# Non-Gaussian shocks and recursive monetary policy SVARs for the Euro Area

Master in Economics

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# SVAR identification problem

Consider a SVAR-B model:

$$\mathbf{y}_t = \boldsymbol{\nu} + \mathbf{A}_1 \mathbf{y}_{t-1} + \cdots + \mathbf{A}_p \mathbf{y}_{t-p} + \mathbf{B}\boldsymbol{\varepsilon}_t \tag{1}$$

With:

$$\mathbb{E}(\boldsymbol{\varepsilon}_t) = \mathbf{0}, \quad \mathbb{E}(\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t') = \mathbf{I}_M \tag{2}$$

Cannot uniquely recover the structural matrix $\mathbf{B}$:

$$\underbrace{\boldsymbol{\Sigma}_{\boldsymbol{u}}}_{\frac{M(M+1)}{2}} = \underbrace{\mathbf{B}\mathbf{B}'}_{M^2} \tag{3}$$

It is necessary (although not sufficient) to impose at least $\frac{M(M-1)}{2}$ restrictions on $\mathbf{B}$.

# SVAR identification problem

A lot of solutions have been proposed: sign restrictions, long-run restrictions, high frequency identification, identification through heteroskedasticity....

Among them all:

- Short-run restrictions on **B** allow to obtain the structural matrix by the Cholesky decomposition of $\boldsymbol{\Sigma}_u$.
- Statistical assumptions on $\boldsymbol{\varepsilon_t}$ and then find **B** relying on ICA.

Statistical identification allow to test short-run restrictions.

# Cholesky SVAR

If $\mathbf{B}$ is lower triangular, then the covariance matrix $\boldsymbol{\Sigma}_{\boldsymbol{u}}$ admits a unique representation through the Cholesky factor:

$$\boldsymbol{\Sigma}_{\boldsymbol{u}} = \mathbf{B}\mathbf{B}' \tag{4}$$

Example in monetary policy SVAR with $M = 3$:

$$\mathbf{B}_{\text{chol}} = \begin{pmatrix} b_{\pi\pi} & 0 & 0 \\ b_{y\pi} & b_{yy} & 0 \\ b_{r\pi} & b_{ry} & b_{rr} \end{pmatrix} \tag{5}$$

**ISSUE**: This basically imposes that some variables in the system do not immediately react to one or more shocks.

# ICA (Comon [1994])

$$\begin{aligned} x_1(t) &= a_{11}s_1(t) + a_{12}s_2(t) + a_{13}s_3(t) \\ x_2(t) &= a_{21}s_1(t) + a_{22}s_2(t) + a_{23}s_3(t) \\ x_3(t) &= a_{31}s_1(t) + a_{32}s_2(t) + a_{33}s_3(t), \end{aligned} \quad , \mathbf{x} = \mathbf{A}\mathbf{s}$$

- The components are assumed statistically independent, i.e.

$$s(s_1, s_2, ..., s_n) = p_1(s_1)p_s(s_2)...p_s(s_n) \tag{6}$$

- The independent components are assumed to have non-Gaussian distribution.
- The mixing matrix **A** is invertible.

**ISSUE**: Identification up to permutation and scaling (sign).

# Why Gaussian distributions are not enough

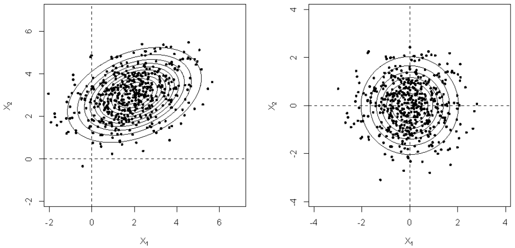
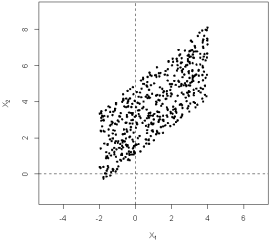


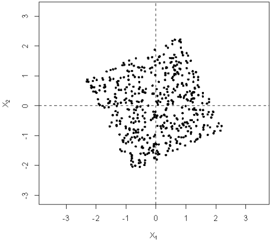

Figure 1: On the left: 500 draws from a $\mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ with $\boldsymbol{\mu} = (2, 3)$ and $\boldsymbol{\Sigma} = \begin{pmatrix} \mathbf{2} & \mathbf{0.5} \\ \mathbf{0.5} & \mathbf{1} \end{pmatrix}$,

On the right: same draws after being center and whitened through $\mathbf{Z} = \mathbf{E}\mathbf{D}^{-1/2}\mathbf{E}'$ obtained via the spectral decomposition of the covariance matrix.

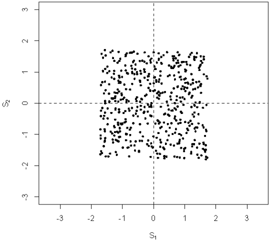
# Why non-Gaussian distributions




(a)
500 draws from a bivariate uniform distribution with linear dependence.




(b) Same draws after centering and whitening: linear dependence wiped out, but non-linear dependence remains.




(c) After a final rotation by the FAST ICA algorithm: original sources are recovered and independent.

Figure 2: An application of ICA with uniform distributions.

## Determine the last rotation

- Go non-parametric:
  - FastICA, maximizes negantropy Moneta et al. [2013]:

$$J(\mathbf{y}) = H(\mathbf{y}_{\text{Gaussian}}) - H(\mathbf{y}), \quad \text{where } H(\cdot) \text{ is the differential entropy} \tag{7}$$

  - CVM statistics Herwartz [2018]:

$$\hat{\boldsymbol{\theta}} = \arg\min_{\theta} \left\{ \mathcal{B}_\theta \,|\, \tilde{\boldsymbol{\varepsilon}}_{\mathbf{t}} = \tilde{\mathbf{B}}_\theta^{-1} \mathbf{u}_t \right\}, \quad \text{where } \mathcal{B}_\theta \text{ is the Cramer-von Mises statistics}$$

  - Distance Covariance Matteson and Tsay [2013]:

$$\hat{\boldsymbol{\theta}} = \arg\min_{\theta} \left\{ \mathcal{D}_\theta \,|\, \tilde{\boldsymbol{\varepsilon}}_{\mathbf{t}} = \tilde{\mathbf{B}}_\theta^{-1} \mathbf{u}_t \right\}, \quad \text{where } \mathcal{D}_\theta \text{ is the distance covariance}$$

- Go parametric:
  - Student-t density ML estimation Lanne, Meitz, and Saikkonen [2017], and PML Gouriéroux, Monfort, and Renne [2017]:

$$\arg\max_{\boldsymbol{\theta}} \left\{ T^{-1} \sum_{t=1}^{T} \ell_t(\boldsymbol{\theta}) \right\}, \quad \text{where } \ell_t(\boldsymbol{\theta}) = \sum_{i=1}^{n} \log f_i \left( \sigma_i^{-1} \iota_i' \mathbf{B}(\boldsymbol{\beta})^{-1} u_t(\boldsymbol{\pi}); \lambda_i \right) - \log |\det(\mathbf{B}(\boldsymbol{\beta}))| - \sum_{i=1}^{n} \log \sigma_i$$

# The model

$$\mathbf{y}_t = \boldsymbol{\nu} + \mathbf{A}_1\mathbf{y}_{t-1} + \cdots + \mathbf{A}_p\mathbf{y}_{t-p} + \mathbf{B}\boldsymbol{\varepsilon}_t, \quad \text{stationarity of } \mathbf{y_t} \text{ implies} \quad \mathbf{y}_t = \mu + \sum_{j=0}^{\infty} \boldsymbol{\Psi}_j \mathbf{B}\varepsilon_{t-j}, \quad \boldsymbol{\Psi}_0 = \mathbf{I}_n,$$

1. The error process $\boldsymbol{\varepsilon_t} = (\varepsilon_{1,t}, \varepsilon_{2,t}, \ldots, \varepsilon_{n,t})$ is a sequence of strictly stationary random vectors, with each component $\varepsilon_{i,t}$, for $i = 1, \ldots, n$, having zero mean and finite unit variance.
2. The component processes $\varepsilon_{i,t}$, for $i = 1, \ldots, n$, are mutually independent, and at most one of them has a Gaussian marginal distribution.
3. For all $i = 1, \ldots, n$, the components $\varepsilon_{i,t}$ are uncorrelated in time, that is: $\mathrm{Cov}(\varepsilon_{i,t}, \varepsilon_{i,t+k}) = 0 \quad \text{for all} \quad k \neq 0$

$$\mathbf{y_t} = \mu + \sum_{j=0}^{\infty} \boldsymbol{\Psi}_j \mathbf{B}\varepsilon_{t-j} = \mu^* + \sum_{j=0}^{\infty} \boldsymbol{\Psi}_j^* \mathbf{B}^* \varepsilon_{t-j}^*, \tag{8}$$

Identification of $\mathbf{B}$ holds up to permutation and scaling (sign) as for some diagonal matrix $\mathbf{D} = \mathrm{diag}(d_1, \ldots, d_n)$ with nonzero diagonal elements, and for some permutation matrix $\mathbf{P}$ $(n \times n)$:

$$\mathbf{B}^* = \mathbf{BDP}, \quad \boldsymbol{\varepsilon}_t^* = \mathbf{P}'\mathbf{D}^{-1}\boldsymbol{\varepsilon}_t, \quad \mu^* = \mu, \quad \text{and} \quad \boldsymbol{\Psi}_j^* = \boldsymbol{\Psi}_j \quad (j = 0, 1, \ldots). \tag{9}$$

# Data

- The sample data ($\pi_t$, $x_t$, $R_t$) contains 92 quarterly observations from December 2000 to September 2023 in the Euro Area
  - Inflation ($\pi_t$): log difference of the production price index
  - Output gap ($x_t$): "Measuring the Euro Area Output Gap" Barigozzi et al. forthcoming
  - Interest rate $(R_t)$: Euribor 3 months

- The Shapiro-Wilk test and the QQ-plot of the resulting residuals show very strong departure from Normality for each equation residual.

Table 1: Coefficients of the reduced form VAR(2) model, significant parameters are in bold.

| | $\boldsymbol{\nu}$ | $\mathbf{A_1}$ | | | $\mathbf{A_2}$ | | |
|---|---|---|---|---|---|---|---|
| $\pi$ | 0.1173 | **0.7261** | **0.2635** | -0.2954 | -0.01145 | **-0.1789** | 0.2545 |
| $x$ | **0.3120** | 0.2099 | **0.9032** | -0.0433 | 0.0524 | 0.0345 | -0.0995 |
| $R$ | -0.0288 | 0.0883 | **0.0828** | **1.4251** | 0.0544 | **-0.0646** | **-0.4601** |

# Cholesky (Inflation first)

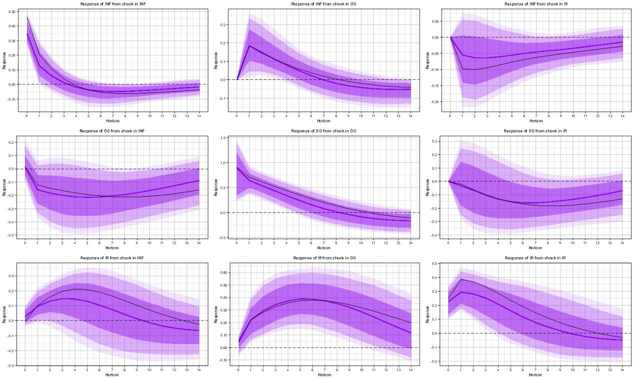



Figure 3: Response to one standard-deviation shock with 68% to 95% confidence intervals, 3000 residual moving block bootstrap median in purple, sample IRF in black

# Dealing with the permutation and sign issue

Some approach in the literature:

1. Minimize the *frobenius norm* from the sample estimates $\hat{\mathbf{B}}$ Herwartz [2018].

2. Herwartz [2018] find:

$$\widehat{\mathbf{B}}_{**} = \mathbf{B}_{\hat{r}} = \text{argmin}_r \, \mathcal{D} = \sum_{i=1}^{K} \sum_{j=1}^{K} \frac{1}{\widehat{\omega}_{ii}} \left( \widehat{b}_{ij} - \widetilde{b}_{ij}^{(r)} \right)^2 I(\widehat{b}_{ij} \widetilde{b}_{ij}^{(r)} < 0),$$

   where: $I(\cdot)$ is the indicator function, $\widehat{\omega}_{ii}$ is i-th diagonal element of $\widehat{\mathbf{\Omega}}$, $\widehat{b}_{ij}$ is an element of $\widehat{\mathbf{B}}$ and $\widetilde{b}_{ij}^{(r)}$ is an element of $\widetilde{\mathbf{B}}_r$, the latter is one of all the possible $(2^K)K! = 16, \; K = 3$ combinations.

3. Choose the permutation that maximizes the absolute value of the product of the diagonal elements and then make the diagonal elements positive Lanne and Luoto [2021], Pham and Garat [1997].

# Bootstrap

**Algorithm 1** Bootstrap Algorithm for ICA-SVAR Model

**for** $i = 1$ to $B$ **do**
    Generate a bootstrap sample through a residual moving block bootstrap
    Refit the VAR model on the bootstrap sample
    Compute the eigen-decomposition of the covariance matrix $\hat{\mathbf{\Omega}}_*$
    Use any ICA estimator to get $\hat{\mathbf{B}}_*$
    Normalize $\hat{\mathbf{B}}_*$ by multiplying each column by the corresponding shock standard deviation i.e. right multiply by $\text{diag}(\sigma_{\varepsilon_{*1t}}, \sigma_{\varepsilon_{*2t}}, \sigma_{\varepsilon_{*3t}})$
    Select the column permutation which maximizes the absolute value of the diagonal elements of $\hat{\mathbf{B}}_*$, then change the column sign in order to have positive diagonal elements and call it $\hat{\mathbf{B}}_{**}$
    Compute and store the IRFs
    Store $\hat{\mathbf{B}}_{**}$
**end for**

# ICA through Distance Covariance

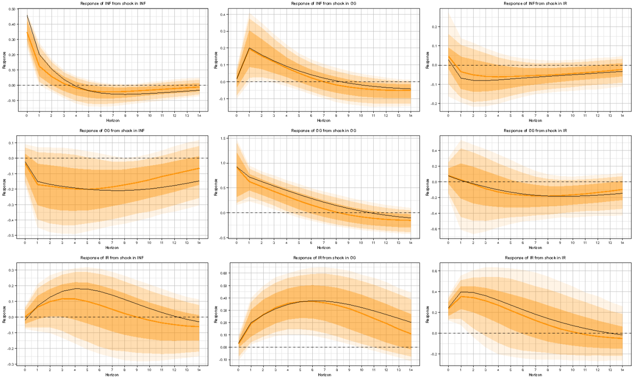



Figure 4: Response to one standard-deviation shock with 68% to 95% confidence intervals, 3000 residual moving block bootstrap median in orange, sample IRF in black

# Testing the recursive identification scheme

Herwartz [2018]:

$$\lambda_{JS} = \left(\mathbf{R}\,\mathrm{vec}(\hat{\mathbf{B}}) - \mathbf{r}\right)' \left(\mathrm{Cov}\left(\mathbf{R}\mathrm{vec}(\hat{\mathbf{B}}^{**})\right)^{-1} \left(\mathbf{R}\,\mathrm{vec}(\hat{\mathbf{B}}) - \mathbf{r}\right) \approx \chi^2_{(J)}$$

where $\mathbf{R}$ is a known selection matrix, $\mathbf{r}$ is a known vector representing the considered restrictions, and the composite null hypothesis is:

$$H_0 : \mathbf{R}\,\mathrm{vec}(\mathbf{B}) = \mathbf{r}.$$

$\hat{\mathbf{B}}_{**}$ is the bootstrap version of the covariance decomposition matrix. $\mathbf{r} = (0, 0, 0)'$ and $\mathbf{R}$ selects the elements above the diagonal.

This test do not reject the recursive identification scheme of the structural matrix at any level of significance.

# Conclusions and ways ahead

- Cannot reject recursive identification even after controlling for sign and column permutation.
- Relaxing the independence assumption:
  - GMM estimation Lanne and Luoto [2021] only needs a number of zero co-kurtosis conditions.
  - Identification of Independent Shocks Under (Co-)heteroskedasticity Herwerts, Wang [2023].
- Do countries react asymmetrically to monetary policy?
  - A cross-country analysis in the EA could potentially reveal asymmetries in the immediate response to monetary policy shocks.

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