

# Advanced Time Series Econometrics

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## 1 Part 1

### 1.1 Introduction

The financial asset selected for our analysis is the EURO STOXX50. This choice is driven by its status as a benchmark for practitioners and traders focused on Euro area financial markets. Our dataset, sourced from Yahoo Finance, spans from March 2007 to 12 December 2023<sup>1</sup>. To calculate the returns, we used the first differences of the logarithmic adjusted closing prices. For the GARCH model computations, we opted for percentage returns for convenience. It's important to note that adjusted closing prices differ from closing prices in that they account for dividends and stock splits. Although this adjustment is not directly relevant for a stock index analysis like ours, we opted for this price type. This decision ensures that our analysis and the associated code are easily transferable to other financial assets.

### 1.2 Sample statistics

It is widely recognized that financial returns exhibit certain distinctive features; they generally have a zero mean, their distribution deviates from normality, and they exhibit 'fat tails' (leptokurtic distribution) with a pronounced peak around the mean. Additionally, their distribution is often slightly asymmetric. As an initial step in our study, we computed the sample statistics for both weekly and daily data. As detailed in Table 1, the mean is approximately zero for both sets of returns. The standard deviation is notably high, translating to about 1.4% for daily returns and 3.0% for weekly returns when expressed in percentages. At both frequencies, the presence of extreme values is evident, as the minimum and maximum values are significantly distant from the first and third quartiles. This observation provides preliminary evidence of fat tails in the distribution. Further, both sets of data exhibit negative skewness and significant excess kurtosis, indicating a leptokurtic empirical distribution. The data are presented in levels in Figure 1 and in their first differences in Figure 2. At first glance, the sample statistics for our specific asset appear to align with the general characteristics commonly associated with returns. However, a more sophisticated analysis will follow to explore these findings in greater depth.<sup>2</sup>

### 1.3 Unit root

We investigate whether the returns follow a *Random Walk*, a *non-stationary* process, by looking for the presence of a unit root. The standard tool for this task is the *Augmented Dickey-Fuller* (ADF) test, whose null hypothesis posits that the time series contains a unit root. The ADF test results reveal that both weekly and daily prices are not stationary, whereas the returns are stationary. This finding aligns intuitively with the visual trends observed in the respective series, as depicted in Figures 1 and 2. Given these insights, and in line with existing literature, our analysis will focus on returns rather than data in levels.

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<sup>1</sup>Daily data start from 28 March, while weekly data are available from 30 March

<sup>2</sup>The first difference of log prices is used to represent returns.

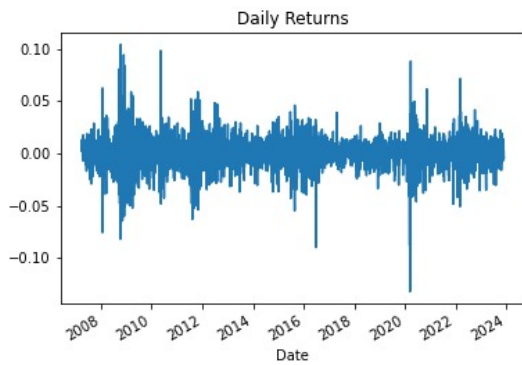


(a) Daily

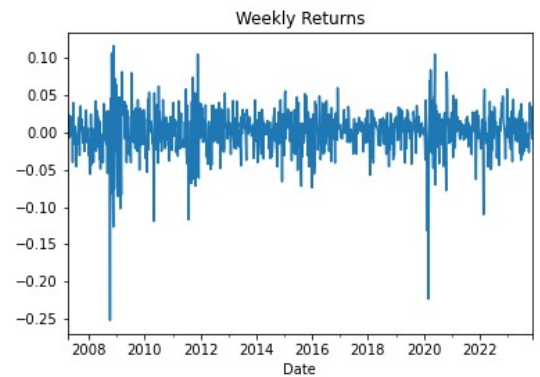


(b) Weekly

Figure 1: Plot of adjusted close price

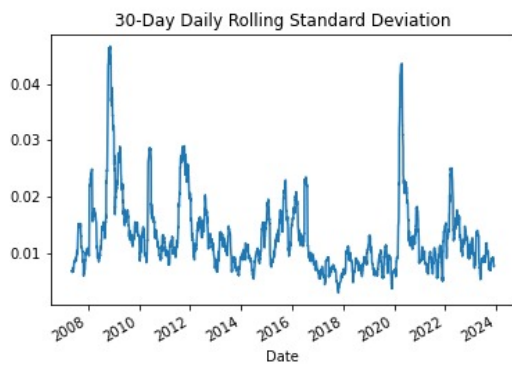


(a) Daily returns

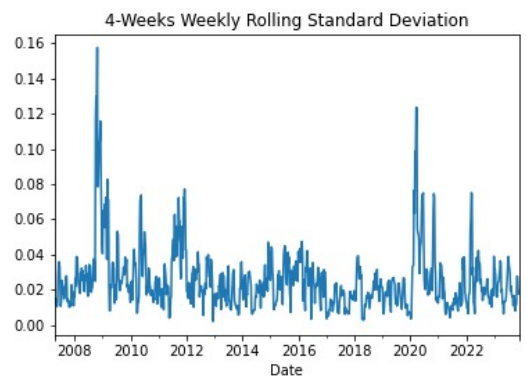


(b) Weekly returns

Figure 2: Plot of the returns



(a) 30-day Daily



(b) 4-weeks Weekly

Figure 3: Plot of rolling standard deviation

Sample statistic	Daily	Weekly
<b>Obs</b>	4191	870
<b>Mean</b>	0.000009	0.000041
<b>Std</b>	0.014271	0.030888
<b>Min</b>	-0.132405	-0.251310
<b>25 %</b>	-0.006317	-0.015045
<b>50 %</b>	0.00412	0.002452
<b>75 %</b>	0.006816	0.017029
<b>Max</b>	0.104376	0.115178
<b>Skewness</b>	-0.290315	-1.333838
<b>Excess Kurtosis</b>	7.445319	9.235805

Table 1: Sample statistics.

ADF Test	Daily	Weekly
<b>test statistic</b>	-2.25..	-2.12..
<b>p-value</b>	0.18..	0.23..

ADF Test	Daily	Weekly
<b>test statistic</b>	-23.6023	-8.7818
<b>p-value</b>	0.0	2.36..e-14

Table 2: ADF tests for Log-prices (left) and Returns (Right)

## 1.4 Correlation of the returns

When considering the prediction of future returns for our asset, a key question is whether past returns hold any information about future ones. A primary relationship that might be investigated is linear dependence. Therefore, we explore the autocorrelation of returns over time. The correlogram, as depicted in Figure 4, suggests that returns are predominantly not significantly autocorrelated, though a few lags are noted as significantly correlated. However, it is important to mention that the confidence intervals in the autocorrelograms are calculated under the assumption of Normality. If this assumption is violated, which we believe to be the case, the confidence intervals may not be accurate. In fact, our subsequent analysis demonstrates that the assumption of normality is not suitable in this context. We hypothesize that if confidence intervals were computed using more appropriate methods, they would reveal that the linear autocorrelation observed in Figure 4 is not statistically significant. Therefore, we conclude that the returns do not exhibit linear dependence over time. We made usage of a *Ljung-Box test/Portmanteau test* whose null hypothesis is that returns are i.i.d.  $Q$  under the null asymptotically follows a  $\chi^2_{(k)}$  with degrees of freedom equals to the maximum lag  $h$ .

$$Q = n(n+2) \sum_{k=1}^h \left( \frac{\hat{\rho}_k^2}{n-k} \right) \quad (1)$$

Where  $n$  is the number of observations,  $h$  the number of maximum lags,  $k$  is the lag and  $\hat{\rho}_k = \frac{\sum_{t=k+1}^n (r_t - \bar{r})(r_{t-k} - \bar{r})}{\sum_{t=1}^n (r_t - \bar{r})^2}$ . As demonstrated in Table 7, we consistently reject the null hypothesis at the 5% significance level for both daily and weekly data. Specifically, for daily data, this holds true across all considered lags: the p-values remain below 5% up to the fourth lag and fall below 1% for subsequent lags. In the case of weekly data, the p-values are under 5% up to the seventh lag and drop below 1% thereafter. Given these results, and although our analysis extends up to 35 lags, we have chosen to present only the first 10 lags in our report, as extending beyond this range offers little additional insight. Remember that the null hypothesis of the Q test is that data are i.i.d.; rejecting this hypothesis is consistent with what we observe in the data. In particular despite the correlogram of returns shows no serial correlation (no linear dependence), it does not tell us about non-linear dependence. Indeed as we observed a lack of linear dependence in returns, this does not necessarily mean that returns are independent over time. Notably, when we perform autocorrelation tests on the squares and cosines of the returns, we obtain statistically significant

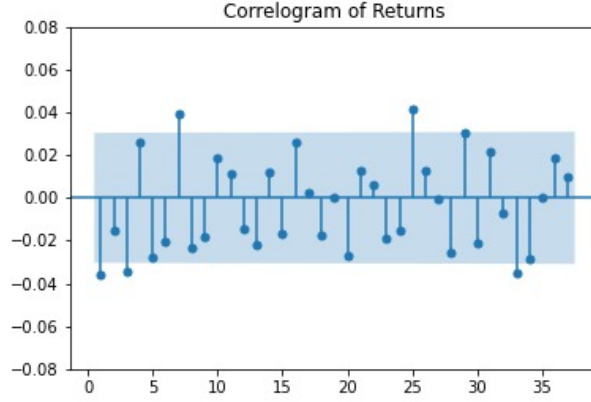
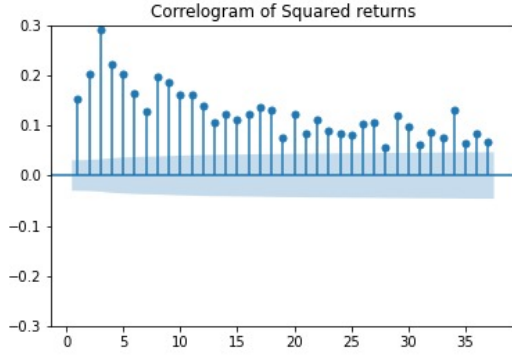
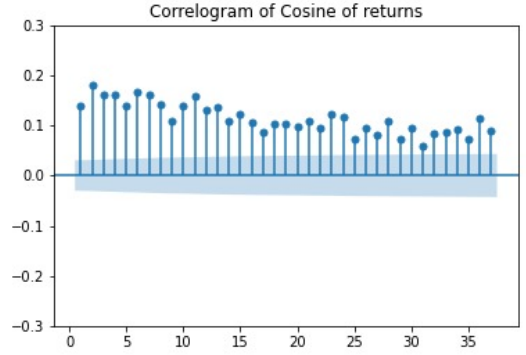


Figure 4: Autocorrelation of the returns



(a) Correlogram of the squared returns



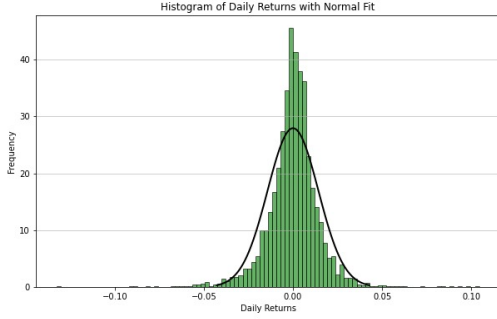
(b) Correlogram of the cosine of the returns

Figure 5: Autocorrelation of the returns

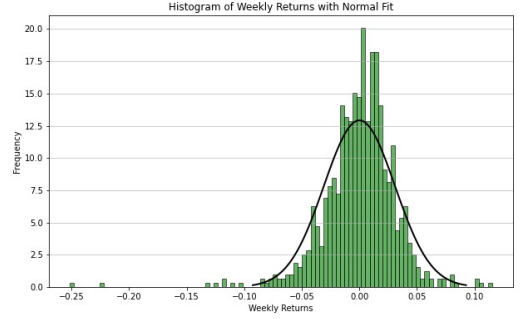
results. Therefore, despite the absence of serial linear dependence, our analysis indicates the presence of non-linear dependence. This is illustrated in Figure 5. Despite the aforementioned downward bias in the confidence intervals, the correlations significantly exceed these intervals, leading us to confidently assert the existence of non-linear dependence. This conclusion is further supported by the rolling window standard deviation presented in Figure 7. The plot clearly demonstrates volatility clustering within the series, characterized by periods of high volatility followed by similarly intense fluctuations, and similarly for periods of low volatility. Such patterns are a classic indication of non-linear dependence, particularly in terms of dependence in squares.

## 1.5 Gaussianity of the returns

For our purposes it is essential to understand if the returns are generated from a Normal distribution. In order to assess this matter we used two different tests for normality: the **Shapiro-Wilk Test** and the **D'Agostino's K-squared test**. Both tests have as null hypothesis that the sample data are generated from a Normal distribution and both strongly reject it at any level of significance for both weekly and daily data. This is shown in Table 3. To show this result visually we fit a Gaussian distribution on our data and compare it with the empirical distribution. The outcome of this exercise is shown in Figure 6. We immediately see that we observe too many realizations in the tails of the distribution, which is poorly consistent with the gaussianity assumption of the return distribution as the Normal is symmetric and has thin tails. Thus, the empirical evidence points towards the idea that the real distribution from which the returns are generated is skewed with more extreme realization on the left tail (negative



(a) Daily returns



(b) Weekly returns

Figure 6: Fit of the normal distribution on the data

Shapiro-Wilk Test		D'Agostino's Test	
test statistic	p-value	test statistic	p-value
0.9236518	8.456836e-42	671.1354431	1.839883e-146
0.9189368	3.458319e-21	313.4827574	8.4739154e-69

Table 3: Normality test of the returns.

skewness). Another feature of our data is that too many observations lie around 0, the mean, which renders the contrast with respect to the normal distribution even starker. Indeed this high peak around the mean is consistent with the general empirical evidence about returns.

## 1.6 Existence of the moments

A specificity of the Gaussian distribution is that it has all moments, but given that we rejected the hypothesis that returns are Normal, one may wonder about how many moments the return's distribution actually has. The Normal distribution is known for having **exponential tails**, meaning that the density of the standard normal  $f_x(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$  goes exponentially fast to zero as  $x$  increases. (i.e. we have very little mass on the tails).

A different kind of tails is the **Pareto tails**, for the distributions that have these  $P(X > x) \propto x^{-\alpha}$  with  $\alpha > 0$ . Those tails are fatter than the exponential ones, because they go to 0 at a way slower rate compared to the exponential tails of the standard normal. Moreover, note that the larger  $\alpha$ , the thinner is the tail because faster is the rate of convergence to 0. We know that for the existence of the  $k_{th}$  moment we need  $\alpha > k$ , hence the mass that we have on the tails is related to the number of moments of the distribution.

$$\mathbb{E}|X|^k < \infty \quad \text{if} \quad k < \alpha \quad (2)$$

We estimated the tail index *alpha* via a non-parametric method: the **Hill estimator**. The Hill estimator for the left tail is given by:

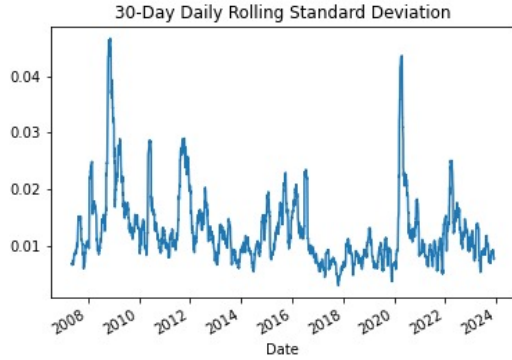
$$\hat{\alpha}_{k,\text{left}} = \frac{1}{k} \sum_{i=1}^k \ln \left( \frac{x_{(i)}}{x_{(k)}} \right), \quad (3)$$

and the Hill estimator for the right tail is given by:

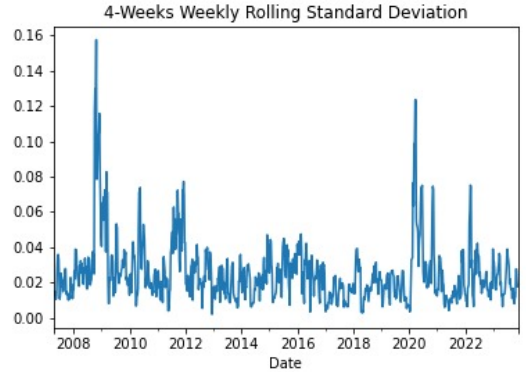
$$\hat{\alpha}_{k,\text{right}} = \frac{1}{k} \sum_{i=0}^{k-1} \ln \left( \frac{x_{(T-i)}}{x_{(T-k)}} \right), \quad (4)$$

Significance Level	Daily Returns		Weekly Returns	
	Left Tail Index	Right Tail Index	Left Tail Index	Right Tail Index
0.01	3.566629	2.775405	3.566629	2.775405
0.02	3.364692	3.105679	3.364692	3.105679
0.03	2.920611	3.394209	2.920611	3.394209
0.04	2.672450	3.010373	2.672450	3.010373
0.05	2.538236	2.920399	2.538236	2.920399
0.10	2.179721	2.336815	2.179721	2.336815

Table 4: Hill Estimator for daily(left) and weekly(right) returns



(a) 30-day daily rolling standard deviation



(b) 4-weeks weekly rolling standard deviation

Figure 7: Rolling standard deviations of the returns

Table 4 shows that as the significance level increases, moving from 0.01 to 0.10, the moments that are implied to exist by the estimator decrease. Across both frequencies and all significance levels the Hill estimator suggests that the variance exists. However, the same cannot be said for the third moment. The Hill estimator suggest the existence of the skewness only for some significance levels, and in an inconsistent way across right and left tail and across frequencies. We see that the results vary significantly between the two tails, this phenomenon is observed at both daily and weekly frequencies. Overall, the values of the Hill estimator are always between 2 and 4, which is what typically happens with stock returns, confirming we are dealing with a leptokurtic distribution. Clearly, this implies that according to the estimator the fourth moment doesn't exist.

## 1.7 Volatility clustering

A remarkable fact that is empirically found by the literature in a consistent manner is the presence of volatility clustering in the returns, meaning that "large changes tend to be followed by large changes, of either sign, and small changes tend to be followed by small changes". This phenomenon could be seen by plotting the 30 days rolling standard deviation of the returns, for daily data, and the 4 weeks one, for weekly data. This is done in 7. In fact, we observe that volatility clusters arise during periods of uncertainty for the euro area and the stock market in general. This is not surprising as it is the typical behaviour of a stock index. The most remarkable examples of this phenomenon are the volatility spikes stemming from the 2008 financial crisis and from the COVID pandemic in 2020. Indeed, in correspondence of these events volatility locally rises at both frequencies.

## 2 Part 2

### 2.1 GARCH model

The GARCH (Generalized Autoregressive Conditional Heteroskedasticity) model is an extension of the ARCH (Autoregressive Conditional Heteroskedasticity) model, specifically designed to capture the typical characteristics observed in financial return series. Unlike the ARCH model, which focuses solely on the lagged conditional variances, the GARCH model incorporates both lagged conditional variances and lagged squared returns. This addition allows the GARCH model to better account for the volatility clustering observed in financial time series. A generic GARCH(p,q) is defined as:

$$x_t = \sigma_t z_t \quad z_t \text{ i.i.d. } \sim N(0, 1) \quad \sigma_t^2 = \omega + \sum_{i=1}^p \alpha_i x_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 \quad (5)$$

$p$  is the number of lags of the returns, while  $q$  is the number of lags for  $\sigma_t^2$  itself.  $\omega$  is a constant and  $x_{t-i}^2$  is the square return at time  $t-i$ , allowing the model to capture the volatility clustering phenomenon in the data.

In this application we decided to employ the AIC, the BIC and the HQC criteria to determine the optimal combinations of  $p$  and  $q$ <sup>3</sup> for our GARCH specification, the training data for our model is 2/3 of the sample, that is between the end of March 2007 and the end of May 2018. We compute the criteria on these data and decide accordingly, the results of this exercise are shown in 8 for both daily and weekly frequencies. Pertaining daily frequencies, the HQC and the BIC select a GARCH(1,1), whilst the AIC selects a GARCH(2,1), we decide to employ a GARCH(1,1) since it ranks second according to the AIC and is preferred by BIC and HQC. At the weekly frequency all three criteria choose the GARCH(1,1) as the best specification. Therefore, we proceed to fit our model to the training sample with the given choices of  $p$  and  $q$ . The model parameters are reported in Table 6.

An heuristic approach to the analysis of the fit of a GARCH model is to plot the standardized residuals it produces and check for clusters. If these are present one should be alarmed as this phenomenon would imply that the model is not capable of capturing the volatility clustering feature of the data. Thus, one could qualitatively conclude that the model is misspecified and further refinements are needed. For our GARCH (1,1) model the results of this exercise are shown in Figure 9. The plot of the residuals for our model doesn't show these clustered residuals and we interpret this as a sign that our model fits the data in a good way.

Given this fact we move on to analyze the forecasts made by our model. These are computed using the model trained on the first 2/3 of the data and by making the 1 and 5 step ahead forecast. Each time a forecast is made, the information set is updated with the realized return for that step. This procedure is iterated up until the remaining third of the data is exhausted. As we can see from Figure 8, at the daily level, both the 1 and 5 step ahead forecasts yield almost identical results, with very minimal differences in the point estimates. Moreover, also the Weekly forecasts yield similar results although the scale is different, especially during the covid period. This is the case given that the weekly data contains very similar information compared to the daily one but is observed at a fifth of the frequency. The most important difference is that the peaks subsequent to the 2020 one are smaller in comparison to it in weekly forecasts with respect to the daily ones. In other words, at the daily frequencies daily peaks after 2020 stick out more whilst weekly ones do less so because they are dwarfed by the 2020 peak. This might be an issue with the model and we will address it when using the EGARCH to make the same estimates. In fact, this phenomenon might be a signal that at the weekly data the GARCH is overshooting the volatility and might yield inconsistent or exaggerated estimates. This will be further analyzed in the section where value at risk implied by these forecasts is presented.

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<sup>3</sup>Specifically, our code fits the model to the training data for each given combination and returns the table referenced in the text. Moreover, computational efficiency is ensured by implementing a custom function that executes parallel computation

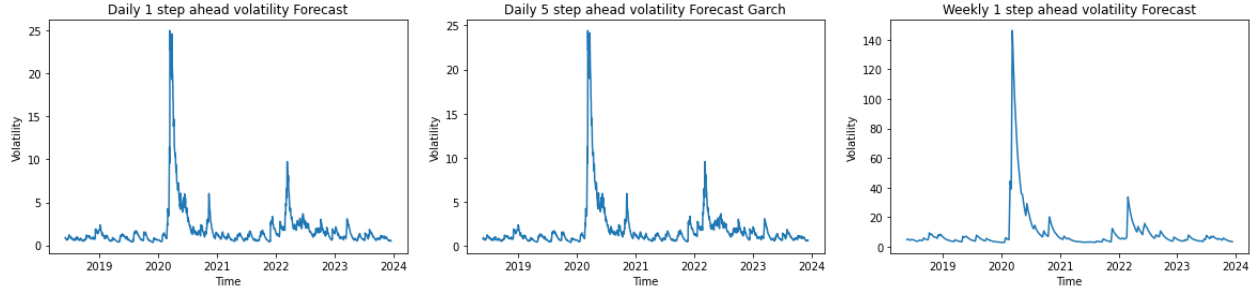


Figure 8: Daily one and five step ahead Garch(1,1) forecasts (Left), Weekly one step ahead Garch(1,1) forecast(Right)

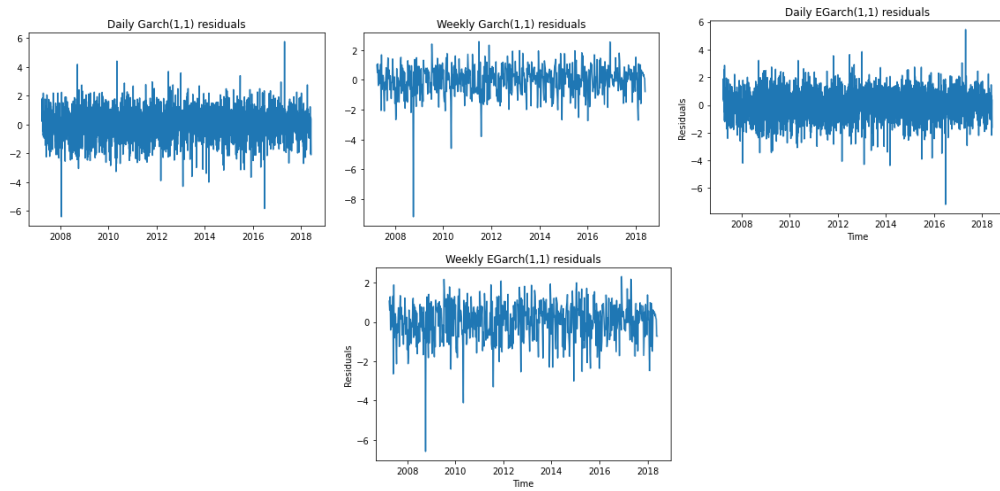


Figure 9: Garch(1,1) and EGarch(1,1) Residuals for both weekly and daily frequencies



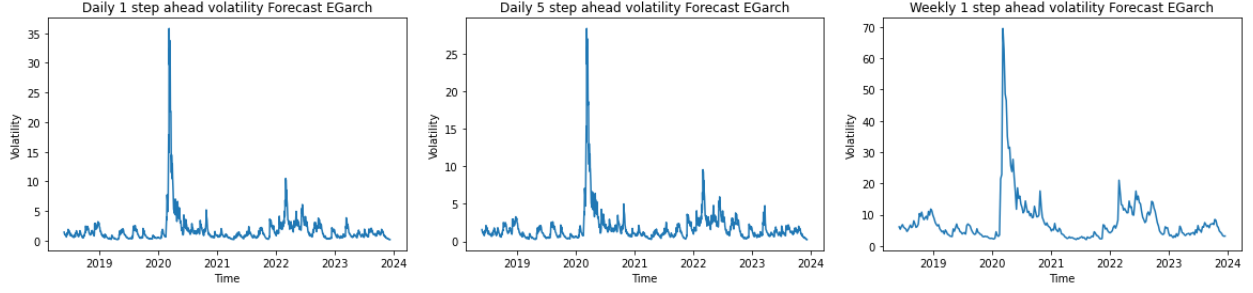


Figure 10: Daily one and five step ahead EGarch(1,1) forecasts (Left), Weekly one step ahead EGarch(1,1) forecast(Right)

### 2.1.1 EGARCH model

The GARCH model does not incorporate the *leverage effect* of returns; indeed if we think that volatility is more affected by bad past returns than good (leverage effect) then a *EGARCH* model is more suited. Moreover, due to its exponential nature the EGARCH might behave better when data is characterized by ample variations in the volatility. We decided to fit an EGARCH(1,1) in order to compare it with the GARCH(1,1) specified below and check whether or not we can get some improvement. As we can see from 9 the discussion done pertaining residuals in the Garch(1,1) case carries over to the EGarch(1,1) one. The forecasts are shown in Figure 10. The specification of the model is outlined below:

$$\log(\sigma_t^2) = \omega + \sum_{i=1}^p \alpha_i (|z_{t-i}| - E|z_{t-i}|) + \sum_{k=1}^p \gamma_k z_{t-k} + \sum_{j=1}^q \beta_j \log(\sigma_{t-j}^2)$$

$$x_t = \sigma_t z_t \quad z_t \text{ i.i.d } (0, 1)$$

The  $\gamma_k$  are called leverage coefficients and capture asymmetry. This is how, as outlined before, this model implements the idea that negative past shocks to volatility have a stronger effect than positive ones.

At the daily frequency we don't observe any relevant difference with respect to the GARCH case. At the weekly frequency we instead observe what we deem as a better performance of the model. Indeed, it seems that the EGARCH doesn't overshoot the volatility forecast, as the GARCH model was doing. This could be imputed to the additional features of the EGARCH.

## 2.2 Value at Risk (VaR)

The Value at risk (VaR) is a very popular and largely used measure of risk in finance. If we express returns as  $x_t$  then the  $\text{VaR}(\alpha)$  is the quantile of the distribution for which the probability of observing a smaller return is equal to  $\alpha$ .

$$P(x_t \leq \text{VaR}(\alpha)) = \alpha \quad (6)$$

Our method is non parametric in that it takes the empirical quantiles of the realized returns distribution in order to construct the VaR. In particular we use the one corresponding to the first 2/3 of the sample. The VaR for a given day is computed as follows:

$$\text{VaR}_{\alpha,t+1} = -\hat{\mu}_{t+1} - \sqrt{\hat{\sigma}_{t+1}^2} * \hat{Q}_{\alpha}$$

Where  $\hat{\mu}$  is the forecasted mean,  $\sqrt{\hat{\sigma}^2}$  is the forecasted variance, and  $\hat{Q}_{\alpha}$  is the value corresponding to the given quantile of the empirical distribution obtained from the training data. Note that the VaR is computed only for the 1 step ahead forecast for both daily and weekly frequencies. Var Estimates for both frequencies and both the GARCH

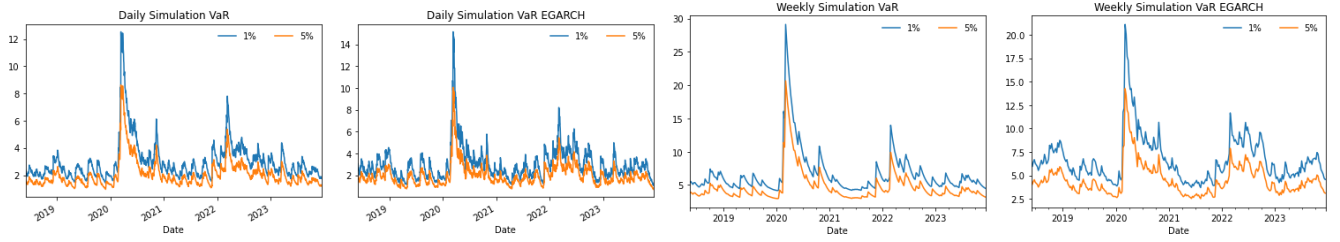


Figure 11: Value at Risk forecast for daily(left) and weekly(right) frequencies according to Garch and EGarch models

and EGARCH are provided in 11. To evaluate the performance of our VaR estimates we compare the estimated VaR with the amount of times it is exceeded by the realized returns. This way we can visualize whether or not our VaR truly reflects the given quantile of the distribution e.g 5% or 1%. This is shown in Table 5, where VaR exceedences both in absolute terms and as percentage of the total are computed for daily and weekly frequencies and for Garch and EGarch methods. As we can see from the table it seems that the daily Garch model better replicates the actual 5% and 1% VaR compared to the EGARCH. Moving at the weekly frequencies we see that the EGarch is slightly more accurate in both the 5% e and the 1% VaR. Indicating that the GARCH is being too extreme in its predictions of the VaR at the weekly frequency<sup>4</sup>.

Overall, this result is coherent with the idea that the EGARCH is more flexible in its computation of the variance and its forecasts of volatility rapidly return to normal after abrupt increases. Indeed, this seems most pronounced in the weekly data where the Garch is too influenced by the Covid pandemic in 2020 and gives really high levels of predicted volatility, more than 3 times that of the EGarch for the same sample. We believe this is the reason why the Garch overestimates the VaR by such a huge margin in the weekly data context. A really interesting fact is that at the daily data this is not observed, or at least not at such an extent. We conjecture that this is due to the fact that with more data points, the magnitude of volatility changes is less pronounced and the Garch can easily adjust itself over time. Therefore, our analysis seems to suggest that the difference between the two methods is accentuated whenever the number of data points is not particularly large.

<sup>4</sup>In interpreting the table, one should sum the exceedence at 5% level and at the 1% as the former only accounts for values between 1% and 5%.

### 3 Appendix

Method	Frequency	No Exceedence	5% Exceedence	1%Exceedence
Garch	Daily	1342(96.06%)	48(3.44%)	7(0.50%)
EGarch	Daily	1362(97.49%)	33(2.54%)	1(0.07%)
Garch	Weekly	286(98.28%)	5(1.71%)	0(0.00%)
EGarch	Weekly	283(97.60%)	5(1.72%)	2(0.68%)

Table 5: VaR Exceedence for Daily and Weekly data. Total daily observations for Garch  $N = 1397$  and for the EGARCH  $N = 1396$ , Total weekly observations for GARCH  $N = 291$  and 290 for EGARCH. The 5%exceedance represents the amount of times realized returns were between the 5% and 1% VaR threshold, whilst the other two

Model	$\mu$	$\omega$	$\alpha$	$\beta$	$\gamma$	$\eta$	$\lambda$
Garch Daily	0.034136	0.01928	0.092109	0.902538	NaN	6.86588	-0.057689
EGarch Daily	-0.02104	0.014308	0.12049	0.974961	-0.179418	8.377382	-0.098961
Garch Weekly	0.047984	0.293319	0.093527	0.872386	NaN	8.501913	-0.256124
EGarch Weekly	-0.08304	0.086345	0.067736	0.957592	-0.193169	13.910965	-0.340309

Table 6: Parameters for Garch and EGarch at weekly and daily frequencies,  $\eta$  represents the degrees of freedom of the skewed t distirbution and  $\lambda$  is the Skewness parameter.

Lag	Daily LB statistic	Daily p-value	Weekly LB statistic	Weekly p-value
1	5.294375	0.021394	5.109463	0.023796
2	6.261415	0.043687	7.834249	0.019898
3	11.120872	0.011090	9.962917	0.018884
4	13.966207	0.007404	11.445544	0.021988
5	17.228778	0.004086	11.464493	0.042910
6	18.924468	0.004293	13.513082	0.035574
7	25.394557	0.000646	21.668690	0.002897
8	27.675977	0.000540	23.517392	0.002760
9	29.045933	0.000637	25.313562	0.002643
10	30.549035	0.000696	26.989150	0.002615

Table 7: Ljung- Box test

Daily Returns					Weekly Returns				
p	q	AIC	BIC	HQC	p	q	AIC	BIC	HQC
1	1	9122.848730	9158.455810*	9135.703394*	1	2	2817.806282	2848.359538	2829.717133
2	1	9121.5739859*	9163.115579	9136.571093	1	1	2816.887261*	2843.075765*	2827.096562*
1	2	9124.848730	9166.390324	9139.845838	2	1	2818.887261	2849.440516	2830.798112
1	3	9126.848730	9174.324837	9143.988282	3	1	2820.887261	2855.805267	2834.499662
3	1	9123.573985	9171.050093	9140.713538	2	2	2819.376651	2854.294657	2832.989052
1	4	9128.848730	9182.259351	9148.130727	1	3	2819.680869	2854.598875	2833.293270
2	2	9123.439857	9170.915965	9140.579410	1	4	2820.445232	2859.727988	2835.759183
4	1	9125.573985	9178.984606	9144.855982	3	2	2821.376651	2860.659408	2836.690603
3	3	9126.742443	9186.087577	9148.166883	2	3	2821.376652	2860.659408	2836.690603
3	2	9124.360510	9177.771131	9143.642506	1	5	2822.445231	2866.092738	2839.460732
1	5	9130.839167	9190.184302	9152.263608	4	1	2822.880270	2862.163027	2838.194221
2	4	9126.294219	9185.639354	9147.718660	3	3	2823.680870	2867.328377	2840.696371
2	3	9125.067745	9178.478366	9144.349742	2	4	2822.021166	2865.668673	2839.036667
2	5	9127.864390	9193.144038	9151.431274	3	4	2824.017637	2872.029895	2842.734689
3	4	9127.867009	9193.146657	9151.433894	2	5	2824.019887	2872.032146	2842.736939
3	5	9129.447568	9200.661730	9155.156897	3	5	2826.015822	2878.392831	2846.434424
4	2	9126.360510	9185.705644	9147.784950	4	2	2823.376651	2867.024159	2840.392153
5	1	9127.573985	9186.919120	9148.998426	5	1	2823.513136	2867.160643	2840.528637
4	3	9128.742443	9194.022091	9152.309327	4	3	2825.376651	2873.388910	2844.093703
4	4	9128.673736	9199.887897	9154.383065	5	2	2824.491645	2872.503903	2843.208696
5	2	9128.360509	9193.640157	9151.927394	4	4	2826.017673	2878.394682	2846.436274
5	3	9130.742443	9201.956604	9156.451771	5	4	2827.725867	2884.467627	2849.846019
4	5	9130.673735	9207.822410	9158.525508	4	5	2828.015822	2884.757582	2850.135974
5	4	9130.355382	9207.504057	9158.207155	5	3	2826.293032	2878.670041	2846.711633
5	5	9132.355382	9215.438570	9162.349599	5	5	2829.777328	2890.883839	2853.599030

Table 8: Garch lag selection for daily (left) and weekly (right) data