Stable coalition formation among energy consumers in the smart grid

Paper 5

ABSTRACT

The vision of the Smart Grid includes demand-side peak shaving strategies, such as real-time pricing or profile's based tariffs, to encourage consumption such that the peaks on demand are flattened. Up to date, most works along this line focused on optimising via scheduling of home appliances or micro-storage the individual user consumption. Alternatively, in this paper we propose to exploit the consumer social side by organising them into coalitions of energy users with complementary needs. To this ends...

1. INTRODUCTION

Since energy cannot be stored efficiently on a large scale, the electricity grid must perfectly balance the demand of all customers at any instant with supply. In all current electricity grids this balance is achieved by varying the supply-side to continuously match demand. The amount of demand required on a continuous basis is usually carried by the baseload stations owing to low cost generation, efficiency and safety. However, these stations are slow to fire up and cool down, so they are not able to match the peakload periods that exceed this baseload that require, in contrast, the use of expensive, carbon-intensive, peaking plants generators. Although only running when there is high demand, these peaking plants generators are responsible of most part of consumers electricity bill.

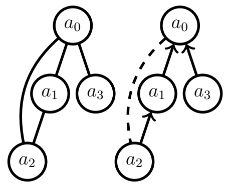
Along this line, the vision of the Smart Grid includes demand-side peak-shaving strategies such as real-time pricing or profile's based tariffs to encourage consumption such that the peaks on demand are *flattened* []. A flattened demand results in a more efficient grid not only with lower carbon emissions but also with lower prices for consumers. Hence, some works [] focused on techniques that flatten individual consumer demand by automatically controlling home domestic or micro-storage devices. Unluckily, since each consumer independently optimises its own consumption, the effectiveness of this approach has a clear limit on the consumer's restrictions and comfort (e.g. it will be unavoidable to get a consumption peak in the non-working hours of consumers).

Against this background ...

2. BACKGROUND

p_F is the unit energy price in the forward market. p_A is the unit energy price in the day-ahead market.

Appears in: *Proceedings of the 11th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2012)*, Conitzer, Winikoff, Padgham, and van der Hoek (eds.), June, 4–8, 2012, Valencia, Spain.



(a) G (b) PT

Figure 1: Example of (a) a graph with a cycle (G); (b) a pseudotree PT of G; and (c) a clique tree of PT.

2.1 Today's electricity market

In most European countries, the current operation of the exchange electricity market is composed of multiples markets available for trading electricity, each with different operation and purpose [2, 1] . In particular, most countries define and distinguishes, at least, between two kinds of markets: the *spot electricity markets* and the *forward electricity markets*. The main goal of spot markets lies in the facilitation of the trading of short-term energy delivery. In a spot market energy is traded independently for each time slot and hence, each time slot may have a different price (e.g. the day-ahead market is a spot market where hourly blocks of electricity are negotiated for the next day). In contrast, forward markets is the venue where forward electricity contracts for long periods (e.g. month, quarter or year) with delivery and withdrawal obligation are negotiated. Thus, the contract in a forward market specifies a single quantity that will be delivered at constant rate for the contract period and a single price.

Up to date, even most countries in the EU are now liberalised, market operation restricts explicitly or implicitly the participation to wholesale companies that subsequently will offer electricity to final consumers in form of standard products. However, in the smart grid . As part of the smart grid community, electricity consumers have already access to smart meters that allow them to monitor its (load) energy profile in an hour-day basis. Thus, the energy profile of a consumer a_i can be represented as a vector $E_i = \{e_i^1, \ldots, e_i^N\}$ where e_i^t is the amount of energy consumed at time slot t. Figure shows an example of energy profile as a graph that plots the variation in the electrical load versus time.

in time units. The most typical type unit is the hour.

MWh.

For electricity, the unit of measurement is the MWh.

2.2 Coalitional games

A coalitional ("transferable utility" or "characteristic function") game is traditionally defined as follows. Let $A = \{a_1, \dots, a_n\}$ be a set of agents. A subset $S \subseteq A$ is termed a coalition. However, depending on the domain not all coalitions may be feasible. In particular, here we are interested on restricting coalitions by a *graph* G: (i) each node of the graph represents an agent; and (ii) a coalition S is allowed to form iff every two agents in S are connected by some path in the subgraph induced by S. We denote the set of graph feasible coalitions as $F(G)$. Then, a coalitional game CG is completely defined by its *characteristic function* $v : F(G) \rightarrow \Re$, which assigns a real value representing (transferable) utility to every feasible coalition [5]. Agents in a coalition are then permitted to freely distribute coalitional utility among themselves. Given a game CG, a *coalition structure* $CS = \{S_1, \dots, S_k\}$ is an exhaustive disjoint partition of the space of agents into feasible coalitions. We overload notation by denoting by $v(CS)$ the (intuitive) worth of a coalition structure: $v(CS) = \sum_{S \in CS} v(S)$.

Then, the coalition formation process can generally be considered to include three differentiated activities: *Coalitional Value Calculation*, *Coalition Structure Generation* and *Payoff Distribution*. First, on *coalitional value calculation*, agents enumerate and evaluate all possible feasible coalitions that can be formed. Next, given the values of feasible coalitions, the key challenge addressed in *coalition structure generation* is to identify the coalition structure CS^* that maximizes *social welfare* - i.e. the coalition structure with maximal value. Finally, *Payoff Distribution* determines the utility that each agent in a coalition should obtain as a result of the actions taken by the coalition as a whole. A vector $\rho = \{\rho_1, \dots, \rho_n\}$ assigning some payoff to each agent $a_i \in A$ is called an *allocation*. We denote $\sum_{i \in S} \rho_i$ by $\rho(S)$. An allocation ρ is an *imputation* for a given CS, if it is efficient ($\rho(S) = v(S)$ for all $S \in CS$), and individually rational (that is, $\rho_i \geq v(\{i\})$ for all a_i). Note that if ρ is an imputation for CS, then $\rho(A) = v(CS)$. A game outcome is a (CS, ρ) pair, assigning agents to coalitions and allocating payoffs to agents efficiently. However, in a selfish environments, agents are only concerned with maximizing their own payoffs. Thus, with the presence of selfish agents we need to the determine *stable* allocations. Here, stability refers to the state where the agents have no incentive to deviate from the coalitions to which they belong. Cooperative game theory provides several stability concepts and one concept, the core which is arguably the most well-studied shall be discussed here. The core is composed of all coalition structure-imputation tuples (CS, ρ) such that no feasible coalition has a deviation incentive. Formally:

$$Core(CG) = \{(CS, \rho) : \rho(A) = v(CS) \ \& \ \rho(S) \geq v(S) \ \forall S \in F(G)\}$$

The core is a strong solution concept, as it is empty in a plethora of games. Moreover, notice that *only optimal coalition structures might admit an element in the core*. Intuitively, if the current structure is suboptimal then a subset of agents can be made strictly better off by moving to an optimal coalition structure. Hence, for any core-pair allocation CS is an optimal coalition structure, CS^*, and the allocation ρ is efficient with respect of the value of the optimal coalition structure ($\rho(A) = v(CS^*)$). In this work, we are interested on the question of how to compute a core member: this includes to solve the CSG problem to get the optimal coalition structure CS^* and compute the stable payoff allocation over CS^* or detect the emptiness of the core if it is the case.

3. THE MODEL

In this section, we model the problem of demand-side coalition formation among energy consumers as a coalitional game. Let $A = \{a_1, \dots, a_n\}$ be the set of agents, each one representing an energy consumer with its associated energy profile E_i. Agents can form energy coalitions $S \subseteq A$, where an energy coalition S stands for the set of consumers S acting as a single virtual consumer in the market along with their joint consumption. Analogously to single consumer coalitions, the (expected) demand of any coalition of consumers S is represented by their joint energy profile $E_S = \{e_S^1, \dots, e_S^N\}$ where $e_S^t = \sum_{i \in S} e_i^t$. By forming coalitions, agents can join other consumers with complementary energy needs, getting a flattened demand for which they can get better prices in the market. Hence, regarding the coalitional value calculation, the value of a coalition among energy customers $v(S)$ needs to codify the expected payment that the coalition of consumers S will need to carry out to get their joint demand. Moreover, we consider that each consumer looks for potential partners for its coalitions through its contacts in a social network. Thus, the set of energy feasible coalitions is constrained by a graph G.

In the next sections, we specify with more detail how we solve the three main activities that underline the coalition formation process for this particular energy domain.

3.1 Coalitional Value Calculation

To capture such complementarities, in Section we define the characteristic function $v(\cdot)$ for this domain, where $v(S)$ stands for the expected payment that the coalition of consumers S will need to carry out to get their joint demand. We address the problem of enumerating and evaluating all energy coalitions in the social network in Section .

3.1.1 Characteristic function definition

To capture such complementarities, in Section we define the characteristic function $v(\cdot)$ for this domain, where $v(S)$ stands for the expected payment that the coalition of consumers S will need to carry out to get their joint demand.

The value of a coalition S, $v(S)$, is the total payment that the set of consumers need to carry out to cover the demand of their joint energy profile. Analogously to the operation of the current grid, we consider that consumers buy directly their electricity in two different markets: the Spot Electricity Market and the Forward Electricity Market. We use the joint average energy profile of a coalition as a prediction of its consumption. Then one question that emerge is how members of a coalition can buy their excepted consumption in the available markets (spot and forward market) in order to minimize the amount to be paid. We denote p_{Spot} and $p_{Forward}$ to the prices of the spot and the forward market respectively. Explain that the price of the spot is the average among hours and the price of the forward is the price for one hour. Agents in a coalition need to decide the quantity they buy in the forward market q^F and the quantity they buy on the spot market q_{Spot}^i for each time slot 1...N. Agents need to decide these quantities such that the demand for each time slot is covered. Formally,

$$e^i \geq q_{Spot}^i + q_{Forward} \ \forall i = 1 \dots N \tag{1}$$

Then, the value of the expected payment for the coalition S is given by:

$$v(S) = -(\sum_{i=1}^{N} q_{Spot}^i \cdot p_{Spot} + q_{Forward} \cdot N \cdot p_{Forward}) \tag{2}$$

Agents need to decide the continuous amount of quantity they buy in the forward market $q_{Forward}$ as well as the quantity they buy in the spot market for each time slot i such that maximise equation 2 whereas satisfying constraints in equation 1.

In the forward market, consumers in a coalition S buy in advance the fix continuous amount of energy of their joint energy profile, $base(S)$, for a better price. To compute the base load from an energy profile you simply compute the minimum of the energy consumption along hours and $base(S) = min * 24h$. $peak(S) = sum_h - base(S)$.

The amount of energy that exceeds this baseload, $peak(S)$, is bought in the day-ahead market.

$$v(S) = -base(S) \cdot p_F - peak(S) \cdot p_A \tag{3}$$

Since $p_F < p_A$, the flattered the energy profile, the most a coalition of consumers can buy in the forward market and the lower the payment of the coalition.

Another measure?

3.2 Network-based coalitions

To generate the set of graph coalitions we observe that generating all feasible graph-based coalitions is analogous to the problem of generating all connected induced subgraphs of a connected undirected graph G. Thus, we formulate an algorithm that allows agents to generate the set of feasible coalitions in a distributed way. Our algorithm is based on a state-of-the-art algorithm formulated in [4] that generates all connected induced subgraphs of a given graph (for further details see [4], section 5). The complexity of the algorithm is $O(n \cdot c(G))$, where n is the number of nodes of G and $c(G)$ is the number of connected induced subgraphs of G. Similarly to Theorem ?, one can prove the following: (Theorem 10) Let $c(G)$ be the number of connected induced subgraph of a connected graph G. Algorithm is correct and its time and space complexities are O(n$\cdot c(G)) and O(n+m), respectively. Instead of using a linear ordering as in [4], we propose to use the partial ordering that defines a pseudotre$

Figure 1(b) shows a pseudotree, rooted at agent a_0, of the cyclic graph G in Figure 1(a). A PT has two kinds of edges: tree-edges (bold lines) that link parent with children (e.g. a_2 is child of a_1); and pseudoedges (dashed lines) that link pseudoparents with pseudochildren (e.g. a_2 is pseudochild of a_0). Let's denote $A(PT)$ the set of agent' nodes in PT and PT_i the subtree of PT rooted at a_i. Thus, in Figure 1(b), PT_1 is a tree rooted at a_1 composed of agents a_1, a_2. Finally, given an agent $a_i \in A(PT)$ we will denote as Ch_i its children, An_i its ancestors (the set composed of its parent and its pseudoparents), and D_i its descendants (the set composed of its children and pseudochildren) in PT. Then, in Figure 1(b), $Ch_2 = D_2 = \emptyset$ and $An_2 = \{a_0, a_1\}$. Then, given a game on a graph $CG = \langle A(G), v, F(G) \rangle$ and a pseudotree PT over G, the partial ordering that PT defines among agents allows us to partition the set of feasible coalitions into $|A|$ disjoint sets $\{\mathbf{L}_i | a_i \in A\}$, one per agent. The set of (leading) coalitions $\mathbf{L}_i$ contains all the feasible coalitions in which a_i is the leader in the hierarchy, that is all coalitions that include agent a_i but no agent up a_i in PT, $\mathbf{L}_i = \{S \in F(G_{A(PT_i)}) | i \in S\}$).

Next, we describe the main steps of a distributed procedure that allow agents to compute the set of leading coalitions on a graph. Thus, at the end of this process, each agent will know its set of leading coalitions $\mathbf{L_i}$.

Each agent a_i uses the PT partial ordering to order its descendants $D_i = \{d^1, \ldots, d^m\}$ from higher to lower. For example, in Figure 1(b), a_0 can order its descendants as a_1,a_2,a_3 (as far as a_1 is placed before a_2 the order is valid). Then, a_i will proceed to generate its set of leading coalitions $\mathbf{L}_i$ into two steps: first, generating a set of basic coalitions, and second, generating a set of composed coalitions, that result from combination of basic ones.

Step 1. (Basic coalitions). In this step, each agent a_i will generate the set of *basic* coalitions. For each descendant $d^{j=1\ldots m}$, a_i generates all coalitions S such that $\{d^j\} \subseteq S \subseteq A(PT_i) \setminus \{E\}$ where $E = \{d_k | k < j\}$ stands for all descendants placed before d^j in the ordering. For example, in Figure 1(b), agent a_0 will generate the set of coalitions that include: (i) a_1 and other agents reachable from a_1 in PT_1 ({1},{12}); (ii) a_2 and other agents reachable from a_2 in PT_1 excluding a_1 ({2}); and (iii) a_3 and other agents reachable from a_3 in PT_1 excluding a_1 and a_2 ({3}).

Each agent a_i records for each coalition S a set of frontiers nodes, F, these are nodes that are reachable from S but not included in S. For example, in Figure 1(b) the set of frontiers for $\{1\}$ is $\{2\}$ since a_2 is reachable from a_1 but not included.

Finally, a_i stores each generated coalition S, adding a_i, $(S \cup \{i\})$, as well as a_i's single coalition ($\{i\}$), as part of its leading coalitions $\mathbf{L}_i$.

Step 2. (Composed coalitions). In this step, each agent a_i will generate the set of *composed* coalitions. For each descendant $d^{j=1\ldots m}$ a_i will combine all coalitions reachable from d^j with all compatible coalitions reachable from d^{j+1}, from d^{j+2}, ..., and until d^m, storing at each step the new coalitions as reachable from d^j. Thus, in Figure 1(b), a_0 will combine coalitions reached from a_1 with coalitions from a_2 (storing any new coalition as reachable from a_1) and coalitions reachable from a_1 with coalitions from a_3. Two coalitions, S reachable from d^j, and S' reachable from $d^{k>j}$, are compatible if S' does not contain any agent in S or in its frontiers ($S' \cap S \cap F_S \neq \emptyset$). Thus, in Figure 1(b), a_1 will not combine $\{1\}$ from a_1 and $\{2\}$ from a_2 because a_2 is a frontier for $\{1\}$. A composed coalition is generated as $\{S \cup S'\}$ with frontier agents $\{F_S \cup F_{S'}\}$. Thus, in Figure 1(b), a_0 the result of combining $\{2\}$ from a_2 with $\{3\}$ from a_3 is a composed coalition $\{2, 3\}$ reachable from a_2 with $F = \{2\} \cup \emptyset$. For each composed coalition resulting from the combination of S with S', a_i computes its set of requiring coalitions $Req_i(S \cup S' \cup \{i\})$ as $\{S, S'\}$ if S is a basic coalition or, alternatively, as $\{Req_i(S), S'\}$ if S is a composed coalition.

Explain the way we generate coalitions. There is a way to count them? [4]

3.3 Coalition Structure Generation

We now show how to map the CSG defined above into integer programming (IP). Our aim is to find .... We define a set of binary decision variables $x_S \in \{0, 1\}$, one per feasible coalition $S \in F(G)$, where x_S takes on value 1 if coalition S is selected and 0 otherwise. Now solving the CSG for the demand-side amounts to solving the following IP:

$$max \sum_{S \in F(G)} v(S) \cdot x_S$$

Subject to:

(1) Each energy customer can join at most one coalition:

$$\forall a_i \in A : \sum_{S \in F(G) | S \ni i} x_S = 1$$

3.4 Payoff Distribution

We now show how to map the Payoff Distribution defined above into a linear program (LP). Following the lemma, x is not blocked if it is not blocked by any B specified in W. The core is not empty if we can find such a payment vector. Therefore, the problem that needs to be solved is:

In this paper we propose a general procedure to compute a core element (or to detect that no core allocation exists) which is based on mathematical programming techniques.

Our aim is to find .... We define a set of binary decision variables $x_S \in \{0, 1\}$, one per feasible coalition $S \in F(G)$, where

x_S takes on value 1 if coalition S is selected and 0 otherwise. Now solving the CSG for the demand-side amounts to solving the following integer program: Find stable payments once CS^* has been found by solving the following LP:

$$min\ \ \rho(A)$$

Subject to:

(1) There are no deviating coalitions for these payments:

$$\forall S \in F(G):\ \rho(S) \geq v(S)$$

(2) Agents can could make a profit

$$\forall a_i \in A:\ \rho_i \leq 0$$

If the value of the objective function of this IP yields $\rho(A) = v(CS^*)$, then the problem has a non-empty core and the values ρ define an allocation in the core. Otherwise, the problem has an empty core. It should be emphasized that the values $v(S)$ as well as a optimal coalition structure CS^* are given parameters which means that this program can not computed the allocation needs as input the results of solving the Coalitional and the CSG.

If $v(CS^*) \neq \rho(A)$ the core is empty. Otherwise the payments are in the core. Notice that (2) ensures that agents do not make a profit expoiling from other agents, getting a positive payoff from payments from other agents in the coalition. So, as maximum agents can get the energy for free in their best deal.

Linear programs can be solved in polynomial time in the number of variables and constraints. In our case, the variables are $x_1, \ldots, x_n$ and there are constraints. The Linear Program will not only specify if the core is not empty, but also provide $\rho \in CORE$ if one exists.

This program has a number of variables equal to the number of agents $N_{variables} = |A|$ and number of constraints equal to the number of feasible coalitions $N_{constraints} = |F(G)|$ which can be very large (in a complete graph is exponential in N).

4. EMPIRICAL EVALUATION

In this section we provide an empirical evaluation of the coalition formation model among energy consumers introduced in Section 3. Firstly, we explain the details of our experimental setup in Section 4.1. Next, we analyse our empirical results in Section 4.2.

4.1 Empirical settings

4.1.1 Problem generation

To evaluate the sensitivity of the coalition formation process with respect to the underlying network topology we evaluate our model on three different network models. Moreover, for each network topology we analyse networks with different density levels. The density of a graph is defined as the ratio between the number of links and the number of agents in the graph ($\frac{|E|}{|A|}$). In more detail, in our experiments we test our model on the following network configurations:

Random Networks. Graphs are created by *randomly* adding a number of links d for each agent. Densities used in this case are : $d = 1$ (low), $d = 2$ (medium) and $d = 3$ (high).

Scale Free Networks. Graphs are created by using an implementation of the Barabasi-Albert model. At each step, a node is added and attached to d neighbours using a biased random selection giving more chance to a node if it has a high degree. Graphs are generated using three different densities: ($d = 0.92$, low), ($d = 1.75$, medium) and ($d = 3.17$, high).

Small-World Networks. Graphs are created by following the Watts and Strogatz model. This model generates a ring of graph where each node is connected to its k nearest neighbours in the ring (k/2 on each side, which means k must be even). Then it process each node of the ring in order following the ring, and "rewiring" each of their edges toward the not yet processed nodes with randomly chosen nodes with a rewiring probability of 0.1. Graphs are generated using three different values for parameter k: $k = 2$ ($d = 1$, low), $k = 4$ ($d = 2$, medium) and $k = 6$ ($d = 3$, high).

Notice that whereas scale free and small-world networks are known to capture some characteristics of social networks, random networks constitute a more synthetic model for our domain. All experiments are run using networks of 12 nodes. For each instance, the energy profile of each node is randomly selected from a real dataset composed of energy profiles characterizing the real domestic electricity consumption of X homes in the United Kingdom.

4.1.2 Market's parameters

As explained in Section , two of the most important parameters that regulate the electricity market are the prices of the electricity in the forward, p_{DA}, and the day-ahead market, p_{DA} (although the price of electricity in the day-ahead market varies on each time slot, we consider here that p_{DA} is calculated by averaging the hourly price of a day). Moreover, in addition to the market prices, another sensitive parameter is the choice of the strategy to decide which is the base amount to buy in the forward market (defined through fraction parameter p). Then, in our experiments, we explore three different market conditions:

M1 $p_F = 70, p_{DA} = 80, p = 1$

M2 $p_F = 70, p_{DA} = 80, p = 0.875$

M3 $p_F = 1, p_{DA} = 2, p = 0.5$

Notice that whereas in M1 agents follow a risk-free strategy ($p = 1$), in M2 and M3 agents takes the maximum much risk that gives them a positive expected gain ($p = \frac{p_F}{p_{DA}}$). Regarding market prices, in $M1$ and $M2$ prices used are those of current electricity markets in Italy[1] whereas $M3$ explores a different scenario in which buying in the forward market is more incentivized with better prices. As a consequence, the maximum relative percent gain[2] an agent can get in M1 and M2 is of 12.5% whereas in M3 is of 50%.

4.2 Results

Using the different configurations explained in section above, we evaluate our model by performing repeated simulations (50 instances per graph configuration) and analysing the results in terms of the individual consumer gain and the structure of the formed coalitions.

[1] Available at: `http://www.mercatoelettrico.org/En/Default.aspx`

[2] We compute the maximum relative percent gain as the difference between the cost of buying all demand in the day-ahead market and the cost of buying all demand in the forward market divided by the cost of buying all demand in the day-ahead market $100 \cdot \frac{p_{DA}-p_F}{p_{DA}}$.

4.2.1 Consumer's gain

In this section we analyse the consumer's gain that agents obtain in our model on the different network and market configurations. Let ρ_i be the payment of agent $a_i \in A$ in a coalition and $v(\{i\})$ the payment of the agent in its single coalition. Then, the average percent consumer gain is assessed as $\frac{\sum_{a_i \in A} \rho_i - v(\{i\})}{|A|}$. Figure 2 show the results for 12 agents on a random, scalefree and small-world networks in the three different market scenarios respectively. We also plotted the standard error of the mean as a measure of the variance in each graph. Only data regarding instances with non-empty core are plotted. Observe that in all configurations, although the average percent user gain is increased with density, this increment is not significant. Moreover, the average percent user gain is much higher (around 10%) in M3 market than not in M1 and M2 (around 1%).

Table 1 shows the percentage of instances under each configuration for which the core was detected as empty. Notice that in all network topologies, the number of instances for which the core is empty increases with the density of the network. These results are coherent with the well-known results that any acyclic network (which has by definition the lowest density) is guarantee to have a non-empty core [3]. As we increase the density the number of cycles also increase and results show that the probability of core emptiness is higher. Regarding different network topologies, we observe that the number of instances with core emptiness is higher in scale free networks, where the links are concentrated on hubs, than not on random and small-world networks, where each node in average have the same degree. Finally, we also observe that the number of instances with core empty is much higher on M1 and M2 than not in M3. Although we need to perform a deeper analysis on these results, they lead to the hypothesis that the more the distance of prices in the market the less the probability of having an empty core in the coalitional game.

4.2.2 Structure of energy coalitions

In this section we analyse the structure of the energy coalitions obtained in the experiments. For each configuration, we plot the mean of the minimum, average and maximum size of coalitions formed. Figure 3 plots the results for networks of 12 agents on a random, scale free and small-world networks in two different market scenarios. We also plotted the standard error of the mean as a measure of the variance in each graph. Market scenario M3 is omitted because in this case we detected that the gran coalition was always formed in all tested instances. In contrast we observe that for markets M1 and M2, the market conditions lead to coalitions of middle size in all network structures. We also observe that as we increase the density of the network, more coalitions of middle size are composed whereas in low densities agents tend to compose larger coalitions.

Topology	**Density**	**% Empty Core**		
		M1	M2	M3
Random	Low	8%	0%	0%
	Medium	50%	26%	6%
	High	56%	44%	10%
ScaleFree	Low	0%	0%	0%
	Medium	52%	22%	2%
	High	46%	38%	12%
SmallWorld	Low	8%	6%	2%
	Medium	46%	18%	8%
	High	46%	48%	6%

Table 1: Percentage of instances with empty core under different configurations.

5. REFERENCES

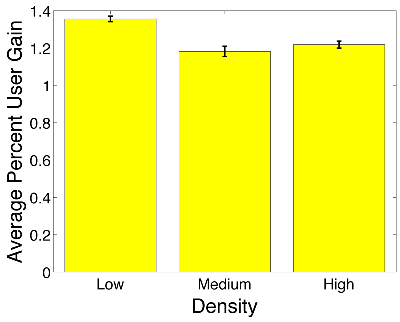
[1] Tradewind, further developing europe's power market for large scale integration of wind power. report d. 4.1., 2007. `http://www.trade-wind.eu/fileadmin/documents/publications/D4.1_Summary_report_of_market_rules.pdf`.

[2] Vademecum of the italian power exchange. Gestore dei mercati energetici S. p. A., 2009. `http://www.mercatoelettrico.org/En/MenuBiblioteca/documenti/20091112VademecumofIpex.pdf`.

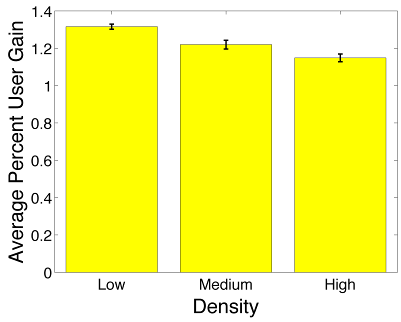
[3] G. Demange. On Group Stability in Hierarchies and Networks. *Journal of Political Economy*, 112(4):754–778, August 2004.

[4] G. Gutin, A. Johnstone, J. Reddington, E. Scott, A. Soleimanfallah, and A. Yeo. An algorithm for finding connected convex subgraphs of an acyclic digraph. In *IN ÔALGORITHMS AND COMPLEXITY*. College Publications, 2008.

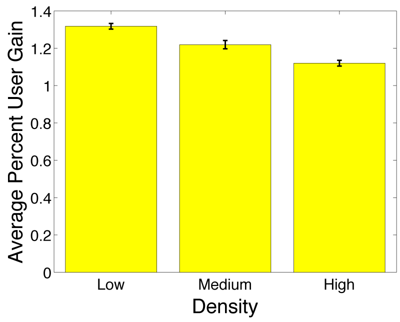
[5] M. J. Osborne and A. Rubinstein. *A Course in Game Theory*. MIT Press, Cambridge MA, USA, 1994.



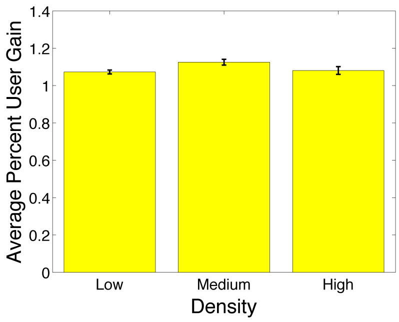
(a) Random Graphs M1.



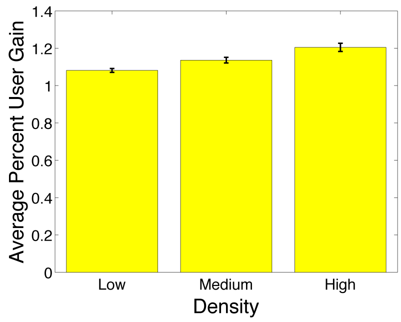
(b) Scale Free M1.



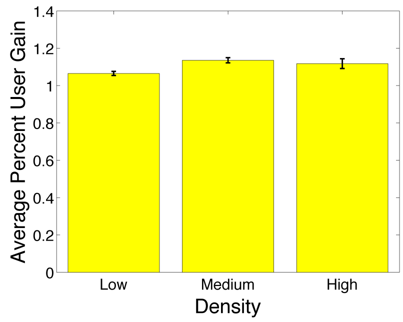
(c) Small World M1.



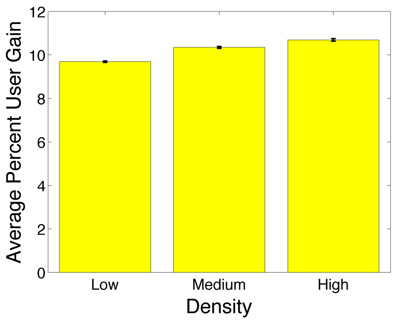
(d) Random Graphs M2.



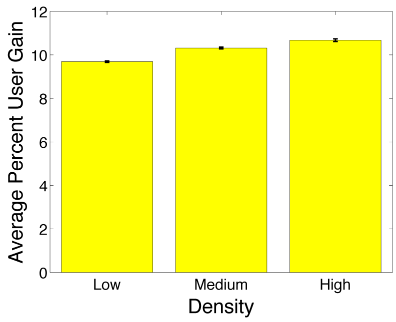
(e) Scale Free M2.



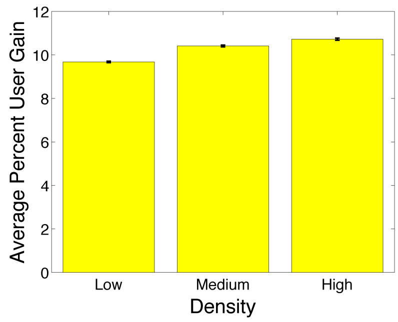
(f) Small World M2.



(g) Random Graphs M3.

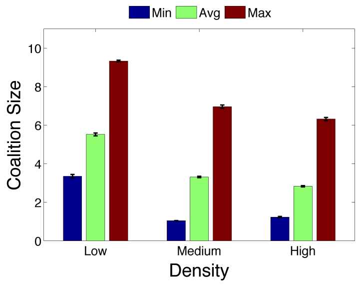


(h) Scale Free M3.

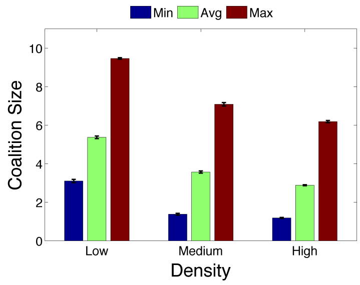


(i) Small World M3.

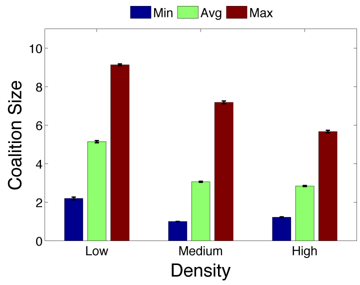
Figure 2: Graphs showing the average percent gain of consumers on different topologies and densities under market conditions M1 (a)-(c), M2 (d)-(f) and M3 (g)-(i).



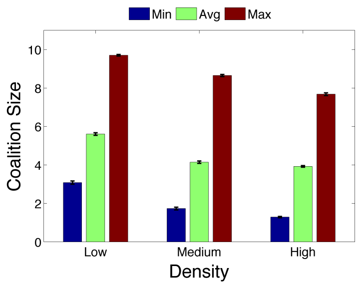
(a) Random Graphs M1.



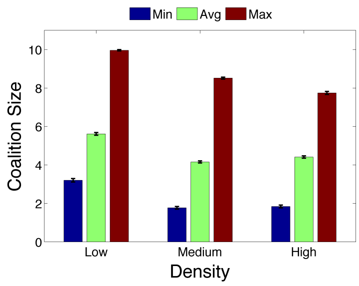
(b) Scale Free M1.



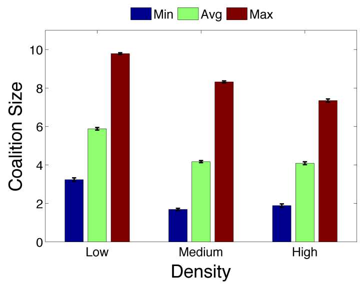
(c) Small World M1.



(d) Random Graphs M2.



(e) Scale Free M2.



(f) Small World M2.

Figure 3: Graphs showing the minimum, average and maximum size of coalitions formed on different topologies and densities under market conditions M1 (a)-(c), M2 (d)-(f).