Online Appendix for "Multiple Testing and the Distributional Effects of Accountability Incentives in Education"

Steven F. Lehrer[*]
Queen's University,
NYU–Shanghai, and NBER

R. Vincent Pohl[†]
Mathematica
Policy Research

Kyungchul Song[‡]
University of
British Columbia

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Abstract

This is the online appendix for Lehrer, Pohl, and Song (2019). Three sections are included, which provide further details regarding (1) the asymptotic validity of the multiple testing procedure, (2) empirical results under block bootstrapping, and (3) an additional application of multiple testing to distributional treatment effects.

[*] School of Policy Studies and Economics Department, email: lehrers@queensu.ca.

[†] Email: vincent.pohl@gmail.com.

[‡] Vancouver School of Economics, email: kysong@mail.ubc.ca.

A Technical Appendix

Our first result is the asymptotic linear representation of $\sqrt{n}(\hat{q}_d(\tau)-q_d(\tau))$ and its bootstrap version that is uniform over $\tau \in [\tau_L, \tau_U]$. Let us introduce some notation. Let

$$a_\tau(Y_i; q) = \tau - 1\{Y_i \leqslant q\}, \tag{1}$$

and

$$\Delta(Y_i; q) = \sqrt{n}\int_0^1 \left(a_\tau(Y_i; q + n^{-1/2}us) - a_\tau(Y_i; q)\right) ds. \tag{2}$$

(Note that the right hand side in (2) does not depend on τ.)

Recall $J_d(\tau_U, \tau_L) = \{q_d(\tau) : \tau \in [\tau_L, \tau_U]\}$. For $q \in J_d(\tau_U, \tau_L)$ and $u \in \mathbb{R}$, let

$$\varphi_n(Y_i; q) = \Delta(Y_i; q)/n^{3/4} = -n^{-1/4}\int_0^1 1\{q < Y_i \leqslant q + n^{-1/2}us\}ds. \tag{3}$$

Let $\mathcal{B}$ be a class of bounded measurable functions $b : \mathbb{R} \times \mathcal{X} \times \{0,1\} \to \mathbb{R}$. Define

$$\mathcal{H}_n = \{\varphi_n(\cdot; q)b(\cdot) : (q, b) \in J_d(\tau_U, \tau_L) \times \mathcal{B}\}. \tag{4}$$

We let $V_i = (Y_i, X_i', D_i)$ for brevity of notation.

We introduce a pseudo-norm $\|\cdot\|_{P,2}$ on the set of measurable functions on $\mathbb{R} \times \mathcal{X} \times \{0,1\}$: $\|f\|_{P,2} = (E|f(V_i)|^2)^{1/2}$, for any measurable map f. For each $\varepsilon > 0$, let $N_{[]}(\varepsilon, \mathcal{H}_n, \|\cdot\|_{P,2})$ denote the ε-bracketing number of $\mathcal{H}_n$ with respect to $\|\cdot\|_{P,2}$ (see van der Vaart and Wellner, 1996, p. 83).

Lemma A.1 *There exist constants $C_1, C_2, C_3, C_4 > 0$ such that for each $\varepsilon \in (0,1)$, there exists a set of brackets $[h_{L,j}, h_{U,j}]$ with $1 \leqslant j \leqslant N(\varepsilon)$, such that the brackets cover $\mathcal{H}_n$ and for each $k \geqslant 2$,*

$$E[|h_{L,j}(V_i) - h_{U,j}(V_i)|^k] \leqslant C_1(C_2 n^{-1/4})^{k-2}\varepsilon^2, \tag{5}$$

and

$$\log N(\varepsilon) \leqslant C_3 - C_3\log(\varepsilon) + C_4 \log N_{[]}(C\varepsilon^2, \mathcal{B}, \|\cdot\|_2). \tag{6}$$

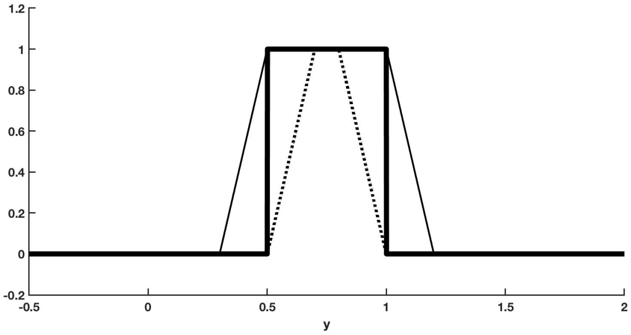


Figure 1: Illustration of $\varphi_{U,\delta}, \varphi_{L,\delta}$: The solid thick line depicts $\varphi(\cdot; q, s)$, the solid thin line $\varphi_{U,\delta}(\cdot; q, s)$ and the dotted line $\varphi_{L,\delta}(\cdot; q, s)$. Here we take $n^{-1/2}us = 0.5, q = 0.5$ and $\delta = 0.2$. The absolute slope of both maps $\varphi_{U,\delta}(\cdot; q, s)$ and $\varphi_{L,\delta}(\cdot; q, s)$ are bounded by $1/\delta$.

Proof: First, define for $\delta > 0$,

$$\begin{aligned} z_\delta(y; q, s) &= (1 - \min\{(y - q - n^{-1/2}us)/\delta, 1\})1\{0 < y - q - n^{-1/2}us\} \\ &\quad +1\{y - q - n^{-1/2}us \leqslant 0\}. \end{aligned} \tag{7}$$

Define

$$\varphi_{U,\delta}(y; q) = \int_0^1 \varphi_{U,\delta}(y; q, s)ds, \text{ and } \varphi_{L,\delta}(y; q) = \int_0^1 \varphi_{L,\delta}(y; q, s)ds, \tag{8}$$

where

$$\varphi_{U,\delta}(y; q, s) = z_\delta(y; q, s) - z_\delta(y + \delta + n^{-1/2}us; q, s), \text{ and} \tag{9}$$

$$\varphi_{L,\delta}(y; q, s) = \min\left\{z_\delta(y + \delta; q, s), z_\delta(-y + 2q + \delta + n^{-1/2}us; q, s)\right\}. \tag{10}$$

Let $\varphi(y; q, s) = 1\{y - q - n^{-1/2}us \leqslant 0\} - 1\{y - q \leqslant 0\}$ and define

$$\varphi_n(y; q) = n^{-1/4} \int_0^1 \varphi(y; q, s)ds. \tag{11}$$

(Note that the definition (3) conforms with this.)

Then, we have for all $y \in \mathbb{R}$, (see Figure 1)

$$\varphi_{L,\delta}(y; q) \leqslant \varphi(y; q) \leqslant \varphi_{U,\delta}(y; q). \tag{12}$$

It is not hard to see that for all $q, q' \in \mathbb{R}$, and all $y \in \mathbb{R}$,

$$\begin{aligned} |\varphi_{U,\delta}(y;q) - \varphi_{U,\delta}(y;q')| &\leqslant |q-q'|/\delta, \text{ and} \\ |\varphi_{L,\delta}(y;q) - \varphi_{L,\delta}(y;q')| &\leqslant |q-q'|/\delta. \end{aligned} \tag{13}$$

Furthermore, for some constant $C > 0$,

$$E\left[(\varphi_{U,\delta}(Y_i;q) - \varphi_{L,\delta}(Y_i;q))^2 \,|D_i = d\right] \leqslant C\delta, \tag{14}$$

and

$$E\left[\varphi^2_{U,\delta}(Y_i;q)|D_i = d\right] \leqslant 1, \text{ and } E\left[\varphi^2_{L,\delta}(Y_i;q)|D_i = d\right] \leqslant 1. \tag{15}$$

Define

$$\mathcal{H}_{L,\delta} = \{\varphi_{L,\delta}(\cdot;q)b(\cdot)/n^{1/4} : (q,b) \in J_d(\tau_U,\tau_L) \times \mathcal{B}\}, \text{ and} \tag{16}$$

$$\mathcal{H}_{U,\delta} = \{\varphi_{U,\delta}(\cdot;q)b(\cdot)/n^{1/4} : (q,b) \in J_d(\tau_U,\tau_L) \times \mathcal{B}\}. \tag{17}$$

From (13) and the fact that $J_d(\tau_U,\tau_L)$ is bounded, and both $\varphi_n(\cdot;q)$ and $b(\cdot)$ are bounded maps, we find that

$$\begin{aligned} N_{[]}(\varepsilon, \mathcal{H}_{L,\delta}, \|\cdot\|_{P,2}) &\leqslant C(\varepsilon\delta)^{-1}N_{[]}(C\varepsilon, \mathcal{B}, \|\cdot\|_{P,2}), \text{ and} \\ N_{[]}(\varepsilon, \mathcal{H}_{U,\delta}, \|\cdot\|_{P,2}) &\leqslant C(\varepsilon\delta)^{-1}N_{[]}(C\varepsilon, \mathcal{B}, \|\cdot\|_{P,2}), \end{aligned} \tag{18}$$

for all $\varepsilon > 0$, for some constant $C > 0$. We take $\delta = \varepsilon^2$ and ε^2-brackets $[h_{L,a,j}, h_{L,b,j}]_{j=1}^N$ and $[h_{U,a,j}, h_{U,b,j}]_{j=1}^N$ such that the former set of brackets cover $\mathcal{H}_{L,\varepsilon^2}$ and the latter $\mathcal{H}_{U,\varepsilon^2}$, both with respect to $\|\cdot\|_{P,2}$. By (12) and (18), we lose no generality by taking brackets so that for each $h \in \mathcal{H}_n$, there exists $j \in \{1, ..., N\}$ such that[1]

$$\min\{h_{U,b,j}, h_{L,a,j}\} \leqslant h \leqslant \max\{h_{L,a,j}, h_{U,b,j}\}, \tag{19}$$

and

$$\log N \leqslant C - C\log\varepsilon + C\log N_{[]}(C\varepsilon^2, \mathcal{B}, \|\cdot\|_{P,2}), \tag{20}$$

[1] Since b can take negative values, the inequality (12) does not necessarily imply that $h_{L,a,j} \leqslant h \leqslant h_{U,b,j}$.

for some $C > 0$. We set

$$h_{L,j} = \min\{h_{U,b,j}, h_{L,a,j}\}, \text{ and } h_{U,j} = \max\{h_{U,b,j}, h_{L,a,j}\}. \tag{21}$$

Therfore,

$$\begin{aligned} E[|h_{L,j}(V_i) - h_{U,j}(V_i)|^k] &\leqslant (Cn^{-1/4})^{k-2} E\left[(h_{L,a,j}(V_i) - h_{U,b,j}(V_i))^2\right] \\ &\leqslant 2(Cn^{-1/4})^{k-2} E\left[(h_{L,a,j}(V_i) - h_{L,b,j}(V_i))^2\right] \\ &\quad +4(Cn^{-1/4})^{k-2} E\left[(h_{L,b,j}(V_i) - h_{U,a,j}(V_i))^2\right] \\ &\quad +4(Cn^{-1/4})^{k-2} E\left[(h_{U,a,j}(V_i) - h_{U,b,j}(V_i))^2\right] \\ &\leqslant C_1(C_2 n^{-1/4})^{k-2} \left(\varepsilon^4 + \varepsilon^2 + \varepsilon^4\right), \end{aligned} \tag{22}$$

for some constants $C, C_1, C_2 > 0$. The terms ε^4 are due to the choice of ε^2-brackets and the term ε^2 comes from (14) and $\delta = \varepsilon^2$. ■

Define for each $\tau \in [\tau_L, \tau_U]$ and $b \in \mathcal{B}$,

$$U(\tau, b; \delta) = \{(\tau_1, b_1) \in [\tau_L, \tau_U] \times \mathcal{B} : |\tau - \tau_1| + \|b - b_1\|_{P,2} \leqslant \delta\}. \tag{23}$$

Lemma A.2 *Suppose that $\mathcal{B}$ and $\mathcal{H}_n$ are as in Lemma A.1. Furthermore, assume that there exists $C > 0$ such that for all $\varepsilon > 0$,*

$$\log N_{[]}(\varepsilon, \mathcal{B}, \|\cdot\|_{P,2}) \leqslant C - C \log \varepsilon. \tag{24}$$

Then the following statements hold.

(i) There exists $s > 0$ such that for all $\delta > 0$,

$$E\left[\sup_{(\tau_1,b_1)\in U(\tau,b;\delta)} \left| \frac{1}{\sqrt{n}} \sum_{i=1}^n \left(a_{\tau_1}(Y_i; q_d(\tau_1)) b_1(V_i) - E\left[a_{\tau_1}(Y_i; q_d(\tau_1)) b_1(V_i)\right]\right)\right|\right] \leqslant C\delta^s.$$

(ii) There exists $C > 0$ such that

$$E\left[\sup_{h\in\mathcal{H}_n} \left|\sum_{i=1}^n \left(h(V_i) - E\left[h(V_i)\right]\right)\right|\right] \leqslant C \log n.$$

Proof: (i) Let

$$\mathcal{B}_1 = \{a_\tau(\cdot; q) : (\tau, q) \in [\tau_L, \tau_U] \times \mathbb{R}\}. \tag{25}$$

Certainly, $\mathcal{B}_1$ is a VC class. Both classes $\mathcal{B}_1$ and $\mathcal{B}$ are classes of bounded functions. Hence, if we let $\mathcal{B}_2$ be the collection of functions $f(\cdot)g(\cdot)$ as we run $f \in \mathcal{B}_1$ and $g \in \mathcal{B}$, we have for some constant $C > 0$,

$$\log N_{[]}(\varepsilon, \mathcal{B}_2, \|\cdot\|_{P,2}) \leqslant C - C\log\varepsilon + \log N_{[]}(C\varepsilon, \mathcal{B}, \|\cdot\|_{P,2}). \tag{26}$$

Using (24), we obtain the finite integral bracketing entropy bound for the left hand side. The desired result of (i) follows by the maximal inequality. (For example, see (1) from van der Vaart (1996).)

(ii) When $u = 0$, we have $\Delta(Y_i; q_d(\tau)) = 0$, a.s. We focus on the case $u \neq 0$. Observe that since b is bounded and $|\varphi_n(Y_i; q)| \leqslant 1$, for some constant $C > 0$,

$$E\left[|\varphi_n(Y_i; q)b(V_i)|^k\right] \leqslant CE\left[|\varphi_n(Y_i; q)|^k\right] \leqslant CE\left[\varphi_n^2(Y_i; q)\right]. \tag{27}$$

Observe that

$$E\left[\varphi_n^2(Y_i; q)\right] \leqslant n^{-1/2}\int_0^1 P\{q \leqslant Y_i \leqslant q + n^{-1/2}us\}ds \leqslant \sup_{q\in\mathbb{R}} f_d(q)n^{-1}. \tag{28}$$

Using this in combination with Lemma A.1, we apply Theorem 6.8 of Massart (2007) (taking $b = 1$ and $\sigma = Cn^{-1/2}$ there) to obtain that

$$\begin{aligned} E\left[\sup_{h\in\mathcal{H}_n}\left|\sum_{i=1}^n (h(V_i) - Eh(V_i))\right|\right] &\leqslant C_1 + C_1\sqrt{n}\int_0^{C_1/\sqrt{n}} \sqrt{\log(1/u)}du + C_1\log n \\ &\leqslant C_2\log n, \end{aligned}$$

for some constants $C_1, C_2 > 0$ from large n on. Thus we obtain the desired result. ■

Theorem A.1 *Suppose that Assumptions 2.2 and 2.3 in the main text hold, and let*

$$\zeta_i = \psi(V_i) - E\psi(V_i), \text{ and } \zeta_i^* = \psi(V_i^*) - E[\psi(V_i^*)|\mathcal{F}_n]. \tag{29}$$

Then the following statements hold.

(i)

$$\begin{aligned}
&\sqrt{n}(\hat{q}_d(\tau) - q_d(\tau)) && (30)\\
= \quad & -\frac{1}{\sqrt{n} f_d(q_d(\tau))} \sum_{j=1}^{n} \frac{a_\tau(Y_j; q_d(\tau)) 1\{D_i = d\}}{p_d(X_i)} \\
& + \frac{1}{\sqrt{n} f_d(q_d(\tau))} \sum_{j=1}^{n} E\left[\frac{a_\tau(Y_j; q_d(\tau)) g_d(X_i; \beta_0)' 1\{D_i = d\}}{p_d^2(X_i)}\right] \zeta_j + o_P(1), && (31)
\end{aligned}$$

uniformly over $\tau \in [\tau_L, \tau_U]$.

(ii)

$$\begin{aligned}
&\sqrt{n}(\hat{q}_d^*(\tau) - \hat{q}_d(\tau)) && (32)\\
= \quad & -\frac{1}{\sqrt{n} f_d(q_d(\tau))} \sum_{j=1}^{n} \left(\frac{a_\tau(Y_j^*; q_d(\tau)) 1\{D_j = d\}}{p_d(X_j^*)} - \frac{1}{n} \sum_{i=1}^{n} \frac{a_\tau(Y_i; q_d(\tau)) 1\{D_i = d\}}{p_d(X_i)} \right) \\
& + \frac{1}{\sqrt{n} f_d(q_d(\tau))} \sum_{j=1}^{n} E\left[\frac{a_\tau(Y_j; q_d(\tau)) g_d(X_i; \beta_0)' 1\{D_i = d\}}{p_d^2(X_i)}\right] \zeta_j^* + o_P(1), && (33)
\end{aligned}$$

uniformly over $\tau \in [\tau_L, \tau_U]$.

Proof: (i) Note that

$$\sqrt{n}(\hat{q}_d(\tau) - q_d(\tau)) = \arg\min_{u \in \mathbb{R}} \left(\hat{Q}_d(q_d(\tau) + n^{-1/2}u; \tau) - Q_d(q_d(\tau); \tau) \right). \tag{34}$$

We write

$$\hat{Q}_d(q_d(\tau) + n^{-1/2}u; \tau) - Q_d(q_d(\tau); \tau) = A_n + B_n, \tag{35}$$

where

$$\begin{aligned}
A_n &= \hat{Q}_d(q_d(\tau) + n^{-1/2}u; \tau) - Q_d(q_d(\tau) + n^{-1/2}u; \tau), \text{ and} && (36)\\
B_n &= Q_d(q_d(\tau) + n^{-1/2}u; \tau) - Q_d(q_d(\tau); \tau).
\end{aligned}$$

We can follow the same arguments as in the proof of Theorem 3 of Kato (2009), and show that

$$B_n = -\frac{u}{\sqrt{n}} \sum_{i=1}^{n} \frac{a_\tau(Y_i; q_d(\tau)) 1\{D_i = d\}}{p_d(X_i)} + \frac{u^2}{2} f_d(q_d(\tau)) + o_P(1). \tag{37}$$

Let us focus on A_n. We write

$$A_n = A_{n,1} + \xi_{n,d}(\tau), \tag{38}$$

where

$$\begin{aligned} A_{n,1} &= \hat{Q}_d(q_d(\tau) + n^{-1/2}u; \tau) - \hat{Q}_d(q_d(\tau); \tau) & (39) \\ &\quad - \left(Q_d(q_d(\tau) + n^{-1/2}u; \tau) - Q_d(q_d(\tau); \tau) \right), & (40) \end{aligned}$$

and

$$\xi_{n,d}(\tau) = \hat{Q}_d(q_d(\tau); \tau) - Q_d(q_d(\tau); \tau). \tag{41}$$

Using Knight's identity (see (15) on page 1855 of Kato (2009)), we write $A_{n,1}$ as

$$uZ_{n,d}^{(1)}(\tau) + Z_{n,d}^{(2)}(u, \tau), \tag{42}$$

where

$$\begin{aligned} Z_{n,d}^{(1)}(\tau) &= \frac{1}{\sqrt{n}} \sum_{i=1}^{n} \frac{a_\tau(Y_i; q_d(\tau)) 1\{D_i = d\}}{p_d(X_i)} \left(\frac{p_d(X_i)}{\hat{p}_d(X_i)} - 1 \right), \text{ and} & (43) \\ Z_{n,d}^{(2)}(u, \tau) &= -\frac{u}{n} \sum_{i=1}^{n} \left(\frac{p_d(X_i)}{\hat{p}_d(X_i)} - 1 \right) \frac{\Delta(Y_i; q_d(\tau)) 1\{D_i = d\}}{p_d(X_i)}. & (44) \end{aligned}$$

We first write

$$Z_{nd}^{(1)}(\tau) = -\frac{1}{\sqrt{n}} \sum_{i=1}^{n} \frac{a_\tau(Y_i; q_d(\tau)) 1\{D_i = d\}}{p_d(X_i)} \frac{p_d(X_i) - \hat{p}_d(X_i)}{p_d(X_i)} + R_n(\tau), \tag{45}$$

where

$$R_n(\tau) = -\frac{1}{\sqrt{n}} \sum_{i=1}^{n} \frac{a_\tau(Y_i; q_d(\tau)) 1\{D_i = d\}}{p_d(X_i)} \frac{(p_d(X_i) - \hat{p}_d(X_i))^2}{p_d(X_i)\hat{p}_d(X_i)}. \tag{46}$$

By expanding $G(x; \hat{\beta})$ around β_0, it is not hard to see that

$$\hat{p}_d(x) - p_d(x) = g_d(X_i; \beta_0)'(\hat{\beta} - \beta_0) + O_P(n^{-1}). \tag{47}$$

Since $|a_\tau(Y_i;q_d(\tau))/p_d(X_i)| \leqslant \varepsilon^{-1}$ for all $\tau \in [\tau_L,\tau_U]$, we find that

$$\sup_{\tau\in[\tau_L,\tau_U]} |R_n(\tau)| = O_P(n^{-1/2}). \tag{48}$$

Applying this and the expansion in (47) to the leading term on the right hand side of (45), we obtain that

$$\begin{aligned} Z_{nd}^{(1)}(\tau) &= -\frac{1}{\sqrt{n}}\sum_{i=1}^n \frac{a_\tau(Y_i;q_d(\tau))1\{D_i=d\}}{p_d(X_i)}\frac{g_d(X_i;\beta_0)'(\hat\beta-\beta_0)}{p_d(X_i)} + o_P(1) \\ &= -\left(\frac{1}{n}\sum_{i=1}^n \frac{a_\tau(Y_i;q_d(\tau))1\{D_i=d\}}{p_d(X_i)}\frac{g_d(X_i;\beta_0)'}{p_d(X_i)}\right)\left(\frac{1}{\sqrt{n}}\sum_{i=1}^n \zeta_i + o_P(1)\right) + o_P(1) \\ &= -E\left[\frac{a_\tau(Y_i;q_d(\tau))1\{D_i=d\}}{p_d(X_i)}\frac{g_d(X_i;\beta_0)'}{p_d(X_i)}\right]\frac{1}{\sqrt{n}}\sum_{i=1}^n \zeta_i + o_P(1), \end{aligned}$$

by Lemma A.2(i). Using the same arguments, we also obtain that

$$Z_{nd}^{(2)}(\tau) = -u\left(\frac{1}{n}\sum_{i=1}^n \frac{\Delta(Y_i;q_d(\tau))1\{D_i=d\}}{p_d(X_i)}\frac{g_d(X_i;\beta_0)'}{p_d(X_i)}\right)\frac{1}{\sqrt{n}}\sum_{i=1}^n \zeta_i + o_P(1) = o_P(1).$$

We define $b_0(V_i) = 1\{D_i = d\}g_{d,k}(X_i;\beta_0)/p_d^2(X_i)$, where $g_{d,k}(X_i;\beta_0)$ is the k-th entry of $g_d(X_i;\beta_0)$, and take $\mathcal{B} = \{b_0\}$, i.e., the singleton of b_0 in the definition of $\mathcal{H}_n$ in (4). We bound

$$\begin{aligned} &\sup_{\tau\in[\tau_L,\tau_U]}\left|\frac{1}{n}\sum_{i=1}^n \frac{\Delta(Y_i;q_d(\tau))1\{D_i=d\}}{p_d(X_i)}\frac{g_{d,k}(X_i;\beta_0)}{p_d(X_i)} - E\left[\frac{\Delta(Y_i;q_d(\tau))1\{D_i=d\}}{p_d(X_i)}\frac{g_{d,k}(X_i;\beta_0)}{p_d(X_i)}\right]\right| \\ \leqslant\ & n^{-1/4}\sup_{h\in\mathcal{H}_n}\left|\sum_{i=1}^n (h(V_i)-Eh(V_i))\right|. \end{aligned}$$

By Lemma A.2(ii), we find that uniformly over $\tau \in [\tau_L,\tau_U]$,

$$\begin{aligned} \frac{1}{n}\sum_{i=1}^n \frac{\Delta(Y_i;q_d(\tau))1\{D_i=d\}}{p_d(X_i)}\frac{g_{d,k}(X_i;\beta_0)}{p_d(X_i)} &= E\left[\frac{\Delta(Y_i;q_d(\tau))1\{D_i=d\}}{p_d(X_i)}\frac{g_{d,k}(X_i;\beta_0)}{p_d(X_i)}\right] \\ &\quad + O(n^{-1/4}\log n). \end{aligned}$$

Therefore, we conclude that

$$A_n = -E\left[\frac{a_\tau(Y_i;q_d(\tau))1\{D_i=d\}}{p_d(X_i)}\frac{g_d(X_i;\beta_0)'}{p_d(X_i)}\right]\frac{1}{\sqrt{n}}\sum_{i=1}^n \zeta_i + \xi_{n,d}(\tau) + o_P(1). \tag{49}$$

Combining this with (37), and applying Theorem 2 of Kato (2009), we obtain the desired result of (i).

(ii) The proof of the bootstrap version is similar to that of (i). First, we write

$$\sqrt{n}(\hat{q}_d^*(\tau) - \hat{q}_d(\tau)) = \arg\min_{u \in \mathbb{R}} \hat{Q}_d^*(\hat{q}_d(\tau) + n^{-1/2}u; \tau) - Q_d^*(\hat{q}_d(\tau); \tau), \tag{50}$$

where

$$\begin{aligned} \hat{Q}_d^*(q;\tau) &= \sum_{i=1}^n \frac{1\{D_i^* = d\}}{\hat{p}_d^*(X_i^*)} \rho_\tau(Y_i^* - q), \text{ and} \qquad (51) \\ Q_d^*(q;\tau) &= \sum_{i=1}^n \frac{1\{D_i^* = d\}}{\hat{p}_d(X_i^*)} \rho_\tau(Y_i^* - q). \qquad (52) \end{aligned}$$

Similarly as before, we write

$$\hat{Q}_d^*(\hat{q}_d(\tau) + n^{-1/2}u; \tau) - Q_d^*(\hat{q}_d(\tau); \tau) = A_n^* + B_n^*, \tag{53}$$

where

$$\begin{aligned} A_n^* &\equiv \hat{Q}_d^*(\hat{q}_d(\tau) + n^{-1/2}u; \tau) - Q_d^*(\hat{q}_d(\tau) + n^{-1/2}u; \tau), \text{ and} \qquad (54) \\ B_n^* &\equiv Q_d^*(\hat{q}_d(\tau) + n^{-1/2}u; \tau) - Q_d^*(\hat{q}_d(\tau); \tau). \end{aligned}$$

We write

$$A_n^* = A_{n,1}^* + \xi_{n,d}^*(\tau), \tag{55}$$

where

$$\xi_{n,d}^*(\tau) = \hat{Q}_d^*(\hat{q}_d(\tau); \tau) - Q_d^*(\hat{q}_d(\tau); \tau), \tag{56}$$

and

$$\begin{aligned} A_{n,1}^* &= \hat{Q}_d^*(\hat{q}_d(\tau) + n^{-1/2}u; \tau) - \hat{Q}_d^*(\hat{q}_d(\tau); \tau) \qquad (57) \\ &\quad -(Q_d^*(\hat{q}_d(\tau) + n^{-1/2}u; \tau) - Q_d^*(\hat{q}_d(\tau); \tau)). \end{aligned}$$

Following the similar arguments as before, we obtain that

$$A^*_{n,1} = -uE\left[\frac{a_\tau(Y_i^*;\hat{q}_d(\tau))1\{D_i^* = d\}}{\hat{p}_d(X_i^*)}\frac{g_d(X_i^*;\hat{\beta})'}{\hat{p}_d(X_i^*)}|\mathcal{F}_n\right]\frac{1}{\sqrt{n}}\sum_{i=1}^n \zeta_i^* + o_P(1). \tag{58}$$

Note that from (i),

$$\sup_{\tau\in[\tau_L,\tau_U]}|\hat{q}_d(\tau) - q_d(\tau)| = o_P(1), \text{ and } \hat{\beta} = \beta_0 + o_P(1). \tag{59}$$

Hence using Lemma A.2(i), we obtain that

$$\begin{aligned} & E\left[\frac{a_\tau(Y_i^*;\hat{q}_d(\tau))1\{D_i^* = d\}}{\hat{p}_d(X_i^*)}\frac{g_d(X_i^*;\hat{\beta})'}{\hat{p}_d(X_i^*)}|\mathcal{F}_n\right] \\ = \ & \frac{1}{n}\sum_{i=1}^n \frac{a_\tau(Y_i;\hat{q}_d(\tau))1\{D_i = d\}}{\hat{p}_d(X_i)}\frac{g_d(X_i;\hat{\beta})'}{\hat{p}_d(X_i)} \\ = \ & \frac{1}{n}\sum_{i=1}^n \frac{a_\tau(Y_i;q_d(\tau))1\{D_i = d\}}{p_d(X_i)}\frac{g_d(X_i;\beta_0)'}{p_d(X_i)} + o_P(1) \\ = \ & E\left[\frac{a_\tau(Y_i;q_d(\tau))1\{D_i = d\}}{p_d(X_i)}\frac{g_d(X_i;\beta_0)'}{p_d(X_i)}\right] + o_P(1). \end{aligned} \tag{60}$$

Let us turn to B_n^* defined in (54). Using Knight's identity, we write B_n^* as

$$uZ_{n,d}^{*(1)}(\tau) + Z_{n,d}^{*(2)}(u,\tau), \tag{61}$$

where

$$Z_{n,d}^{*(1)}(\tau) = \frac{1}{\sqrt{n}}\sum_{i=1}^n \frac{a_\tau(Y_i^*;\hat{q}_d(\tau))1\{D_i^* = d\}}{\hat{p}_d(X_i^*)}, \text{ and} \tag{62}$$

$$Z_{n,d}^{*(2)}(u,\tau) = -\frac{u}{n}\sum_{i=1}^n \frac{\hat{\Delta}(Y_i^*;\hat{q}_d(\tau))1\{D_i^* = d\}}{\hat{p}_d(X_i^*)}, \tag{63}$$

with

$$\hat{\Delta}(Y_i^*;\hat{q}_d(\tau)) = \sqrt{n}\int_0^1 \left(a_\tau(Y_i^*;\hat{q}_d(\tau) + n^{-1/2}us) - a_\tau(Y_i^*;\hat{q}_d(\tau))\right) ds. \tag{64}$$

We write

$$Z_{n,d}^{*(1)}(\tau) = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} \left(\frac{a_\tau(Y_i^*; q_d(\tau)) 1\{D_i^* = d\}}{G_d(X_i^*; \beta_0)} - \frac{1}{n} \sum_{i=1}^{n} \frac{a_\tau(Y_i; q_d(\tau)) 1\{D_i = d\}}{G_d(X_i; \beta_0)} \right) + R_{n,1}(\tau),$$

where

$$R_{n,1}(\tau) = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} \left(\eta_\tau(V_i^*; \hat{q}_d(\tau), \hat{\beta}) - \frac{1}{n} \sum_{i=1}^{n} \eta_\tau(V_i; \hat{q}_d(\tau), \hat{\beta}) \right), \tag{65}$$

with

$$\eta_\tau(V_i^*; \hat{q}_d(\tau), \hat{\beta}) = \frac{a_\tau(Y_i^*; \hat{q}_d(\tau)) 1\{D_i^* = d\}}{G_d(X_i^*; \hat{\beta})} - \frac{a_\tau(Y_i^*; q_d(\tau)) 1\{D_i^* = d\}}{G_d(X_i^*; \beta_0)}. \tag{66}$$

Using Lemma A.2(i) again, we can show that $R_{n,1}(\tau) = o_P(1)$ uniformly over $\tau \in [-\tau_L, \tau_U]$.

We turn to $Z_{n,d}^{*(2)}(u, \tau)$ which we write as

$$-\frac{u}{n} \sum_{i=1}^{n} \left(\tilde{\eta}(V_i^*; \hat{q}_d(\tau), \hat{\beta}) - \frac{1}{n} \sum_{i=1}^{n} \tilde{\eta}(V_i; \hat{q}_d(\tau), \hat{\beta}) \right) \tag{67}$$

$$-\frac{u}{n} \sum_{i=1}^{n} \left(\tilde{\eta}(V_i; \hat{q}_d(\tau), \hat{\beta}) - \int \tilde{\eta}(v; \hat{q}_d(\tau), \hat{\beta}) dF_V(v) \right)$$

$$-u \left(\int \tilde{\eta}(v; \hat{q}_d(\tau), \hat{\beta}) dF_V(v) - \int \tilde{\eta}(v; q_d(\tau), \beta_0) dF_V(v) \right)$$

$$-uE\left[\frac{\Delta(Y_i; q_d(\tau)) 1\{D_i = d\}}{p_d(X_i)} \right], \tag{68}$$

where F_V is the CDF of V_i, and

$$\tilde{\eta}(V_i^*; \hat{q}_d(\tau), \hat{\beta}) = \frac{\Delta(Y_i^*; \hat{q}_d(\tau)) 1\{D_i^* = d\}}{\hat{p}_d(X_i^*)}. \tag{69}$$

We show that the first two terms in (67) are $o_P(1)$ uniformly over $\tau \in [\tau_L, \tau_U]$. We will deal with the first term in (67). By Assumptions 2.1(ii) and 2.3(i) in the main text, we can find $\varepsilon > 0$ such that $G_d(x; \beta) > 0$ for all $x \in \mathcal{X}$ and all $\beta \in B(\beta_0; \varepsilon)$, where $B(\beta_0; \varepsilon) = \{\beta \in \Theta : \|\beta - \beta_0\| \leqslant \varepsilon\}$. Define

$$b_\beta(V_i) = \frac{1\{D_i = d\}}{G_d(X_i; \beta)}, \tag{70}$$

and let

$$\mathcal{B} = \{b_\beta : \beta \in B(\beta_0; \varepsilon)\}, \tag{71}$$

and define $\mathcal{H}_n$ in (4) using this $\mathcal{B}$. Since the set $B(\beta_0; \varepsilon)$ is bounded in $\mathbb{R}^{d_\beta}$, by Assumptions 2.1(ii) and 2.3(i) in the main text, we find that the bracketing condition in (24) is satisfied for this set $\mathcal{B}$. Furthermore, by Assumption 2.2(i) in the main text, we have $\hat{\beta} \in B(\beta_0; \varepsilon)$ with probability approaching one. Now, observe that

$$\begin{aligned}
&\sup_{\tau \in [\tau_L, \tau_U]} \left| \frac{1}{n} \sum_{i=1}^n \left(\frac{\Delta(Y_i^*; \hat{q}_d(\tau)) 1\{D_i^* = d\}}{\hat{p}_d(X_i^*)} - E\left[\frac{\Delta(Y_i^*; \hat{q}_d(\tau)) 1\{D_i^* = d\}}{\hat{p}_d(X_i^*)} | \mathcal{F}_n \right] \right) \right| && (72) \\
\leqslant \; & n^{-1/4} \sup_{h \in \mathcal{H}_n} \left| \sum_{i=1}^n (h(V_i^*) - E[h(V_i^*) | \mathcal{F}_n]) \right| = O_P(n^{-1/4} \log n),
\end{aligned}$$

by Lemma A.2(ii). Thus the first term in (67) is $o_P(1)$ uniformly over $\tau \in [\tau_L, \tau_U]$. The second term can be dealt with in the same way.

Let us turn to the third term in (67). This term is also $o_P(1)$ because $\hat{q}_d(\tau) = q_d(\tau) + o_P(1)$ and $\hat{\beta} = \beta_0 + o_P(1)$. Following precisely the same argument in the proof of Theorem 3 in Kato (2009) used to deal with B_n in (36), we can show that

$$-uE\left[\frac{\Delta(Y_i; q_d(\tau)) 1\{D_i = d\}}{p_d(X_i)} \right] = \frac{u^2}{2} f_d(q_d(\tau)) + o(1). \tag{73}$$

Hence we conclude that

$$\begin{aligned}
&\hat{Q}_d^*(\hat{q}_d(\tau) + n^{-1/2} u; \tau) - Q_d^*(\hat{q}_d(\tau); \tau) && (74) \\
&= -u \frac{1}{\sqrt{n}} \sum_{i=1}^n \left(\frac{a_\tau(Y_i^*; q_d(\tau)) 1\{D_i^* = d\}}{G_d(X_i^*; \beta_0)} - \frac{1}{n} \sum_{i=1}^n \frac{a_\tau(Y_i; q_d(\tau)) 1\{D_i = d\}}{G_d(X_i; \beta_0)} \right) && (75) \\
&+ \frac{u^2}{2} f_d(q_d(\tau)) + o_P(1). && (76)
\end{aligned}$$

Now the desired result follows from Theorem 2 of Kato (2009). ■

Let us define

$$q^\Delta(\tau) = q_1(\tau) - q_0(\tau), \text{ and } \hat{q}^\Delta(\tau) = \hat{q}_1(\tau) - \hat{q}_0(\tau). \tag{77}$$

Similarly, we define a bootstrap version $\hat{q}_1^*(\tau) - \hat{q}_0^*(\tau)$. The following theorem gives the weak convergence of the process $\{\sqrt{n}(\hat{q}^\Delta(\tau) - q^\Delta(\tau)) : \tau \in [\tau_L, \tau_U]\}$. Let $\ell^\infty([\tau_L, \tau_U])$ be the collection of bounded and measurable functions on $[\tau_L, \tau_U]$. Let BL_1 be the bounded Lipschitz functionals on $\ell^\infty([\tau_L, \tau_U])$, i.e.,

$$\mathrm{BL}_1 = \{h \in \ell^\infty([\tau_L, \tau_U]) : |h(\tau_1) - h(\tau_2)| \leqslant |\tau_1 - \tau_2|, \tau_1, \tau_2 \in [\tau_L, \tau_U]\} . \tag{78}$$

For a sequence of stochastic processes $\mathbb{G}_n$ and a process $\mathbb{G}$ on $[\tau_L, \tau_U]$, we write

$$\mathbb{G}_n \rightsquigarrow \mathbb{G}, \text{ in } \ell^\infty[\tau_L, \tau_U]), \tag{79}$$

as $n \to \infty$, if

$$\sup_{h \in \mathrm{BL}_1} |E^* [h(\mathbb{G}_n)] - E[h(\mathbb{G})]| \to 0, \tag{80}$$

as $n \to \infty$, where E^* denotes the outer expectation. Let $\mathbb{G}_n^*$ be a stochastic process on $[\tau_L, \tau_U]$ such that for each $\tau \in [\tau_L, \tau_U]$, $\mathbb{G}_n^*(\tau)$ is a measurable map of the bootstrap sample (Y_i^*, X_i^*). Then if for any $\varepsilon > 0$,

$$P^* \left\{ \sup_{h \in \mathrm{BL}_1} |E [h(\mathbb{G}_n^*)|\mathcal{F}_n] - E[h(\mathbb{G})]| > \varepsilon \right\} \to 0, \tag{81}$$

as $n \to \infty$, for some we write $\mathbb{G}_n^* \rightsquigarrow_* \mathbb{G}$ in $\ell^\infty([\tau_L, \tau_U])$. Here P^* denotes the outer probability.

Theorem A.2 *Suppose that Assumptions 2.2 and 2.3 in the main text hold. Then the following statements hold.*

(i)

$$\sqrt{n}(\hat{q}^\Delta - q^\Delta) \rightsquigarrow \mathbb{G}, \textit{ in } \ell^\infty([\tau_L, \tau_U]). \tag{82}$$

(ii)

$$\sqrt{n}(\hat{q}^{\Delta *} - q^{\Delta *}) \rightsquigarrow_* \mathbb{G}, \textit{ in } \ell^\infty([\tau_L, \tau_U]). \tag{83}$$

Proof: Define

$$\mathcal{G} = \{\xi(\cdot; q, \tau) : (q, \tau) \in J_d([\tau_U, \tau_L]) \times [\tau_L, \tau_U]\}, \tag{84}$$

where

$$\xi(V_j; q, \tau) = -\frac{a_\tau(Y_j; q)1\{D_i = d\}}{f_d(q)p_d(X_i)} \tag{85}$$

$$+E\left[\frac{a_\tau(Y_j; q)g_d(X_i; \beta_0)'1\{D_i = d\}}{f_d(q)p_d^2(X_i)}\right]\zeta_j + o_P(1). \tag{86}$$

For (i), it suffices to show that $\mathcal{G}$ is P-Donsker. This also implies (ii) by Theorem 2.2 of Giné (1997, p. 104). Define

$$\nu_n(\xi) = \frac{1}{\sqrt{n}}\sum_{i=1}^{n}(\xi(V_i) - E\xi(V_i)). \tag{87}$$

The convergence of finite dimensional distributions of $\{\nu_n(\xi) : \xi \in \mathcal{G}\}$ follow by the usual central limit theorem. In order to show that $\mathcal{G}$ is P-Donsker, it suffices to show that $\mathcal{G}$ is totally bounded with respect to a pseudo-norm ρ and $\{\nu_n(\xi) : \xi \in \mathcal{G}\}$ is asymptotically equicontinuous with respect to ρ. We take the norm ρ to be $\|\cdot\|_{P,2}$. The total boundedness follows by the same arguments as in the proof of (i) of Lemma A.2. It remains to show asymptotic equicontinuity of the process ν_n. For this, we write

$$\sqrt{n}(\hat{q}^\Delta(\tau) - q^\Delta(\tau)) = -A_{n,1}(q_d(\tau), \tau) + A_{n,2}(q_d(\tau), \tau) + o_P(1), \tag{88}$$

where

$$A_{n,1}(q, \tau) = \frac{1}{\sqrt{n}f_d(q)}\sum_{j=1}^{n}\frac{a_\tau(Y_j; q)1\{D_j = d\}}{p_d(X_j)}, \text{ and} \tag{89}$$

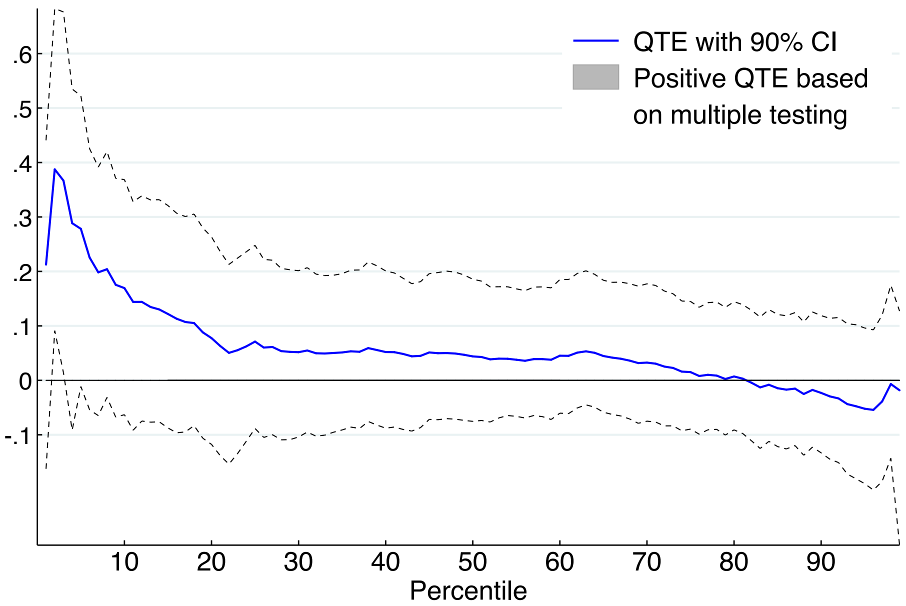
$$A_{n,2}(q, \tau) = \frac{1}{\sqrt{n}f_d(q)}\sum_{j=1}^{n}E\left[\frac{a_\tau(Y_j; q)g_d(X_i; \beta_0)'1\{D_i = d\}}{p_d^2(X_i)}\right]\zeta_j. \tag{90}$$

Stochastic equicontinuity of $\{A_{n,1}(q, \tau) : (q, \tau) \in J_d([\tau_L, \tau_U]) \times [\tau_L, \tau_U]\}$ obviously follows from Lemma A.2(i) and the Lipshitz continuity of $1/f_d(q)$ in $q \in J_d(\tau_U, \tau_L)$. It is not hard to show similarly that $A_{n,2}$ is stochastically equicontinuous as well. ■

We are prepared to prove Theorem 2.1in the main text. Define for any $W' \subset W$,

$$\hat{c}_{1-\alpha}(W') = \inf\left\{c \in \mathbb{R} : \frac{1}{B}\sum_{b=1}^{B}\mathbf{1}\left\{T_b^*(W') \leqslant c\right\} \geqslant 1 - \alpha\right\}.$$

Proof of Theorem 2.1 in the main text: In light of Theorem 2.1 of Romano and Shaikh (2010) and the fact that the functional $\sup_{w \in W'} \Gamma(\cdot, S_w)$ is increasing in W', it suffices to show



Note: Multiple testing results show quantiles for which the QTE is positive at a FWER of 5% (see hypothesis (H.3) in Table 1).

Figure 2: Quantile Treatment Effects and Multiple Testing Results, No Subgroups, Under Block Bootstrapping

that

$$\limsup_{n\to\infty} P\left\{\sup_{w\in W_P} \Gamma(\sqrt{n}(\hat{q}^{\Delta} - q^{\Delta}); S_w) \geqslant \hat{c}_{1-\alpha}(W_P)\right\} \leqslant \alpha.$$

However, this follows immediately from Theorem A.2 and the Continuous Mapping Theorem, as $\sup_{w\in W_P} \Gamma(\cdot, S_w)$ is a continuous functional. ■

B Empirical Results Under Block Bootstrapping

In our empirical application, we follow Andrabi, Das, and Khwaja (2017) and resample individuals in our bootstrapping procedure. Here, we report pointwise confidence intervals and multiple testing results for QTEs for the full sample under block bootstrapping. The blocks used in this analysis correspond to villages. The results in Figure 2 show that only one percentile (the second) has a pointwise statistically significant QTE. When applying our multiple testing procedure, we do not find any signifiant QTEs. In this empirical application, the within village correlation structure is such that estimating QTE leads to completely uninformative results.

C Additional Application: Connecticut's Jobs First Welfare Experiment

In this section, we present results from an application of our multiple testing procedure to a welfare experiment carried out in Connecticut in the mid-1990s. Bitler, Gelbach, and Hoynes (2006, 2017) analyze the distributional effects of this welfare reform. In the following, we describe the policy background and data, provide a simple model of labor supply that predicts heterogeneous treatment effects, and discuss the multiple testing results. This section and analyses are motivated in footnote 19 of the main text where we illustrate the broader applicability of not only the tests that we develop, but how the hypotheses themselves are motivated by economic theory.

C.1 Policy Background and Data

Following years of debate and after President Clinton vetoed two earlier welfare reform bills, the federal Personal Responsibility and Work Opportunity Reconciliation Act (PRWORA) was passed in 1996.[2] PRWORA provided a major change in how federal cash assistance would be provided by requiring each state to replace their Aid to Families with Dependent Children (AFDC) program with a Temporary Assistance to Needy Families (TANF) program. In addition, PRWORA gave state governments more autonomy over welfare delivery. Several states, including Connecticut, conducted randomized experiments to provide an evidence base for subsequent reforms as well as to receive a waiver from the federal government which allowed state governments to implement their own version of TANF.

Connecticut's Job First experiment was carried out by the Manpower Demonstration and Research Corporation and involved about 4,800 women residing in New Haven and Manchester in 1996 and 1997 who were either new welfare applicants or had applied for a continued receipt of benefits. Participants were randomly assigned to either receive a new program called Jobs First, which was the basis of the subsequent TANF program, while participants assigned to the control group received the original AFDC benefits. In contrast to AFDC, Jobs First imposed a time limit of 21 months on welfare receipt. In addition, participants assigned to the Jobs First group were required to attend job training programs or provide proof of job search activities to remain eligible for benefits. On the other hand, Jobs First included more generous earnings disregards. Specifically, under Jobs First, all earnings up to the federal poverty level (FPL) were disregarded, whereas AFDC participants faced an

[2] Haskins (2006) details the political battles underlying the passage of this act.

implicit tax rate of 49 percent during the first three months of employment and 73 percent thereafter.[3] Participants and their families were followed up until 2001 via surveys and administrative records from multiple sources including unemployment insurance earnings, food stamps, and AFDC/TANF benefits. The Jobs First experiment is well-studied (Bitler, Gelbach, and Hoynes, 2006, 2017; Kline and Tartari, 2016, among others). We use it to illustrate the methods proposed in the paper because it facilitates comparisons with the existing literature that used the same data.

Summary statistics are reported in Table 1 where the second and third columns present characteristics of those women respectively assigned to the Jobs First and AFDC groups. On average, the single mothers in this sample have lower educational attainment and are much more likely to be part of a minority than the general population. About 60 percent of the sample have a child under the age of six, indicating that there may be additional constraints on their labor supply decisions. The women in this sample earn less than \$800 per quarter before random assignment and therefore rely heavily on welfare and food stamps. The standard deviation of earnings is high relative to the mean, suggesting heterogeneous responses to different welfare policies across the earnings distribution. The last column in Table 1 contains p-values for the test that individuals assigned to the Jobs First and AFDC groups do not differ in observed characteristics. For most characteristics and as shown in Bloom et al. (2002) and Bitler, Gelbach, and Hoynes (2006), we cannot reject the null hypothesis of no difference. There are small but statistically significant differences in a few variables, and two differences are particularly surprising given the random assignment protocol. Specifically, and as also noted by Bitler, Gelbach, and Hoynes (2006), we observe that women assigned to the control group (AFDC) have significantly higher earnings and hence receive significantly lower welfare benefits before random assignment. To ensure covariate balance we make adjustments via propensity score weighting in our analyses.

C.2 Economic Model Predicting Heterogeneous Treatment Effects

A simple static labor supply model motivates our investigation of treatment effect heterogeneity.[4] Individuals maximize their utility over consumption (C) and earnings (E) subject

[3] Other differences include a \$3,000 asset disregard and two years of transitional Medicaid for Jobs First and a \$1,000 disregard and one year of transitional Medicaid for AFDC (see Bloom et al., 2002; Bitler, Gelbach, and Hoynes, 2006).

[4] Static models are commonly used in the literature on single mothers' labor supply (Keane, 2011, p. 1070). Our discussion follows earlier work on static labor supply models including Kline and Tartari (2016). We extend this literature by considering differences across subgroups.

Table 1: Summary Statistics by Experimental Group, Jobs First Experiment

	Jobs First	AFDC	Difference
	Mean	Mean	*p*-value
	(Std.dev.)	(Std.dev.)	
Mother's age < 20	0.089	0.086	0.684
Mother's age 20 to 29	0.214	0.216	0.898
Mother's age $\geqslant 30$	0.497	0.488	0.537
White	0.362	0.348	0.307
Black	0.368	0.371	0.836
Hispanic	0.207	0.216	0.423
Never married	0.654	0.661	0.624
Separated/divorced/living apart	0.332	0.327	0.715
No educational degree	0.350	0.334	0.242
High school degree/GED or more	0.650	0.666	0.242
Youngest child < 6	0.605	0.614	0.520
Youngest child $\geqslant 6$	0.395	0.386	0.520
Number of children	1.649	1.591	0.037
	(0.932)	(0.944)	
Mean quarterly earnings pre-RA	682.7	796.0	0.006
	(1304.1)	(1566.0)	
Mean quarterly welfare benefits pre-RA	890.8	835.1	0.015
	(806.0)	(784.8)	
Mean quarterly foods stamp benefits pre-RA	352.1	339.4	0.156
	(320.0)	(303.9)	
Fraction of quarters employed pre-RA	0.327	0.357	0.006
	(0.370)	(0.379)	
Fraction of quarters welfare receipt pre-RA	0.573	0.544	0.026
	(0.452)	(0.450)	
Fraction of quarters food stamps receipt pre-RA	0.607	0.598	0.486
	(0.438)	(0.433)	
Observations	2396	2407	4803

Source: Manpower Demonstration Research Corporation's study of Connecticut's Jobs First Program. Note: *p*-values are obtained from two sided *t*-tests of the equality of means between the Jobs First and AFDC groups.

to a budget constraint:

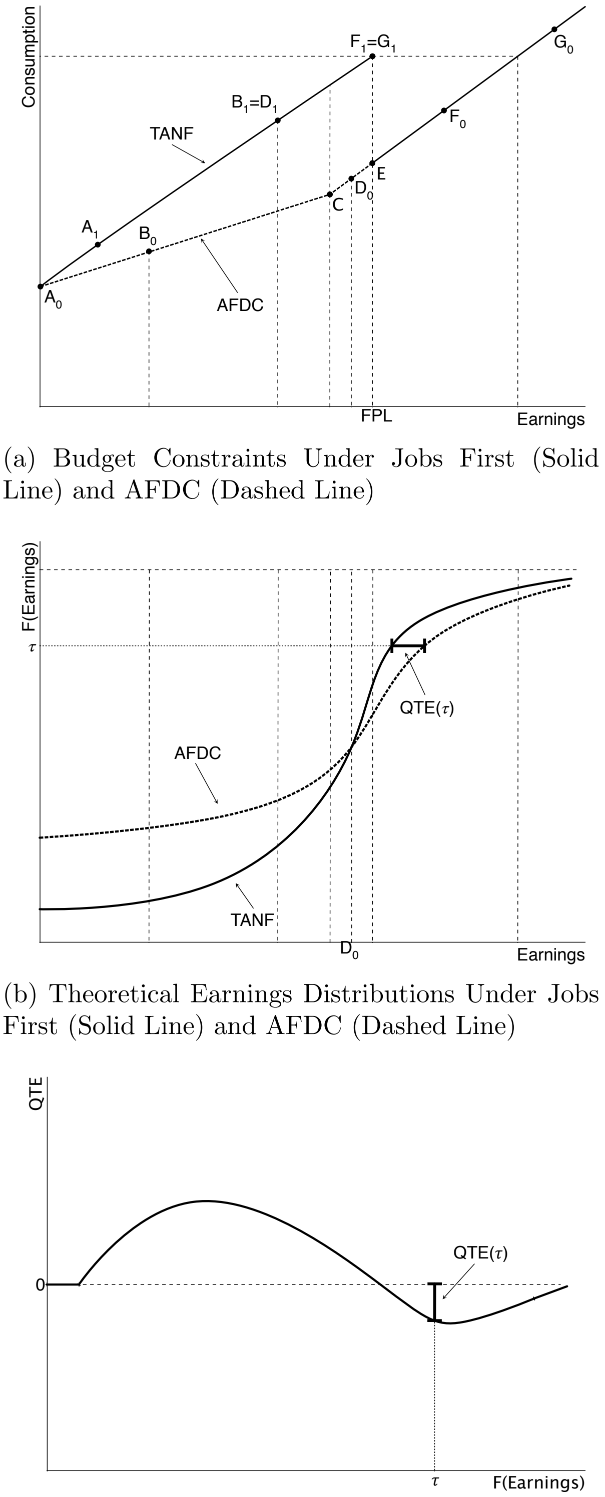
$$\begin{aligned}\max_{C,E} U &= U\left(C, E(X_1); X_2\right) \qquad (91)\\ \text{s.t. } C &= E(X_1) + W\left(E(X_1); X_3, Z^t\right) \qquad (92)\end{aligned}$$

where X_1 and X_2 denote characteristics that may affect earnings and individual preferences, respectively, and $W(\cdot)$ denotes the welfare benefit function, which depends on the level of earnings $E(\cdot)$, household characteristics X_3, and policy parameters Z^t with $t = \{AFDC, JF\}$. The vector Z^t includes the base grant amount, earnings disregards, and time limits, so it traces out the budget constraint faced by a welfare participant in the AFDC or Jobs First group. Following Saez (2010), we assume that the marginal utility of consumption is positive ($\frac{\partial U}{\partial C} > 0$) and the marginal utility of earnings is negative ($\frac{\partial U}{\partial E} < 0$).

We use the panels of Figure 3 to demonstrate how economic theory predicts treatment effect heterogeneity. This heterogeneity arises because there is a differential labor supply response on both the intensive and extensive margins between the AFDC and the Jobs First program due to different budget constraints and earnings distributions in the two experimental groups.[5] The solid line in the top panel of Figure 3 illustrates the budget constraint faced by Jobs First participants and is defined by the points A_0, F_1, E and G_0. A_0 denotes the base grant amount. The segment A_0F_1 is parallel to the 45 degree line due to the implicit tax rate of 0 for welfare recipients with earnings below the FPL. The dashed line represents the budget constraint faced under AFDC and is defined by the points A_0, C and G_0, where C corresponds to the eligibility threshold, which is below the FPL. In particular, the segment A_0C represents the earnings disregard under AFDC with a positive implicit tax rate. The middle panel of Figure 3 presents hypothetical cumulative distribution functions of earnings for those in AFDC (dashed) and Jobs First (solid) groups that are the result of different welfare program parameters. QTEs are presented in the bottom panel and equal the horizontal distance at each quantile between the two earnings distributions in the middle panel.

To provide intuition for the shape of the QTEs presented, we consider the thought experiment of moving individuals from AFDC to Jobs First. First, consider an individual located at point A_0 under AFDC. Supposing she is assigned to receive Jobs First, she can now either remain at point A_0 or can move along the budget constraint to point A_1. The

[5] We abstract from welfare stigma and the hassle associated with not working while on welfare modeled by Kline and Tartari (2016). We are interested in the distribution of earnings and how it varies by subgroup, but not in the welfare participation decision or decomposing labor supply responses into the extensive and intensive margin here.



(a) Budget Constraints Under Jobs First (Solid Line) and AFDC (Dashed Line)

(b) Theoretical Earnings Distributions Under Jobs First (Solid Line) and AFDC (Dashed Line)

(c) Theoretical Quantile Treatment Effects

Figure 3: Theoretical Predictions of a Static Labor Supply Model

observed choice depends on her preferences over consumption and earnings. In particular, women with steeper indifference curves at point A_0 are less likely to change their decisions between AFDC and Jobs First. The potential move from A_0 to A_1 corresponds to less mass at zero in the earnings distribution under TANF compared to AFDC in panel (b).

We next consider a woman on AFDC at point B_0 who works while receiving welfare. Transitioning to Jobs First lowers her implicit tax rate from either 49 or 73 percent to 0, therefore boosting her net wage.[6] If the substitution effect exceeds the income effect, the labor supply response moves her to point B_1 and leads to a rightward shift in the earnings distribution. Hence, for workers whose earnings lie in the segment between A_0 and C, theory predicts positive QTEs.

The QTEs in panel (c) shift from positive to negative around the point D_0, which corresponds to the earnings of women who are ineligible for welfare under AFDC but who would be eligible under Jobs First. Theory predicts moving to Jobs First would lead to a reduction in labor supply to point D_1, if we make the standard assumption that leisure is a normal good. Intuitively, by moving fro AFDC to Jobs First at point D_0 (where welfare benefits are not available under AFDC), women now gain the base grant, resulting only in an income effect.[7] Similarly, negative QTEs arise for women located at point F_0. These women would neither qualify for AFDC nor Job First at point F_0. However, the generous earnings disregard under Jobs First would incentivize women to reduce their labor supply to qualify for the new benefits leading to a movement to point F_1 that is characterized by higher consumption and lower earnings.

Last, women with relatively high earnings under AFDC at point G_0 face a trade-off when moving to Jobs First, since the point $G_1 = F_1$ does not strictly dominate point G_0. For women whose marginal disutility from earnings outweighs the marginal utility from consumption, labor supply will fall from G_0 to G_1, whereas women with different preferences may choose to remain at point G_0 and not change their labor supply. Thus, we predict the QTEs to be negative around G_0 but at higher quantiles, the QTEs may become zero. Taken together, theory predicts that the earnings QTEs of Job First will start at zero and be positive for a range of quantiles before becoming negative and eventually reaching zero again as illustrated in panel (c) of Figure 3.

The above discussion concerned the general shape of treatment effect heterogeneity but did not consider subgroups. Subgroup membership denoted by X_1, X_2, and X_3 in equations (91) and (92) affects preferences and the budget constraint. Hence, the parameters in this

[6] Note that we will use changes in labor supply and earnings interchangeably here because the gross wage is assumed to be constant.

[7] Note, to avoid clutter, we set $D_1 = B_1$ without loss of generality.

optimization problem vary, so the resulting QTEs could be shifted to the left or right, be compressed or stretched, or otherwise be transformed without losing their overall shape depicted in panel (c) of Figure 3. To illustrate, consider subgroups defined by maternal education. We ignore the potential effect of education on preferences, but assume that women with more education receive higher wage offers. Therefore, we expect a larger fraction of women with higher educational attainment to be located around the points F_0 and G_0 and correspondingly less mass around A_0, B_0, and D_0 compared to women with less education.[8] Thus, we expect an overall shift of the QTEs to the left with less mass in the lower tail of the distribution where the QTEs equal zero for higher educated women.

A similar shift is anticipated for subgroups defined by earnings and welfare history, where we also expect qualitative differences in the shape of the QTEs. Recent welfare recipients have little if any positive earnings in the period before the experiment, so there is little mass around points D_0, F_0, and G_0 relative to A_0 and B_0. Thus, we expect more positive QTEs (i.e. moves from B_0 to B_1) for these individuals and more negative QTEs (i.e. switches from D_0 to D_1 or from F_0 to F_1) for individuals with less recent welfare participation and higher previous earnings.

Finally, we consider subgroups defined by either the age or number of children. Additional children will mechanically influence the size of benefits because the latter increase with family size. Yet, under Jobs First the potential loss of welfare benefits when time limits are imposed might be higher for women with additional children. While it is not possible to predict differences in the range of positive and negative QTEs by the number of children, it is reasonable to expect larger QTEs among women with more children. Similarly, women with older children may exhibit a similar pattern of larger QTEs. This arises since young children impose a higher opportunity cost of work for mothers relative to older children and this cost is fixed independent of receiving AFDC or Jobs First. In summary, economic theory predicts treatment effect heterogeneity both within and between subgroups, motivating the development of tools to assess its extent in general, as well as in the specific context of the Jobs First experiment.

C.3 Results

In this section, we use data from the Jobs First experiment to conduct the battery of tests presented in the preceding section. Following Bitler, Gelbach, and Hoynes (2006) we use

[8] The average level of education is much lower in our sample of welfare recipients than in the general population. Therefore, we split the sample into high and low education subgroups by whether individuals have either a high school degree or a GED versus no degree at all.

quarterly earnings pooled over the seven quarters after random assignment as our outcome variable and estimate QTEs for percentiles 1 to 97.[9] To balance covariates between the Jobs First and AFDC groups, we estimate the propensity score $\hat{p}(x)$ using a series logit specification.[10] For the results that follow, we set the level of each test to $\alpha = 0.05$. The test results for the whole sample are based on bootstraps with $B = 9999$ replications while we use $B = 999$ for the subgroup-specific tests.

Figure 4 shows our estimated QTEs for the full sample along with pointwise 90 percent confidence intervals.[11] Similar to Bitler, Gelbach, and Hoynes (2006) we find pointwise significant treatment effects extending from the 48th to the 80th percentile.[12] Above the 86th percentile the point estimates for treatment effects become negative but the pointwise confidence intervals mostly include zero. Hence, the shape of the estimated QTEs aligns with the theoretical prediction in Section C.2.

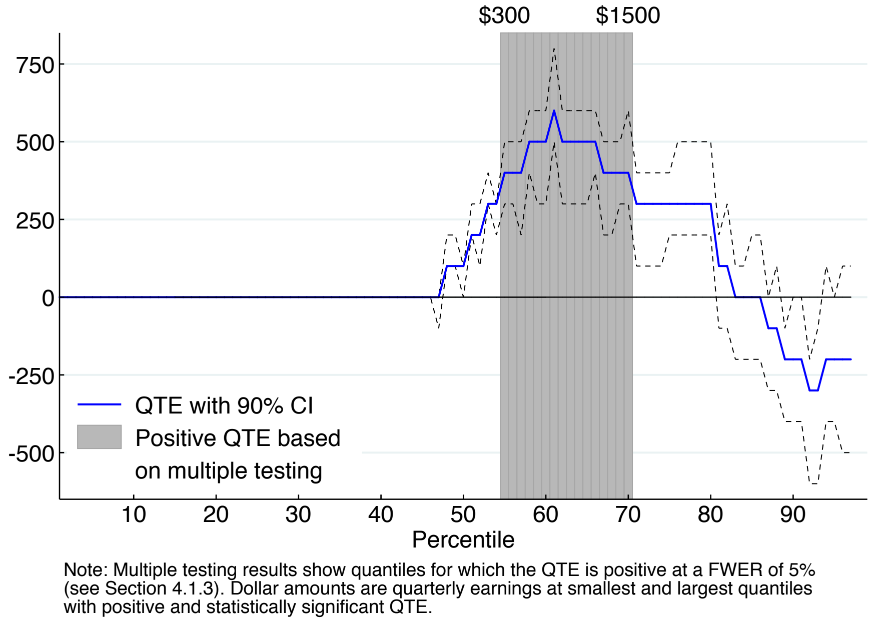
Table 2 summarizes the test result for hypotheses (H.1) and (H.2) in the main text proposed in Section 3.1 in the main text. First, we test the null hypothesis of no positive treatment effect at any percentile. As shown in Figure 4, the largest QTE (which occurs at the 61st percentile) equals 600, so this value becomes the test statistic in the first row of Table 2. Comparing this test statistics to the bootstrap critical value of 300 indicates that we can reject the null hypothesis. The associated p-value equals 0.0003. Thus, there is clear evidence that the Jobs First experiment had the desired effect of increasing earnings for at least some individuals. Next, we present results from the test of no treatment effect heterogeneity across quantiles, i.e. hypothesis (H.2). The test statistic, which is calculated as the largest deviation from the mean estimated QTE ($\bar{q}^{\Delta} = 100$), equals 500. With a bootstrap critical value of 294.85, we also reject this null hypothesis at a p-value of 0.0017. This result implies that

[9] Hence, we have a total of $7 \times 4803 = 33621$ observations. To infer treatment effects for specific individuals from QTEs we have to assume that there are no rank reversals in the earnings distribution between the Jobs First and AFDC groups. This assumption is likely violated and even predicted not to hold by labor supply theory (see Section C.2). However, positive QTEs imply that the treatment has a positive effect for some interval of the earnings distribution (Bitler, Gelbach, and Hoynes, 2006).

[10] We use a nonparametric approach since the tests are also nonparametric. That said, the vast majority of our results are robust to using a parametric logit estimator to calculate the weights via the propensity score. We include the following covariates into the logit specification: age, race, education, marital status, number of children, and employment and welfare histories.

[11] We show 90 percent CI because they corresponds to a one-sided test with a level of five percent, and we implement one-sided tests that hold the FWER at that level.

[12] Our results look slightly different from the QTEs shown in Bitler, Gelbach, and Hoynes (2006, Figure 3) because we use Firpo's (2007) check function approach as described in Section 3.1 of the main text instead of estimating empirical cumulative distribution functions of Jobs First and AFDC earnings. The QTEs are in multiples of 100 because the quarterly earnings data are rounded to the closest $100, which does not affect the validity of the results (Gelbach, 2005).



Note: Multiple testing results show quantiles for which the QTE is positive at a FWER of 5% (see Section 4.1.3). Dollar amounts are quarterly earnings at smallest and largest quantiles with positive and statistically significant QTE.

Figure 4: Quantile Treatment Effects and Multiple Testing Results, No Subgroups

treatment effects are heterogenous across quantiles, thereby indicating that individuals vary in their response to welfare reform.

Having rejected the null hypothesis of no treatment effect heterogeneity, we now identify the range of the earnings distribution where positive treatment effects are located, i.e. we test hypothesis (H.3) in the main text. As described in Section 3.1, this test accounts for potential dependencies across quantiles of the same outcome variable and the number of individual hypotheses ($|\mathcal{T}| = 97$). The shaded area in Figure 4 corresponds to the set $\mathcal{T} \backslash \mathcal{T}_k$, i.e. the percentiles where the treatment effect remains significant using a FWER

Table 2: Testing for Presence of Positive QTEs and QTE Heterogeneity Without Subgroups

	Test statistic	Critical value	p-value
Test of (H.1)	600	300	0.0003
Test of (H.2)	500	294.85	0.0017

Notes: This table shows test results for hypotheses (H.1) and (H.2) in the main text, i.e. we test that there is no positive treatment effect for all quantiles and that the treatment effect is the same for all quantiles, respectively.

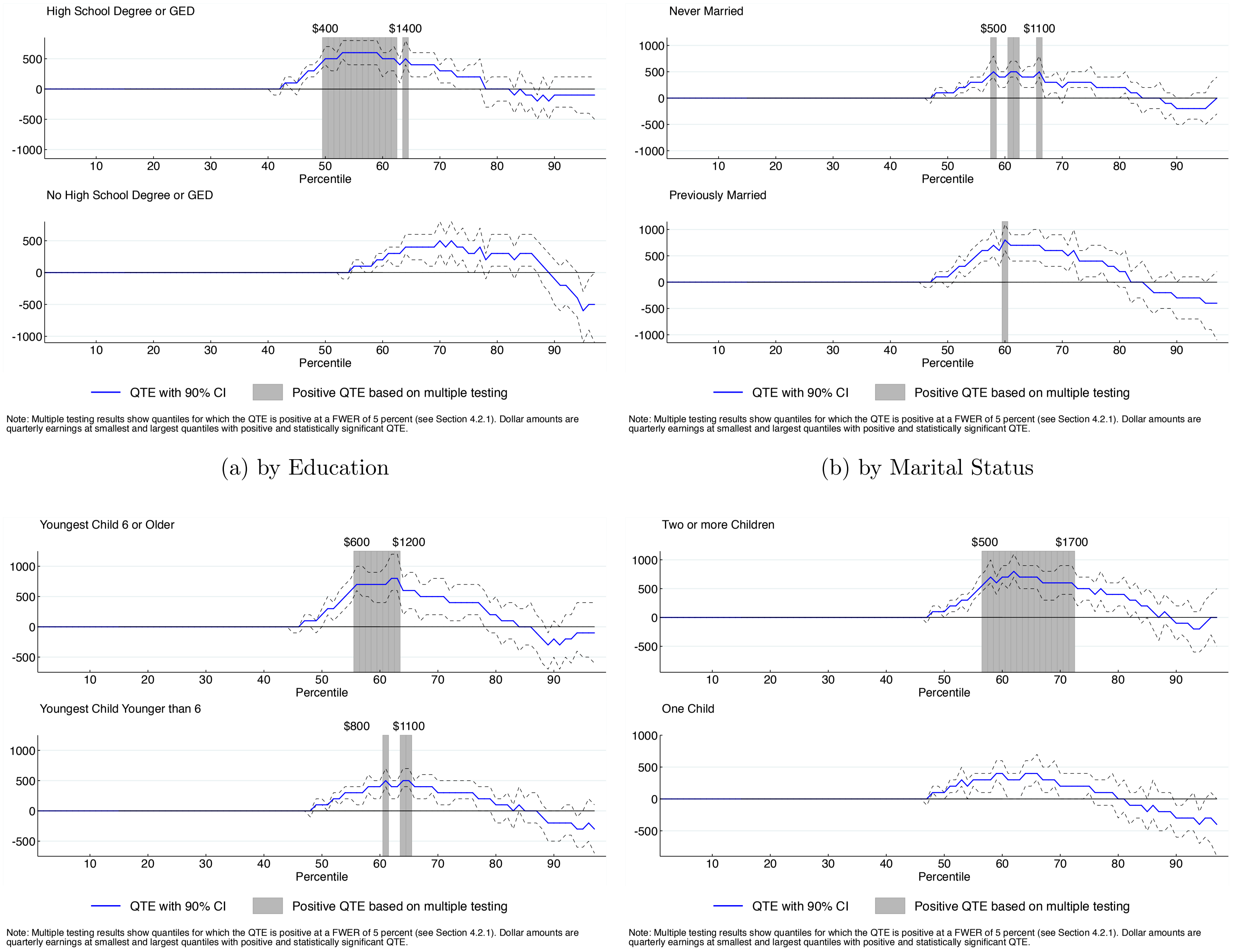
of $\alpha = 0.05$. Examining the plot we observe that the set of significantly positive QTEs supports the distributional effects predicted by labor supply theory. However, we find that individuals located between the 48th and 54th and the 71st and 80th percentiles of the earnings distribution do not exhibit significant QTEs once we adjust for multiple testing. The smallest and largest quantiles at which QTEs are significantly positive correspond to quarterly earnings of \$300 and \$1,500, respectively. Hence, we can conclude that the benefits of this particular welfare reform are more confined than one would otherwise find based on traditional statistical inference that ignores potential dependencies and testing at multiple percentiles. Given the predictions derived in Section C.2, we find that there is a more limited range of individuals who increase their labor supply when assigned to the Jobs First group.

Next, we present results incorporating subgroups using the tests described in Section 3.2 in the main text. As discussed in Section C.2, labor supply theory predicts that individuals with different observed characteristics may react differently to the same welfare rules. In particular, characteristics such as age and number of children, and prior earnings and welfare receipt may determine for which range of the earnings distribution we observe an increase or decrease in labor supply. Following Bitler, Gelbach, and Hoynes (2017) and informed by the model presented in Section C.2, we consider subgroups defined by proxies for standard demographics, wage opportunities, fixed costs of work, preferences for income versus leisure, and employment and welfare histories.[13]

Figures 5 and 6 present QTEs conditional on demographic observables and individuals' labor market and welfare histories. Shaded areas denote significant QTEs based on our multiple testing procedure of testing hypothesis (H.4) in the main text. These figures provide an easy and intuitive way to check which subgroups benefit from the welfare reform (heterogeneity across subgroups). In addition, we can inspect the figure for each subgroup to determine the range of the earnings distribution in which individuals exhibit positive subgroup-specific QTEs (heterogeneity within subgroup).

First, we split the sample by observable characteristics that may determine single mothers' wage offers (education) and labor supply (marital status, age and number of children). The multiple testing results illustrated in Figure 5 show that women with a high school degree or GED, who were never married, those with older and with two or more children, respectively, have higher earnings under Jobs First than AFDC over a wider range of the earnings distribution. These results confirm the theoretical predictions from Section C.2.

[13] Note that in our application the number of hypotheses being tested is quite small particularly relative to genomic studies from genome wide association studies. If the number of hypotheses were large it is well known that FWER controlling procedures typically have low power, and in response **?** propose an optimal false discovery rate controlling method.



Note: Multiple testing results show quantiles for which the QTE is positive at a FWER of 5 percent (see Section 4.2.1). Dollar amounts are quarterly earnings at smallest and largest quantiles with positive and statistically significant QTE.

(a) by Education

Note: Multiple testing results show quantiles for which the QTE is positive at a FWER of 5 percent (see Section 4.2.1). Dollar amounts are quarterly earnings at smallest and largest quantiles with positive and statistically significant QTE.

(b) by Marital Status

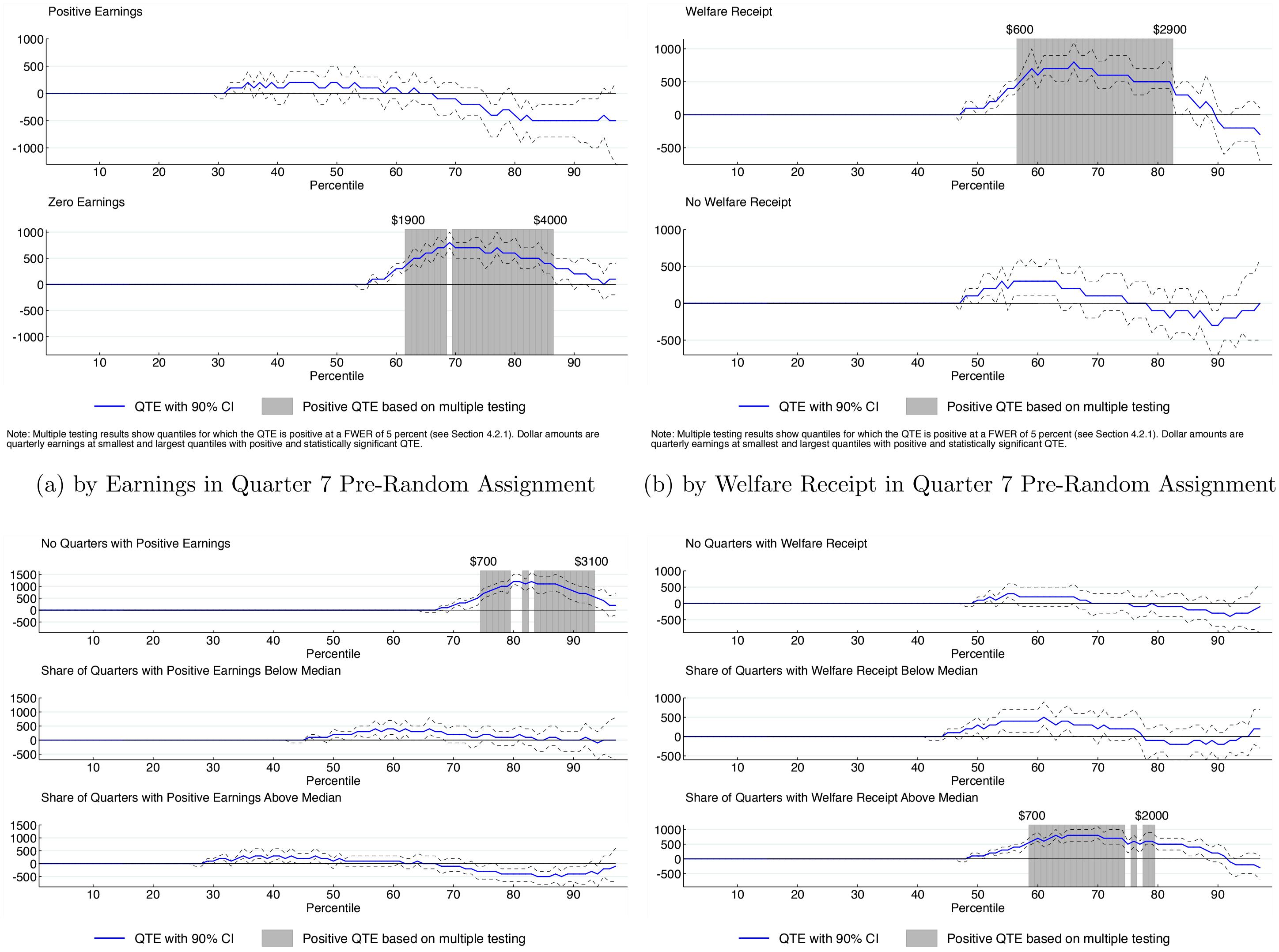
Note: Multiple testing results show quantiles for which the QTE is positive at a FWER of 5 percent (see Section 4.2.1). Dollar amounts are quarterly earnings at smallest and largest quantiles with positive and statistically significant QTE.

(c) by Age of Youngest Child

Note: Multiple testing results show quantiles for which the QTE is positive at a FWER of 5 percent (see Section 4.2.1). Dollar amounts are quarterly earnings at smallest and largest quantiles with positive and statistically significant QTE.

(d) by Number of Children

Figure 5: Quantile Treatment Effects and Multiple Testing Results, Demographic Subgroups



Note: Multiple testing results show quantiles for which the QTE is positive at a FWER of 5 percent (see Section 4.2.1). Dollar amounts are quarterly earnings at smallest and largest quantiles with positive and statistically significant QTE.

(a) by Earnings in Quarter 7 Pre-Random Assignment

Note: Multiple testing results show quantiles for which the QTE is positive at a FWER of 5 percent (see Section 4.2.1). Dollar amounts are quarterly earnings at smallest and largest quantiles with positive and statistically significant QTE.

(b) by Welfare Receipt in Quarter 7 Pre-Random Assignment

Note: Multiple testing results show quantiles for which the QTE is positive at a FWER of 5 percent (see Section 4.2.1). Dollar amounts are quarterly earnings at smallest and largest quantiles with positive and statistically significant QTE.

(c) by Share of Quarters with Positive Earnings

Note: Multiple testing results show quantiles for which the QTE is positive at a FWER of 5 percent (see Section 4.2.1). Dollar amounts are quarterly earnings at smallest and largest quantiles with positive and statistically significant QTE.

(d) by Share of Quarters with Welfare Receipt

Figure 6: Quantile Treatment Effects and Multiple Testing Results, Earnings and Welfare History Subgroups

Better educated women who receive higher wage offers may benefit more from the generous earnings disregards under Jobs First and therefore increase their labor supply more. At the same time, women without a high school degree or GED are more likely to lower their labor supply in order to become eligible for Jobs First benefits, leading to negative QTEs in the upper range of the earnings distribution. Single mothers with young children are more restricted in their time allocation, so they are less likely to change their labor supply in response to different welfare rules. The wider range of significant QTEs among mothers with two or more children may be due to the welfare rules that make benefits a function of family size. These results are important because they can show policymakers which subgroups should be targeted with a welfare reform such as Jobs First.

We now move to individual characteristics that reflect outcomes before random assignment, in particular past earnings and welfare receipt.[14] Bitler, Gelbach, and Hoynes (2017) find that subgroup-specific constant treatment effects conditional on previous earnings and welfare receipt come closest in explaining the observed QTEs in the entire sample. Figure 6 shows the QTEs and multiple testing results for subgroups defined by earnings and welfare receipt before the experiment. The only subgroups that exhibit jointly significant QTEs are those with either no earnings or with the highest levels of welfare receipt before random assignment. Compared to the results for the whole sample in Figure 4, women in these subgroups benefit from the reform in higher ranges of the earnings distribution, roughly between the 60th and 80th percentile when considering pre-random assignment welfare receipt and between the 75th and 95th percentile for single mothers who had no positive earnings in the seven quarters before the experiment. These percentiles correspond to quarterly earnings up to $2,000 to $3,800 depending on the subgroup category.

The results for subgroups defined based on prior earnings and welfare receipt are consistent with a static labor supply model. Welfare recipients who were not employed before participating in the Jobs First experiment, but instead relied on welfare, benefit the most from this policy. They move from non-employment to a point on the budget constraint where they have positive earnings and may take advantage of the generous earnings disregards under the new welfare rules. On the other hand, individuals who had positive earnings before the experiment are located further to the right on the budget constraint and may increase their labor supply only a little. Those with high earnings may even reduce their labor supply to become eligible for Jobs First. These predictions are clearly borne out by the results in Figure 6. For example, among women with an above-median share of pre-random

[14] Heckman and Smith (1998) provide evidence that groups based on pre-treatment earnings are a better predictor of treatment effect heterogeneity than groups based on standard demographic variables.

Table 3: Testing For Treatment Effect Heterogeneity Between Subgroups

Subgroup category	Test statistic	Critical value	p-value
Education	477.32	567.11	0.093093
Marital status	672.16	587.58	0.028028
Age of youngest child	653.61	525.98	0.021021
Number of children	623.71	497.22	0.02002
Earnings in quarter 7 pre-treatment	616.49	705.26	0.089089
Welfare receipt in quarter 7 pre-treatment	617.53	508.81	0.022022
Share of quarters with positive earnings	979.38	721.19	0.007007
Share of quarters on welfare	589.69	766.65	0.13113

Notes: This table shows test results for hypothesis (H.5) in the main text, i.e. these tests show for which subgroups categories we can reject treatment effects that are homogenous within subgroups for some subgroups.

assignment quarters with positive earnings, the range of negative QTEs is largest, because many of them reduce their labor supply in response to the Jobs First rules. Overall, our multiple testing results have clear policy implications as they show that a substantial share of the most disadvantaged women benefit from this reform.

We now formally test for treatment effect heterogeneity between and within subgroups. Table 3 presents the results for hypothesis (H.5) in the main text for the same subgroups as above. This null hypothesis posits that there are no differences across subgroups that can explain the observed heterogeneity of QTEs in the full sample. We can reject (H.5) for all but two sets of subgroups at a level of five percent. The p-value is largest for subgroups defined by education and the share of quarter on welfare before random assignment. Hence, for these two subgroup categories, we cannot reject the null hypothesis that the treatment effect is constant across earnings percentiles for all subgroups. Overall, however, we conclude that differences across subgroups do not explain the observed distributional treatment effects in the whole sample. While this result may appear similar to Bitler, Gelbach, and Hoynes (2017), our test relaxes the strong assumption of treatment effect homogeneity within subgroups that is implicit in their test.

The tests of hypothesis (H.6) in the main text shown in Table 4 additionally account for potential dependencies within and across subgroups. These test results provide additional insight beyond testing (H.5) because they identify the individual subgroups that

Table 4: Testing Which Subgroups Exhibit Treatment Effect Heterogeneity

Subgroup category	Test statistic	p-value
Education		
High School Degree or GED	477.32	0.995
No High School Degree or GED	424.74	0.995
Marital Status		
Never Married	411.34	0
Previously Married	672.16	0
Age of Youngest Child		
Youngest Child 6 or Older	653.61	0
Youngest Child Younger than 6	418.56	0.005
Number of Children		
Two or more Children	623.71	0
One Child	358.76	0.045
Earnings in Quarter 7 Pre-Treatment		
Positive Earnings	278.35	0
Zero Earnings	616.49	0.095
Welfare Receipt in Quarter 7 Pre-Treatment		
Welfare Receipt	617.53	0
No Welfare Receipt	274.23	0.005
Share of Quarters with Positive Earnings		
No Quarters with Positive Earnings	979.38	0
Below Median	311.34	0.675
Above Median	337.11	0.675
Share of Quarters on Welfare		
No Quarters with Welfare Receipt	306.19	0.87
Below Median	422.68	0.42
Above Median	589.69	0.01

Notes: This table shows test results for hypothesis (H.6) in the main text, i.e. these tests show for which subgroups in each subgroup category we can reject homogenous treatment effects. p-values are calculated using a grid with step size 0.005. Hence an entry of zero indicates that the corresponding p-value is below 0.005.

exhibit treatment effect heterogeneity. In these results, a p-value below 0.05 indicates that the corresponding subgroup exhibits a statistically significant amount of treatment effect heterogeneity across the earning distribution. The only subgroup categories for which we do not find evidence of treatment effect heterogeneity are share of quarters with positive earnings and welfare receipt, respectively. These results confirm the findings in Figure 6. In particular, they indicate that individuals with little past welfare receipt of positive past earnings generally do not increase their labor supply, so we also do not find any heterogeneity in the QTEs for these subgroups. Overall, however, our results clearly suggest a substantial amount of treatment effect heterogeneity between subgroups and across the earnings distribution within subgroups.

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