

## RESEARCH

# Methods to compute the composite log-likelihood (CLL) of allelic frequencies for the detection of signatures of selection in diploid genomes

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## Abstract

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## Background

Selection, both natural and artificial, is one of the major forces that can shape the genome of living organisms and change allele frequencies. A mutation that is beneficial for the adaptation of an organism to its environment, or that is of agricultural or industrial interest, tends to increase in frequency in the population, together with neighbouring genomic regions which are dragged along (“hitch-hiked”) in the process ([1]). Through the last decade, the on-going genomic revolution has been making available hundreds of thousands of genetic markers for several animal, microbial and plant species. By looking at the allele frequency at marker loci along the genome of populations experiencing different selective pressures, it is possible to identify genomic regions -and ultimately genes- involved in processes such as domestication, adaptation, evolution and artificial selection. There are a number of methods based on allele frequencies that have been developed to detect such signatures of selection. A popular method is Wright’s  $F_{ST}$  ([2, 3]) that has been applied to studies in humans ([4]), plants ([5]) and animals ([6]). Alternatively, the likelihood of the difference between allele frequencies in different populations can be estimated and used to detect the presence of signatures of selection ([7, 8]). However, there are several ways in which such likelihoods can be computed, and these might differ in computation requirements and statistical properties, such as the sensitivity to detect signals of selection or the behaviour along the margins of the dimensional space. It may therefore be of interest to investigate the statistical properties of different estimators for the likelihood of the difference between allele frequency, and assess how well they are capable of detecting actual signals of selection.

In this study we evaluated 4 different ways to estimate the likelihood of the difference in allele frequency between populations. The logarithm of the likelihoods thus calculated were then computed and combined across sliding windows along the genome (CLL, composite log-likelihood) in order to detect signatures of selection. The proposed CLL approaches were compared among them and with approaches

based on  $F_{ST}$  and on simple squared distances between genotypes. All methods were first analysed numerically by simulating scenarios along the entire dimensional space (frequency range and population size ratios between the test and null populations) and then tested with real data where strong signals of selection are known to be present. The methods hereby presented have been developed having in mind biallelic genetic markers (namely SNPs), but can in principle be adapted to any kind of markers and applied to any diploid organism.

## Methods

Let's assume we have a test population  $T$  (the population on which we want to detect signatures of selection) and a null or reference population  $N$  (the contrasting population(s)). On these populations  $F_{ST}$ , the squared difference of the allelic frequencies, and four different likelihood measures of allele frequency differences have been computed and evaluated in terms of their ability to detect signatures of selection. First, we present numerical details of the different calculations, then examples with real data on which the different estimators were tested.

### Detecting signatures of selection

#### $F_{ST}$

The  $F_{ST}$  (Wright's  $F$  statistics for *Subpopulation* vs *Total*), a.k.a. *fixation index*, is a measure of the genetic differentiation between (sub)populations (e.g. the test and reference populations in our case). For any given locus,  $F_{ST}$  was calculated as described by Nei ([3]); partitioning the total expected heterozygosity ( $H_T$ , or gene diversity) into the *interpopulation* diversity ( $D_{ST}$ ) and the *intrapopulation* diversity ( $\overline{H}_S$ ), the  $F_{ST}$  is defined as the proportion of gene diversity due to differentiation among subpopulations:

$$F_{st} = \frac{D_{st}}{H_T} = \frac{H_T - \overline{H}_S}{H_T} = 1 - \frac{\overline{H}_S}{H_T} \quad (1)$$

where  $\overline{H}_S = \sum_{i=1}^k w_i H_{S_i}$  is the weighted average of the expected heterozygosities within  $k$  subpopulations each accounting for a proportion  $w_i$  of the total population size ( $H_{S_i} = 1 - \sum_{j=1}^m p_j^2$  is the expected heterozygosity within subpopulation  $i$  -with  $m$  alleles at the given locus);  $H_T = 1 - \sum_{i=1}^m \overline{p}_i^2$  is the expected heterozygosity in the overall population, with  $\overline{p}_i = \sum_{k=1}^k w_k p_k$ , the average allele frequency at the given locus across  $k$  subpopulations each accounting for a proportion  $w_i$  of the total sample size.

$F_{ST}$  measures the differentiation between the null and reference populations, and was used in this study as primary benchmark against which other methods were evaluated.

#### Squared difference ( $D^2$ )

A very simple and naive approach for the estimation of the genetic distance between populations is the squared difference of their allele frequencies. For any given locus, the allele frequencies of the first allele in the test and null populations ( $p_1$  and

$p_2$ ) were calculated, and their squared difference used to measure the degree of differentiation between the two populations:

$$D^2 = (p_1 - p_2)^2 \quad (2)$$

While  $F_{ST}$  was chosen as benchmark due to its long-standing theoretical development and its wide-spread use in the detection of population substructure,  $D^2$  has been used as additional naive benchmark against which more sophisticated methods can be compared.

#### *Binomial log-likelihood (BLL)*

Allele frequencies at a given locus ( $A/a$ ) between the test and null populations are compared. Let  $p_N$  be the frequency of allele  $A$  in the null population, and  $n_T$  and  $k_T$  the total number of alleles and the number of  $A$  alleles in the test population respectively. Under the null hypothesis that  $p_T = p_N$ , the allele count in the test population can be thought of as a random sample from the same reference population. The following binomial function measures the likelihood that such a sample actually comes from the reference population:

$$LL_{ij} = \binom{n}{k} (1-q)^k q^{n-k} \quad (3)$$

where  $LL_{ij}$  is the probability to observe the test allele distribution at locus  $j$  on chromosome  $i$  in the null population. The smaller  $LL_{ij}$  the less likely it is that the two populations are one and the same, and the more justifiable it is to accept the alternative hypothesis that the test and null populations are actually different (for instance because of some genetic processes such as natural or artificial selection).

Unless the test and null populations have the same size and their allele frequencies are complementary ( $p_N = p_T$ ), the binomial log-likelihood is not symmetric. Therefore, three cases can be envisaged: 1) comparison of N vs T (*BLL1*); 2) comparison of T vs N (*BLL2*); 3) comparison of T vs N+T (*BLL3*). The latter is what was implemented in [8], and works well with multiple comparisons, when one population has to be compared against many other populations: it has the advantage of creating fewer numerical problems (lack of symmetry, falling outside of the computational space etc ...), but has the shortcoming of shrinking the power of the comparison (since  $T \subset N$  -the test population is included in the null population). Additionally, it is often the case that two populations have to be compared against each other. For the reasons above, all three scenarios (*BLL1*, *BLL2*, *BLL3*) were evaluated in the present study.

#### *Multiplicative log-likelihood (MLL)*

To overcome the issue of the lack of symmetry with BLL, a possibility is to combine the likelihood of *BLL1* and *BLL2* (T vs N and N vs T). A natural approach to combining probabilities is through multiplication. Therefore the multiplicative log-likelihood was defined as:

$$MLL_{ij} = BLL_{ij(TvsN)} * BLL_{ij(NvsT)} \quad (4)$$

where  $MLL_{ij}$  is the multiplicative log-likelihood at locus  $j$  on chromosome  $i$  and  $BLL_{ij(TvsN)}$  and  $BLL_{ij(NvsT)}$  are the two binomial log-likelihoods (BLL1 and BLL2).

#### *Hypergeometric log-likelihood (HGLL)*

An alternative approach is to look at the test population as a random sample from the reference population and calculate the probability of obtaining the observed allele counts. Under the null hypothesis that  $p_N = p_T$ , the two populations can be combined into one and such probability would follow a hypergeometric distribution:

$$HGLL_{ij} = \frac{\binom{n_{A_N} + n_{A_T}}{n_{A_T}} \cdot \binom{(N_N + N_T) - (n_{A_N} + n_{A_T})}{N_T - n_{A_T}}}{\binom{N_N + N_T}{N_T}} \quad (5)$$

where  $n_{A_N}$ ,  $n_{A_T}$ ,  $N_N$  and  $N_T$  are the number of A alleles and the total number of alleles in the null and test population respectively. HGLL is symmetric with respect to the population chosen as test or reference ([9]), and assumes that sampling is without replacement, which may be closer to the allele sampling process involved in the biological differentiation of populations.

#### *Distribution of the differences (DIFF)*

Instead of comparing the allele frequency observed in the test population against that of the null population, one could look directly at the difference between the two allele frequencies. First, the absolute difference between allele frequencies is calculated:  $d = |p_1 - p_2|$ . In order to obtain a likelihood value for  $d$ , both an analytical (*DIFF1*) and numerical (*DIFF2*) approach were adopted. In *DIFF1*,  $d$  was standardized ( $z = \frac{d}{\sqrt{\hat{p}(1-\hat{p})(1/N_N + 1/N_T)}}$ ) and compared against a normal distribution. In *DIFF2*, genotypes were randomly permuted between the null and test population, and the difference between the allele frequencies calculated each time. A distribution of values of  $d$  under  $H_0$  ( $d = 0$ ) was thus produced and its empirical probability distribution function used to obtain a likelihood value for the allele frequency difference between the original populations,  $d$ . Different numbers of permutations were tested in terms of results obtained and computation time.

#### *Simulated scenarios*

For all methods for the detection of signatures of selection, different scenarios were simulated, in order to evaluate the statistical behaviour and computational requirements for changing values of the parameters used. Specifically, the methods were evaluated against different values of allele frequency and different sample sizes in the null and test populations. The whole range of allele frequency (from 0 to 1) was simulated, and three different population size ratios were considered: balanced design (equally sized null and test population), mildly unbalanced design (1:2 population size ratio), and extremely unbalanced design (1:10 population size ratio).

#### *“Ballons d’essai”*

Data from a population of xxx Holstein-Frisian (zzz males and xxx females) and yyy Piedmontese (males and females?) cattle were available. The first breed has

been long selected for dairy production, the latter for meat production. All animals were genotyped with the Bovine 54k SNP-chip. Genotypes were edited for individual and marker call-rate ( $> 95\%$  ?) and for MAF ( $> 0.05$  ?). Remaining missing genotypes were imputed using (check this!). Xxx SNPs were eventually available for analysis. We selected the zzz SNPs on chromosome 3 (BTA-3) as working example to test the different estimators of the CLL of the allele frequency difference between populations and the reference methods ( $F_{ST}$  and Euclidean distances) for the detection of signatures of selection. BTA-3 is known to host, within the gene *SLC35A3* (position: 43400328-43445390 bps, approximately halfay along the chromosome), the point mutation responsible for CVM (Complex Vertebral Malformation) in Holstein-Frisian cattle ([10]). CVM is a recessive inherited disorder that frequently causes abortion or perinatal death in Holstein-Frisian calves. Other cattle breeds -including Piedmontese- are not affected by the condition. A strong signal is therefore expected to be found at this site, making this an ideally suited comparison to test different methods for the detection of signatures of selection.

¿ Also DGAT1, Myostatin, Caseins ?

### Sliding windows

When analysing actual data,  $F_{ST}$  and  $D^2$  values, and the likelihoods obtained with all the methods presented, were combined in sliding windows based on the base-pair distance (bps) between SNPs, in order to reduce the influence of spurious signals. A fixed sliding window of 200 kbps (check this!) was used, and composite log-likelihoods (CLL) were thus obtained:

$$CLL_{i\bar{j}} = \frac{1}{w} \sum_{j=1}^{j+w-1} LL_{ij} \quad (6)$$

where  $CLL_{i\bar{j}}$  is the composite log-likelihood at position  $\bar{j}$  (midpoint of the sliding window) on chromosome  $i$ , and  $LL_{ij}$  are the log-likelihood values calculated for all the SNPs between SNP  $j$  and 200 kbps ahead of it.

### Software

Code in C/C++ (check with Nelson!) was written to implement all of the statistical methods to detect signatures of selection presented above. The R programming environment and Octave ([11]) for statistical analysis were used to prototype some of the tests and produce graphical visualization of the results.

## Results and discussion

All methods were evaluated in terms of their statistical properties (sensitivity to detect signatures of selection, symmetry around population subdivision, robustness to unbalanced data, behaviour at the margin of the dimensional space) and computation requirements.

For biallelic loci,  $D^2$  is intrinsically symmetric:  $(p_1 - p_2)^2 = ((1 - p_1) - (1 - p_2))^2$

Is Fst obsolete?

Use haplotypes (Gianola et al.)?

## Numerical illustration

## Application to real data

## Testing significance

When analysing tens or hundreds of thousands of markers the problem of multiple comparisons is incurred. This is a known statistical problem in general ([12]), and specifically in the field of genetic studies ([13, 14]). As the number of independent tests increases, the probability of obtaining at least one false positive, under  $H_0$ , approaches one. This probability is a function of the number of tests  $n$  and of the chosen significance threshold  $\alpha$ :  $P(\text{false\_positive}) = 1 - (1 - \alpha)^n$ . When, for instance, 50000 SNP markers are tested along the genome in search of signatures of selection, for  $\alpha = 0.01$ , 500 false positive associations are expected on average, just by mere chance.

- Bonferroni correction: very strict; implicit prior assumption that the probability that *all* tests are null ( $H_0$  is true) is not small. If we believe that all tests could be null, then aiming to make the number of false positives zero is justifiable ([15])
- FDR
- Permutation test

## Conclusions

### Competing interests

The authors declare that they have no competing interests.

### Author's contributions

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### Acknowledgements

Data (Marras ...) ? ...

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Figures

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Tables

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