Linear Mixed Models

Introduction

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Our questions

If we want to answer the following questions….

Time effect: What is the shape of the trajectory of the mean response over time?

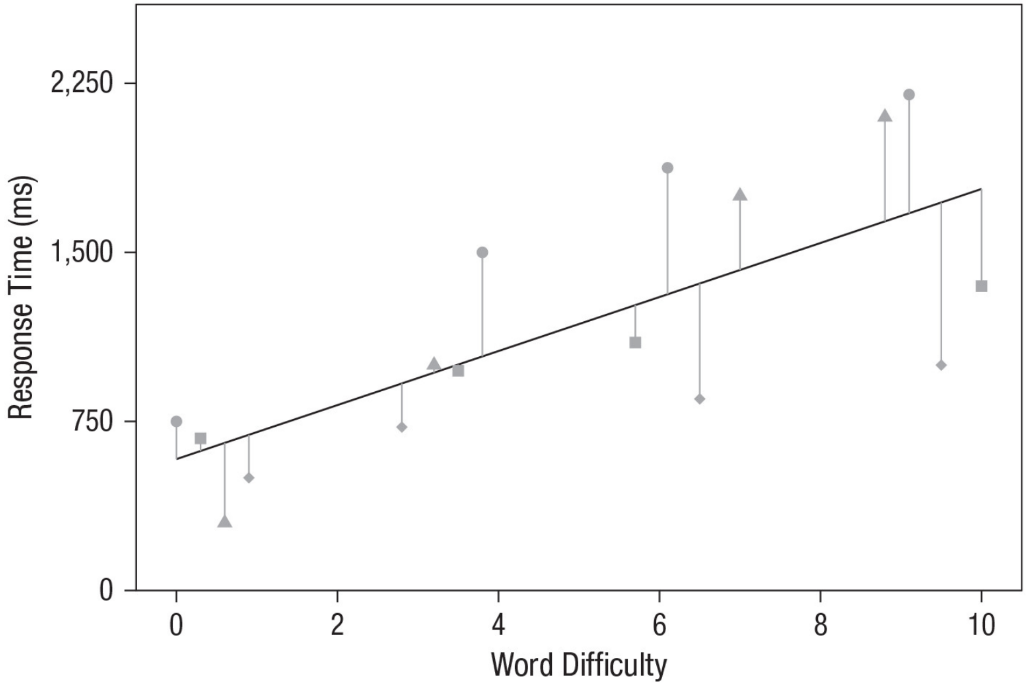
Group effect: What is the average difference between groups of individuals?

Interaction between time and group: How does the relationship between the response and time vary according to groups of individuals?

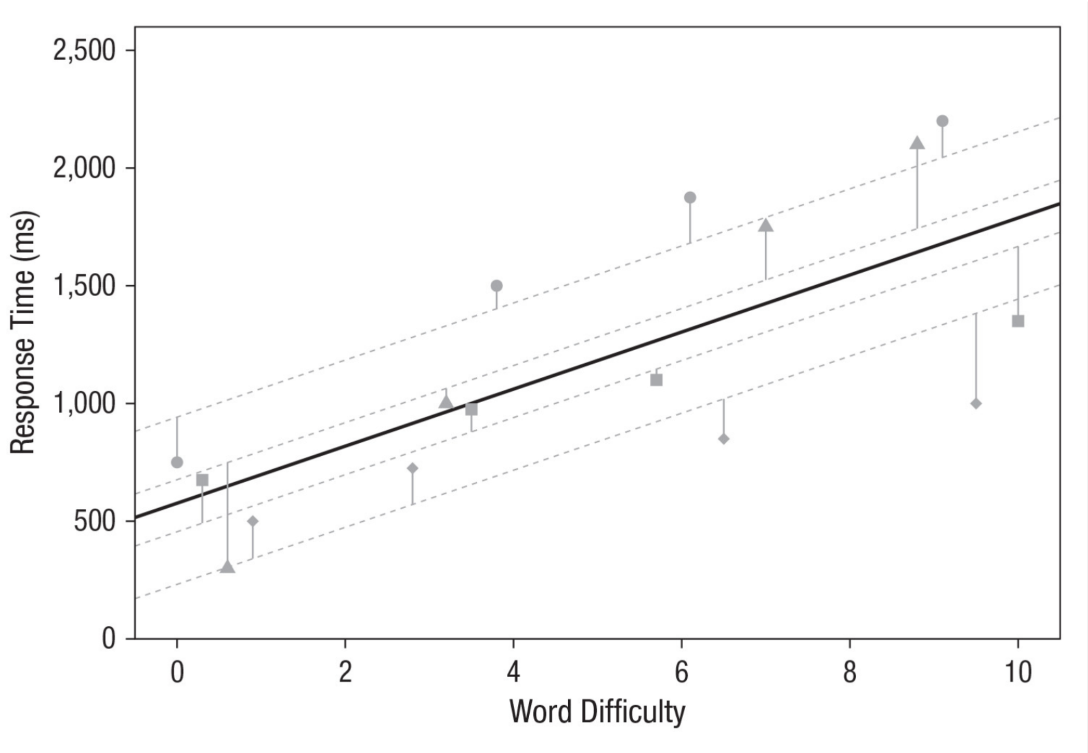
Linear Mixed Models

- Natural heterogeneity across subjects → Some subset of the regression coefficients vary randomly from one individual to another
- Individuals in population are assumed to have their own subject-specific mean response trajectories over time.

Fixed effects only



Mixed – fixed + random



Fixed and Random effects

The mean response is modeled as a combination of population characteristics (***fixed effects***) assumed to be shared by all individuals, while subject-specific effects (***random effects***) are unique to a particular individual.

Mixed Models

Linear Mixed Models are a particular type of hierarchical models which contain both fixed and random effects.

one-way ANOVA we have a single 'treatment' factor with several levels **(= groups),** and replicated observations at each level.

Empty model

one-way random-effect ANOVA model, is **$Y_{ij}=\beta_0+u_{0i}+\epsilon_{ij}$**

where β_0 is the intercept, u_{0i} the random effect, and ϵ_{ij} is the residual error terms.

The model has the following distributional assumptions:

- $u_{0i} \sim N(0,\sigma^2_{u0})$
- $\epsilon_{ij} \sim N(0, \sigma^2)$
- $u_{0i} \coprod \epsilon_{ij}$

Interpretation

β_0 is the fixed effect representing the overall mean of the response variable Y_{ij}

- u_{0i} is a random variable of which we estimate the variance. It is the individual-specific deviation from β_0 that can be predicted from the model and observed data.

- The *conditional mean response* for a specific individual is $E[Yij|u0i]=\beta0+u0i$

- The *marginal mean response* in the population (i.e., averaged over all individuals in population) is $E[Yij]=\beta0$

Estimation

- β coefficients can be estimated using *Maximum Likelihood* (ML) estimation
- ML does not take into account the degrees of freedom used for estimating fixed effects when estimating variance components
- This can lead to bias estimates (underestimation) for the variance components (small number of individuals). *Restricted Maximum Likelihood* (REML) can overcome this limitation

Literature

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