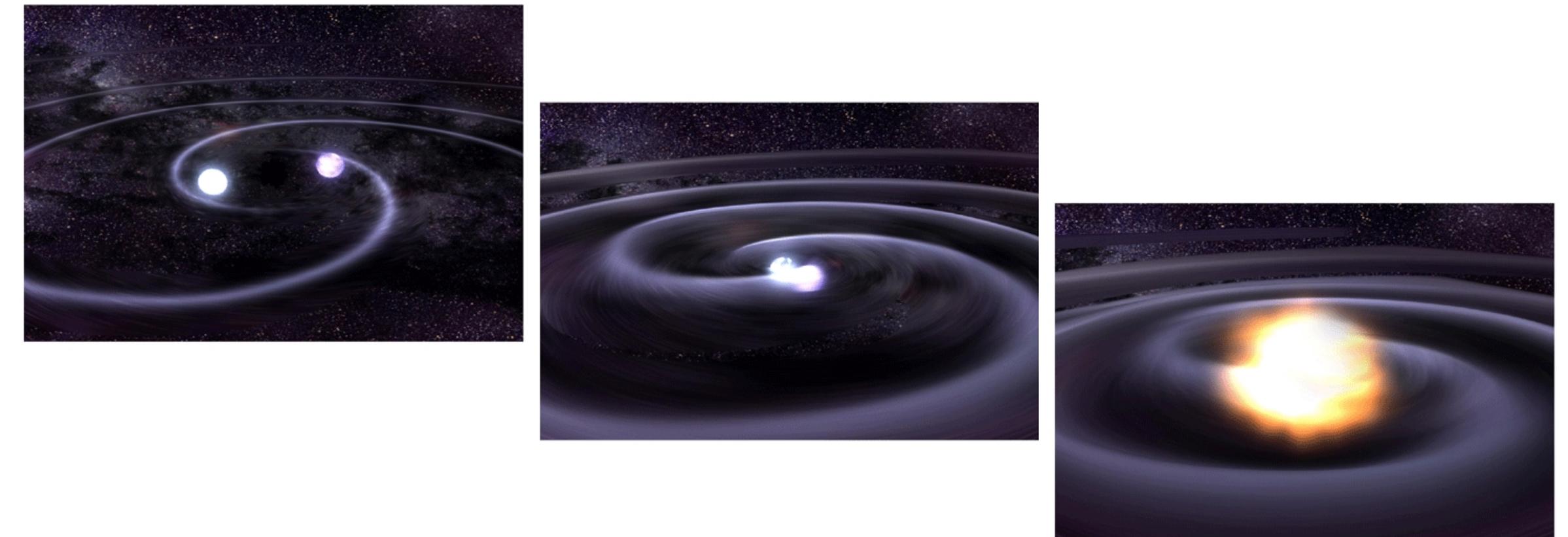
Credits: LIGO-Virgo-KAGRA collaboration

- *O4b will now end 9 June 2025.*
- [Public alerts](#)



Detected signals

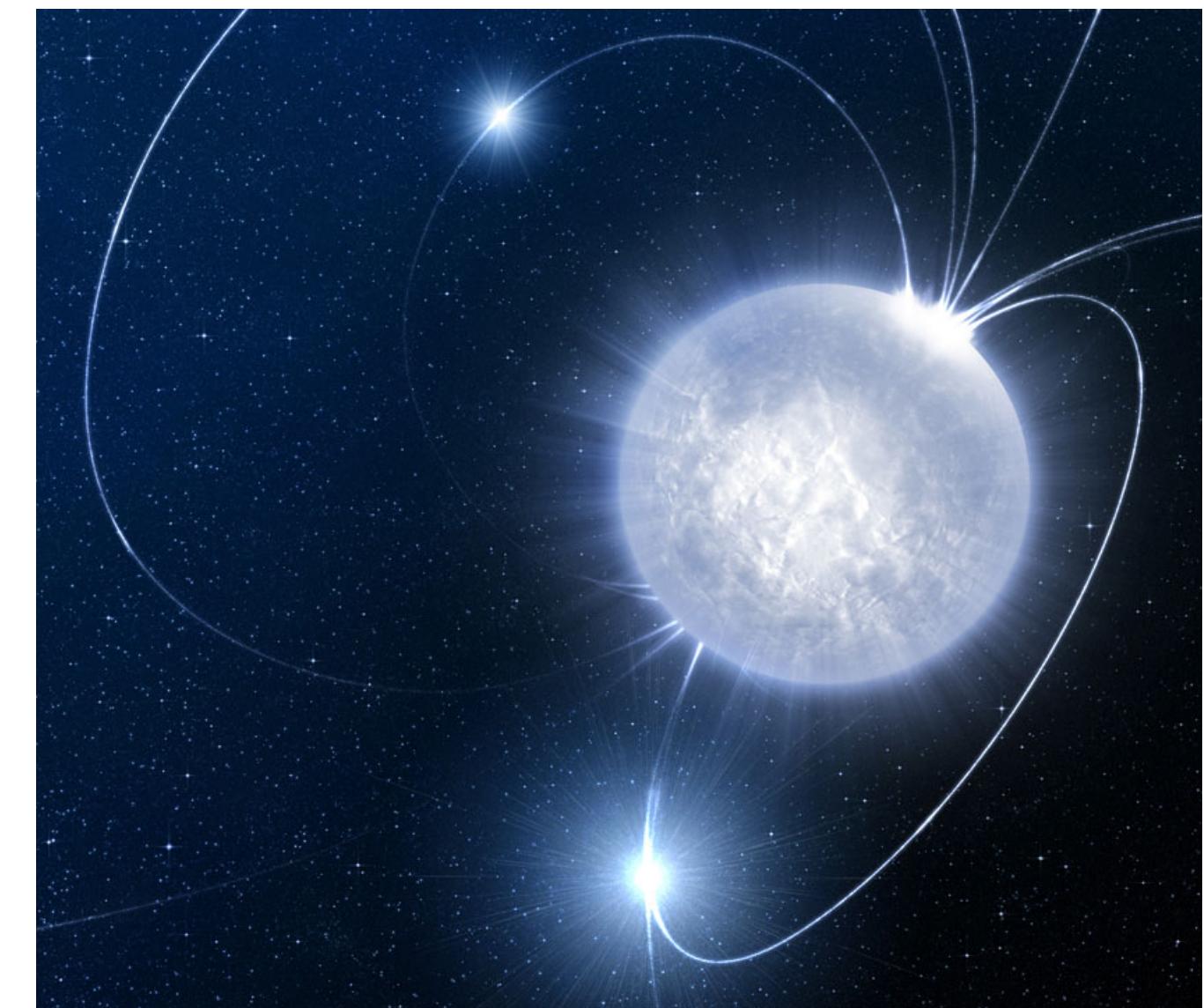
- CBC = compact binaries coalescence
 - Binary Black Holes (**BBHs**)
 - Binary Neutron Stars (**BNSs**)
 - Black hole-neutron star binaries (**BHNSs**)



Credits: LIGO-Virgo-KAGRA collaboration

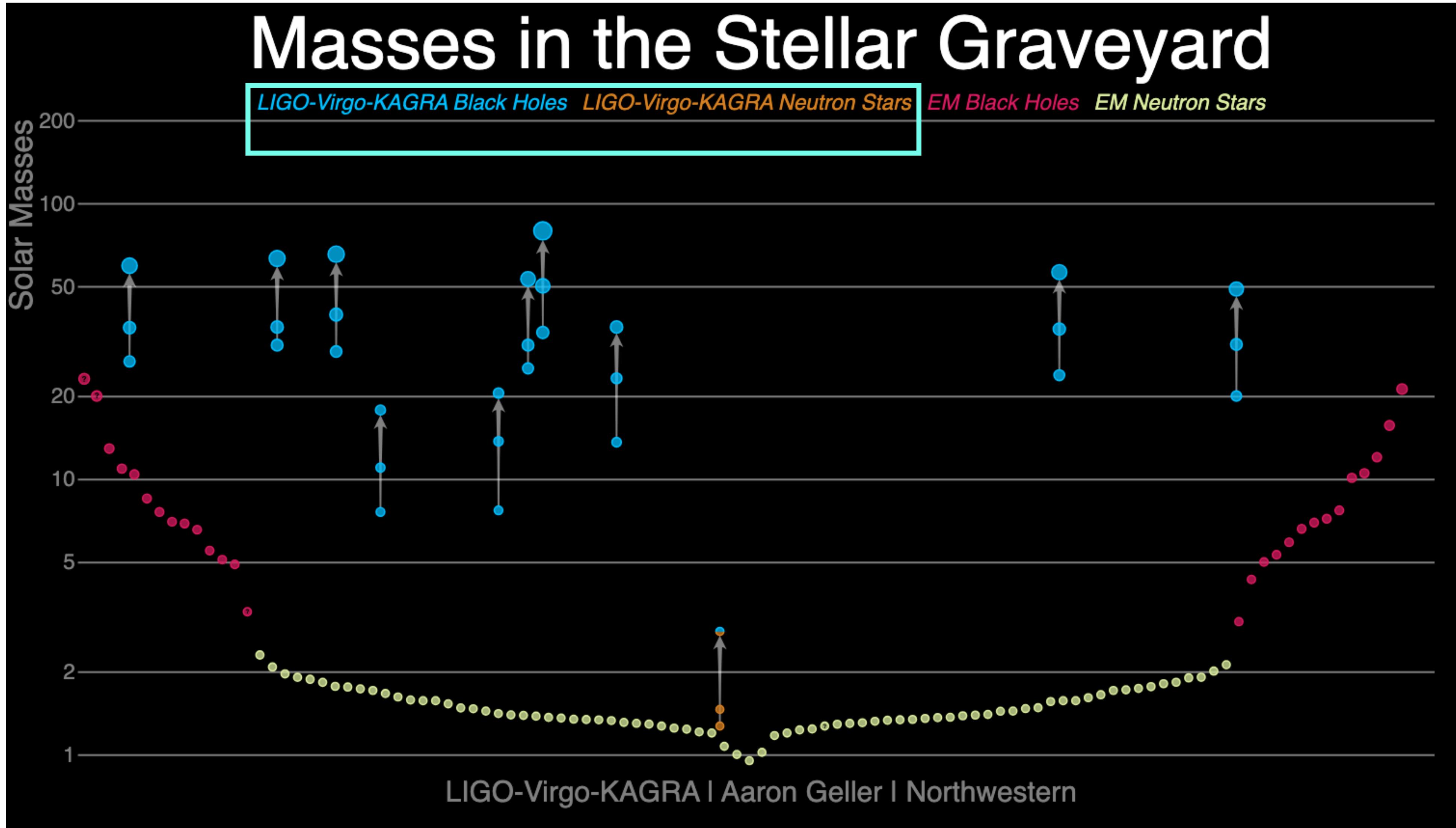
Detected signals

- CBC = compact binaries coalescence
 - Binary Black Holes (**BBHs**)
 - Binary Neutron Stars (**BNSs**)
 - Black hole-neutron star binaries (**BHNSs**)
- Other (undetected) sources:
 - short-duration burst
 - Continuous waves
 - Stochastic Background

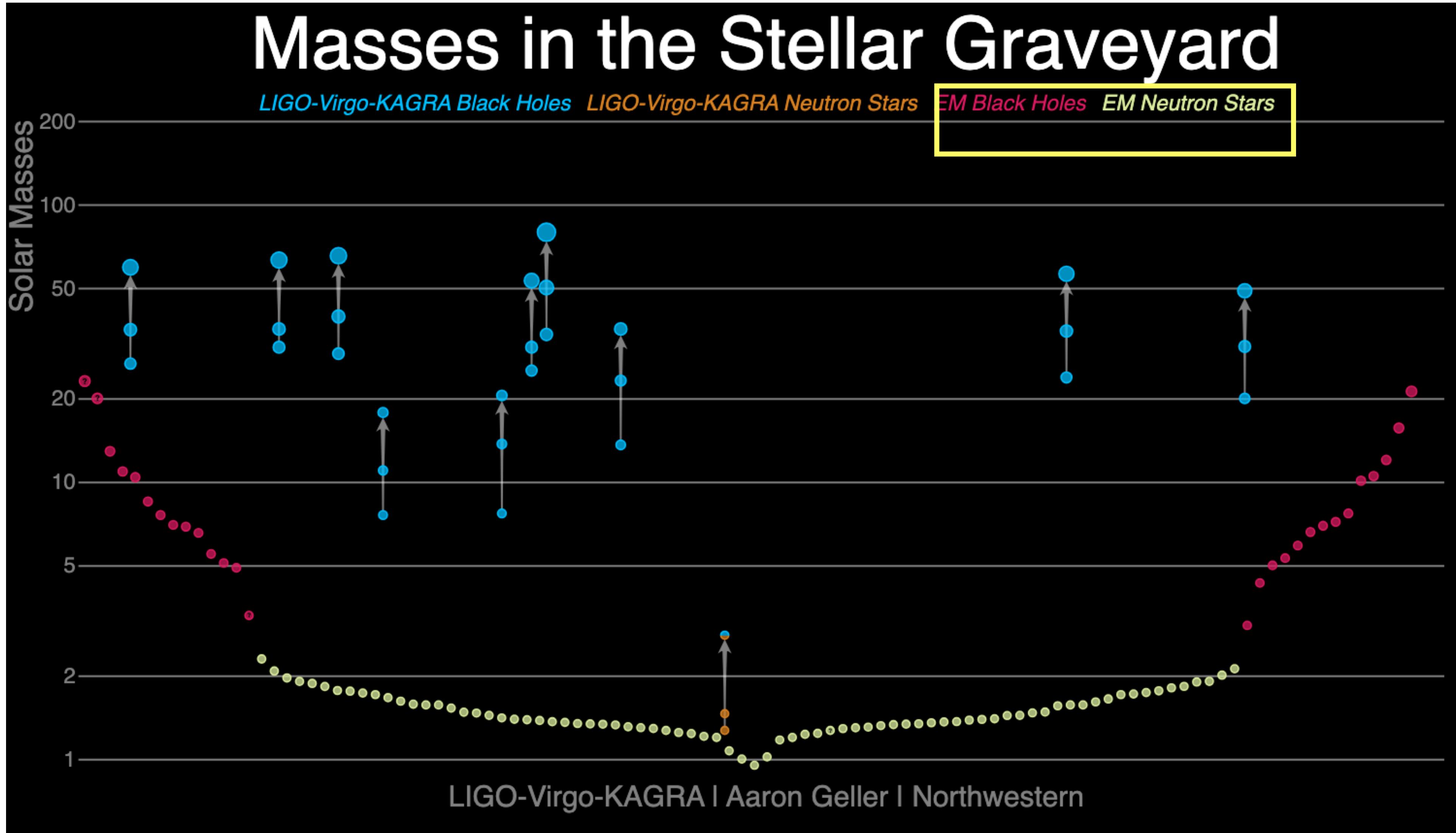


Credits: [ESO](#)

First Gravitational Wave Transient Catalog

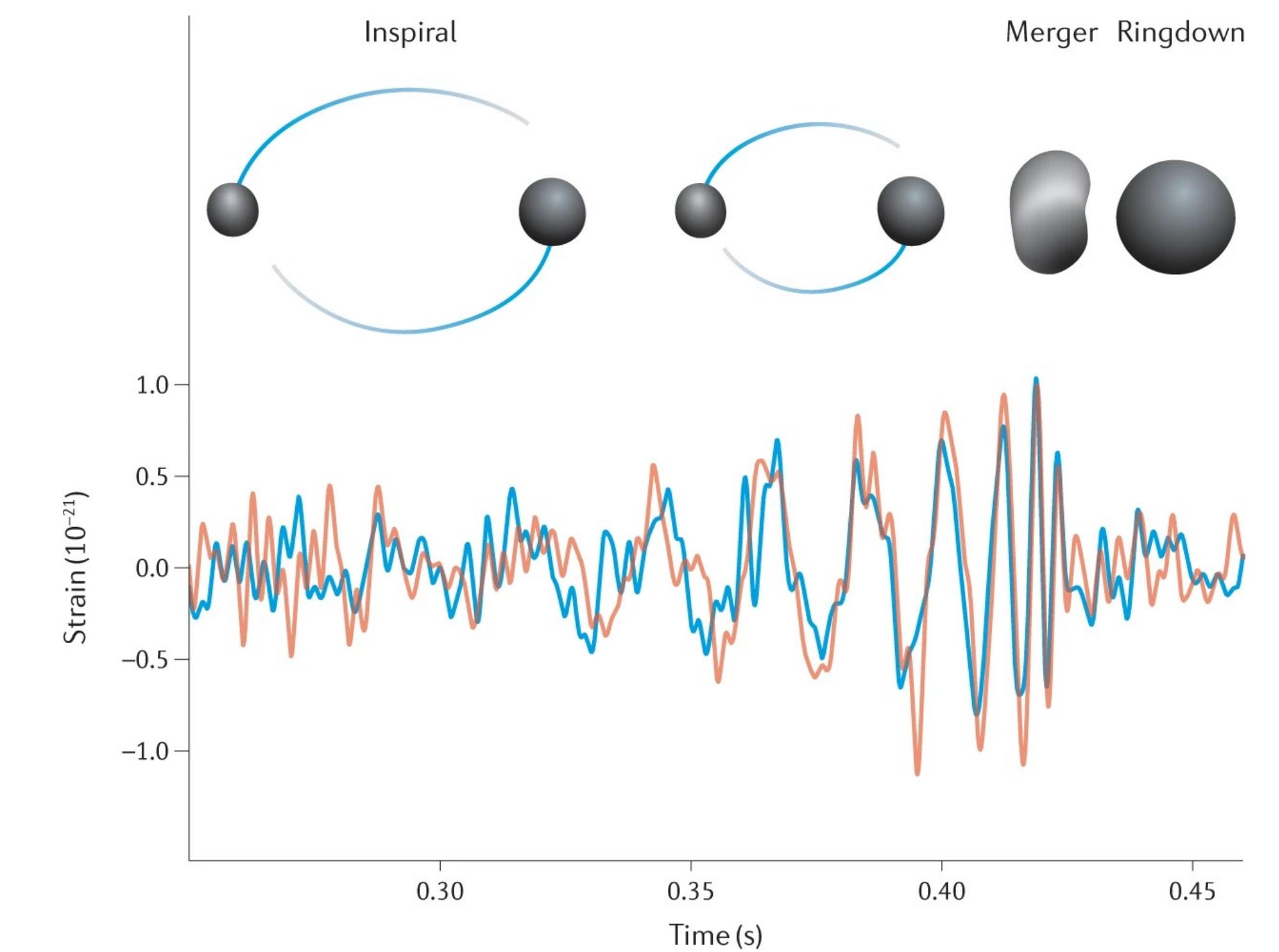
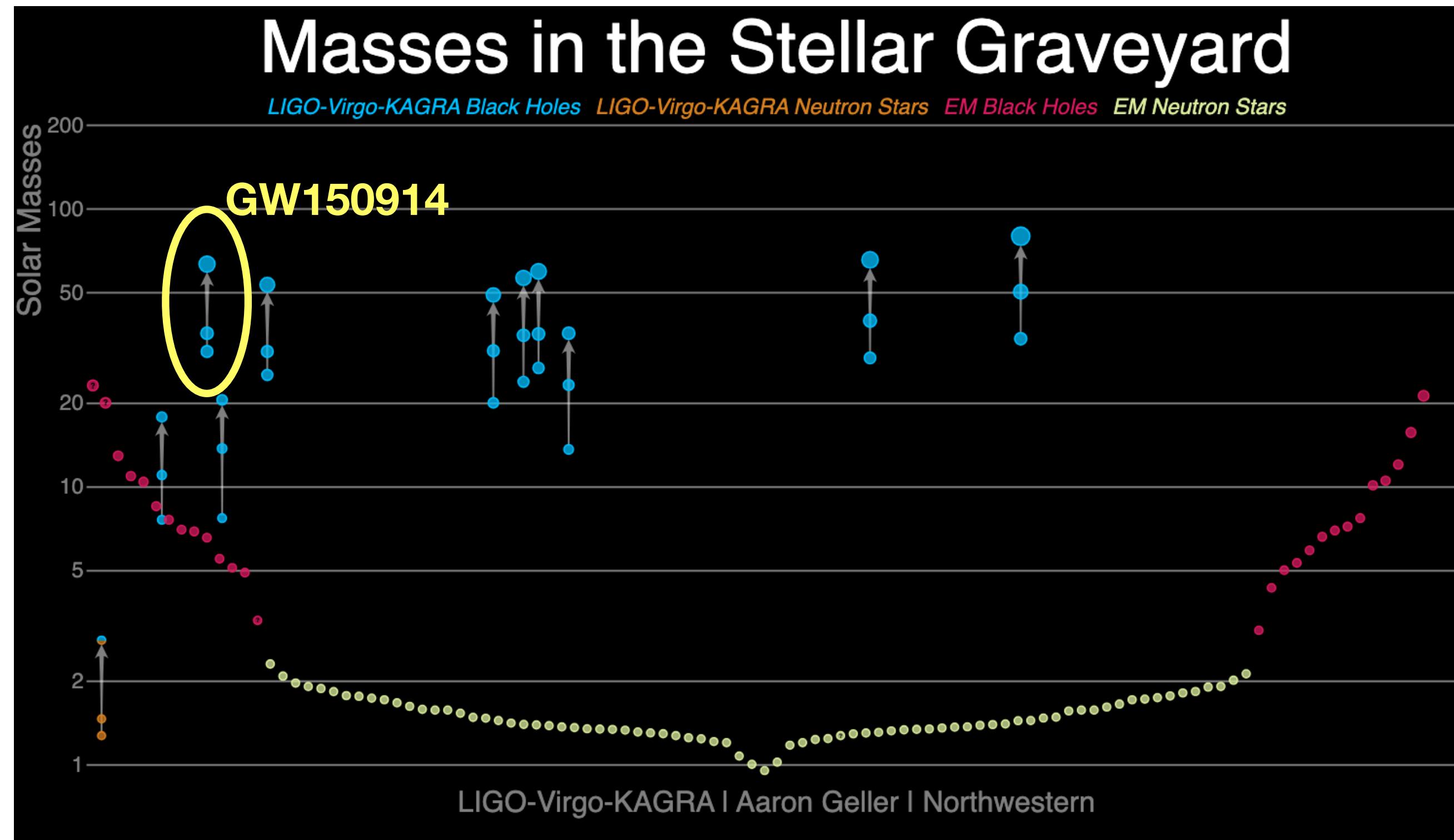


First Gravitational Wave Transient Catalog



GWTC-1

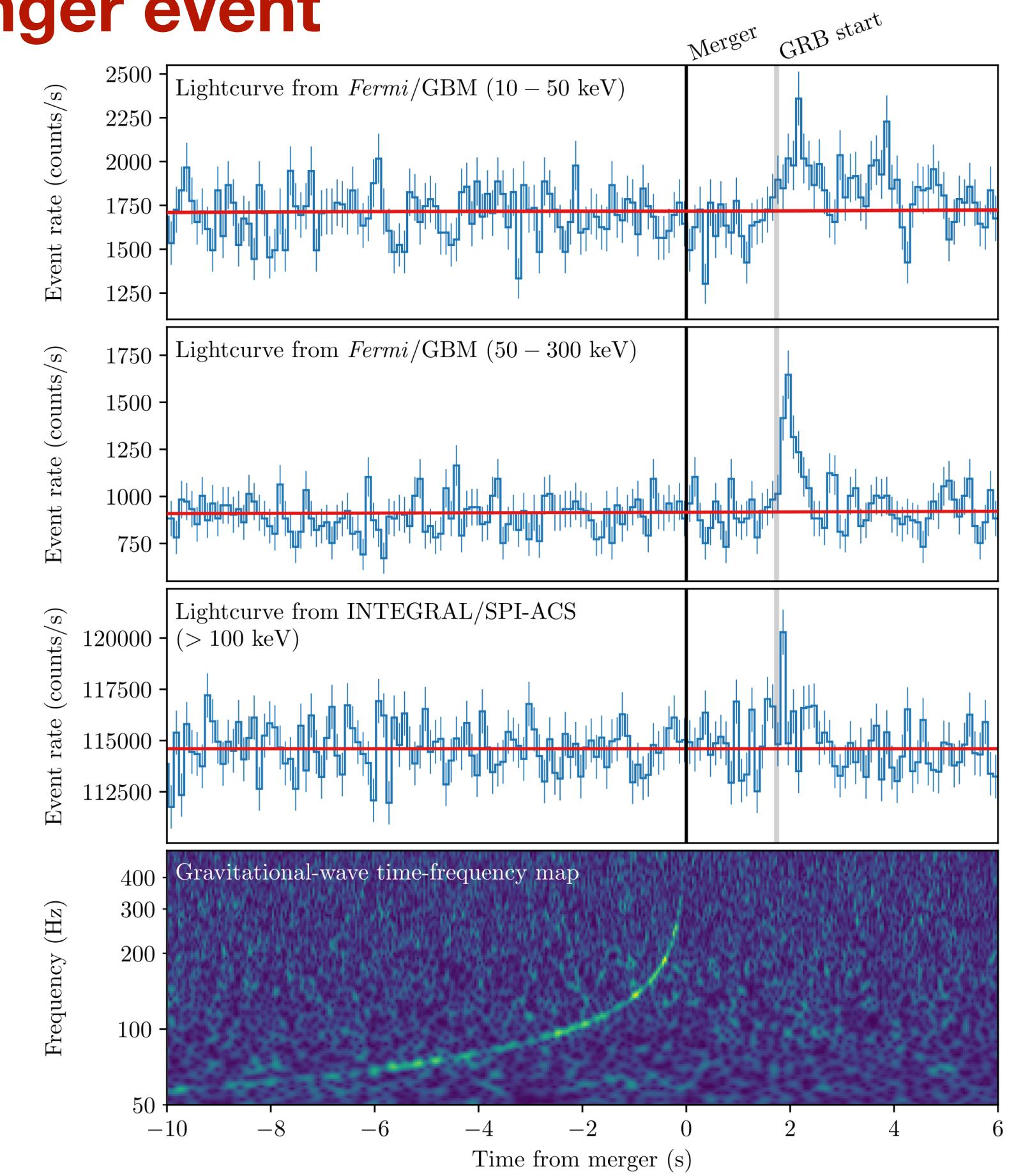
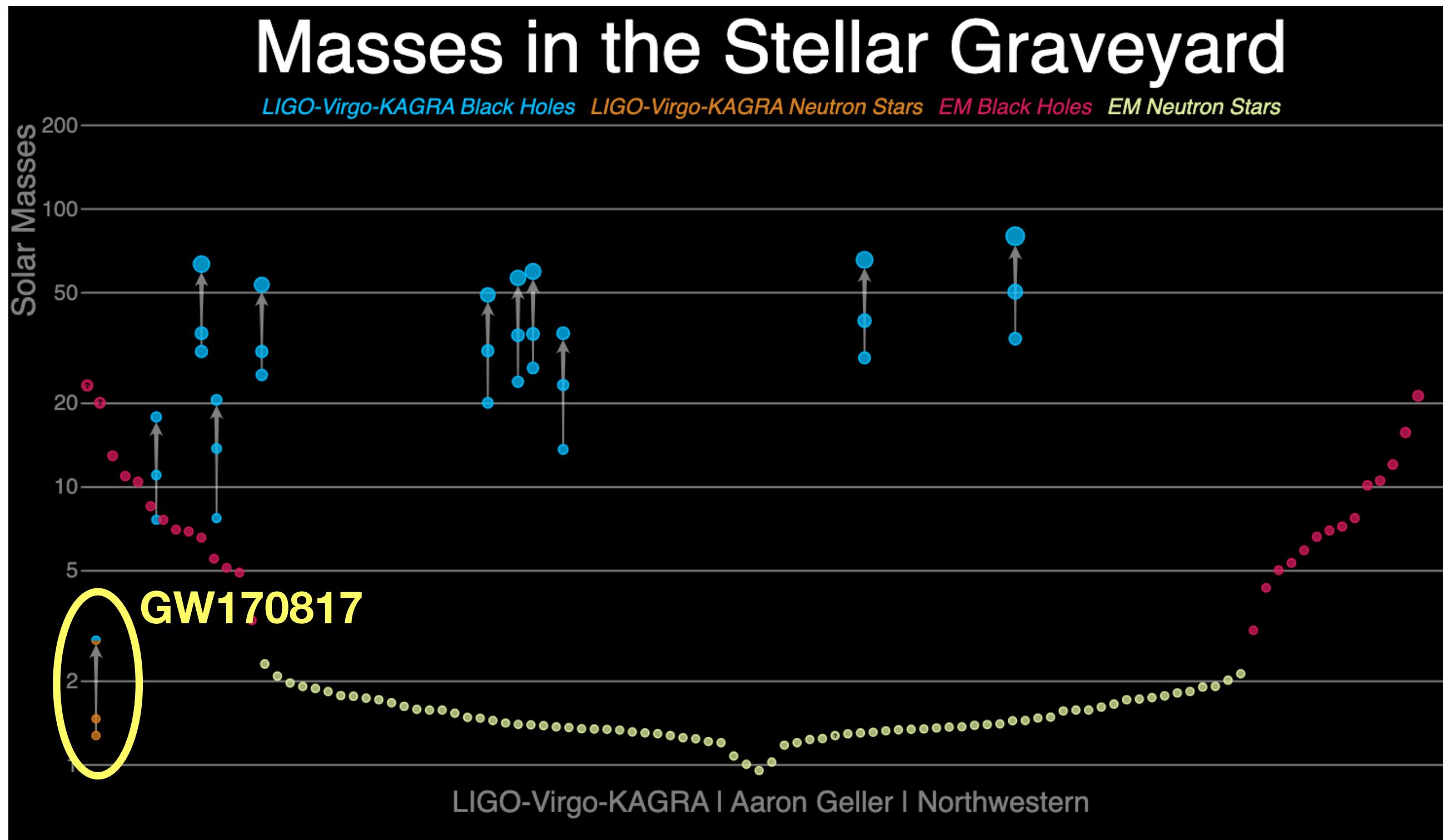
1st milestone: first ever detected direct GW signal



Credits: [Bailes et al. 2021](#)

GWTC-1

2nd milestone: first multimessenger event



Credits: Abbott et al. 2017

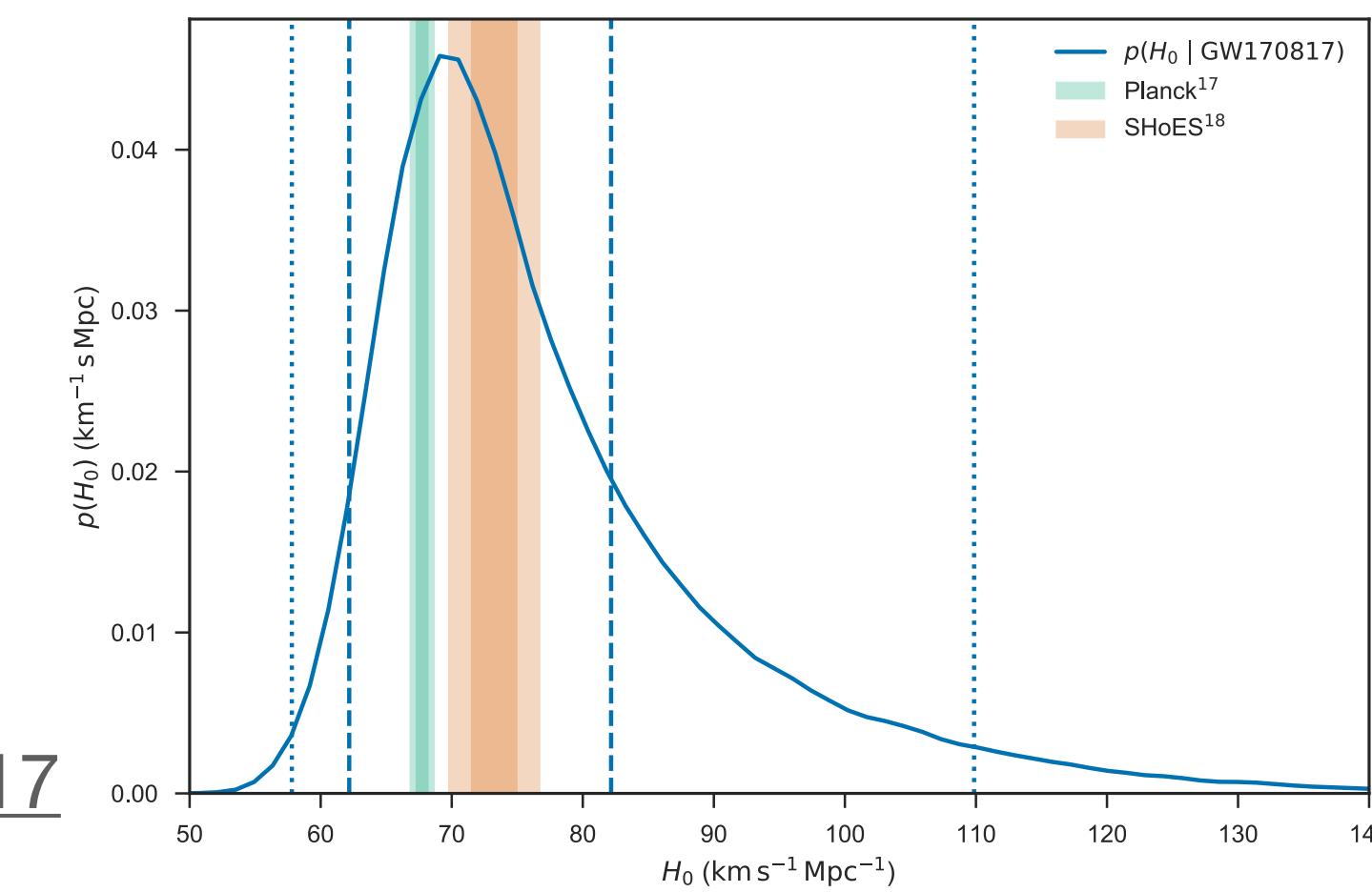
Multimessenger astrophysics

- Definitive link between BNS merger and short GRB
- observation of a kilonova
- BNS mergers produce heavy elements
- GWs travel at the same speed as light
- Indipendente measure of the Hubble constant

Credits: [Abbott et al. 2017](#)

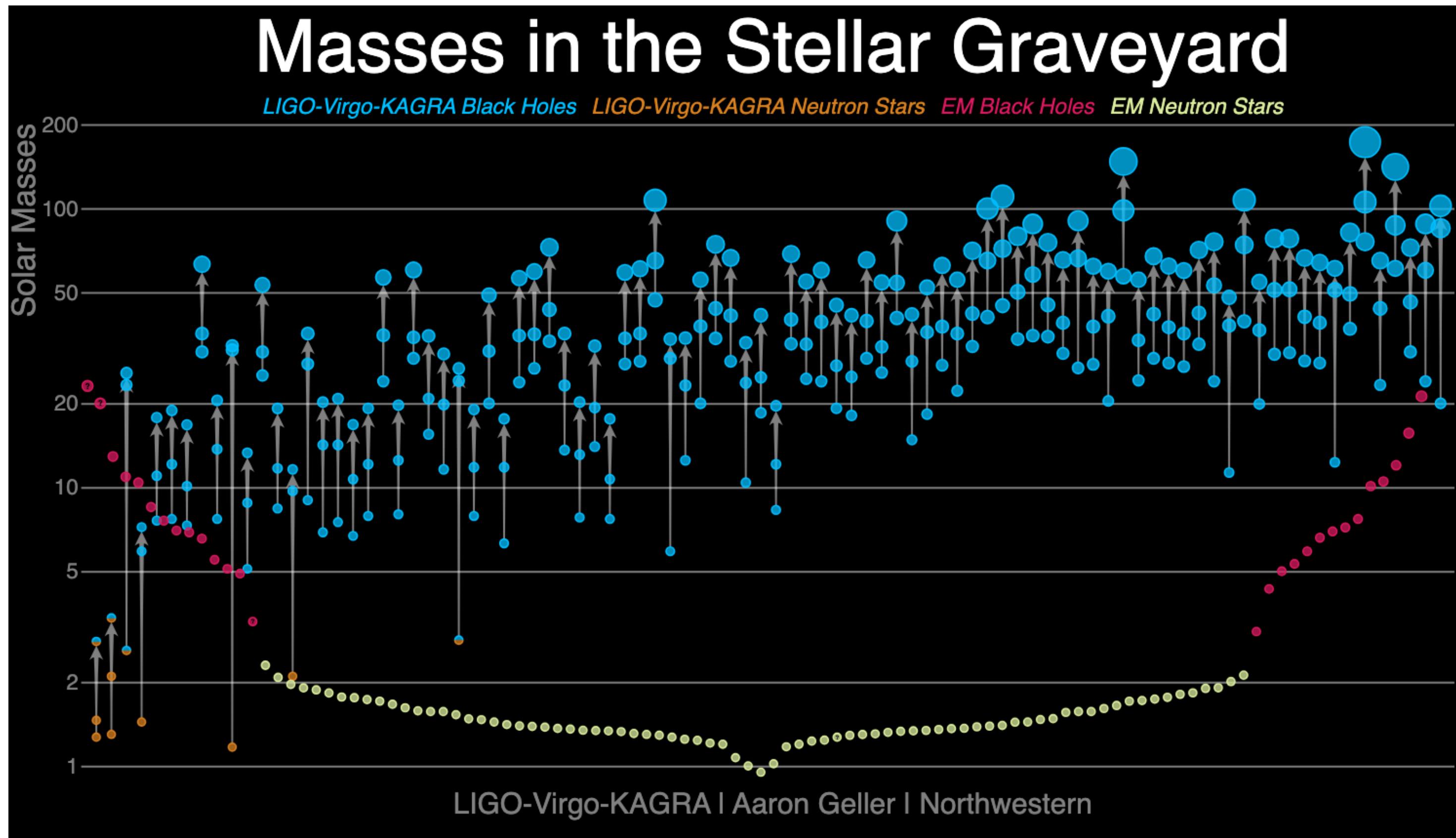


Credits: NASA



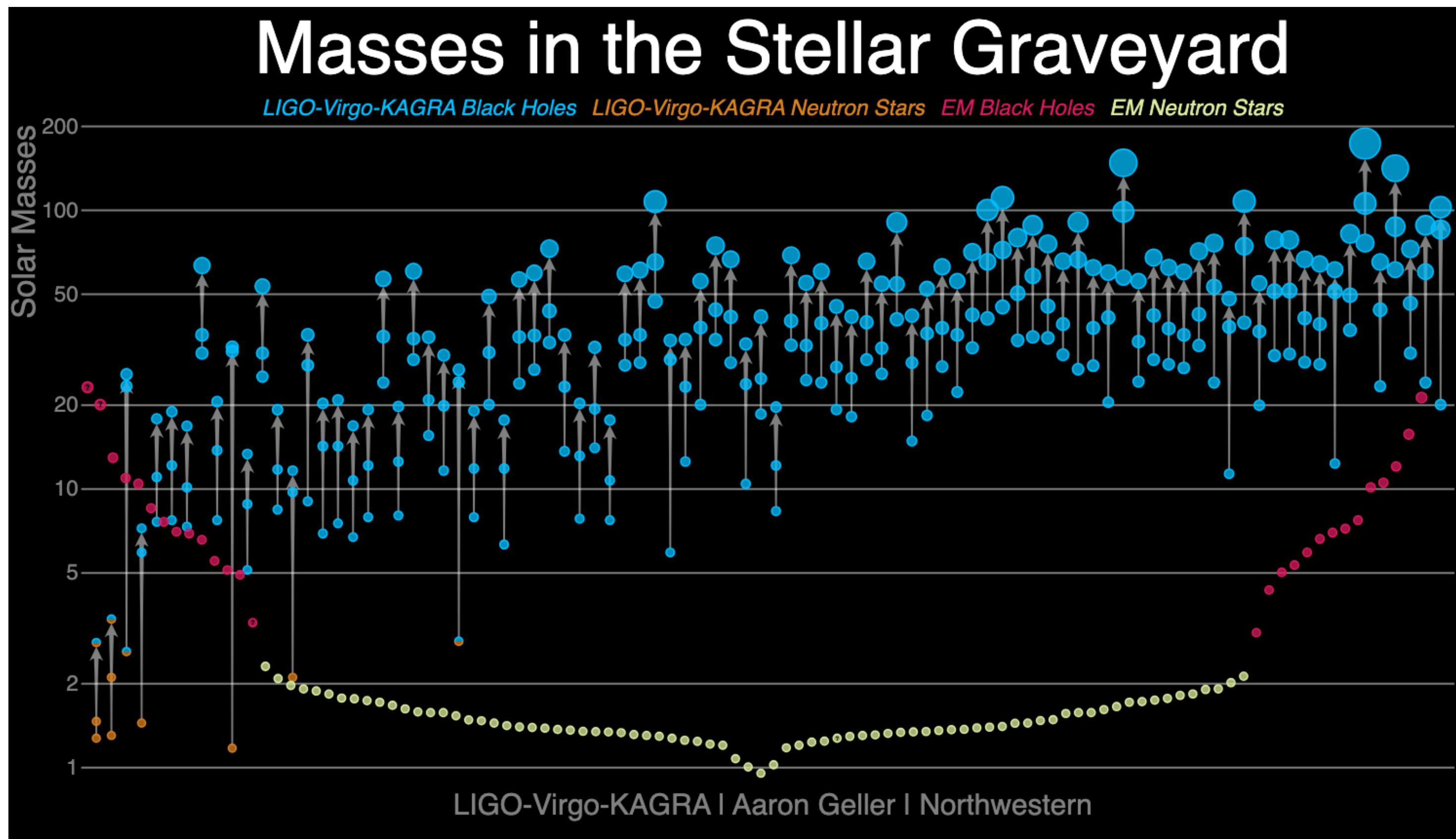
Credits: NASA

GWTC-2.1 and GWTC-3



What does this chart already tell you about GW detections ?

GWTC-2.1 and GWTC-3



Population studies

Frequency

- Dynamical frequency: $f_{dyn} = \sqrt{\frac{3GM}{16\pi^2 R^3}}$
- Schwarzwild radius: $r_s = \frac{2GM}{c^2}$
- Examples:
 - **neutron star:** $M = 1.4 M_\odot$, $R = 10$ km, $f_{dyn} \sim 1.9$ kHz
 - **black hole:** $M = 10 M_\odot$, $R = 30$ km, $f_{dyn} \sim 1$ kHz
 - **super massive black holes:** $M = 4.5 \times 10^6 M_\odot$, $R = 1.3 \times 10^7$ km, $f_{dyn} \sim 2$ mHz

Amplitude

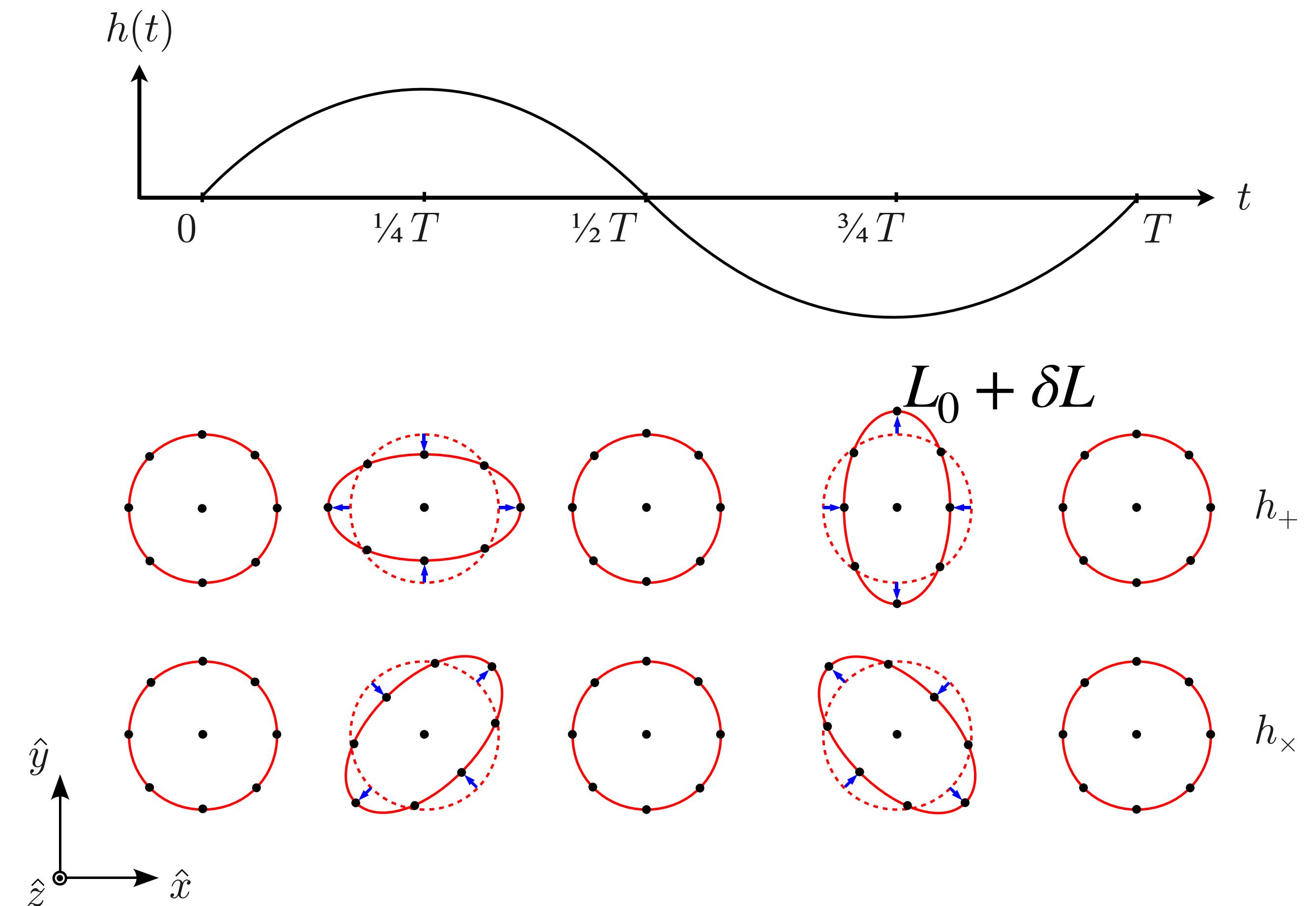
- Variation of quadrupole moment: $|\ddot{Q}_{ij}| < \frac{d^2}{dt^2} \int \rho x^3 dx \sim (2\pi f_{dyn})^2 M R^2$
- Quadrupole formula: $h_{ij} = \frac{G}{c^4} \frac{2}{r} \left[\ddot{Q}_{ij} \right]_{ret}^{TT}$ (plane waves)
- GW amplitude: $h \sim 2 \left(\frac{2GM}{Rc^2} \right) \left(\frac{GM}{rc^2} \right)$ where r is the distance from the source

Amplitude

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- How do you expect a CBC GW signal to be like?

Polarisation

- Gravitational waves have two polarisations, + and ×
- $\omega = 2\pi/T$
- Effect on a ring of freely falling particles
- $\delta L \sim \frac{1}{2} h L_0$
- This means for a detector on Earth of $L_0 = 1 \text{ km}$, $\delta L \sim 10^{-18} \text{ m}$ due to GW. Size of an atomic nucleus $\sim 10^{-15} \text{ m}$



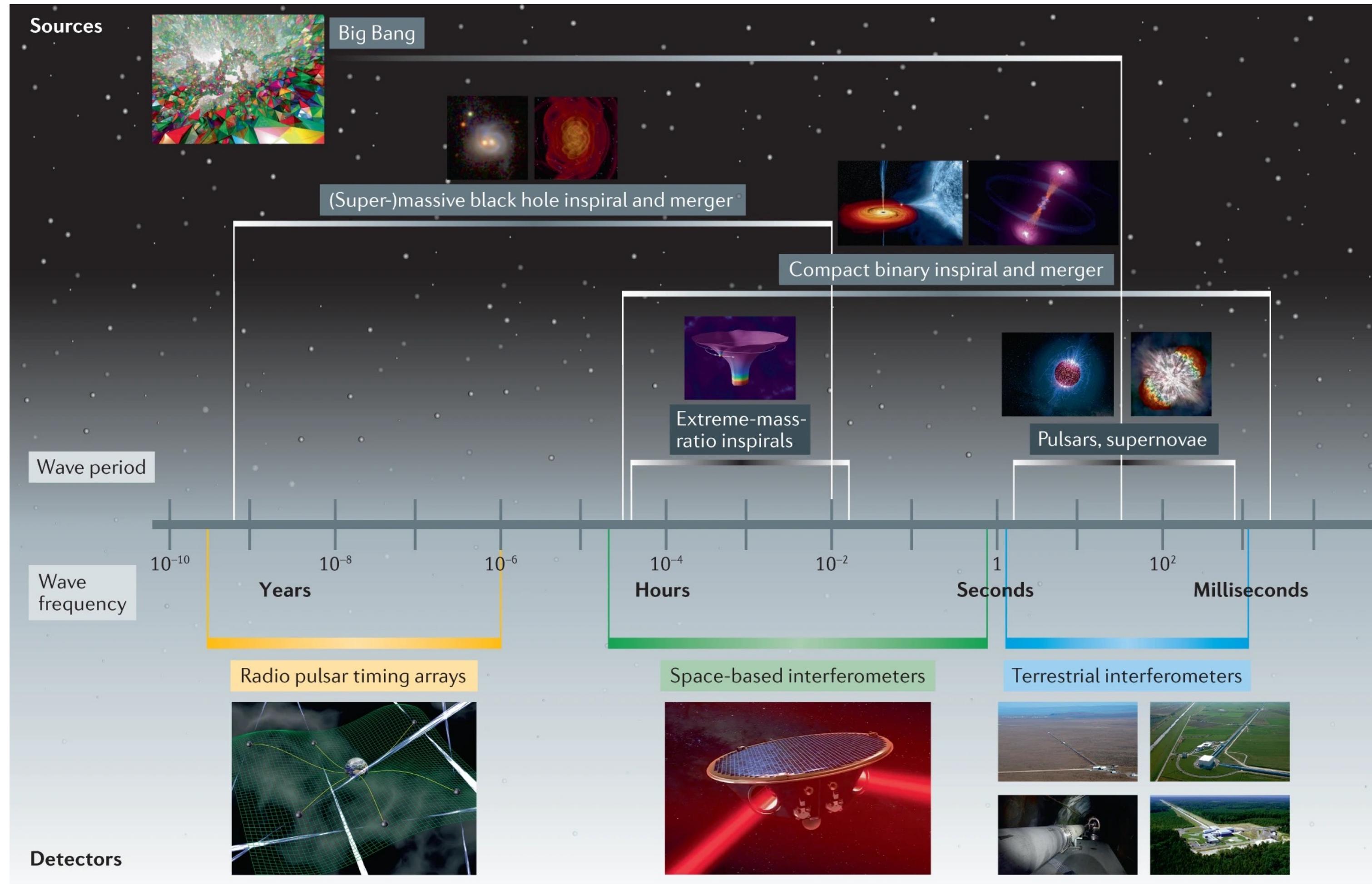
BNS GW

- Binary system consisting of two neutron stars of equal mass of $1.4 M_{\odot}$ separated by 90 km at a distance of 15 Mpc

- $$h \sim 10^{-21} \left(\frac{15 \text{ Mpc}}{r} \right) \left(\frac{M}{2.8 M_{\odot}} \right)^2 \left(\frac{90 \text{ km}}{R} \right)$$

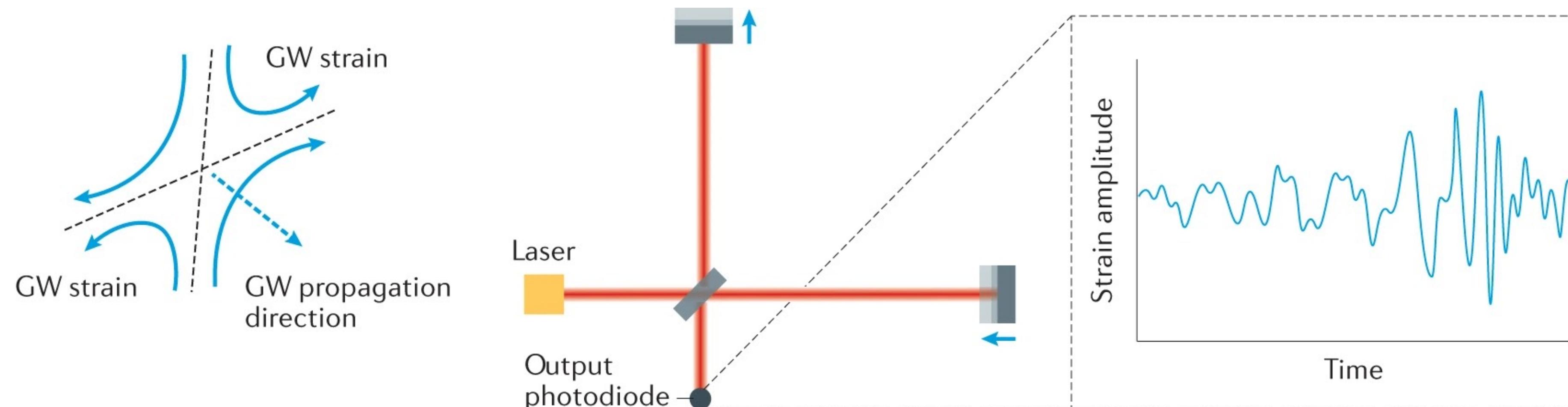
- $$f = \left(\frac{M}{2.8 M_{\odot}} \right)^{1/2} \left(\frac{90 \text{ km}}{R} \right)^{3/2} \quad 100 \text{ Hz}$$

GW observation band



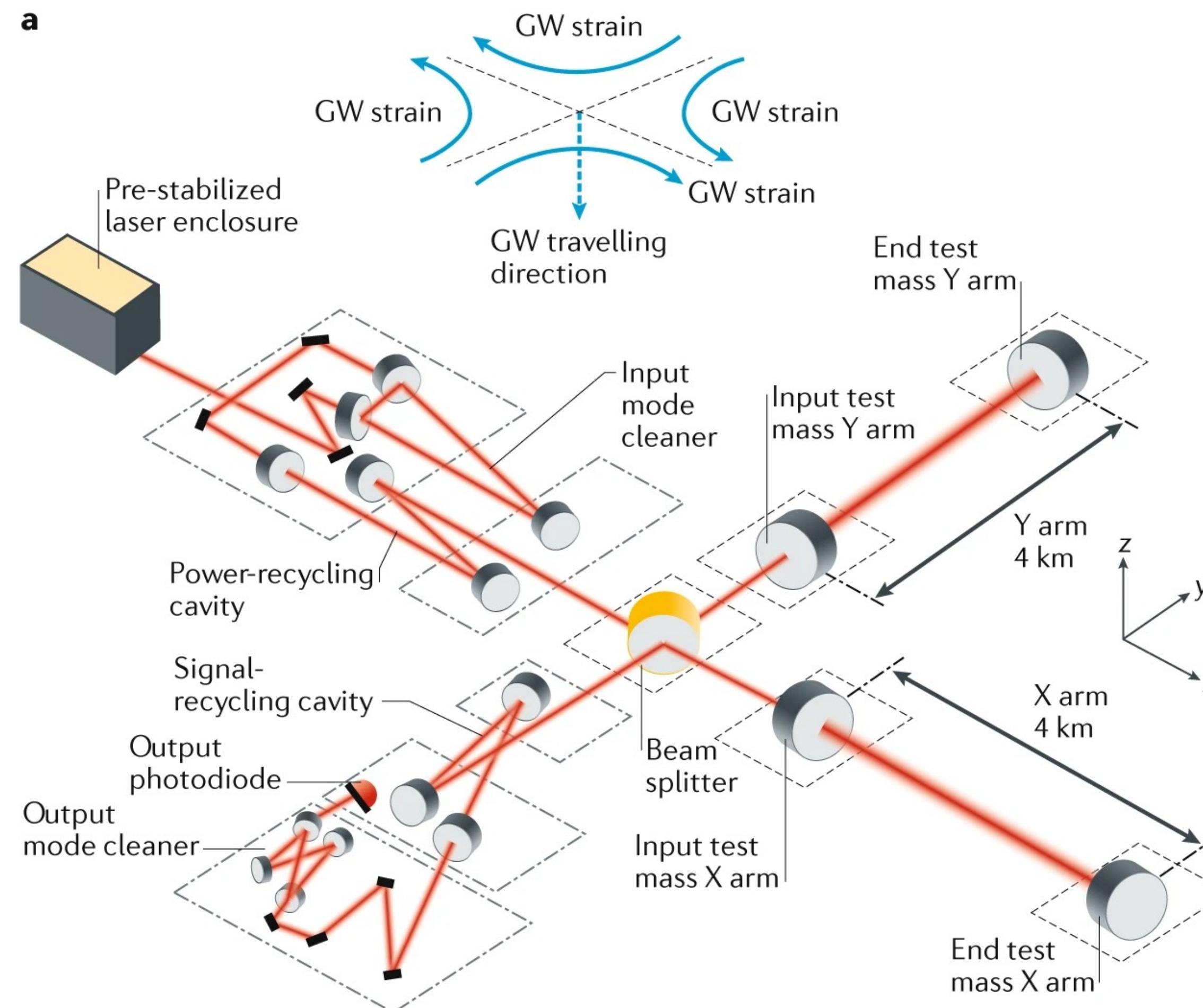
Credits: [Bailes et al. 2021](#)

GW detection



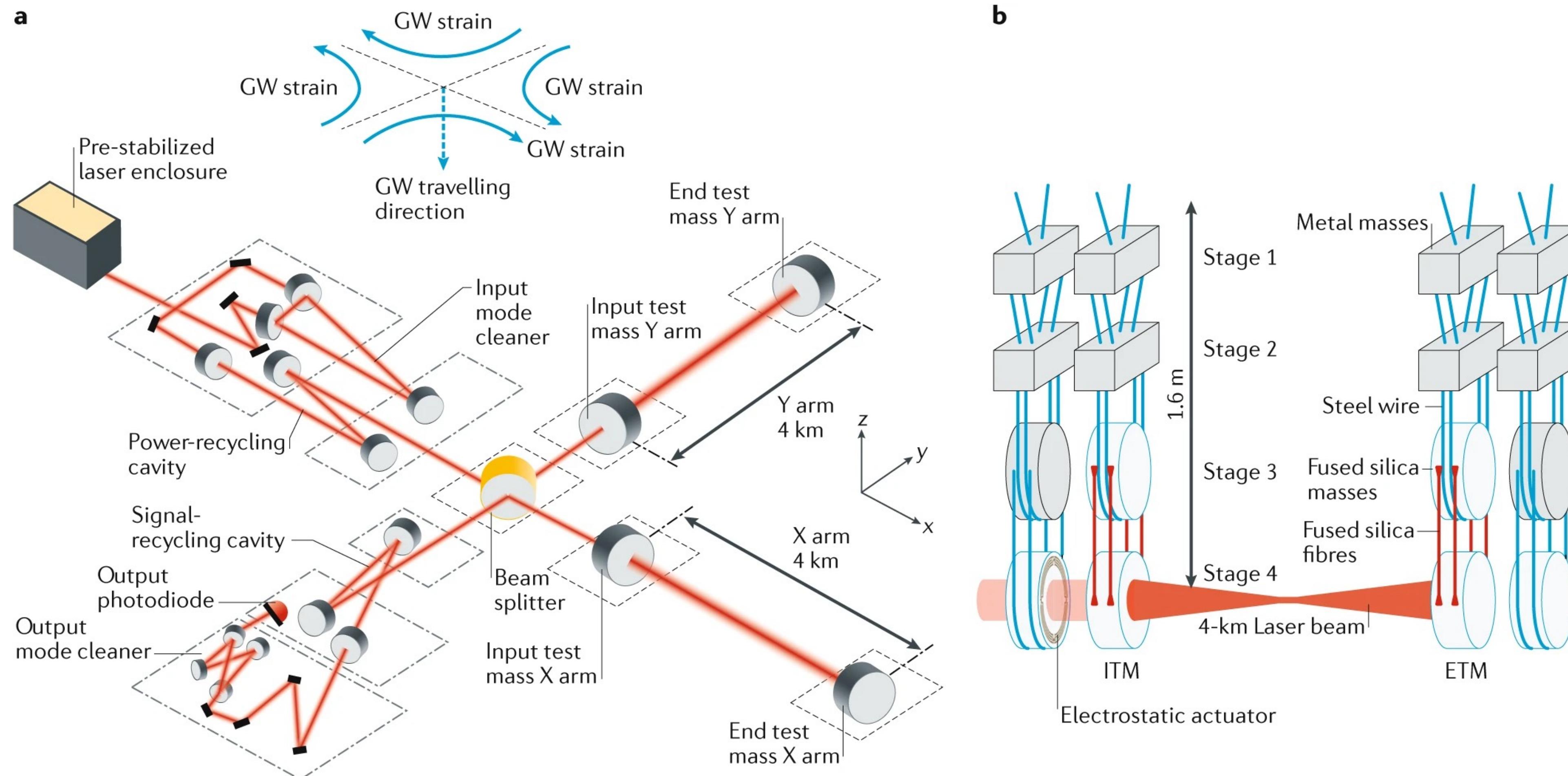
Credits: [Bailes et al. 2021](#)

Detector: Michelson interferometer



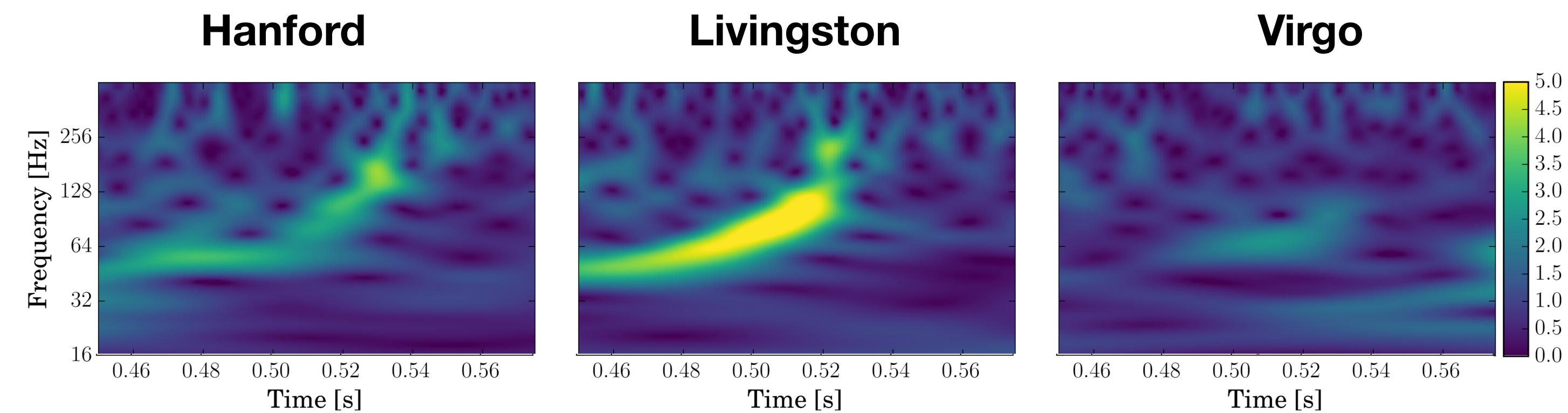
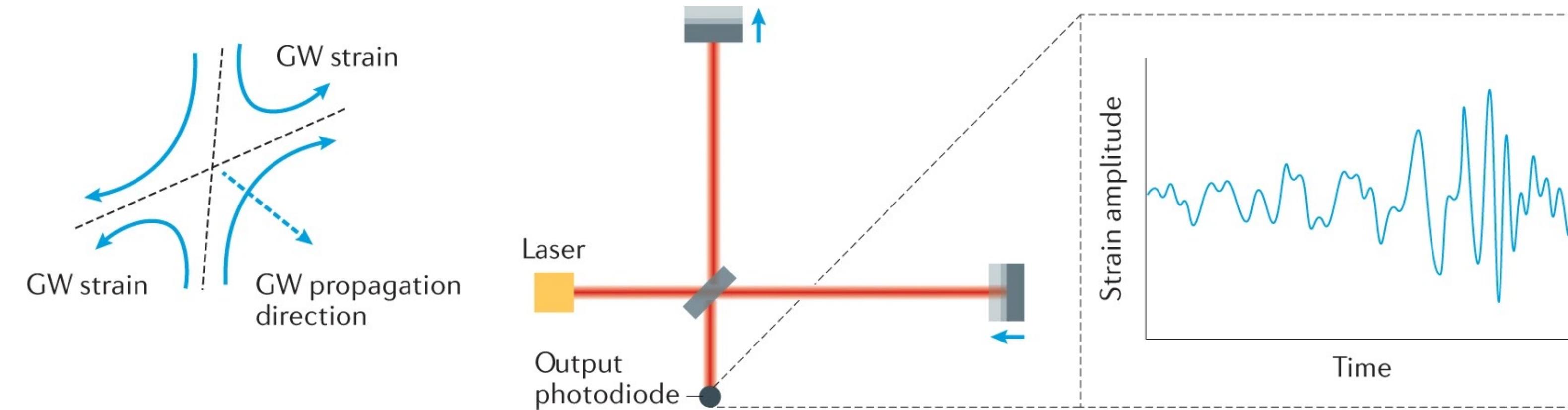
Credits: [Bailes et al. 2021](#)

Detector : super attenuator



For a more interactive explanation see [here](#)

GW signal

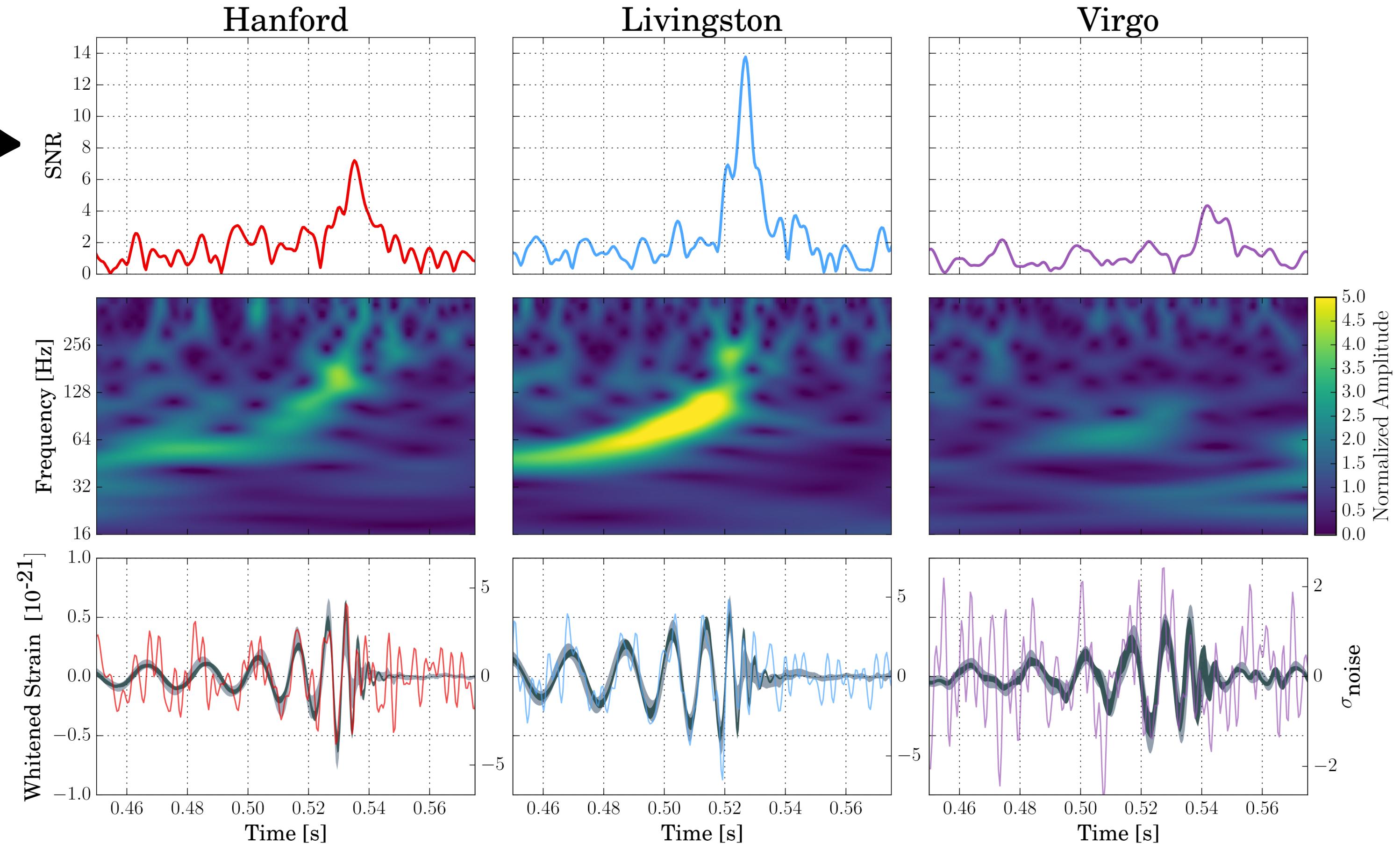


Credits: [Ricci 2019](#)

Matched filtering

$$\rho \sim \int \frac{\tilde{s}(f)\tilde{h}^*(f)}{S_n(f)} df \quad \rightarrow$$

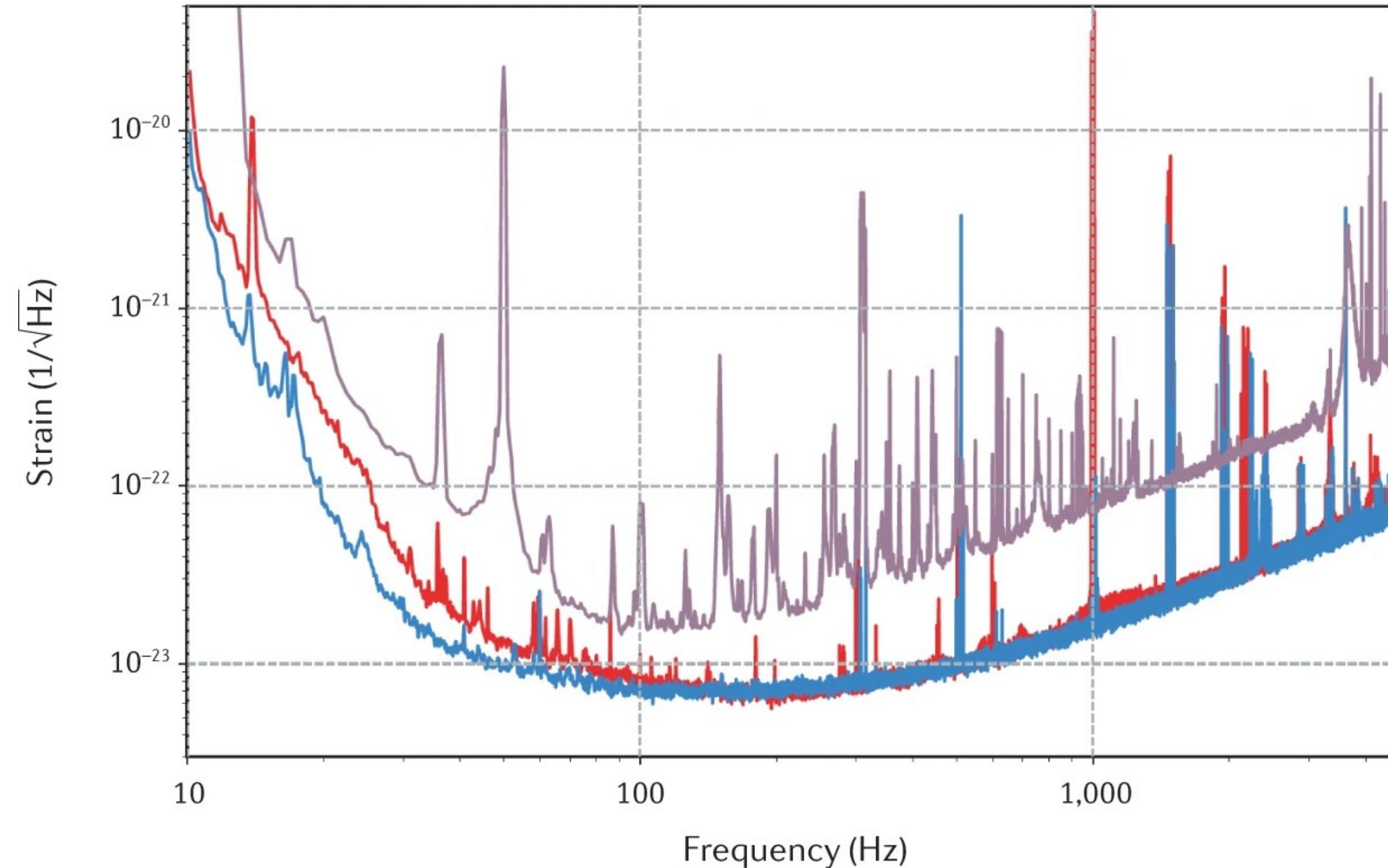
Finding the maximum of SNR



Credits: [Ricci 2019](#)

Matched filtering

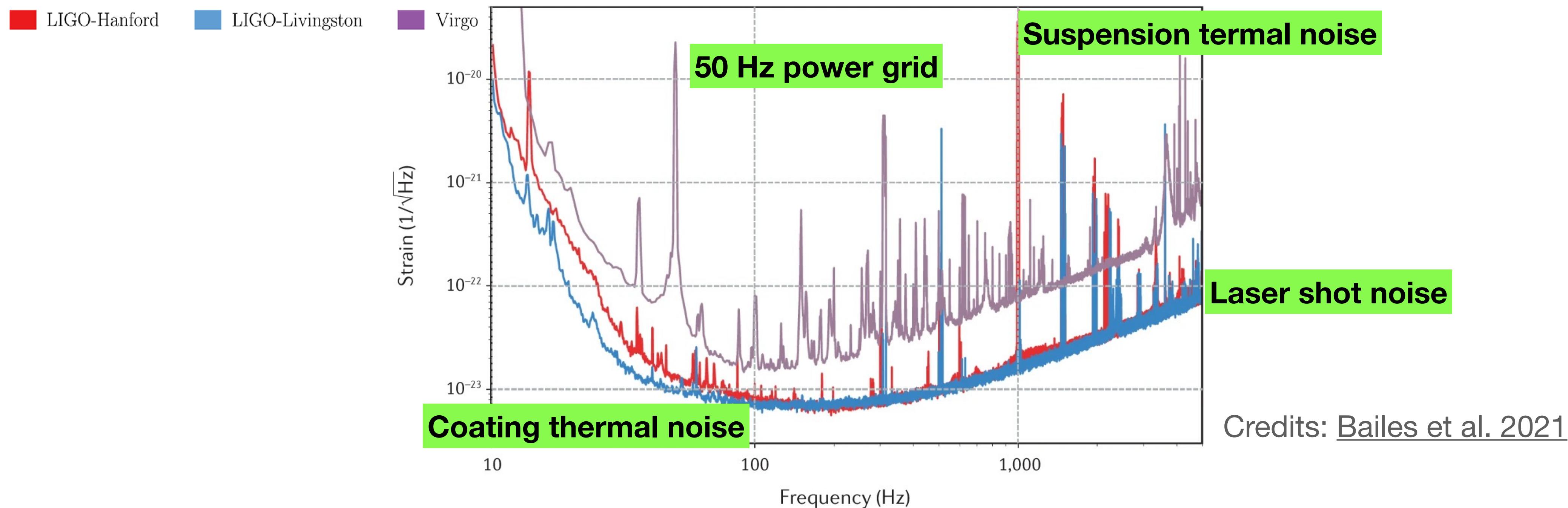
$$\rho \sim \int \frac{\tilde{s}(f)\tilde{h}^*(f)}{S_n(f)} df$$



Credits: [Bailes et al. 2021](#)

Matched filtering

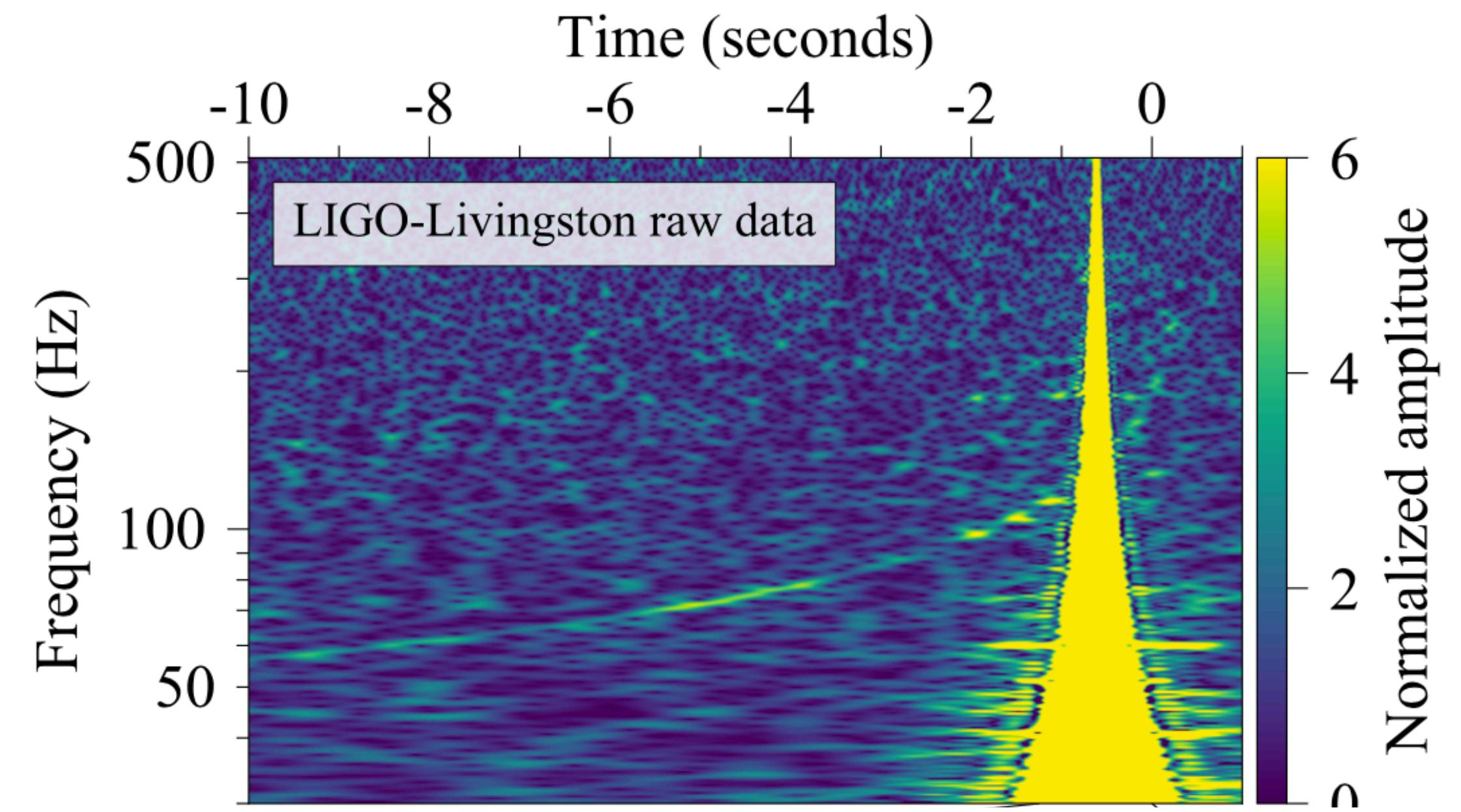
$$\rho \sim \int \frac{\tilde{s}(f)\tilde{h}^*(f)}{S_n(f)} df$$



Matched filtering

$$\rho \sim \int \frac{\tilde{s}(f)\tilde{h}^*(f)}{S_n(f)} df$$

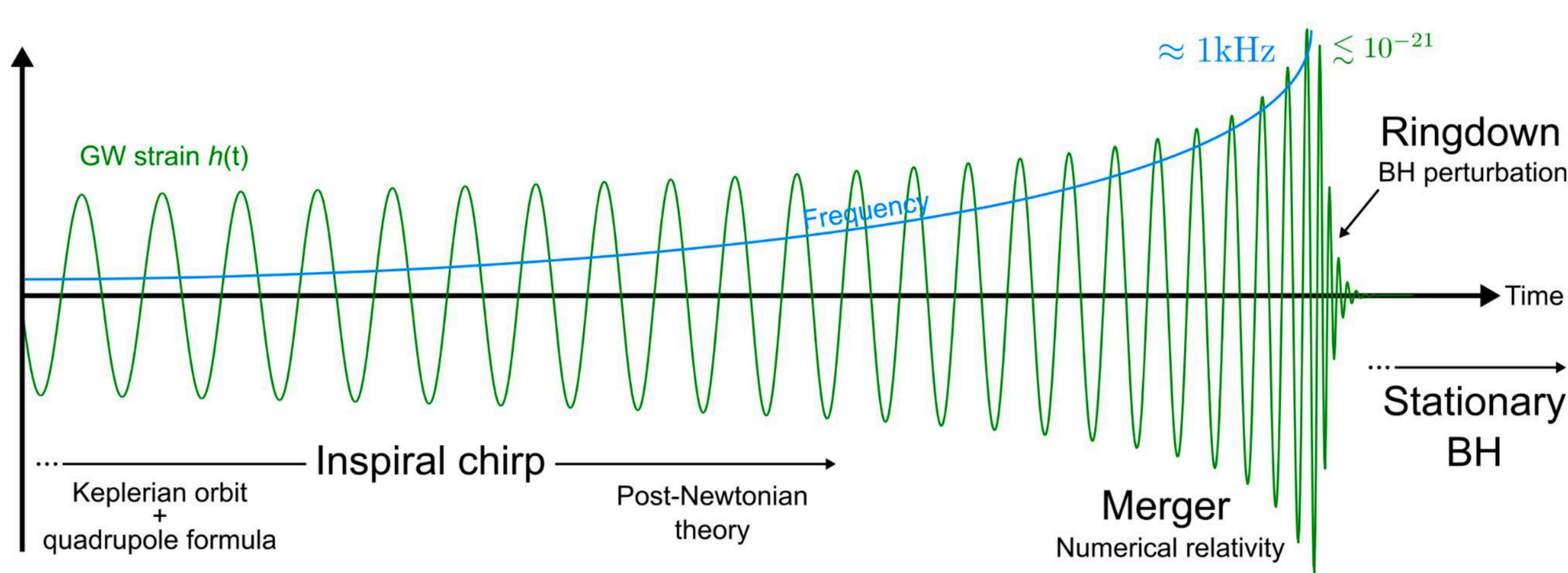
$$s(t) = n_{\text{Gaussian}}(t) + n_{\text{non-Gaussian}}(t) + h(t)$$



GW170817 Credits: [Abbott et al. 2017](#)

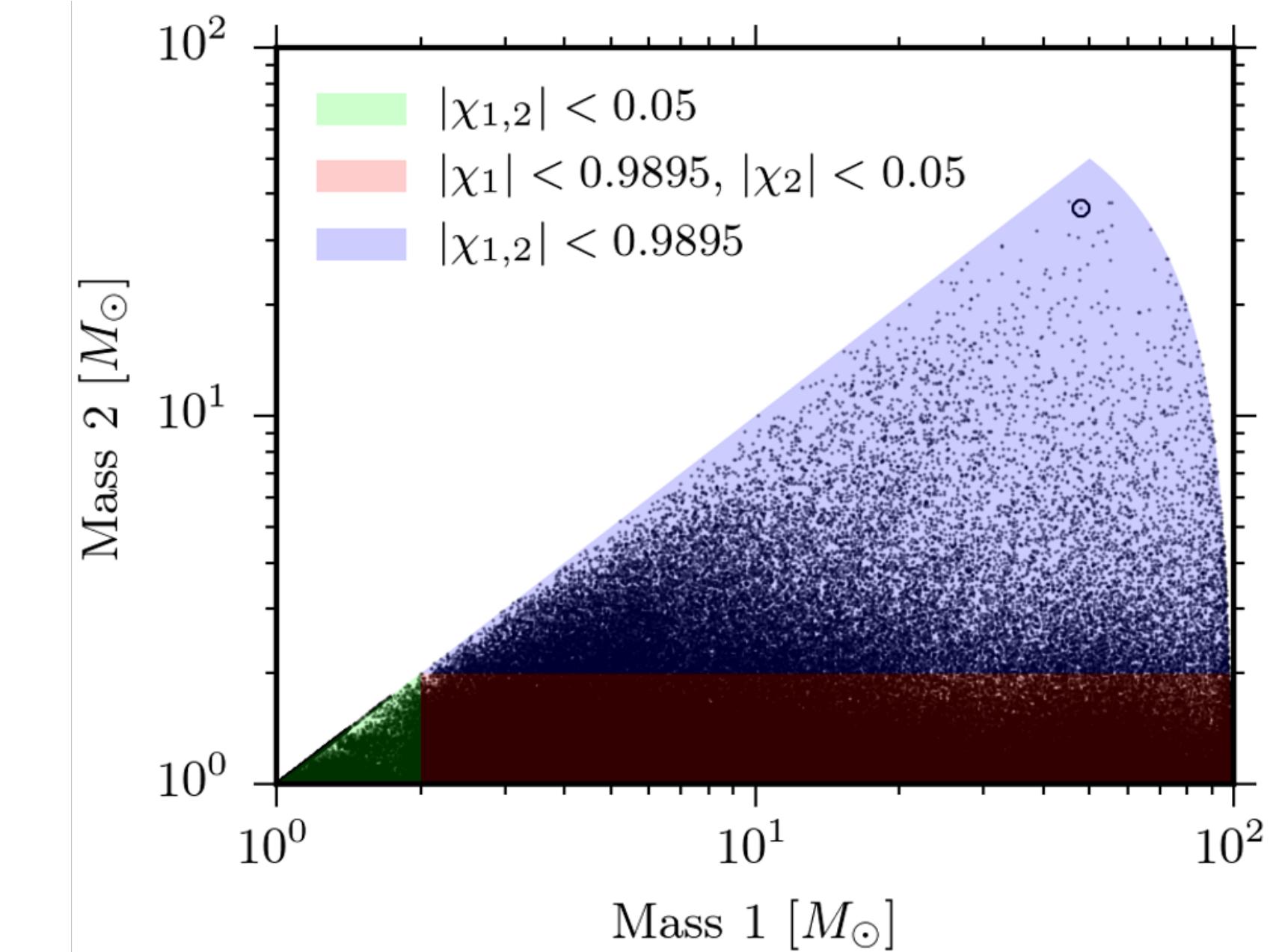
Matched filtering

$$\rho \sim \int \frac{\tilde{s}(f) \tilde{h}^*(f)}{S_n(f)} df$$



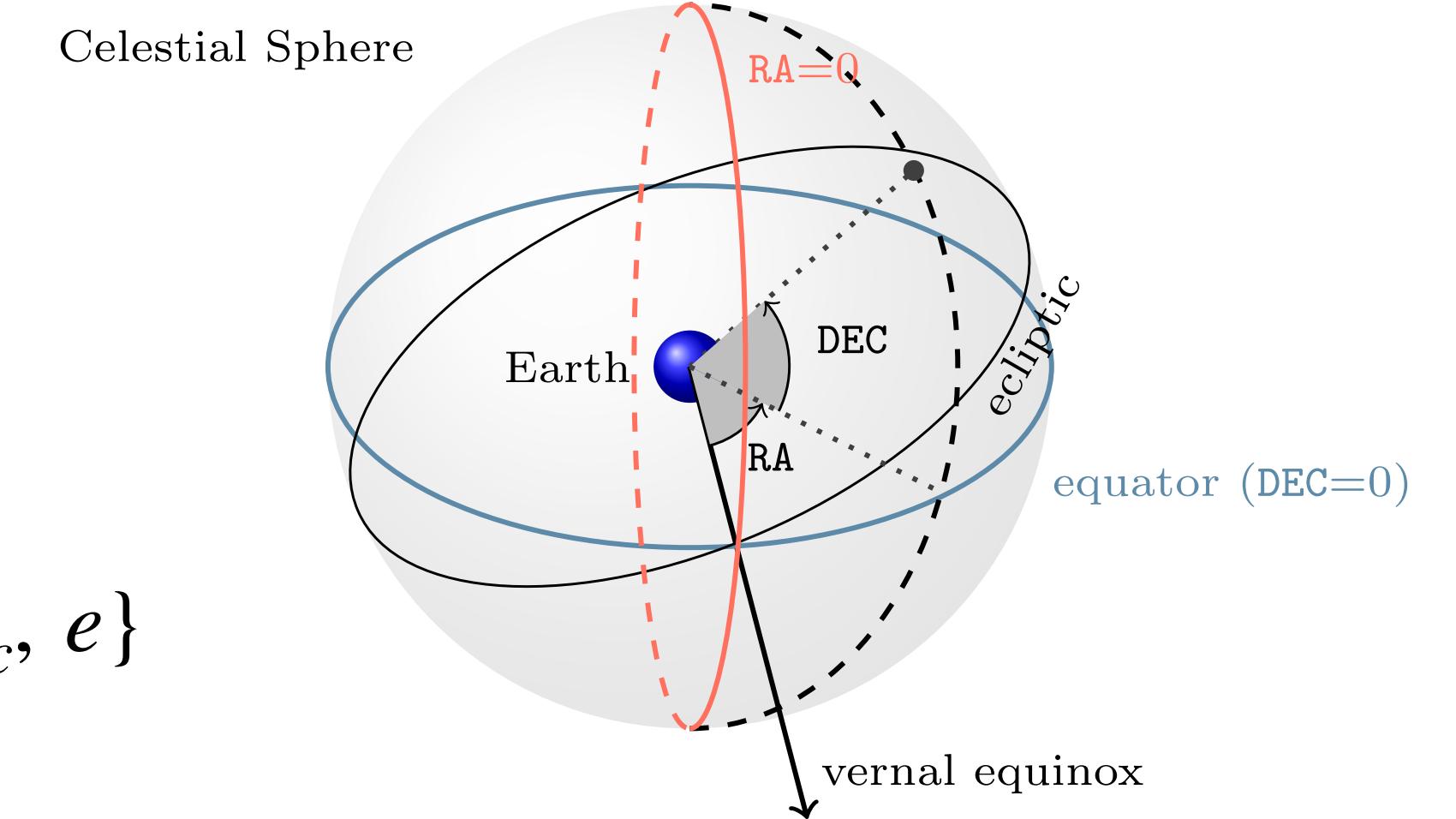
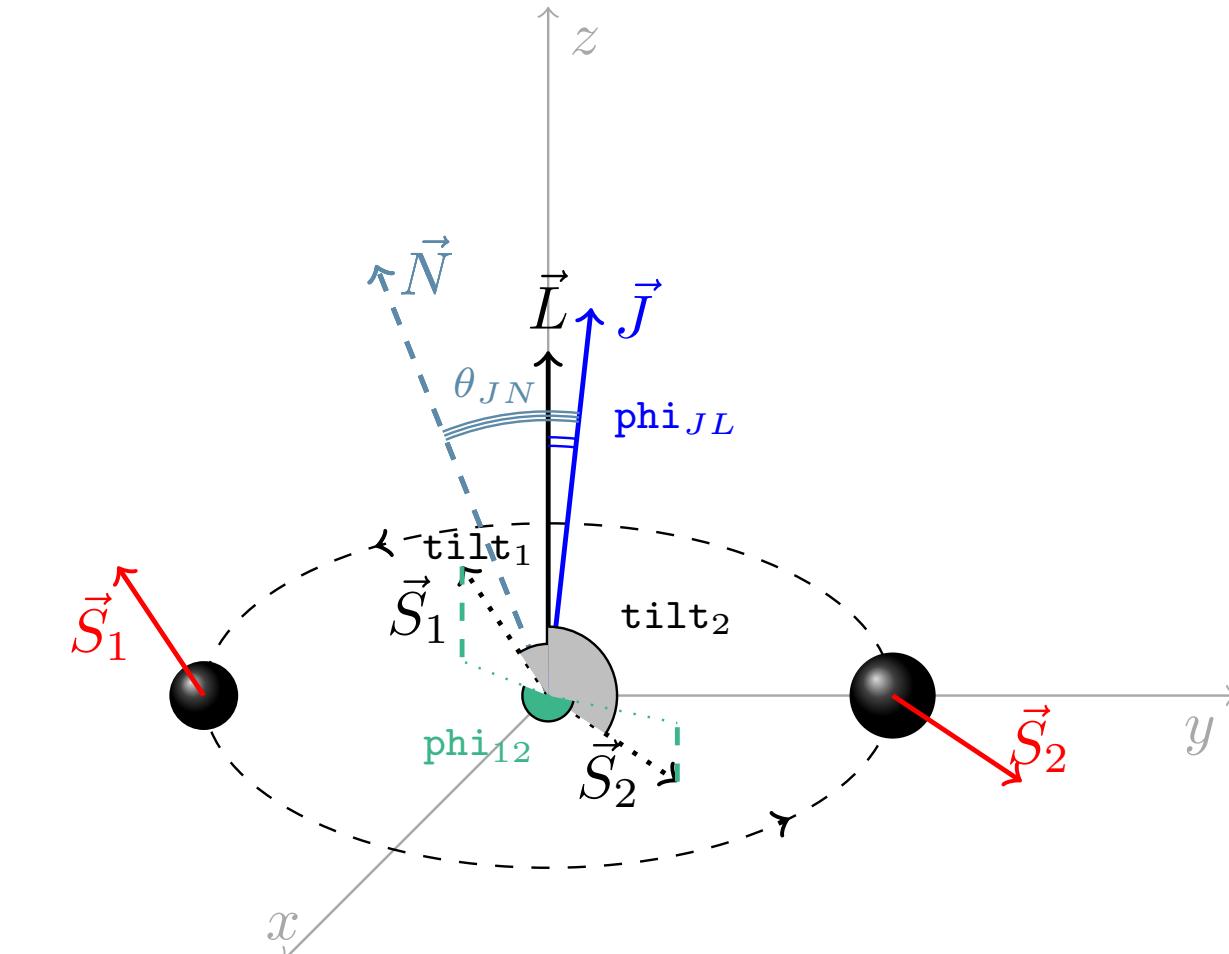
Credits: Tito Dal Canton

See [here](#) why it is called chirp

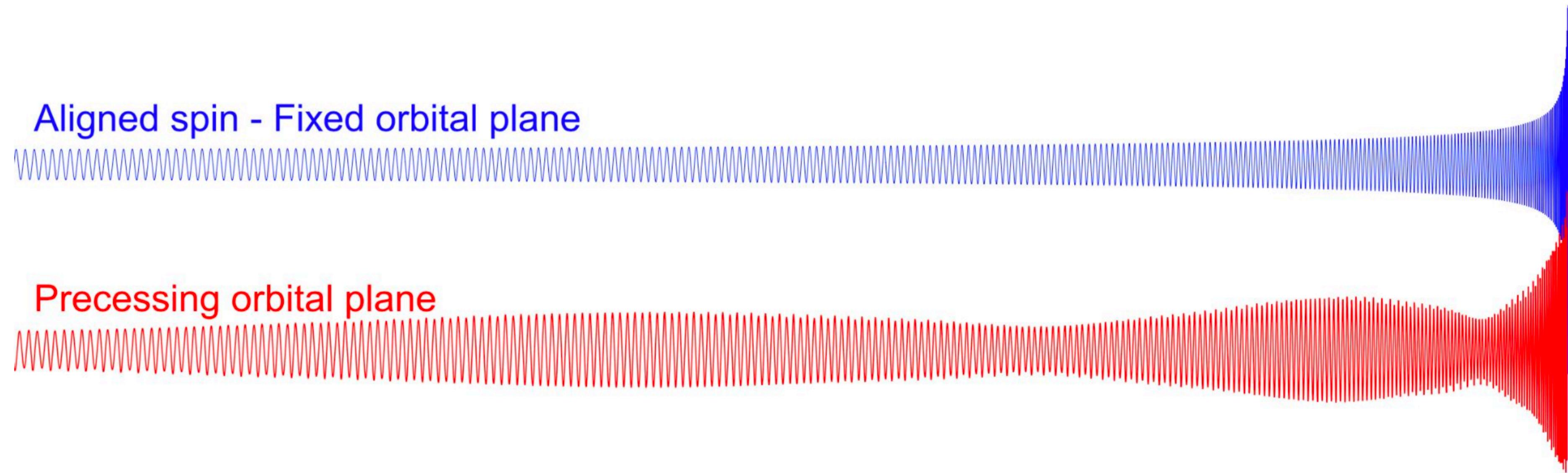


Parameters

- A BBH waveform has in total 16 parameters
 - **8 intrinsic:** masses and spins
 - **8 extrinsic:** source location, inclination angle, luminosity distance, polarisation angle, orbital phase, merger time and eccentricity
 - $\theta = \{m_1, m_2, \chi_{1,x}, \chi_{1,y}, \chi_{1,z}, \chi_{2,x}, \chi_{2,y}, \chi_{2,z}, \text{ra, dec, } \iota, d_L, \psi, \phi_c, t_c, e\}$

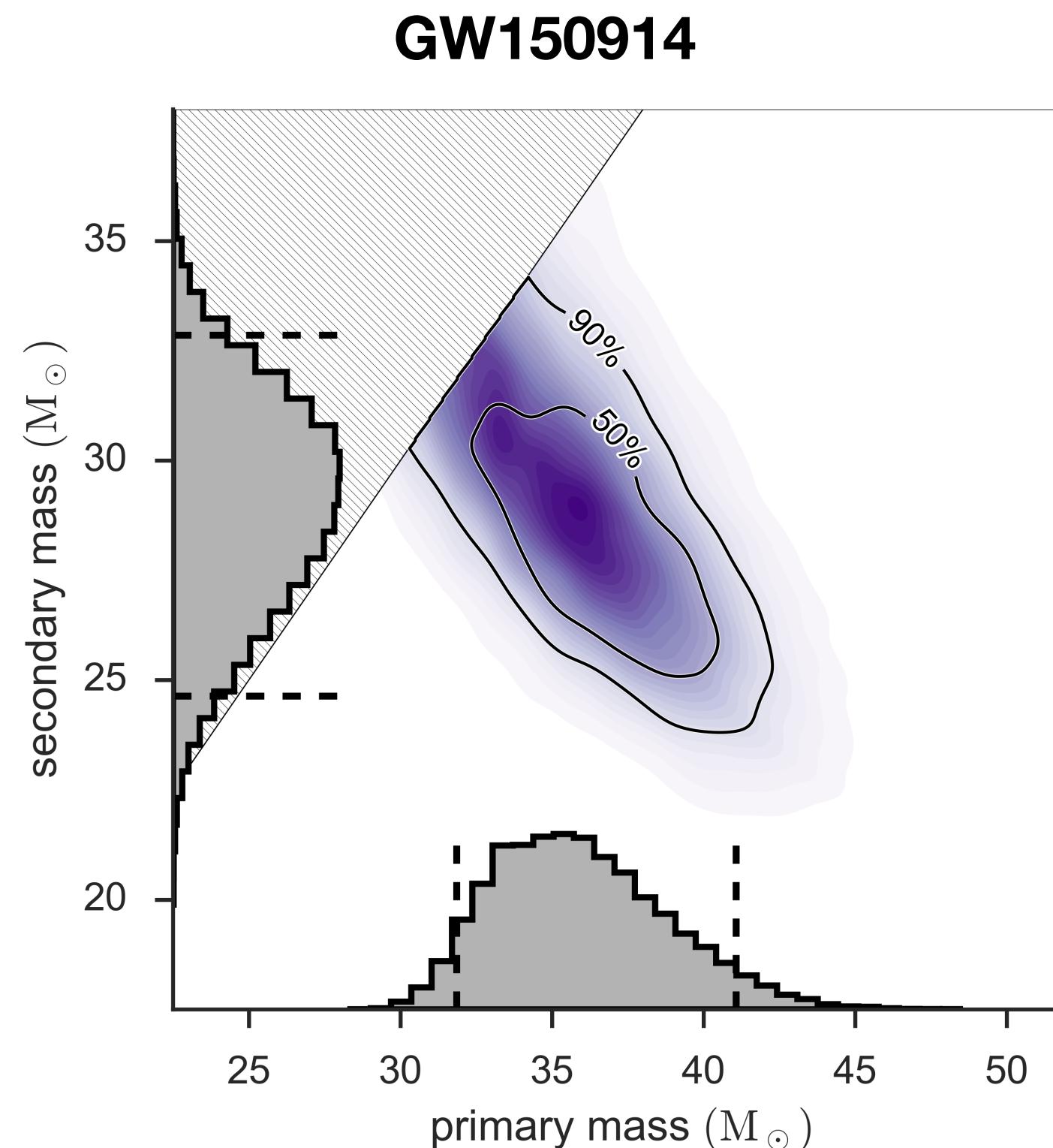


Effect of parameters



Credits: Tito Dal Canton

Parameter estimation

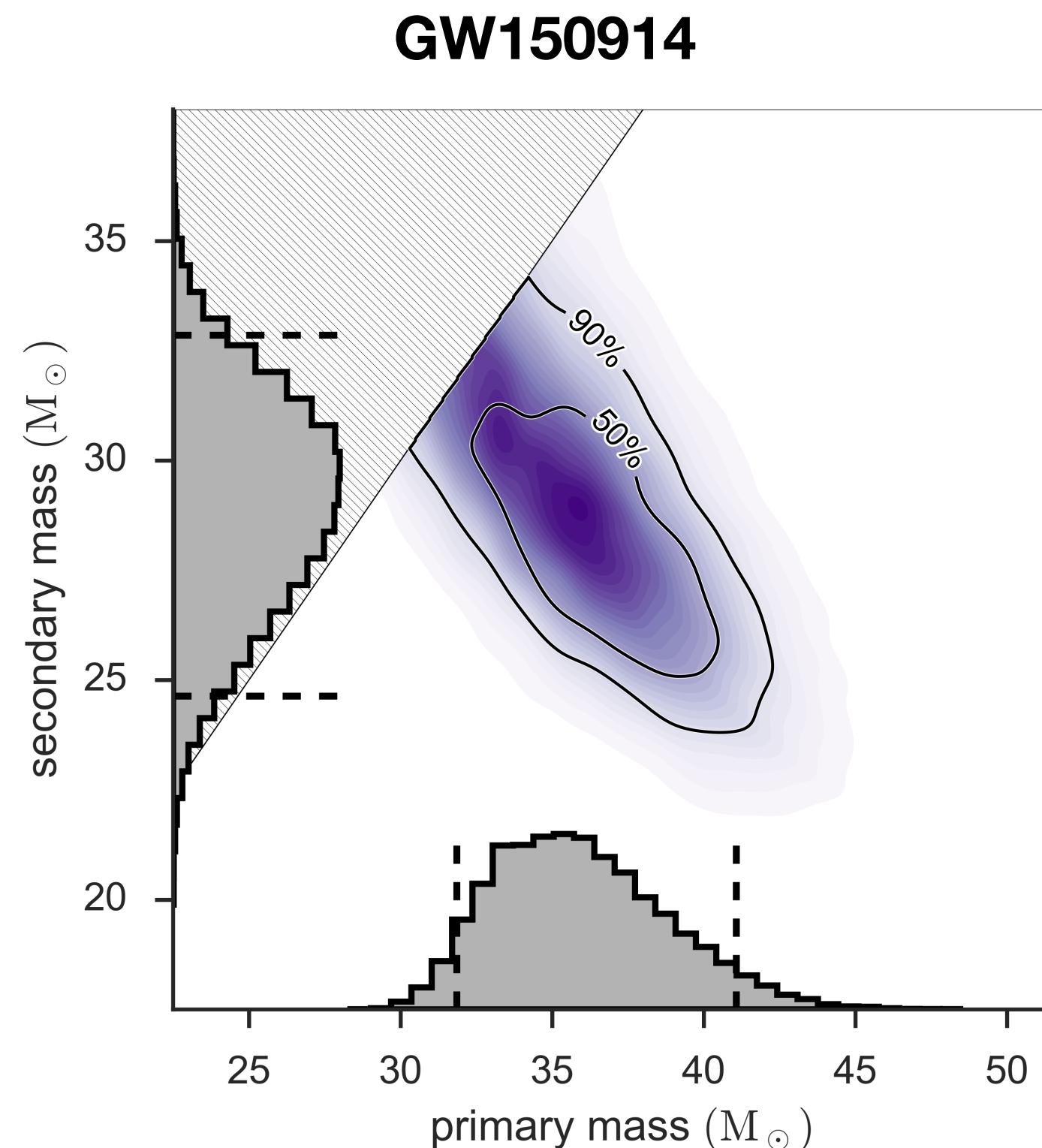


Posterior

$$p(\theta | d) = \frac{\mathcal{L}(d | \theta)p(\theta)}{p(d)}$$

Credits: [LIGO-Virgo-KAGRA collaboration](#)

Parameter estimation



Posterior

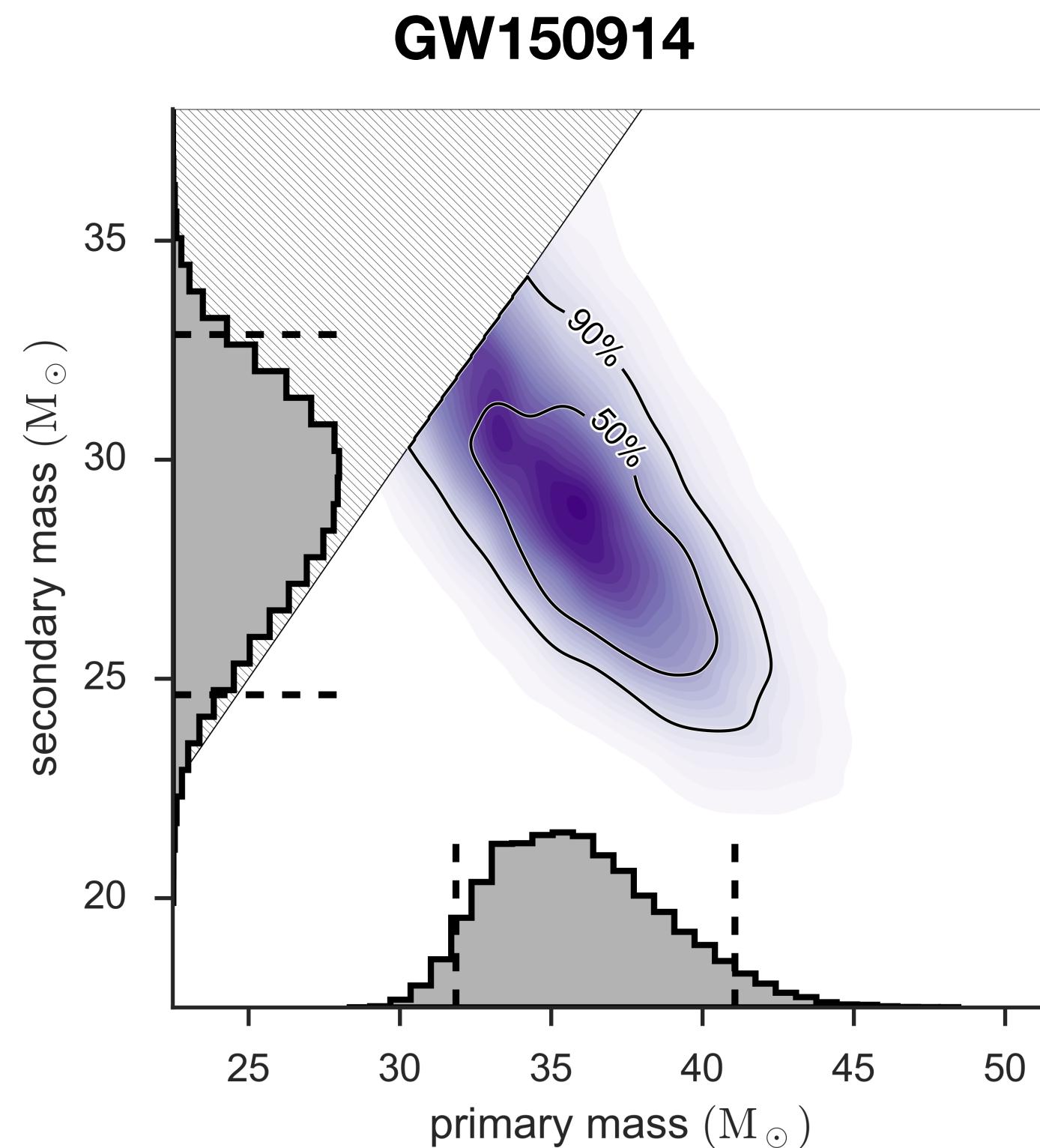
$$p(\theta | d) = \frac{\mathcal{L}(d | \theta)p(\theta)}{p(d)}$$

Prior

What is a good choice for priors?

Credits: [LIGO-Virgo-KAGRA collaboration](#)

Parameter estimation



$$p(\theta | d) = \frac{\mathcal{L}(d | \theta)p(\theta)}{p(d)}$$

Posterior

Likelihood Prior

Evidence

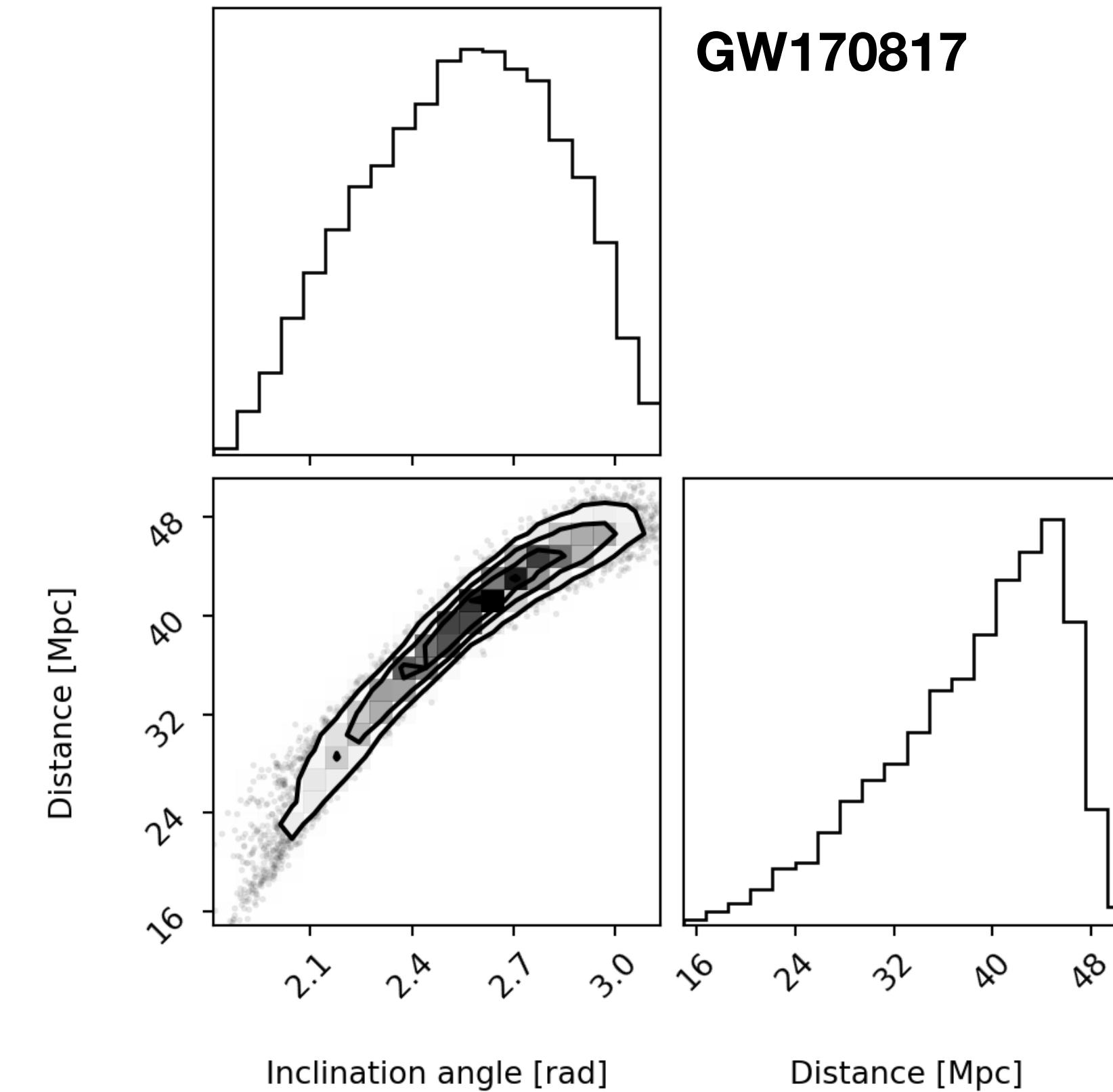
$$\mathcal{L} \propto \exp \left[-(d | h(\theta)) - \frac{1}{2}(h(\theta) | h(\theta)) \right]$$

$$(a | b) = 4\Re \int_0^\infty \frac{a(f)b^*(f)}{S_n(f)} df$$

Credits: [LIGO-Virgo-KAGRA collaboration](#)

Parameter Estimation challenge

- $p(\theta | d)$ has no analytical solution
- Necessary to **sample** from it (MCMC, nested sampling)
- Once PE is done, necessary do deal with **degeneracies**



Fisher approximation matrix



- Introducing today's hands-on on
- Approximation of the likelihood $\mathcal{L}(d|\theta) \propto \exp\left[\frac{1}{2}\Delta\theta^i \mathcal{F}_{ij} \Delta\theta^j\right]$
- Where $\mathcal{F}_{ij} = (\partial_i h | \partial_j h) = 4\Re \int \frac{1}{S_n(f)} \frac{\partial h}{\partial \theta_i} \frac{\partial h^*}{\partial \theta_j} df$

What you did (not) learn today

Tomorrow

- GW detection
- Match filtering
- Parameter estimation of single events
- Population studies
- Astrophysical formation of GW sources

Further reading

- This lecture is based on lecture materials from Marica Branchesi, Jan Harms, Tito Dal Canton, Michela Mapelli, Giuliano Iorio and Eleonora Loffredo
- Books: Gravitational Waves: Volume 1, Gravitational Waves: Volume 2
- **See you this afternoon for the hands-on section!**