A gentle intro to Meta-analysis 🚀

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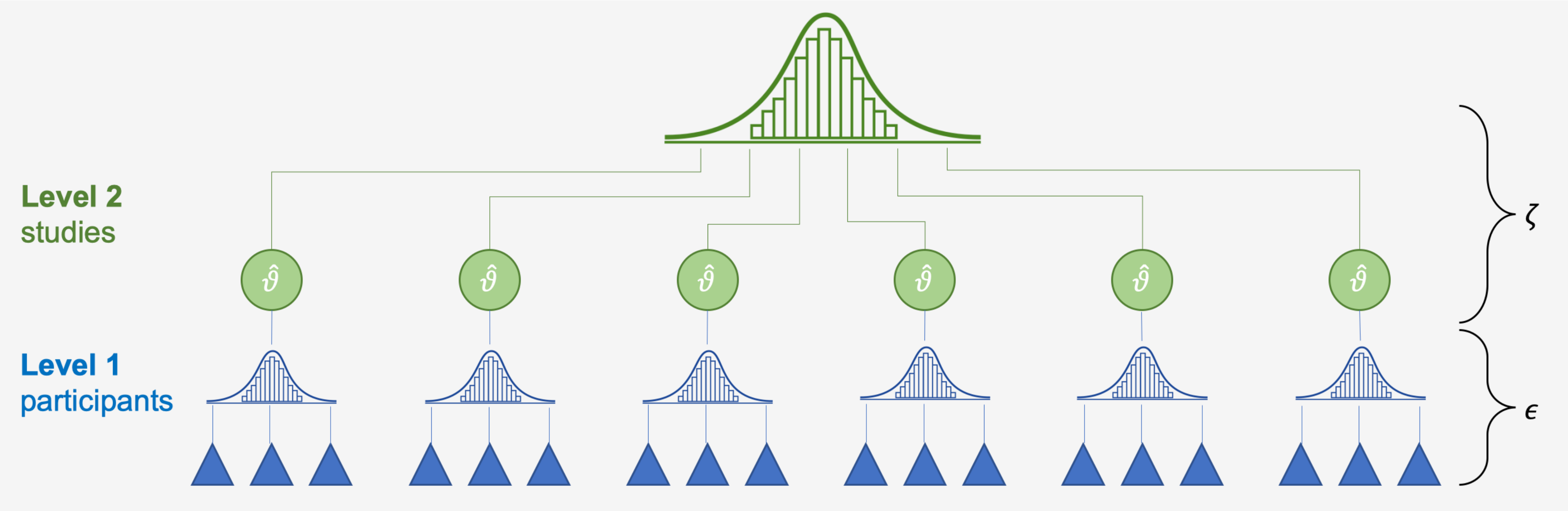
General Intro

General Intro

- A meta-analysis is a **quantitative** way to summarize research result about a specific topic, research question or research area.
- Shares the **systematic literature search** with narrative or systematic reviews
- Beyond "simply" quantifying a phenomenon, the meta-analysis can be considered as a **radiography** of the published literature
- Gives insights about holes in literature, could suggest new research topics or support a new experiment

The Meta perspective



Source: Doing meta-analysis in R

Meta-analysis steps

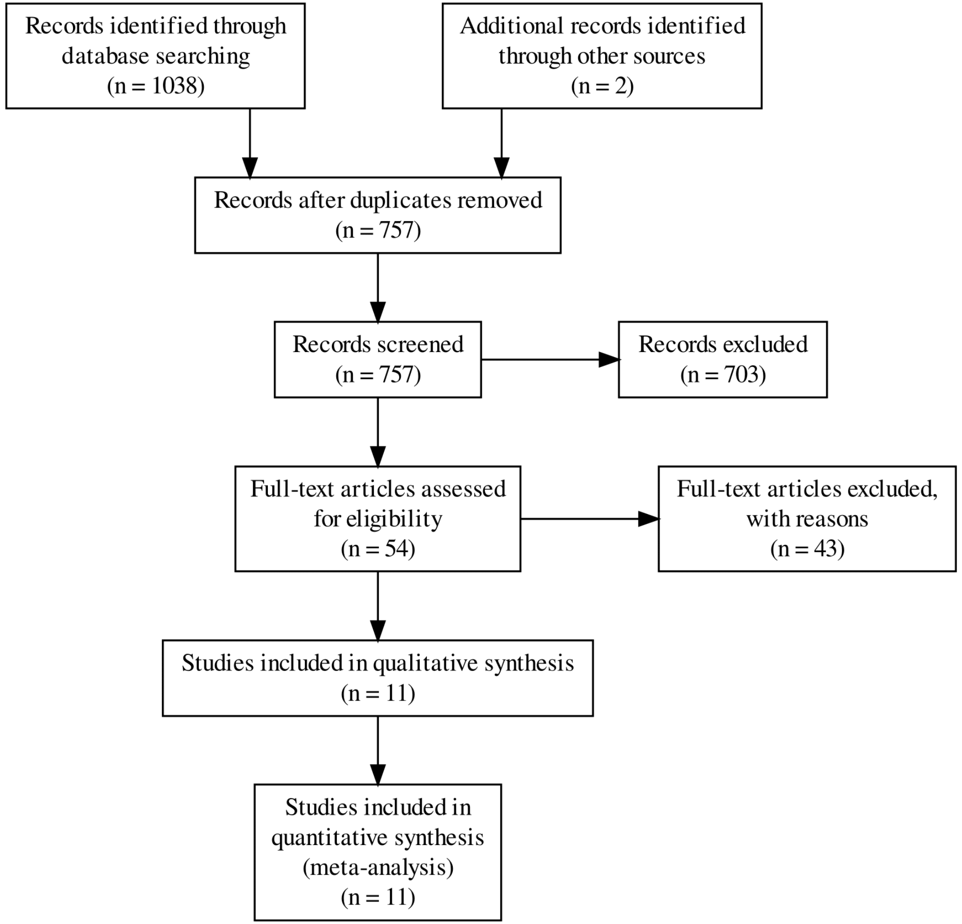
Research Question

- Not all topics can be managed using a meta-analytic approach
  - **Too broad**: *Efficacy of psychotherapy*
  - **Too narrow**: Not a problem, but probably a little amount of available papers

- *Mixing apple and oranges*
  - Does makes sense to pull together these n articles?

Literature Search

- This is a very crucial and complicated step
    - Keywords
    - Systematic research across *databases*
    - Ideally performed at least by **2 peoples**
    - A lot of guidelines such as the **PRISMA** approach



Extracting Data

- The first step towards the statistical modeling

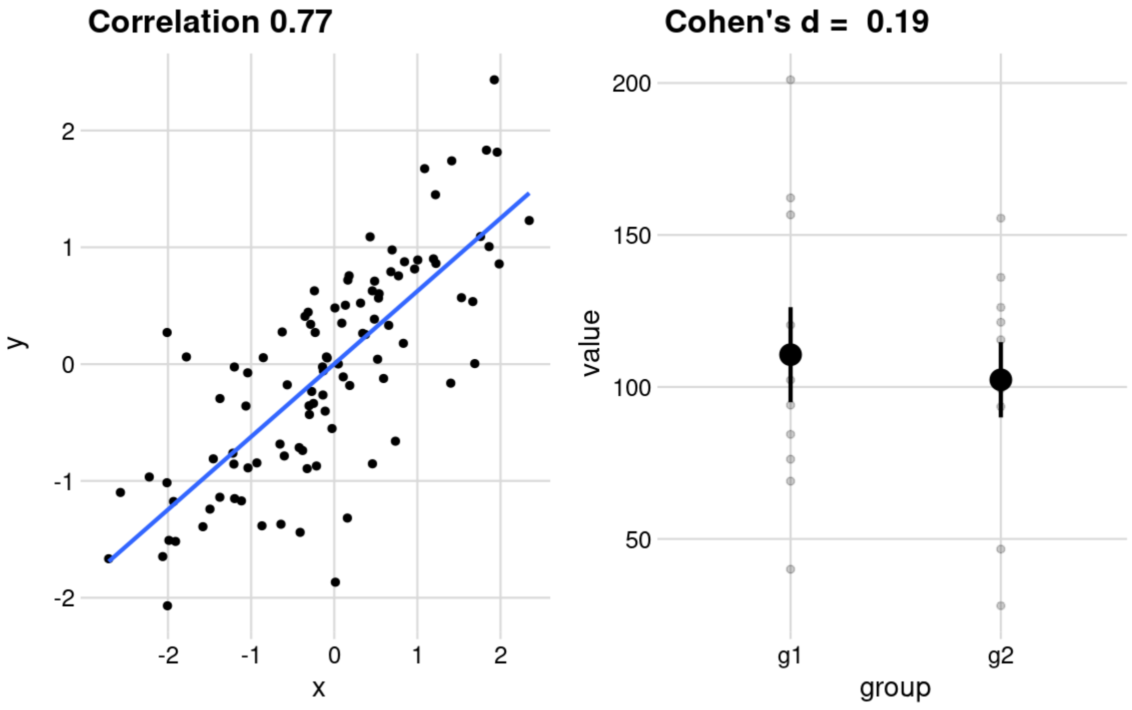
- Extract relevant data in order to:

  - Compute **outcome** (e.g., effect size) measures
  - Select relevant **moderators** for the meta-regression

- These moderators need to be theoretically relevant and should be selected before starting the meta-analysis. At the same time exploratory analysis is perfectly fine.

Effect Size Computation

The effect size is number that quantify the **strength of an effect** in the population that is estimated from our sample



Effect Size Computation

Correlation

$$r = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2}\sqrt{\sum_{i=1}^{n}(y_i - \bar{y})^2}}$$

Standardized Mean Difference (e.g., Cohen's d)

$$d = \frac{M_1 - M_2}{SD_{pooled}}$$

Effect Size Standard Error

In statistics, every population parameter estimation has an associated uncertainty measure: **Standard Error**. Given that our sample size is always a subset of the population, our estimation is not perfect.

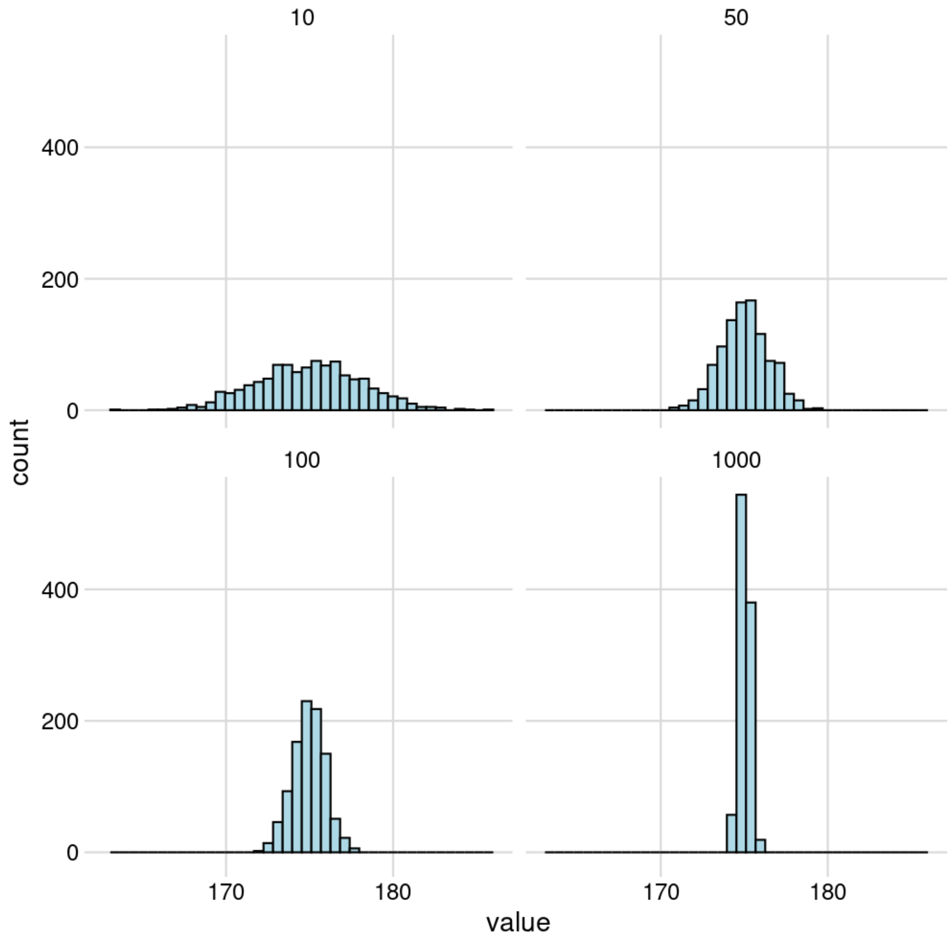
Example: estimation of the males height in Italy

- θ = real population height (175cm; but unknown in real research)
- We took a sample S of size n, with observed mean $\bar{X}$ and standard deviation s
- From statistical theory we know that $\bar{X}$ is a good estimator of θ but with a given uncertainty:

$$SE = \frac{s}{\sqrt{n}}$$

Effect Size Standard Error

- The estimated mean is the same but the **uncertainty vary as a function of sample size**
- A study with an **higher sample size** has a **greater estimation precision**



Quick Recap

We have:

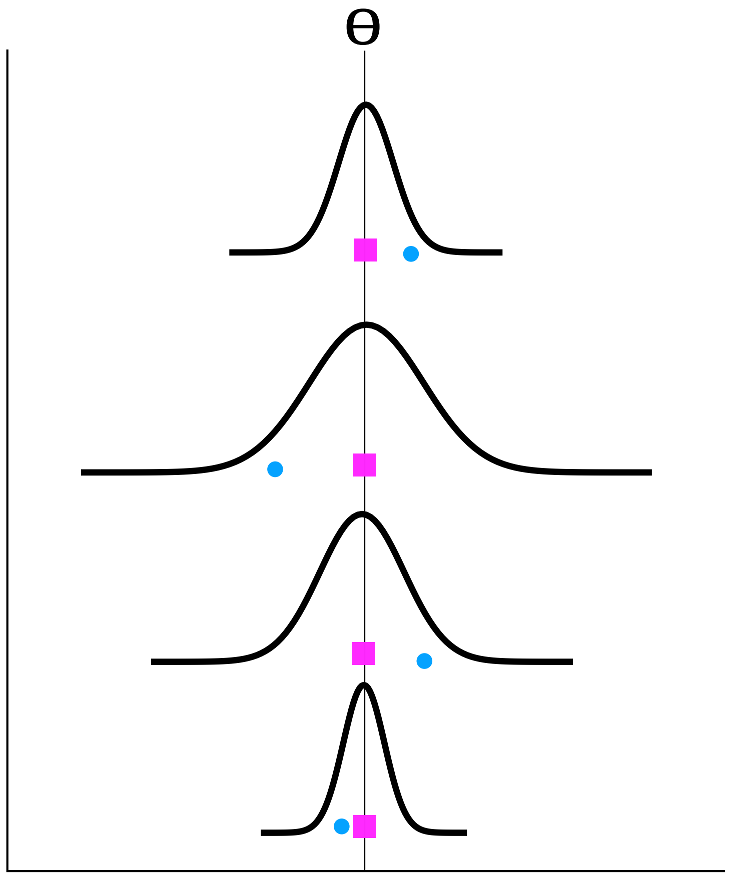
- Defined our research question
- Found relevant literature
- Gathered important data
- For each study:
    - Outcome measure
    - Measure uncertainty (i.e., Standard Error)

Meta-analysis models

Meta-analysis models

- The basic meta-analytic model can be considered a weighted average where all information is combined giving **more weight** to **more precise** studies
- From a linear regression point of view, is the simplest model (aka null model) where only the (weighted) mean is estimated
- Two main meta-analytic models:
  - Fixed-effect or Equal effect model
  - Random-effect model

Equal-effect model



Equal-effect model

Model:

$$y_i = \theta + \epsilon_i$$

$$\epsilon_i \sim Normal(0, v_i)$$

$$\hat{\theta} = \frac{\sum w_i y_i}{\sum w_i}$$

$$w_i = \frac{1}{v_i}$$

- Weighted average where the weight is the inverse of the study precision (i.e., inverse variance weight)
- More precision = more weight

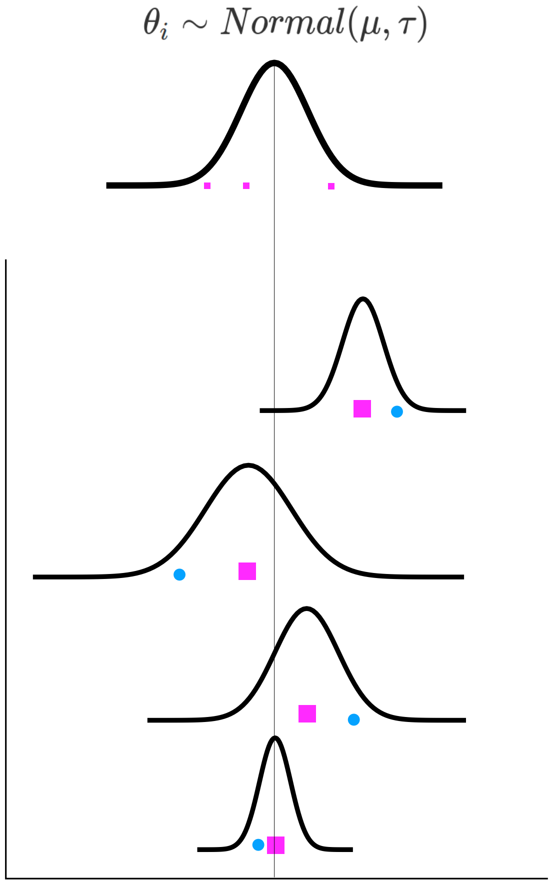
Equal-effect model

The statistical assumption: the **true** (latent) effect is fixed and observed variability is due to different study precision

- Only 1 parameter to estimate and the associated standard error
- No real between-study variability
- Each study is considered like a replication of the same effect
- Meta-regression is not considered --> no variability to explain

Often (in psychology) not appropriate!!

Random-effect Model



Random-effect Model

Model:

$$y_i = \mu + \theta_i + \epsilon_i$$

$$\theta_i \sim Normal(0, \tau)$$

$$\hat{\mu} = \frac{\sum w_i y_i}{\sum w_i}$$

$$w_i = \frac{1}{v_i + \hat{\tau}}$$

- Weighted average where the weight is the inverse of the study precision (i.e., inverse variance weight) and the between-study heterogeneity
- More precision = more weight BUT extreme studies (very low/high precison) are smoothed in terms of final weight

Equal-effect model

The statistical assumption: the **true** (latent) effect is fixed and observed variability is due to different study precision

- Given that we are estimating a distribution of effect and not a single value:
  - Estimation of μ
  - Estimation of τ
- τ is the between-study heterogeneity that can be explained using moderators

Fixed vs Random-effect model

Important!

These two models are not estimating the same quantity from both the statistical and theoretical point of view:

- **Fixed-effect model**: We are estimating the **true underlying effect**
- **Random-effect model**: We are estimating the **true average effect** along the effect variability. The same effect, under specific conditions could be higher or lower/absent

Practical Example

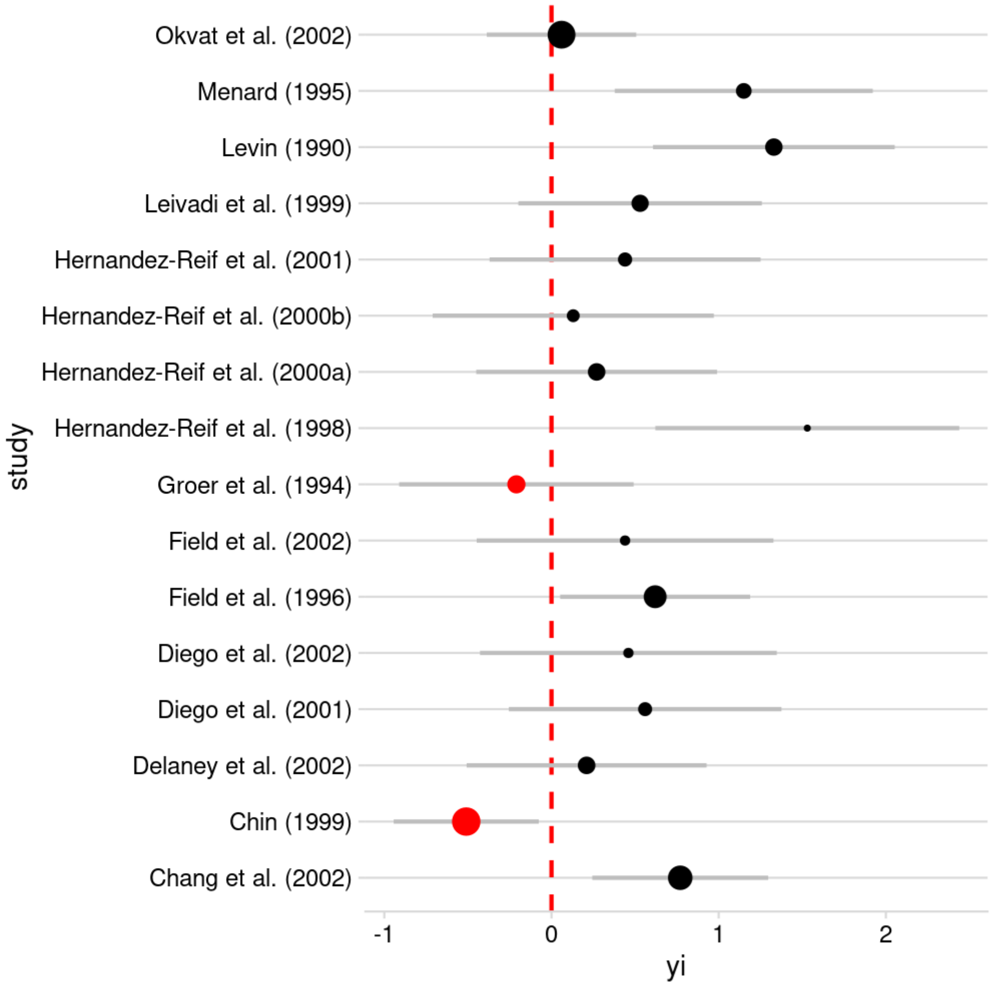
Analysis Steps

- Effect size computation
- Model fitting
    - Model diagnostic
    - Parameters Intepretation
    - Forest Plot
- Publication Bias
- Meta-regression

Data

study	minutes	yi	vi
Chang et al. (2002)	30	0.77	0.072
Chin (1999)	10	-0.51	0.049
Delaney et al. (2002)	20	0.21	0.134
Diego et al. (2002)	40	0.46	0.205
Diego et al. (2001)	20	0.56	0.173
Field et al. (2002)	30	0.44	0.205
Field et al. (1996)	15	0.62	0.084
Groer et al. (1994)	10	-0.21	0.128
Hernandez-Reif et al. (1998)	45	1.53	0.215
Hernandez-Reif et al. (2001)	30	0.44	0.171

Data


Okvat et al. (2002)
Menard (1995)
Levin (1990)
Leivadi et al. (1999)
Hernandez-Reif et al. (2001)
Hernandez-Reif et al. (2000b)
Hernandez-Reif et al. (2000a)
Hernandez-Reif et al. (1998)
Groer et al. (1994)
Field et al. (2002)
Field et al. (1996)
Diego et al. (2002)
Diego et al. (2001)
Delaney et al. (2002)
Chin (1999)
Chang et al. (2002)
study
-1
0
1
2
yi

Model - Metafor

Fixed Effect Model

```r
fit.fe <- rma(yi, vi, method = "FE", data = massage)
```

Random Effect Model

```r
fit.re <- rma(yi, vi, method = "REML", data = massage)
```

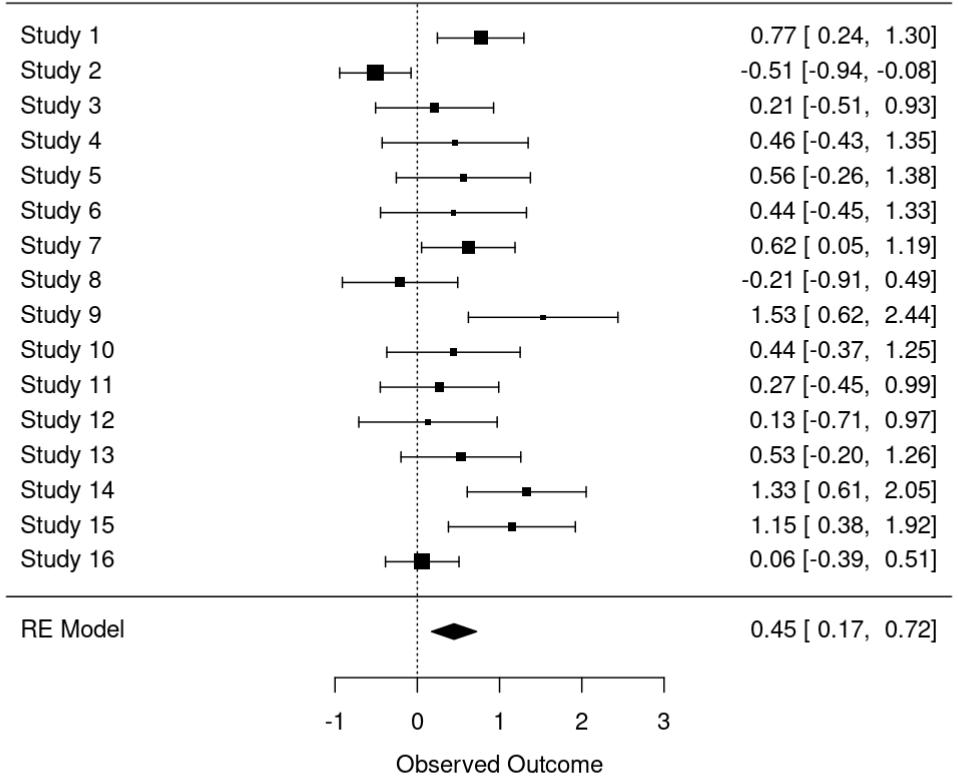
Fixed Effect

```
##
## Fixed-Effects Model (k = 16)
##
## I^2 (total heterogeneity / total variability):   63.62%
## H^2 (total variability / sampling variability):  2.75
##
## Test for Heterogeneity:
## Q(df = 15) = 41.2360, p-val = 0.0003
##
## Model Results:
##
## estimate      se    zval    pval   ci.lb   ci.ub
##   0.3542  0.0843  4.2002  <.0001  0.1889  0.5194  ***
##
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Random Effect

```
##
## Random-Effects Model (k = 16; tau^2 estimator: REML)
##
## tau^2 (estimated amount of total heterogeneity): 0.1813 (SE = 0.1126)
## tau (square root of estimated tau^2 value):      0.4258
## I^2 (total heterogeneity / total variability):   60.96%
## H^2 (total variability / sampling variability):  2.56
##
## Test for Heterogeneity:
## Q(df = 15) = 41.2360, p-val = 0.0003
##
## Model Results:
##
## estimate      se    zval    pval   ci.lb   ci.ub
##   0.4468  0.1397  3.1992  0.0014  0.1731  0.7205  **
##
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Forest Plot



Study 1
Study 2
Study 3
Study 4
Study 5
Study 6
Study 7
Study 8
Study 9
Study 10
Study 11
Study 12
Study 13
Study 14
Study 15
Study 16
RE Model
0.77 [0.24, 1.30]
-0.51 [-0.94, -0.08]
0.21 [-0.51, 0.93]
0.46 [-0.43, 1.35]
0.56 [-0.26, 1.38]
0.44 [-0.45, 1.33]
0.62 [0.05, 1.19]
-0.21 [-0.91, 0.49]
1.53 [0.62, 2.44]
0.44 [-0.37, 1.25]
0.27 [-0.45, 0.99]
0.13 [-0.71, 0.97]
0.53 [-0.20, 1.26]
1.33 [0.61, 2.05]
1.15 [0.38, 1.92]
0.06 [-0.39, 0.51]
0.45 [0.17, 0.72]
-1
0
1
2
3
Observed Outcome

Publication Bias

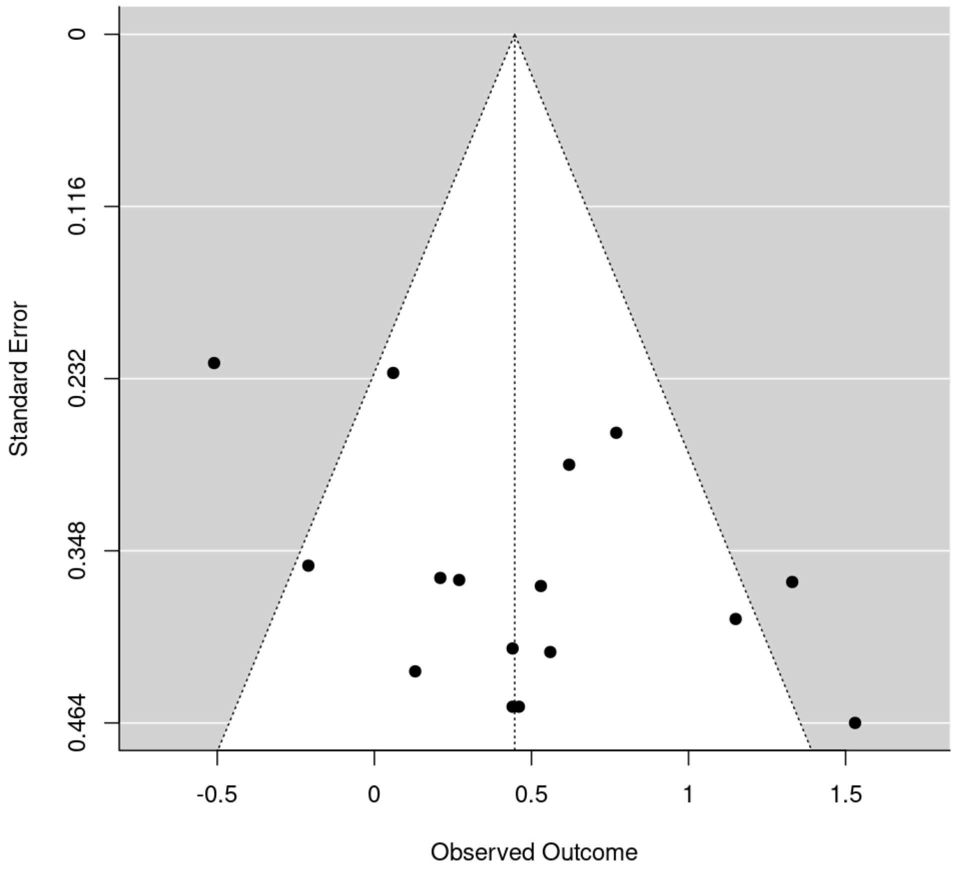
Published articles are only a (biased) subset of the research conducted on a particular phenomenon

- Only "significant" results are published
- Replication studies are not catchy

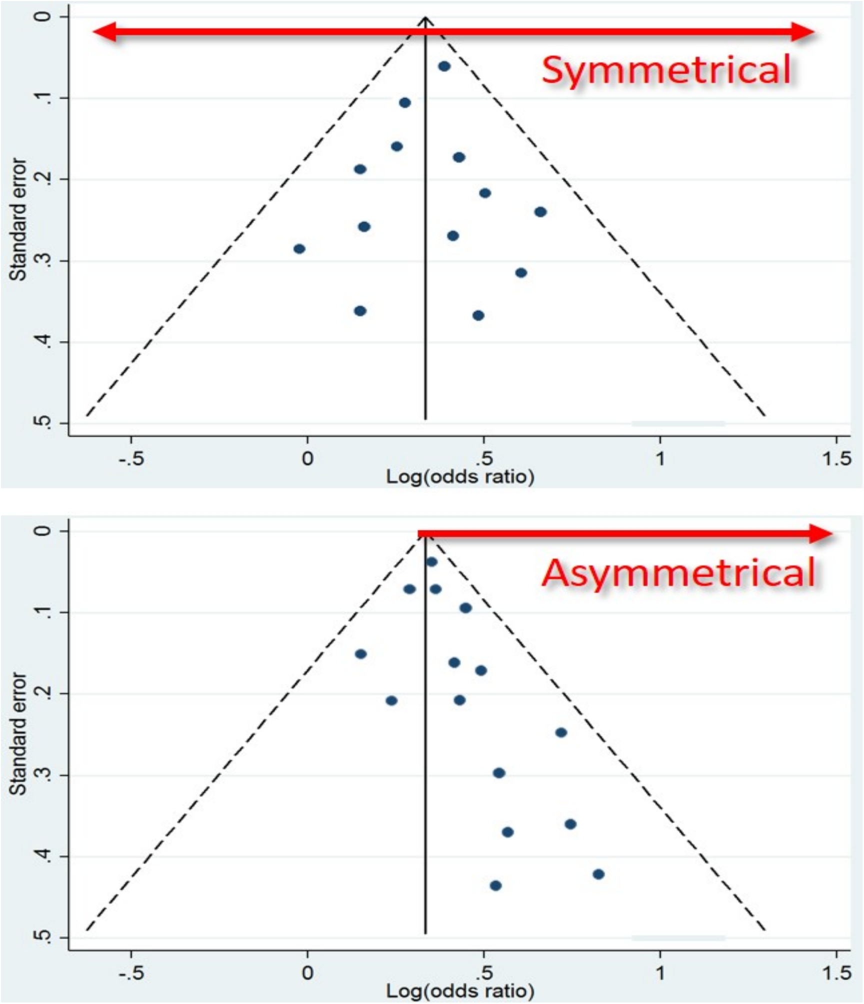
Approaches to Publication Bias

- Graphical evaluation
- Test for the presence of publication bias
- Effect size correction

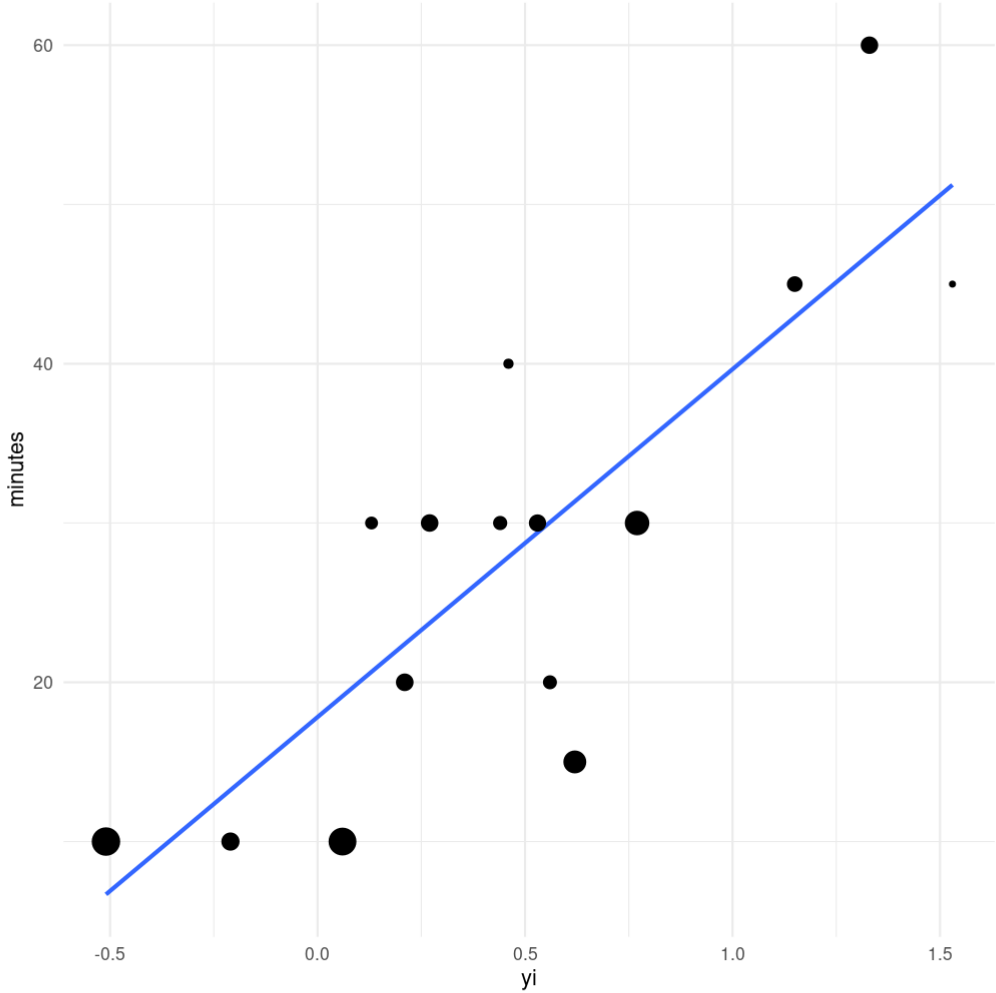
Publication Bias - Funnel Plot



Publication Bias - Funnel Plot



Meta-regression



Meta-regression

The impact of **minutes** on the estimated effect size:

```
fit.meta <- rma(yi, vi, method = "REML", data = massage,
                mods = ~minutes)
```

Meta-regression

```
##
## Mixed-Effects Model (k = 16; tau^2 estimator: REML)
##
## tau^2 (estimated amount of residual heterogeneity):     0.0187 (SE = 0.0493)
## tau (square root of estimated tau^2 value):             0.1366
## I^2 (residual heterogeneity / unaccounted variability): 13.63%
## H^2 (unaccounted variability / sampling variability):   1.16
## R^2 (amount of heterogeneity accounted for):            89.71%
##
## Test for Residual Heterogeneity:
## QE(df = 14) = 13.0270, p-val = 0.5244
##
## Test of Moderators (coefficient 2):
## QM(df = 1) = 22.9524, p-val < .0001
##
## Model Results:
##
##          estimate      se     zval
## intrcpt   -0.4028  0.1881  -2.1415
## minutes    0.0314  0.0066   4.7909
##             pval    ci.lb    ci.ub
## intrcpt  0.0322  -0.7715  -0.0341    *
## minutes  <.0001   0.0186   0.0443  ***
```

Final considerations

Final considerations

- We know the average effect of interest
- We know which moderators explain our effect variance
- We know which effect we should expect for a future study (i.e., power analysis)
- We know the literature structure about our phenomenon:
  - missing studies?
  - future studies?

What else?

What else?

- Bayesian meta-analysis
- More complex models (multilevel, multivariate)
- Effect size calculation from computed statistics, p-values, etc.
- Diagnostic: outliers, influential points
- Power of meta-analysis

Some Materials

Some Materials

- notion.so/filippogambarota/Meta-analysis
- Metafor Website
- Wolfgang Viechtbauer Course
- Doing Meta-analysis in R

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Slides made with the Xaringan package by Yihui Xie