A Bayesian Non-Parametric Framework for Modelling Implied Volatility

BEMACS Thesis Fair

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Motivation & Research Gap

Why this matters

- Options require an accurate implied-volatility (IV) surface across strike & maturity.
- **Black–Scholes**: constant vol; **Heston / SVI**: rigid parametric shapes.
- Outcome: smile/skew mispricing $\Rightarrow$ weaker pricing/hedging and risk metrics.

Goal (one-liner)

- Learn the IV surface directly from data with uncertainty bands, then plug into standard pricing.

Method

Two-stage, interpretable, fast

- Stage 1 — BNP clustering on $\log(\mathrm{IV})$: Dirichlet-Process Gaussian mixture discovers ∼3–5 *regimes* (levels + spreads).
- Stage 2 — Local tilt inside each regime: OLS on {Moneyness, DTE} to bend the surface ⇒ piecewise-affine IV surface.
- Output: point forecasts & 90% predictive bands for any (moneyness, DTE).

Why it's better

- BNP flexibility with interpretability and speed (no heavy black-box).

Method: BNP baseline

Goal: learn the baseline structure of $\log(\mathrm{IV})$ directly from data.

- For each option quote i: $\log \mathrm{IV}_i$ is drawn from a Dirichlet–Process Gaussian Mixture

$$\log \mathrm{IV}_i \mid c_i = k \sim \mathcal{N}(\mu_k, \sigma_k^2), \qquad G \sim \mathrm{DP}(\alpha, H)$$

- The DP prior lets the data decide the number of **volatility regimes** (typically 3–5).
- Each regime has its own mean level μ_k and spread σ_k^2.
- After MCMC we obtain a **consensus clustering** and take the regime's **median level** m_k as its baseline.

Key point: flexible piece-wise constant "surface skeleton" that captures level-shifts (ATM vs OTM, short vs long maturities).

Method: Local linear tilt & prediction

Step 2: refine each regime with a simple linear slope.

For option i in regime k:

$$\log \mathrm{IV}_i = m_k \; + \; \beta_{1,k}\,\textsf{Moneyness}_i \; + \; \beta_{2,k}\,\textsf{DTE}_i \; + \; \varepsilon_i, \qquad \varepsilon_i \sim \mathcal{N}(0, \tau_k^2)$$

- m_k acts as the **cluster-specific intercept** (baseline level from Stage 1).
- $\beta_{1,k}, \beta_{2,k}$ give the **local linear tilt** inside each regime.
- Prediction for a new quote uses the **most likely regime** $k^\star$ and its coefficients.
- **Uncertainty bands** come from posterior predictive draws; for the talk we show the central 90 % interval.

Result: a **piece-wise affine IV surface with quantified uncertainty**, simple enough to update live.

Data & Setup

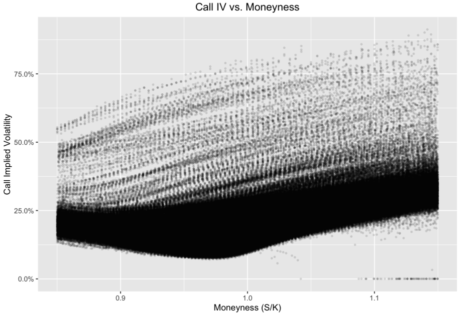

Fig. 1 — SPY Calls: Implied-Volatility vs Moneyness, showing the characteristic smile across 2020–22.

Dataset highlights

- **Underlying:** SPY options (proxy for S&P 500)
- **Period:** Jan 2020 – Dec 2022 (COVID shock $\rightarrow$ recovery $\rightarrow$ 2022 turbulence)
- **Size:** over **500 k quotes** after cleaning

Filtering for modelling

- Moneyness window: $0.85 \leq m \leq 1.15$
- Time-to-expiry: 20 – 60 calendar days

Validation protocol

- Rolling CV: train on past 50 trading days $\rightarrow$ test on following month

Key Results vs Benchmarks

Predictive performance (OOS)

- BNP achieves **NRMSE** ≈ 0.14 **(Calls),** ≈ 0.19 **(Puts)**.
- **SVI benchmark:** NRMSE ≈ 0.23 – less accurate, especially in wings.
- **Heston (calibrated):** NRMSE ≈ 0.28 – struggles with smile/skew.
- **Coverage of 90% PI:** BNP 88–91% (close to nominal) vs SVI 73%.

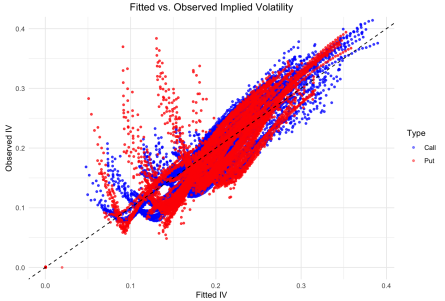

Fig. 2 — Predicted vs Observed IV: BNP close to diagonal, parametric benchmarks show heavier dispersion.

Take-away

- BNP provides better fit in skewed wings and more reliable uncertainty bands than popular parametric surfaces.

BNP outperforms SVI & Heston in accuracy and UQ.

Thank you for your attention!

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