

# Introduction to GWAS Linkage Disequilibrium and Linear Regression

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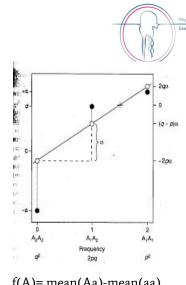
- What marker is associated with the phenotype?
- What individual is genetically more susceptible?

trait id	snp1	snp2	snp3	snp4	snp5	snp6
9.616 1	Aa	Aa	AA	Aa	Aa	AA
10.140 2	aa	AA	AA	AA	AA	AA
10.687 3	AA	AA	aa	AA	aa	Aa
9.906 4	Aa	aa	Aa	AA	AA	aa

- What marker is associated with the phenotype?
- What individual is genetically more susceptible?

## Allele substitution effect

The effect that the presence of a copy of an allele has on the phenotype (regarding the reference allele).



f(A) = mean(Aa) - mean(aa)





- What marker is associated with the phenotype?
- What individual is genetically more susceptible?

## Allele substitution effect

snp1	snp2	snp3	snp4	snp5	snp6
-0.4	0	0	0.2	-0.4	0.4

GWAS aims to discover the loci with causal effect and their magnitude



- What marker is associated with the phenotype?
- What individual is genetically more susceptible?

id (pop. µ=10)	Genetic merit	trait	id
1	0.20	9.616	1
2	0.40	10.140	2
3	0.00	10.687	3
4	-0.80	9.906	4
	Predictin	g the genetic respon	nse, the phenotype or th

Predicting the genetic response, the phenotype or the disease susceptibility is approached in Genome-Wide Prediction

## Genome-wide association studies (GWAS)



- Based on linear regression models (mathematics, linear algebra, statistics)
- Uses linkage disequilibrium between genomic markers and genes



## Genome-wide association studies (GWAS)

- Linkage disequilibrium
- Linear regression



## Linkage disequilibrium recap



Linkage	SNP1	SNP2	SNP3	
	[A / T]	[C / G]	[A / G]	Equilibrium
	A	_ c	G	Equilibrium
	A	_ c	А	
	_т	_ G	G	
	- T	– G	A	<u> </u>
				J

**Haplotype:** specific combination of alleles that appear in the same chromosome or segment (*in-cis*) and are inherited together from a single parental.





S	SNP1[A/a]	SNP2	[B/b]
Major allele frequency:	p(A)	p(	B)
Minor allele frequency:	p(a)	p(	b)

### Marker segregation:

Haplotype frequency  $p(ab) = p(a) \times p(b)$  Linkage equilibrium Haplotype frequency  $p(ab) \neq p(a) \times p(b)$  Linkage disequilibrium





### SNP2 Allele

4.		В	b	
Allele	Α	AB p(A)p(B)	p(A)p(b) Ab	p(A)
SNP1 /	а	aB p(a)p(B)	p(a)p(b) ab	p(a)
SN		p(B)	p(b)	

Example:

p(ab)=p(a)p(b) Expected Haplotype Frequencies

 $p(A)p(B)+p(a)p(B)=p(B)\{p(A)+p(a)\}=p(B)$ 





### SNP2 Allele

4)		В	b	
Allele	Α	AB p(A)p(B)+D	p(A)p(b)-D	b p(A)
SNP1 /	а	aB p(a)p(B)-D	p(a)p(b) + D	b p(a)
S		p(B)	p(b)	

Example:

$$p(ab)\neq p(a)p(b) \rightarrow p(ab)=p(a)p(b)+D$$
Non-Expected Haplotype
Frequencies (D is LD degree)



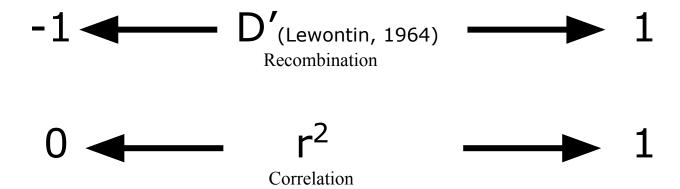


Question: What markers are in LD?

- A frequency=0.6; B frequency=0.3; haplotype ab frequency =0.40
- a frequency=0.4; C frequency=0.3; haplotype AC frequency=0.18
- haplotype bC frequency=0.49



Most common measurements







**D**'(Lewontin, 1964)

D' = D / D<sub>max</sub>  

$$D_{max} = min (p(A)p(B), p(a)p(b))$$
 D < 0  
 $D_{max} = min (p(A)p(b), p(a)p(B))$  D > 0

D' sign depends on the reference allele

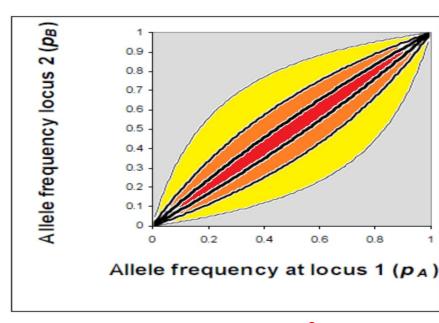




$$r^{2} = \underline{\qquad}$$

$$p(a)p(b)p(A)p(B)$$

- Between 0 1
- Depends on allele frequency
- Most common metric



r<sup>2</sup>>0.80 r<sup>2</sup>>0.50 r<sup>2</sup>>0.20 r<sup>2</sup><0.20



- Non-random allele segregation between 2 or more loci

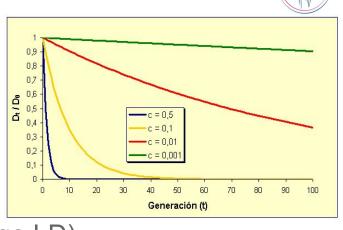
- Loci on the same chromosome with LD have larger probability of being transmitted together (less likely recombination)
- Physically close loci tend to be in higher LD, as recombination rates are low





### Possible causes of LD

- Close location
- Low recombination rates
- Recent populations with low founder size
- Selective breeding (e.g. Holstein Long range LD)
- New mutations
- Inbreeding
- Population bottlenecks
- Population stratification
- Asexual reproduction





### **Applications**

- We can infer the genotype of one locus given we know the genotype of another locus with LD (Imputation)
- We can predict the effect of a gene, given we know a marker genotype in LD (GWAS – GWP)
- Caution
  - LD may disappear in future generations due to new recombinations. Consider this cautiously when applying results to further apart generations or low LD markers
  - Differentiate between statistical linkage and physical linkage





Why do we care about LD?

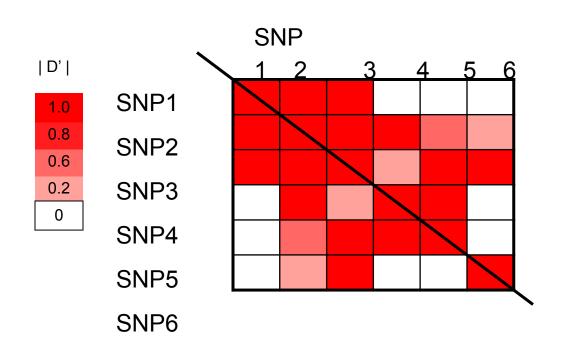
The power of an indirect association study depends on 4 parameters:

- Disease-allele effect size
- Disease-allele frequency
- Marker-allele frequency
- Extent of LD between marker and disease locus

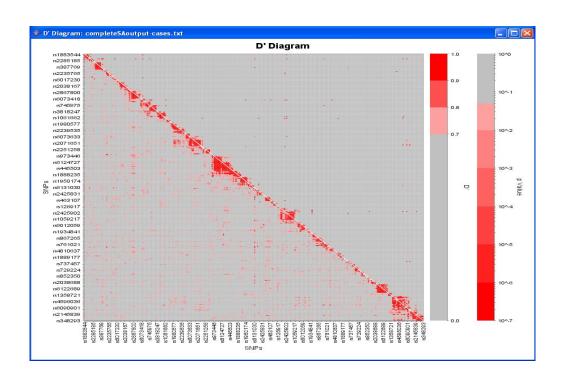


## **Visualizing LD**



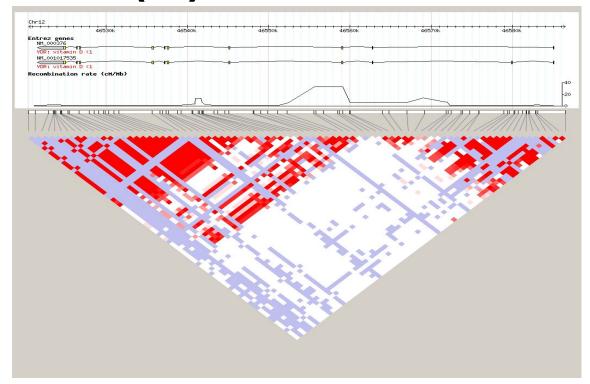








## Haploview: TCN2 (r<sup>2</sup>)





## Linear regression recap

## **Linear regression**



$$\mathbf{y} = a + b\mathbf{x} + \mathbf{e}$$

 Estimate the line that minimizes the MSE Sum(y-ÿ)<sup>2</sup>.

$$y = \hat{a} + \hat{b}x$$

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$$00 - \hat{b} = \hat{b}x$$

$$00 - \hat{b$$

$$\hat{b} = \text{cov}(\mathbf{x}, \mathbf{y}) / Var(\mathbf{x})$$
  
 $\hat{a} = \overline{\mathbf{y}} - \hat{b}\overline{\mathbf{x}}$ 

Is the *b* estimate statistically significant?  $\rightarrow$  Real effect (association)

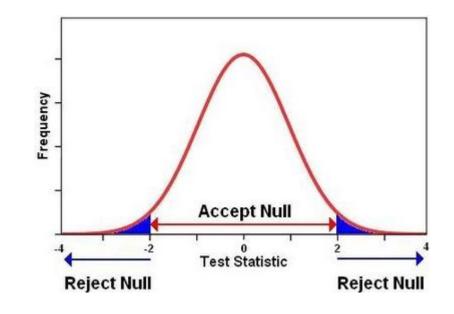




$$H_0:\beta_1=0$$

$$H_1:\beta_1\neq 0$$

- Continuous variable → t-Student
- Binary variable → Chi-squared, log-ratio test



Significance level α (e.g. 0.05, 0.01,...)



$$H_0: \beta_1 = 0$$

 $H_1:\beta_1\neq 0$ 

	One-tail (left) $H_0: \beta_+ = h_1$			Two-tails	One-tail (right)
Contrast	$H_0: \beta_1 = b_1$			$\beta_1 = b_1$	$H_0: \beta_1 = b_1$
	$H_1: \beta_1 < b_1$		$\Pi$	$_1:\beta_1\neq b_1$	$H_1:\beta_1>b_1$
Statistic (t-Student)		$t = \frac{\hat{\beta_1} - b_2}{\sqrt{\hat{s_R}^2/S}}$	$S_{xx}$	$ con \hat{s_R}^2 = \frac{ss}{s} $	$\frac{y_y - \hat{\beta_1} S S_{xy}}{n-2}$
Reject Region	$t < t_{a,n-2}$		t  >	$> t_{1-a/2,n-2}$	$t > t_{1-a,n-2}$

$$SS_{xy} = \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y}) = \sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y},$$

$$SS_{xx} = \sum_{i=1}^{n} (x_i - \bar{x})^2 = \sum_{i=1}^{n} x_i^2 - n\bar{x}^2 = n\sigma_x^2$$

$$t = \frac{\beta_1 - b_1}{\sqrt{\hat{s}_R^2 / S S_{xx}}}$$

Does my *t-statistic* value belong to the null hypothesis distribution

$$\hat{s_R}^2 = \frac{SSE}{n-2} = \frac{\sum_{i=1}^n e_i^2}{n-2} = \frac{SS_{yy} - \beta_1 SS_{xy}}{n-2} \quad SSE = \sum_{i=1}^n e_i^2 = \sum_{i=1}^n y_i - (\beta_0 + \beta_1 x_i)^2$$

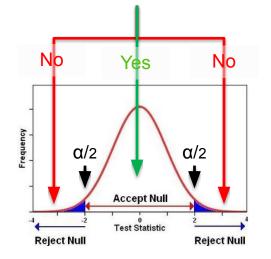


$$H_0: \beta_1 = 0$$

$$H_1: \beta_1 \neq 0$$

$$t = \frac{\beta_1 - b_1}{\sqrt{\hat{s_R}^2 / S S_{xx}}}$$

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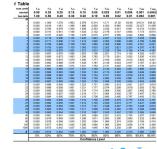


Calculate the <u>p-value</u>: probability of obtaining a value of the statistic at least as extreme as the one obtained in the experiment, if we repeat the experiment an infinite number of times

i.e. two-tails test

$$2 \cdot (1 - p(z \le t))$$

If *p-value*  $\alpha \rightarrow \text{reject}$  null-hypothesis









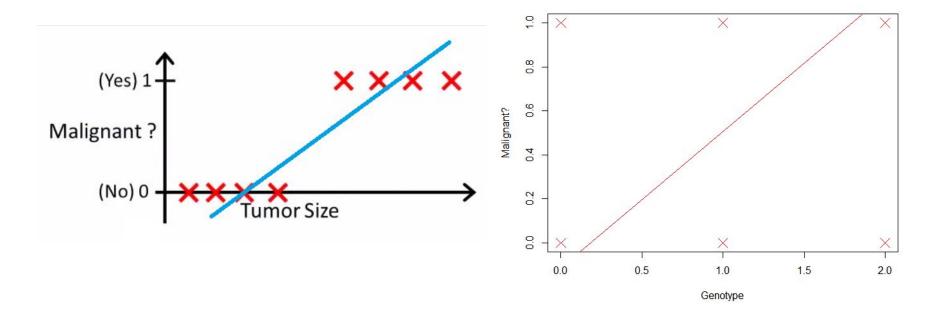






## **Logistic regression**







## **Logistic regression** y = a + bx + e



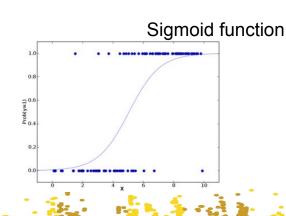
Uses the sigmoid function

$$\log_{base} \frac{P(Y=1)}{1 - P(Y=1)} = a_0 + a_1 x$$

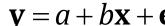
$$\frac{P(Y=1)}{1 - P(Y=1)} = base^{\left(a_0 + b_1 x\right)} \Rightarrow P(Y=1) = base^{\left(a_0 + b_1 x\right)} - P(Y=1) base^{\left(a_0 + b_1 x\right)} \Rightarrow$$

$$\left(1 + base^{\left(a_0 + b_1 x\right)}\right) P(Y=1) = base^{\left(a_0 + b_1 x\right)} \Rightarrow$$

$$P(Y=1) = \frac{base^{(a_0 + b_1 x)}}{1 + base^{(a_0 + b_1 x)}} = \frac{1}{1 + base^{-(a_0 + b_1 x)}}$$



## **Logistic regression** $\mathbf{y} = a + b\mathbf{x} + \mathbf{e}$





Estimation of coefficients a and b

The intercept a is the log of the odds for the control group (AA)

	0(control)	1(case)
AA	$AA_{o}$	AA <sub>1</sub>
Aa	Aa <sub>o</sub>	Aa <sub>1</sub>

$$a = \log \frac{AA_1/AA_{totales}}{AA_0/AA_{totales}} = \log \frac{AA_1}{AA_0}$$

The regression coefficient b is the log of the odds ratio between the Aa group and the AA group:

$$b = \log \frac{\frac{Aa_{1}/Aa_{0}}{AA_{1}/AA_{0}}}{\frac{Aa_{1}/AA_{0}}{Aa_{0}/AA_{1}}} = \frac{Aa_{1}/AA}{Aa_{0}/AA}$$

## **Logistic regression** y = a + bx + e



- How to interpret the Odd Ratio
  - It is a measurement of the degree of association between
     2 variables, and is estimated with respect to a given base
  - Ranges between 0 and ∞
  - OR=1→ lack of association; OR<1 → negative association; OR>1→ positive association
  - If OR<1, estimate the inverse relationship is preferred for easiness of interpretation

	0(control)	1(case)
AA	$AA_{o}$	AA <sub>1</sub>
Aa	Aa <sub>o</sub>	Aa <sub>1</sub>



## Logistic regression

$$\mathbf{y} = a + b\mathbf{x} + \mathbf{e}$$



- Two examples
  - Colorectal cancer and meat consumption.
     USA male (30-55yr) and male (40-75yr)
     High vs low levels of processed meat intake

Population prevalence: 1%

OR=1.22

Prevalence in the high intake levels of meat?

Meta-Analysis > Int J Cancer. 2006 Dec 1;119(11):2657-64. doi: 10.1002/ijc.22170.

### Meat consumption and risk of colorectal cancer: a meta-analysis of prospective studies

Susanna C Larsson <sup>11</sup>, Alicja Wolk
Affiliations + expand
PMID: 16991129 DOI: 10.1002/ijc.22170
A Sprint
Free article

#### Abstract

Accumulating epidemiologic evidence indicates that high consumption of red meat and of processed meat may increase the risk of colorectal cancer. We quantitatively assessed the association between red meat and processed meat consumption and the risk of colorectal cancer in a meta-analysis of prospective studies published through March 2006. Random-effects models were used to pool study results and to assess dose-response relationships. We identified 15 prospective studies on red meat (involving 7,367 cases) and 14 prospective studies on processed meat consumption (7,903 cases). The summary relative risks (RRs) of colorectal cancer for the highest vs. the lowest intake categories were 1.28 (95% confidence interval (CI) = 1.15-1.42) for red meat and 1.20 (95% CI = 1.11-1.31) for processed meat. The estimated summary RRs were 1.28 (95% CI = 1.18-1.39) for an increase of 120 q/day of red meat and 1.09 (95% CI = 1.05-1.13) for an increase of 30 q/day of processed meat. Consumption of red meat and processed meat was positively associated with risk of both colon and rectal cancer, although the association with red meat appeared to be stronger for rectal cancer. In 3 studies that reported results for subsites in the colon, high consumption of processed meat was associated with an increased risk of distal colon cancer but not of proximal colon cancer. The results of this meta-analysis of prospective studies support the hypothesis that high consumption of red meat and of processed meat is associated with an increased risk of colorectal cancer.



## Logistic regression

$$\mathbf{y} = a + b\mathbf{x} + \mathbf{e}$$



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Prevalence in the high intake levels of meat?

answer: 1.22%

## **Logistic regression** $\mathbf{y} = a + b\mathbf{x} + \mathbf{e}$

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Article Published: 11 December 2020

#### Genetic mechanisms of critical illness in COVID-19

Erola Pairo-Castineira, Sara Clohisey, [...]J. Kenneth Baillie

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Two examples

COVID-19 and Neardenthal gene (Chr3).

UK Biobank (679531 controls), severe COVID (1128

cases)

Table 1 | Lead variants from independent genome-wide significant regions

			1000000			_	_		19070	10.1	1111111
SNP	Chr.: pos.	Risk	Alt.	RAF <sub>gee</sub>	RAFukb	OR	CI	P <sub>gcc.ukb</sub>	P <sub>gcc.gs</sub>	P <sub>goc.100k</sub>	Locus
rs73064425	3: 45,901,089	Т	С	0.15	0.07	2.1	1.88-2.45	4.8 × 10 <sup>-30</sup>	$2.9 \times 10^{-27}$	3.6 × 10 <sup>-32</sup>	LZTFL1
rs9380142	6: 29,798,794	А	G	0.74	0.69	1.3	1.18-1.43	3.2×10 <sup>-8</sup>	0.00091	1.8 × 10 <sup>-8</sup>	HLA-G
rs143334143	6: 31,121,426	А	G	0.12	0.07	1.8	1.61-2.13	8.8 × 10 <sup>-18</sup>	2.6 ×10 <sup>-24</sup>	5.8 × 10 <sup>-18</sup>	CCHCR1
rs3131294	6: 32,180,146	G	Α	0.9	0.86	1.5	1.28-1.66	2.8 × 10 <sup>-8</sup>	1.3 × 10 <sup>-10</sup>	2.3×10 <sup>-8</sup>	NOTCH4
rs10735079	12: 113,380,008	Α	G	0.68	0.63	1.3	1.18-1.42	1.6 × 10 <sup>-8</sup>	2.8 × 10 <sup>-9</sup>	4.7 × 10 <sup>-6</sup>	OAS1-OAS3
rs2109069	19: 4,719,443	А	G	0.38	0.32	1.4	1.25-1.48	4 × 10 <sup>-12</sup>	4.5 × 10 <sup>-7</sup>	2.4 × 10 <sup>-8</sup>	DPP9
rs74956615	19: 10,427,721	A	T	0.079	0.05	1.6	1.35-1.87	2.3 × 10 <sup>-8</sup>	2.2 × 10 <sup>-13</sup>	3.9 × 10 <sup>-6</sup>	TYK2
rs2236757	21: 34,624,917	А	G	0.34	0.28	1.3	1.17-1.41	5×10 <sup>-8</sup>	8.9 × 10 <sup>-5</sup>	8.3 × 10 <sup>-7</sup>	IFNAR2

As this is a meta-analysis of all available data, external replication cannot be attempted, so SNPs are included if they meet a more stringent P-value threshold of P < 10° 5. SNP, the strongest SNP in the locus. Chr.: pos., chromosome and position of the top SNP (build 37); Risk, risk allele; Alt., alternative allele; RAF, risk allele frequency; OR, effect size (odds ratio) of the risk allele in the GenOMICC European analysis; CI, 95% confidence interval for the odds ratio in the GenOMICC European cohort; P, P value; Locus, gene nearest to the top SNP. Subscript identifiers indicate the cohorts used for cases (gcc, GenOMICC European cohort) and controls (ukb, UK Biobank; gs, Generation Scotland; 100k, 100,000 Genomes Project).

Population prevalence: 0.16%

OR = 2.1

Prevalence in the Risk Allele group ?



## **Logistic regression** $\mathbf{y} = a + b\mathbf{x} + \mathbf{e}$

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Population prevalence: 0.16%

OR = 2.1

Prevalence in the Risk Allele group ?

answer: 0.34%

(needs to be considered carefully, because control group did not report COVID19; we don't know what would have been the progress of the disease)







# YES Now you!

Compute the t-student statistic given some data, and accept or reject the null-hypothesis

- 1.Basis of linear regression.R
- 2.Basis of logistic regression.Rmd
- 3. Homework: Analyze data in Exercise 1. example Data.xlsx,
- Determine SNPs associated to the phenotype, and calculate the Odds-Ratio
- b) If you got 2 significant SNPs, that's wrong. Figure out why.

