Theoretical Foundations of a Risk-Aware FX Carry Strategy

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Abstract

This document develops the theoretical underpinnings of a professional FX carry strategy with risk-aware overlays. We cover carry premia intuition, the role of (un)covered interest parity, portfolio construction via cross-sectional ranking and L1 budgeting, USD beta neutralization, EWMA volatility targeting, momentum and macro overlays, central-bank event controls, an ex-ante FX risk index with hysteresis-based throttling, execution costs, and performance evaluation (Sharpe, volatility, drawdown, event studies).

Contents

1 Carry Premia in FX

1.1 Definitions and Parity Relations

Let S_t be the spot price in USD per unit of foreign currency (XXXUSD convention), and let i_t^{USD} and i_t^{FX} be short rates (e.g., 3M money-market proxies) for USD and the foreign currency. The *covered interest parity* (CIP) implies the forward rate

$$F_t \approx S_t \frac{1 + i_t^{\mathrm{USD}} \Delta}{1 + i_t^{\mathrm{FX}} \Delta},$$

for accrual Δ (year fraction). The *uncovered interest parity* (UIP) posits

$$\mathbb{E}_t \left[\frac{S_{t+\Delta}}{S_t} \right] \approx \frac{1 + i_t^{\mathrm{USD}} \Delta}{1 + i_t^{\mathrm{FX}} \Delta}.$$

Empirically, UIP fails on average: high-yield currencies tend not to depreciate sufficiently to offset their rate advantage. The residual is interpreted as a *carry risk premium*.

1.2 Why Carry May Pay

Several complementary explanations exist:

- **Risk compensation:** carry trades load on global risk, funding liquidity, and crash risk; premia compensate for bearing these risks.
- **Intermediary constraints:** balance-sheet limits and margin constraints lead to time-varying arbitrage capacity; premia rise when risk capital is scarce.
- **Behavioral & flow effects:** slow-moving capital and liability-hedging flows can support persistent differentials.

2 Signal Construction and Cross-Sectional Weights

2.1 Carry Differential

We define daily carry signal for currency i as the interest differential vs USD:

$$\mathrm{Carry}_i(t) \equiv i_i^{\mathrm{FX}}(t) - i^{\mathrm{USD}}(t).$$

After aligning series on a business-daily calendar and forward-filling, we obtain a panel $\mathrm{Carry}(t) \in \mathbb{R}^{1 \times N}$ (across N currencies, excluding USD).

2.2 Ranking to Scores and L1 Budgeting

To obtain stable, scale-free cross-sectional weights from carry:

$$\begin{aligned} \mathrm{pct}_i(t) &= \mathrm{rank}_i\big(\mathrm{Carry}(t)\big)/(N+1), && (1) \\ s_i(t) &= 2\,\mathrm{pct}_i(t) - 1 \in [-1, 1], && (2) \\ \tilde{w}_i(t) &= \frac{s_i(t)}{\sum_j |s_j(t)|}, \qquad w_i(t) = \mathrm{clip}\big(\tilde{w}_i(t), -w_{\max}, +w_{\max}\big), && (3) \\ \text{then } w(t) &\leftarrow \frac{w(t)}{\sum_j |w_j(t)|} \quad \text{to enforce } \sum_j |w_j(t)| = 1. && (4) \end{aligned}$$

Equation (4) imposes an L1 budget (*risk budget*) and a per-asset cap $w_{\max}$, controlling concentration and tail exposure.

2.3 Top/Bottom Selection (3/3 Basket)

For a more interpretable portfolio, we can keep the top-K and bottom-K names by carry (e.g., $K = 3$) and re-budget within the L1 constraint. Optionally, emphasize relative strength via z-scores and a nonlinearity:

$$z_i(t) = \frac{\text{Carry}_i(t) - \bar{\text{Carry}}(t)}{\sigma_{\text{Carry}}(t)}, \qquad \text{weight strength} \propto |z_i(t)|^\alpha, \ \alpha > 1.$$

3 USD Beta Neutralization

3.1 Motivation

FX baskets can inherit *USD beta* through systematic dollar moves. To reduce unwanted USD exposure, we orthogonalize the weight vector to a dollar factor while preserving same-day L1.

3.2 Rolling Beta Estimation and Soft Orthogonalization

Let r_i be daily FX returns in XXXUSD convention and $f_t = \frac{1}{N}\sum_i r_i(t)$ the cross-sectional mean as a USD factor proxy. Using a rolling window of length L,

$$\beta_i(t) = \frac{\text{Cov}(r_i, f)_t}{\text{Var}(f)_t + \varepsilon}, \qquad \boldsymbol{\beta}(t) \in \mathbb{R}^N.$$

Given target weights $\boldsymbol{w}(t)$, we remove a fraction $\kappa \in [0, 1]$ of USD beta:

$$\alpha(t) = \frac{\boldsymbol{w}(t)^\top \boldsymbol{\beta}(t)}{\|\boldsymbol{\beta}(t)\|_2^2 + \varepsilon}, \quad \boldsymbol{w}_\perp(t) = \boldsymbol{w}(t) - \kappa\, \alpha(t)\, \boldsymbol{\beta}(t).$$

We then clip per-asset to $w_{\max}$ and re-pack to the same-day L1 to maintain implementability.

4 EWMA Volatility Targeting

4.1 RiskMetrics-Style Estimation

Let $r_p(t)$ be daily portfolio returns from pre-targeting weights. The EWMA volatility is

$$\sigma^2_{\text{EWMA}}(t) = \lambda\, \sigma^2_{\text{EWMA}}(t-1) + (1-\lambda)\, r_p(t)^2,$$

with decay $\lambda \in (0, 1)$ (e.g., 0.94). Annualized vol is $\sigma_{\text{ann}}(t) = \sqrt{252}\, \sigma_{\text{EWMA}}(t)$.

4.2 Scaling to Target

For a target $\sigma^\star$ and leverage cap $L_{\max}$,

$$\text{scale}(t) = \min\left(L_{\max},\ \frac{\sigma^\star}{\sigma_{\text{ann}}(t) + \epsilon}\right), \qquad \boldsymbol{w}^{(\text{vt})}(t) = \text{scale}(t) \cdot \boldsymbol{w}_\perp(t).$$

This stabilizes ex-ante risk and mitigates volatility clustering.

5 Overlays: Momentum, Macro, and Central Banks

5.1 Momentum Filter

Define $M_i(t)$ as m-day price momentum, e.g.,

$$M_i(t) = \prod_{s=t-m+1}^{t} (1 + r_i(s)) - 1.$$

To avoid "catching falling knives", down-weight names with $M_i(t) < 0$ via a halving factor $h \in (0, 1)$ (optionally stronger in recent regimes). Re-budget to preserve L1.

5.2 Macro Tilt (CLI/PMI)

Let $z_i^{\text{macro}}(t)$ be standardized macro signal (e.g., OECD CLI z-score). Apply a mild linear tilt:

$$\boldsymbol{w}^{(\text{macro})}(t) = \boldsymbol{w}^{(\text{vt})}(t) \odot \big(1 + \lambda\, \boldsymbol{z}^{\text{macro}}(t)\big),$$

clip per-asset, and re-pack to L1.

5.3 Central-Bank Windows

Define a boolean flag around policy meetings (e.g., $\pm N$ business days) and shrink exposure by a stop-out scale $\gamma \in (0, 1)$ during flagged dates. Re-pack L1 post-shrink.

6 FX Risk Index and Hysteresis Throttling

6.1 Constructing a Synthetic FX Volatility Index

Compute per-currency EWMA vols $\sigma_i(t)$ from daily returns, aggregate across the cross-section (e.g., median or mean), and optionally blend with an external currency volatility index:

$$\text{FXVol}(t) = \text{median}_i\big(\sqrt{252}\,\sigma_i(t)\big), \qquad \text{RiskIdx}(t) = \eta \cdot \text{FXVol}(t) + (1 - \eta) \cdot \text{ExtIdx}(t),$$

with blend $\eta \in [0, 1]$.

6.2 Rolling Quantile Thresholds and Hysteresis

Let $Q_p(t)$ be the rolling p-quantile of RiskIdx over a lookback L (e.g., $p = 95\%$ and 97.5%). We define states with hysteresis counters (ON/OFF days) and map state to a target L1 shrink:

$$\text{TargetL1}(t) = \begin{cases} s_{\text{ext}} & \text{if RiskIdx}(t) > Q_{97.5\%}(t) \text{ (after ON hysteresis)} \\ s_{\text{main}} & \text{else if RiskIdx}(t) > Q_{95\%}(t) \text{ (after ON hysteresis)} \\ 1.0 & \text{else (after OFF hysteresis)} \end{cases}$$

with $0 < s_{\text{ext}} < s_{\text{main}} \leq 1$. Daily weights are scaled to respect the time-varying L1 cap without exceeding per-asset limits.

7 Execution, Turnover, and Costs

7.1 Operational Clock and Drift Bands

Trade only on a chosen clock (e.g., weekly Friday). Between rebalances, stick to prior weights unless *deviation* exceeds a drift band δ and minimum holding days H elapsed:

$$\max_i \left| w_i^{\text{target}}(t) - w_i^{\text{held}}(t-1) \right| > \delta \quad \wedge \quad \text{days since last trade} \geq H.$$

Use a minimum trade size (L1) to ignore micro-churn and optional weight quantization.

7.2 Transaction Costs and Returns

Daily portfolio return combines price and carry legs:

$$r_p(t) = \sum_i w_i^{\text{exec}}(t) \left(r_i^{\text{px}}(t) + \frac{\text{Carry}_i(t)}{\text{PPY}} \right) - \text{TC} \cdot \text{Turnover}(t),$$

where turnover is $0.5 \sum_i |w_i^{\text{exec}}(t) - w_i^{\text{exec}}(t-1)|$, and TC is cost per unit notional.

8 Performance Metrics and Event Studies

8.1 Sharpe, Volatility, and Drawdown

For daily net returns $r(t)$:

$$\text{SR} = \frac{\mathbb{E}[r]}{\sqrt{\text{Var}(r)}} \sqrt{252}, \qquad \sigma_{\text{ann}} = \sqrt{\text{Var}(r)}\,\sqrt{252}.$$

Let $\text{NAV}(t) = \prod_{s \leq t}(1 + r(s))$. The drawdown is

$$\text{DD}(t) = \frac{\text{NAV}(t)}{\max_{s \leq t} \text{NAV}(s)} - 1, \qquad \text{MDD} = \min_t \text{DD}(t).$$

8.2 Event Study Methodology

Given an anchor date τ and window W business days, rebase NAVs to 1 at $t = \tau$ and compare $\{\text{Pure}, \text{Hedged}\}$ trajectories over $[\tau - W, \tau + W]$. Report window total return and worst drawdown. This highlights crisis resilience.

9 Robustness and Overfitting Control

9.1 Parameter Grids and Sub-Periods

Evaluate stability across:

- Lookbacks (L), thresholds (p), momentum windows, overlay intensities.
- Sub-periods: Pre-GFC, GFC, Post-GFC, COVID, Recent.
- Cost stress (e.g., double TC), lag stress (T+1 vs T+2).

Prefer simple rules that generalize; avoid tailoring to a single event.

9.2 Data Hygiene

- Align series on a business-daily calendar; forward-fill macro rates and monthly indicators.
- Resample prices for weekly clocks using last available close; no use of future information.
- Lag weights by at least one business day (signal at t applies from $t + 1$).

10 Limitations and Practical Considerations

10.1 Model Risks

- Regime shifts (pegs, negative rates, capital controls).
- Liquidity drying up in crises (slippage > assumed TC).
- Over-shrinking in protracted high-vol regimes (opportunity cost).

10.2 Capacity and Implementation

- G10 FX is deep; capacity mainly limited by desired turnover and gross leverage.
- Prefer forwards/futures for operational simplicity and funding treatment.
- Continuous monitoring of realized trading costs vs backtest assumptions.

11 Why Hedged Typically Outperforms Pure on a Risk-Adjusted Basis

- **Drawdown control:** volatility targeting and L1 throttling reduce tail losses.
- **Crisis adaptivity:** risk index with hysteresis activates only in persistent stress, avoiding whipsaw.
- **Cleaner exposures:** USD beta soft-neutralization removes unintended macro dollar bets.
- **Turnover discipline:** bands, minimum trade size, and quantization reduce cost drag.

The net effect is a smoother NAV path and higher realized SR despite occasional underperformance in violent carry rallies.

A Notation Summary

Symbol	Meaning
S_t	Spot (USD per unit of foreign currency)
$i^{\text{USD}}, i^{\text{FX}}$	Short rates (USD and foreign)
Carry_i	Interest differential vs USD
r_i^{px}	Price return of currency i (XXXUSD)
$\boldsymbol{w}$	Cross-sectional weight vector (L1 budgeted)
$w_{\max}$	Per-asset cap
PPY	Periods per year (e.g., 252 daily)
λ	EWMA decay
σ_{ann}	Annualized volatility
κ	USD beta neutralization strength
$s_{\text{main}}, s_{\text{ext}}$	L1 throttles for stress states
TC	Transaction cost per unit notional

B EWMA and Annualization Details

With daily returns r_t, the EWMA variance satisfies $\sigma_t^2 = \lambda\sigma_{t-1}^2 + (1-\lambda)r_t^2$. The unconditional mean of σ_t^2 converges to $\text{Var}(r)$ if r_t is stationary. Annualization by $\sqrt{252}$ assumes i.i.d. daily increments; in practice volatility clustering makes this an approximation.

C Rolling Beta Estimation Stability

To improve numerical stability:

- Use ridge regularization ε in denominators $(\text{Var}(f) + \varepsilon)$.
- Enforce minimum window size (e.g., 60 days) before using estimates.
- Replace infinities and NaNs by 0 and re-pack L1 to preserve implementability.