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The footprint of a tennis ball

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The footprint of a tennis ball

Abstract

The Hawk-Eye line calling system in tennis uses a footprint of the ball to determine whether the ball was in or out. However, the footprint itself is not measured. It is the ball trajectory through the air that is measured. Measurements of the footprint are presented in this paper, showing how the high initial rate of expansion of the footprint and other technical difficulties can lead to potential errors in the Hawk-Eye system. The footprint was measured for vertical and oblique impacts and is compared with recent results presented at a tournament where Hawk-Eye was installed.

1 Introduction

A tennis ball impacting obliquely on a tennis court slides for a short distance before bouncing off the court. It can slide for the whole impact duration, or it can slide for a short distance and then grip the court, depending on the incident angle, incident spin and coefficient of friction between the ball and the court [1]. The resulting mark left on the court is sometimes used by players or umpires to decide whether the ball was in or out. If the ball was well in or well out, then there is no need for a player or umpire to check the mark. If the ball lands close to a line then it is common to see players inspect the mark closely, even in major tournaments. The mark itself may not fully delineate the actual contact region between the ball and the court, since light contact near the edge of the ball may not leave any visible mark. The mark itself is more clearly visible on clay courts, less so on hard courts and may not be visible at all on a grass court.

At major tennis tournaments, a video-based line calling system known as Hawk-Eye [2, 3] is usually available on the main show courts so that players can challenge a line call made by a lines person or the umpire. The Hawk-Eye line calling system does not rely on the visible mark at all. Instead, an estimate is made of the landing point on the court, based on the measured trajectory of the ball. The footprint itself is not measured with Hawk-Eye so the decision on whether the ball was in or out must be made using an assumed footprint. The method used to estimate the footprint with the Hawk-Eye system has not been published as far as the author is aware, but is potentially subject to significant error.

The ITF (International Tennis Federation) specifies that an approved line-calling system must have an accuracy of 5 mm or better [3]. In this paper, measurements are described of the footprint of a tennis ball incident on a horizontal surface. Consequences for the accuracy of the Hawk-Eye system are also described.

Of particular interest in this paper is the distance between the back edge of the footprint and the initial contact point. That distance depends on the impact speed and the impact angle of the ball. The overall length of the footprint also depends on the spin of the incident ball. The nature of the problem is illustrated in Fig. 1.

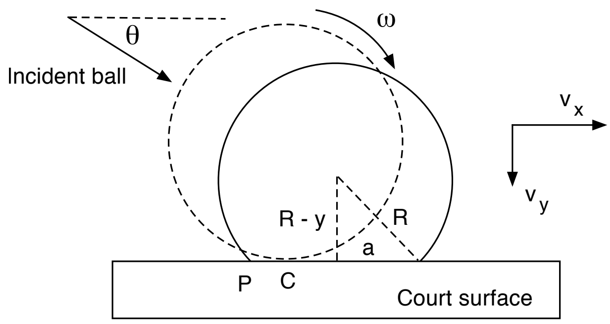

FIG. 1: A tennis ball of radius R incident at angle θ on a court surface, with topspin at angular velocity ω. The dashed circle shows the position of the ball when it first contacts the surface, at point C. The solid circle shows the position of the ball a short time later, compressed vertically by a distance y. The contact region of the ball forms a circle of radius a. Point P denotes the back edge of the ball.

In Fig. 1, the first point of contact of the ball on the surface is the point C, located at distance R directly below the centre of the ball. If the ball is incident at speed v_1, angle θ to the horizontal and with topspin at angular velocity ω, then the contact point will start sliding forward along the surface at speed $v_{x1} = v_1 \cos\theta - R\omega$, while the centre of the ball moves forward at speed $v_x = v_1 \cos\theta$. Simultaneously, the ball will start compressing in the vertical direction, with the result that the contact radius, a, increases with time. In Fig. 1, the back edge of the circular contact region is denoted by point P.

If the ball happens to be incident at right angles to the court with $\omega = 0$, then $v_x = 0$ and point P will move rapidly backwards due to rapid expansion of the contact radius. In that way, a ball that makes first contact at a point outside a line can expand backwards (or sideways) and touch the line. Similarly, if v_x is relatively small, as in a topspin lob, then P will also move backwards while the ball is compressing. More generally, the centre of the ball moves forward in time if $v_x > 0$, but point P moves backwards with respect to the centre of the ball while the ball is compressing. If $da/dt < v_x$, then the contact point P at the rear edge of the ball will slide forward on the surface. If $da/dt > v_x$ then P will slide backwards. Figure 1 shows a situation where $da/dt > v_x$ and hence the point P lies to the left of point C, despite the fact that the ball as a whole moves to the right. If P slides backwards, then the back edge of the footprint will not be coincident with the initial point of contact C.

An approximate estimate of da/dt can be obtained from the geometry of Fig. 1 where $R^2 = (R-y)^2 + a^2$. Then $a = (2Ry - y^2)^{1/2}$ and $da/dt = [(R-y)/a]dy/dt$, indicating that $da/dt = \infty$ when $a = 0$. For small t, while the vertical speed v_y remains approximately constant and while $y \ll R$, it is easy to show that $y = v_y t$, $a = (2Rv_y t)^{1/2}$ and $da/dt = [Rv_y/(2t)]^{1/2}$. During the initial compression phase, a and da/dt are therefore independent of the mass and stiffness of the ball but depend on the ball radius and the vertical speed of the ball. From these expressions we find that da/dt decreases to 33 m/s and a increases to 6.8 mm after 0.1 ms for a 33 mm radius tennis ball incident with $v_y = 7$ m/s. If the ball is incident with $v_x = v_1 \cos\theta = 30$ m/s, then the centre of mass will move forward by 3 mm in the first 0.1 ms, P will move 6.8 mm horizontally backward with respect to the centre of mass, and hence the back edge of the footprint will be 3.8 mm behind the initial contact point. The latter result is typical of a first or second serve.

The time at which point P reaches its maximum backward displacement can be estimated by assuming that v_x and v_y remain constant during the initial stages of ball compression, in which case the x coordinate of P is given by $x_P = v_x t - (2Rv_y t)^{1/2}$. For example, if $v_x = 30$ m/s and $v_y = 7$ m/s then x_P has a minimum value of -3.80 mm at $t = 0.1$ ms. At a larger angle of incidence, x_P is a minimum at a later time. For example, if $v_x = 20$ m/s and $v_y = 7$ m/s then x_P has a minimum value of -5.77 mm at $t = 0.29$ ms.

Given the very high initial rate of increase of the contact area, it is possible that a lines person would not notice any backward movement of the back edge of the ball, especially if the whole ball is moving forward at high speed [4]. The effect may not even be observable with high speed video unless the frame rate is greater than 10,000 frames/s. In that case, a ball could touch the line and still be called out, unless physical contact with the line is detected by some other means [5]. In the case of a topspin lob where v_x is relatively small, the back edge of the footprint could well be 20 mm or so behind the initial contact point. It is clear from these estimates that the initial contact point cannot be used, on its own, to determine accurately whether a ball is in or out. In cases where the ball trajectory is parallel or nearly parallel to a line, the back edge of the footprint is irrelevant. The relevant factor is then the maximum width of the footprint.

Several different experiments are described in the following sections, all related to measurements of the footprint. The first experiment involved measurements of the rate of increase of the contact radius when a ball is incident vertically on a horizontal surface at speeds between 7 m/s and 8 m/s. The second experiment involved measurements of the maximum contact diameter when a ball is incident vertically at speeds from 1 m/s to 11 m/s. The third experiment describes oblique impacts at two different angles of incidence. The results are then compared with recent Hawk-Eye data obtained at a tournament in Brisbane.

2 Vertical bounce experiment

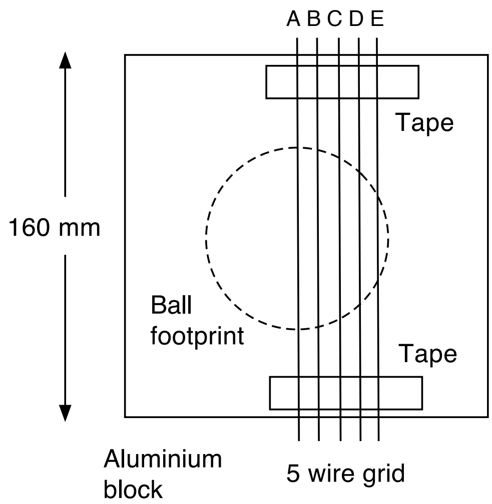

FIG. 2: Arrangement used to measure the speed of the contact edge of the ball. As the ball compresses, each wire is pressed in turn against the aluminium block and closes an additional switch, as indicated by the circuit in Fig. 3.

Measurements of da/dt for a vertical bounce were made using the apparatus shown in Figs. 2 and 3. The apparatus consists of a 20 mm thick, 160 mm square aluminium block with five, 0.6 mm diameter copper wires spaced 5 mm apart and mounted about 1 mm above the block.

A ball incident at right angles on the block pressed each wire firmly onto the block, one at a time, as the contact area expanded from an initial contact point into a circle of increasing radius. Each wire made initial contact with the block at a time that was measured using the voltage divider circuit shown in Fig. 3. The measured voltage, V, was 1.5 V before the ball impacted the first wire, and then decreased in increments of 0.3 V as each additional wire made contact with the block. This technique was used since the author did not have access to a video camera capable of recording at 10,000 frames/s. Even if access was available, an accurate determination of the initial contact time would be difficult visually, given the soft and furry surface of a tennis ball. In fact, the technique used had much better time resolution, limited only by the 1 MHz sampling rate of the data acquisition system. Time resolution better than 0.01 ms was essential in order to measure high expansion speeds over a distance of only 5 mm. Even at 10,000 frames/s, the time resolution of a video camera is at best 0.1 ms. The spatial resolution was worse, limited by the 5 mm separation between copper wires. Nevertheless, it was possible to estimate the contact radius at any given time to within about 1.0 mm by curve fitting the resulting position vs time data. Up to ten data points were fit in that manner in order to determine the maximum contact radius.

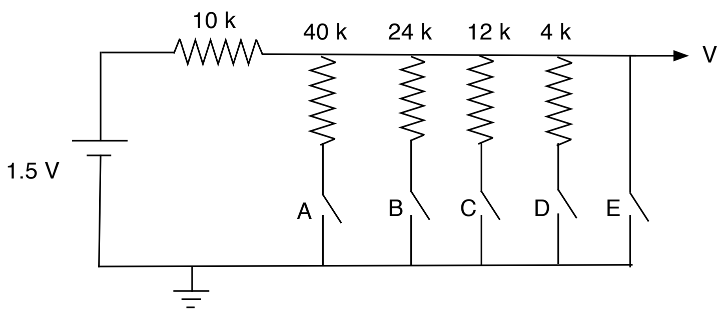

FIG. 3: Circuit used to detect the time at which each of the grid wires in Fig. 2 makes contact with the aluminium block. If all switches are open, $V = 1.5$ volt. When switch A closes, $V = 1.2$ volt. When A and B are both closed, $V = 0.9$ volt. V decreases in increments of 0.3 volt as each additional switch closes.

Measurements were made using two new balls. One was a pressureless (Tretorn TXT) ball of mass 58.1 g and diameter 66.0 mm, and the other was a pressurised (Wilson Australian Open) ball of the same mass and diameter. Both were ITF approved balls. The Tretorn ball was slightly stiffer than the Wilson ball, both in terms of the observed impact duration and the observed compression in a materials testing machine. The incident speed of each ball, and its alignment with respect to the wire grid, were measured by filming the ball at 300 frames/s with a Casio EX-F1 video camera. Each ball was thrown by hand to impact vertically on the wire grid. Many such results were obtained, with balls impacting at various speeds and at various points across the grid. All showed that the contact radius increased very rapidly during the first 0.1 ms, at speeds typically between 40 and 70 m/s.

Typical results are shown in Fig. 4, for cases where (a) the Tretorn ball was incident vertically at 7.1 ± 0.1 m/s and (b) the Wilson ball was incident vertically at 7.7 ± 0.1 m/s. In both cases, the centre of the ball impacted wire A at time $t = 0$. The Tretorn ball then impacted wire B at $t = 0.125$ ms, wire C at $t = 0.334$ ms and wire D at $t = 1.280$ ms. It did not impact wire E. From this data we find that a increased from 0 to 5 mm at an average speed 40 m/s. It then increased from 5 to 10 mm at an average speed of 23.9 m/s, and it increased from 10 to 15 mm at an average speed of 5.3 m/s. The maximum contact diameter was 34 ± 1 mm, and the total impact duration was 4.28 ± 0.01 ms.

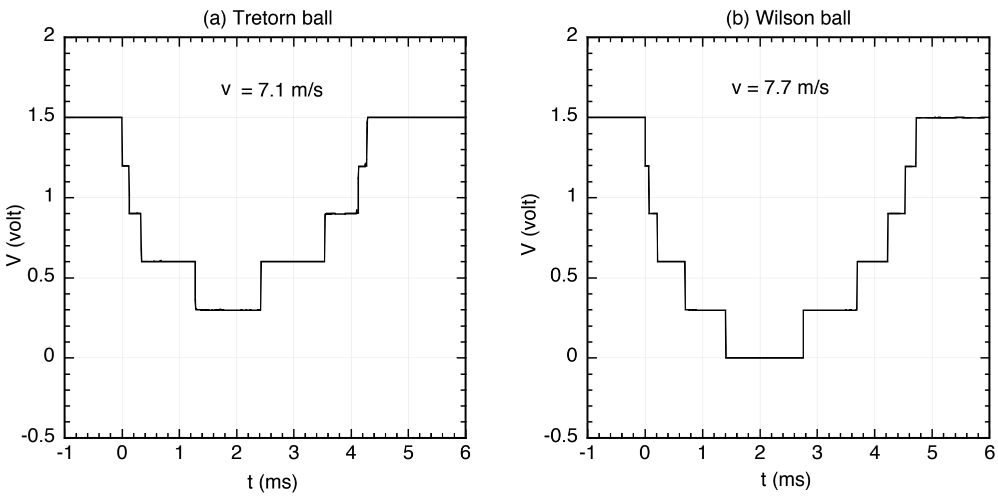

FIG. 4: Results obtained with the wire grid apparatus shown in Figs. 2 and 3, when (a) the Tretorn ball and (b) the Wilson ball were incident vertically on wire A.

The Wilson ball was slightly softer, with a total impact duration of 4.72 ± 0.01 ms, consistent with previous measurements of the impact duration of a tennis ball [6]. It impacted wire B at $t = 0.070$ ms, wire C at $t = 0.218$ ms, wire D at $t = 0.698$ ms and wire E at $t = 1.408$ ms. a increased from 0 to 5 mm at an average speed of 71 m/s. It then increased from 5 to 10 mm at 33.8 m/s, from 10 to 15 mm at 10.4 m/s and from 15 to 20 mm at 7.0 m/s. The maximum contact diameter was 42 ± 1 mm. For both balls, the duration of the compression phase was shorter than the duration of the expansion phase, as can be seen by visual inspection of the results in Fig. 4. For example, if we assume that maximum compression of the Wilson ball occurred at $t = 2.0$ ms, then the expansion phase lasted 2.72 ms.

It can be concluded from these results that the contact radius indeed increases at high speed when the ball first impacts the surface, that the rate decreases with time and that the rate increases when the incident vertical speed increases. The measured expansion rate of the contact area is consistent with the simple impact model shown in Fig. 1.

3. Maximum contact diameter experiment

The maximum contact width of a ball on the court depends primarily on its incident vertical speed. In order to measure the width, a new ball was projected vertically at speeds up to 11 m/s on a sheet of paper taped to a heavy, polished granite slab. The vertical speed of the ball in tennis is typically less than 11 m/s, even for a fast serve. The footprint was made visible by painting the lower half of the ball lightly with acrylic paint. The edge of the footprint was fuzzy, as expected with a cloth cover, but the diameter could still be estimated to within about 1 mm. It was found that the maximum contact width of the ball was not strongly affected by painting the ball, or by varying the angle of incidence, since similar results were obtained with a dry ball impacting on the wire grid or on chalk dust. Chalk dust was used in the oblique impact experiment, as described below, but it was not used for normal impacts since the footprint was then too faint and irregular to obtain a reliable measure of its diameter. The new Tretorn ball and the new Wilson ball were both painted and projected by hand onto the sheet of paper. The incident ball speed was measured to within $\pm 2\%$ by filming at 300 frames/s with the Casio camera.

Results obtained with the two painted balls are shown in Fig. 5, indicating that the footprint diameter increased with ball speed by almost the same amount for each ball, despite the fact that the Wilson ball was slightly softer. When a tennis ball is at rest on the surface, the soft cloth cover compresses slightly under the ball's own weight, with the result that the diameter of the footprint is about 10 - 15 mm rather than zero. At an incident vertical speed of 9.5 ± 0.5 m/s, the diameter of the footprint was 44 ± 1 mm for both balls. A typical footprint mark is shown as an inset in Fig. 5(b).

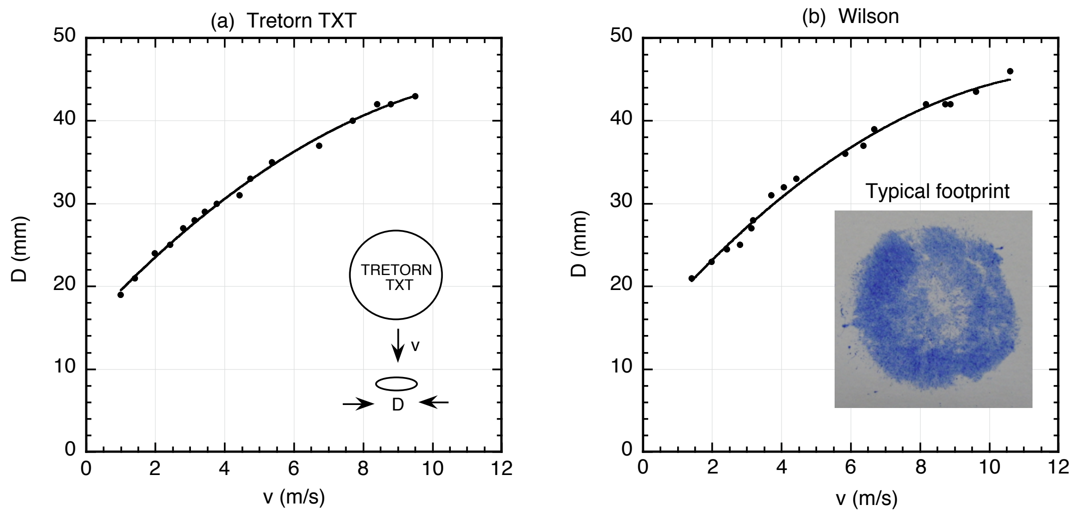

FIG. 5: Measured footprint diameter, D, vs incident ball speed, v for (a) the Tretorn ball and (b) the Wilson ball, both incident normally. The curved lines are quadratic fits to the experimental data. For the Tretorn ball, $D = 15.2{+}4.51v{-}0.17v^2$. For the Wilson ball, $D = 14.2{+}4.89v{-}0.19v^2$.

The results in Fig 5 are directly relevant to the dimensions of the footprint of a ball, and are also relevant in terms of measuring the stiffness of the ball. The quasi-static stiffness of a ball can be easily measured in a materials testing machine, but the dynamic stiffness is more difficult to determine. The standard approach is to measure the displacement of the centre of mass in terms of the measured impact force, and then model that displacement in terms of the compression of the contact region of the ball [6-10]. Alternatively, the dynamic compression can be determined from the results in Fig. 5, using a compression model such as that shown in Fig. 1. The maximum ball compression, y, can be obtained from the relation $y = R - (R^2 - a^2)^{1/2}$ where $a = D/2$.

The one-sided stiffness of a ball for a rapid impact can also be estimated from the mass, m, of the ball and the impact duration, τ [7, 11]. Using the result in Fig. 4(a) as an example, where $\tau = 4.28$ ms, gives $k = m(\pi/\tau)^2 = 31$ kN/m, essentially the same as that obtained in the slow compression test. For the Wilson ball, $\tau = 4.72$ ms, giving an impact stiffness $k = 26$ kN/m, also very similar to the slow compression result.

4 Oblique bounce experiments

Oblique bounce experiments were conducted by filming the bounce of the new Wilson ball at several different speeds and angles of incidence, without spin, and repeating each bounce five or six times at each of the selected speeds and angles. At low ball speeds, the ball was projected by hand. At high speeds, the ball was projected using a Tennis Tutor ball launcher. In order to observe the footprint, the ball was projected onto a small, horizontal blackboard rigidly attached to a table top. The blackboard was uniformly coated with chalk dust to enhance the image of the footprint, using an eraser to spread the dust evenly. The dust stuck to the board sufficiently well not to be blown away by the blast of air from the impacting ball, even at high ball speeds. Photographs of the footprint were recorded with the aid of a bright light to illuminate the white dust in order to increase the contrast between the dust and the blackboard.

The ball was filmed either at 300 frames/s or at 600 frames/s (with the Casio EX-F1 camera) to measure the incident speed and angle, aligning the camera at the same height as the table top in order to view the bottom of the ball. Marked parallel lines on the blackboard were used to calibrate the distance scale, to define the coordinate origin and to measure the location of the footprint. These lines were not clearly visible when filming the bounce of the ball, so a separate calibration was filmed after each shot, without moving the camera, using a vertical grid located in the same plane as the footprint and having the same coordinate origin. The camera was zoomed in to enlarge the ball in order to measure its coordinates as accurately as possible.

A typical footprint of the ball is shown in Fig. 6. The ball was incident at a speed of 22 m/s, without spin, and at an angle of incidence of 17.7° to the horizontal. The footprint has a poorly defined edge, especially at the back edge, especially at low angles of incidence, and especially at high ball speeds. These are precisely the conditions of greatest interest to players in terms of challenging line calls.

Although the footprint is approximately elliptical, the footprint is relatively pointy at the end where the ball bounces off the surface. The thin white band at the pointed end represents an accumulation of chalk dust swept forward by the ball. Of greater interest, in terms of whether a ball lands in or out, are the marks trailing backwards from the dark, back

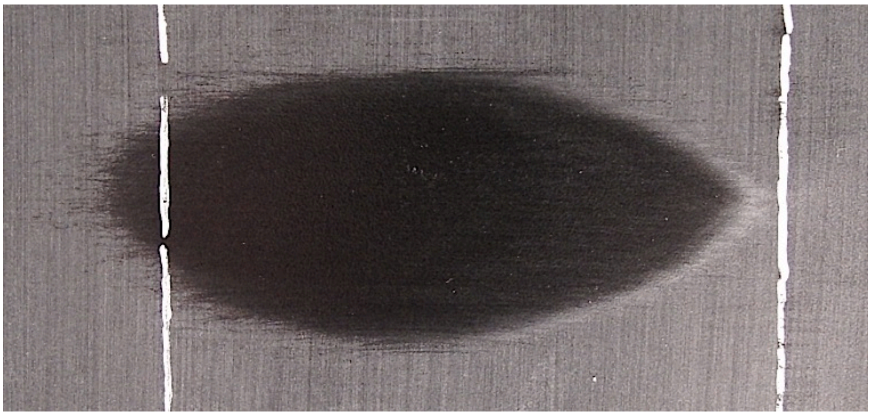

FIG. 6: Footprint of the Wilson tennis ball incident from the left at 22 m/s and at an angle of incidence of 17.7°, showing trailing marks at the left end and a pointed right end. The two white lines on the blackboard are 100 mm apart.

edge of the footprint. The trailing marks are due to brushing of the surface by cloth fibres extending out from the ball. Those fibres contact the surface lightly before the ball begins to compress, and effectively increase the overall length of the footprint by up to about 10 mm. The question as to whether a ball lands in or out therefore depends on whether the trailing marks are considered to be part of the footprint. From a practical point of view, fibres extending beyond the edge of a ball are probably not clearly visible to the eye when a ball is incident at high speed, and are not clearly visible even when filmed with a video camera. They only become obvious when the ball is back-lit by a bright light [12]. For the measurements described below, the back edge of the footprint was therefore taken as the start of the dark region of the footprint, ignoring the trailing brush marks.

For comparison, a much cleaner footprint is shown in Fig. 7. The latter footprint was obtained by projecting a 60 mm diameter, hollow rubber ball onto the blackboard at a speed of 22.7 m/s, without spin and at an angle of incidence of 18.5°. The rubber ball slid for a short distance then gripped the board, bouncing in an overspinning mode (ie with $R\omega > v_x$). Evidence of the grip phase can be seen in the remarkably clear chalk dust pattern. During the grip phase, the ball rolls over the chalk dust rather than sweeping it forward. The ball rolls forward when it grips, despite the fact that the bottom of the ball is flattened as a result of ball compression. The rolling process was observed clearly when filming at 600 frames/s. The clarity of the edge of the footprint in Fig. 7 confirms that the ill-defined edge in Fig. 7 is a real effect caused by fibres extending outwards from the ball surface.

FIG. 7: Footprint of a 60 mm diameter rubber ball incident from the left at 22.7 m/s and an angle of incidence of 18.5°. Unlike the tennis ball, the edge of the footprint is very clear, the left end is pointed (rather than the right end) and the ball gripped the surface during the latter half of the bounce.

The result in Fig. 7 demonstrates an effect described by Maw et al [13]. When a ball grips the surface it does not necessarily grip over the whole contact area. There may be local regions that slip and other regions that simultaneously grip, depending on the magnitude of the friction force and the magnitude of elastic deformation of the ball in each region. As shown in Fig. 7, the central region of the ball grips before the outer edge grips, presumably because the normal reaction force and hence the friction force is relatively small at the outer edge of the contact region. The effect is of incidental interest in this paper, but further studies of the effect could prove to be rewarding. Similarly, the overall shape of the two footprints was not investigated in this paper, but they contain detailed information that could provide useful tests of the validity of ball impact models. As far as the author is aware, the footprint of an obliquely bouncing ball has not previously been published or modelled for any ball type.

Typical footprints of the Wilson ball measured at the same vertical speed but at two different horizontal speeds and angles of incidence are shown in Fig. 8. The coordinates of the bottom of the ball are also shown in Fig. 8(a), at intervals of 1/300 s just before and after impact, in order to compare with the start and end positions of the footprint. In Fig. 8(b) the coordinates are shown at intervals of 1/600 s. In Fig. 8(a), the footprint starts at $x = 0 \pm 1$ mm and ends at $x = 42 \pm 1$ mm. Extrapolation of the ball coordinates to $y = 0$ using a quadratic fit indicates that the bottom of the ball first impacts the surface at $x = 10.0 \pm 0.5$ mm and the last point of contact is at $x = 33 \pm 0.5$ mm.

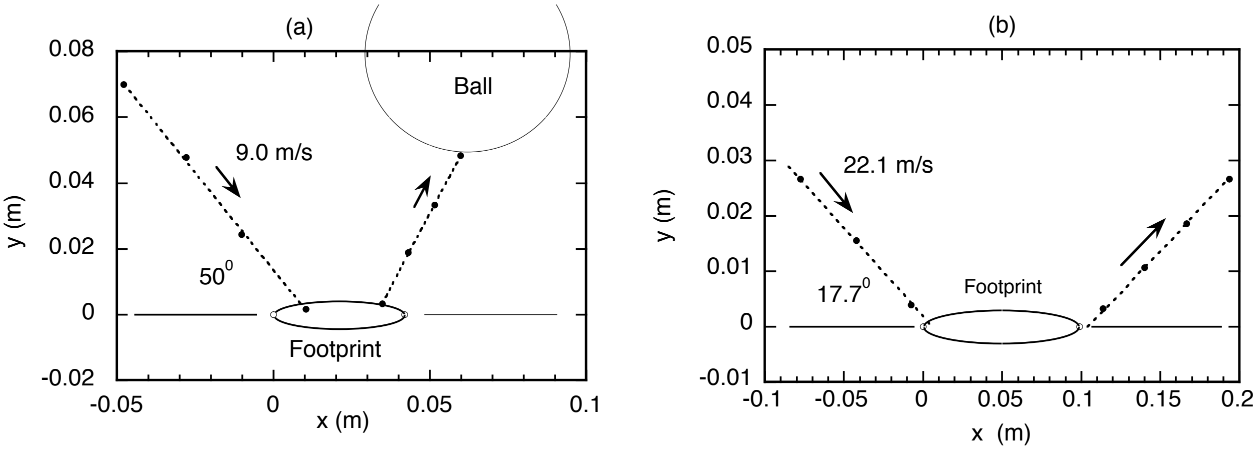

FIG. 8: Measured footprints for the Wilson ball incident at (a) 50° and (b) 17.7° to the horizontal. Black dots show the bottom of the ball at intervals of 1/300 s in (a) and at intervals of 1/600 s in (b). Dashed curves are quadratic fits to the data. Open circles show the start (at $x = 0$) and end of the footprint on the horizontal surface at $y = 0$. In both cases, the ball was incident at a vertical speed of 6.8 m/s.

The result in Fig. 8(b) was obtained at a higher ball speed, at a lower angle of incidence and at an incident vertical speed $v_y = 6.8$ m/s. The footprint starts at $x = 0 \pm 1$ mm. The bottom of the ball, when extrapolated to $y = 0$, intersects the surface at $x = 5.5 \pm 1$ mm. The footprint ends at $x = 99 \pm 1$ mm. Extrapolation of the coordinates of the bottom of the ball indicates that the bottom of the ball departed the surface at $x = 104 \pm 1$ mm, 5 mm beyond the end of the footprint. If that were the case, then the footprint would be expected to end at $x = 104$ mm rather than at 99 mm. The implication of the latter result is that the ball bounced off the surface while it was still compressed and that it regained its spherical shape only after it bounced off the surface. The ball was observed to be spherical 1 ms after the bounce, and could be accurately tracked after that time so the recovery to a spherical shape occurred within 1 ms of the ball leaving the surface. Post-impact recovery has previously been observed when measuring the impact dynamics of bouncing balls [6-10].

The results in Fig. 8 can be compared with the ball compression results shown in Fig. 5(b). In Fig. 8(a), the ball was incident with $v_x = 5.8$ m/s and with $v_y = 6.8$ m/s. The maximum footprint width, from Fig. 5(b), should therefore be 39 ± 1 mm, assuming that a painted ball incident vertically has the same footprint width as a ball landing obliquely on chalk dust. The observed width of the footprint was 40 ± 1 mm. The length of the footprint was 42 mm, while the horizontal displacement of the centre of mass of the ball was 23 mm, giving an average horizontal speed of 4.9 m/s assuming a 4.7 ms impact duration. During the first 0.3 ms, the sliding distance was 1.7 mm (assuming a constant horizontal speed of 5.8 m/s), the vertical compression was 2.0 mm (assuming a constant vertical speed of 6.8 m/s), and hence a increased to 11.3 mm according to the Fig. 1 model calculations. These results indicate that the start of the footprint should be located $11.3 - 1.7 = 9.6$ mm before the initial contact point, approximately as observed. Similar results were obtained for other bounces at about the same speed and angle of incidence, all showing that the back edge of the footprint was typically 8 to 10 mm behind the initial contact point, at least on the chalked blackboard.

In Fig. 8(b), the ball was incident with $v_x = 21.1$ m/s and with $v_y = 6.8$ m/s. From Fig. 5(b), the maximum footprint width can be estimated as 39 ± 1 mm. In fact, the observed width of the footprint was 44 ± 1 mm, averaged over five different bounces, indicating that the width of the footprint was affected either by the high incident speed or by the extension of cloth fibres from the surface of the ball, or both. Painting the ball tended to hold the fibres together and may have prevented them extending outwards when the ball bounced vertically. During the first 0.3 ms, the sliding distance of the ball was 6.4 mm and the footprint expanded in radius to $a = 11.3$ mm according to the Fig. 1 model. In that case, the start of the footprint should be located 4.9 mm before the initial contact point. The footprint was observed to start 5.5 ± 2 mm before the initial contact point. Similar results were obtained with all five bounces, the footprint length being 100 ± 1 mm in all cases, the width being 44 ± 1 mm in all cases and the start of the footprint being 5 ± 2 mm before the initial contact point in all cases.

5 Hawk-Eye footprints

Hawk-Eye footprints recorded from a live broadcast at the Brisbane International tennis tournament on 2nd Jan 2014 are shown in Fig. 9. The results in Fig. 9 are identical to those shown to the television viewing audience. They were obtained by recording the broadcast on a computer as a movie file and then copying part of one frame of the movie file as a jpg image. The length and width of the footprint for any given shot can be deduced from the width of the lines on the court, which are typically 100 mm for the baselines and 50 mm for all other lines. Most disputed line calls involve either a fast first serve or a fast groundstroke. The footprint shown in the Hawk-Eye system is an ellipse that is typically about 126 mm long and about 45 mm wide for a fast first serve. If the ball slides throughout the bounce, then a 126 mm long footprint implies that the average horizontal speed of the ball during a 4.5 ms bounce [6, 14] is about 28 m/s, approximately as expected. The 45 mm width of the footprint is larger than one might expect from the data in Fig. 5, given that the vertical speed of the ball in a fast serve is typically about 7 to 8 m/s, so the expected width is about 40 to 42 mm. However, a wider footprint was observed in Fig. 6 and in Fig. 8(b) for the unpainted ball, probably because cloth fibres were then able to extend further beyond the edge of the ball.

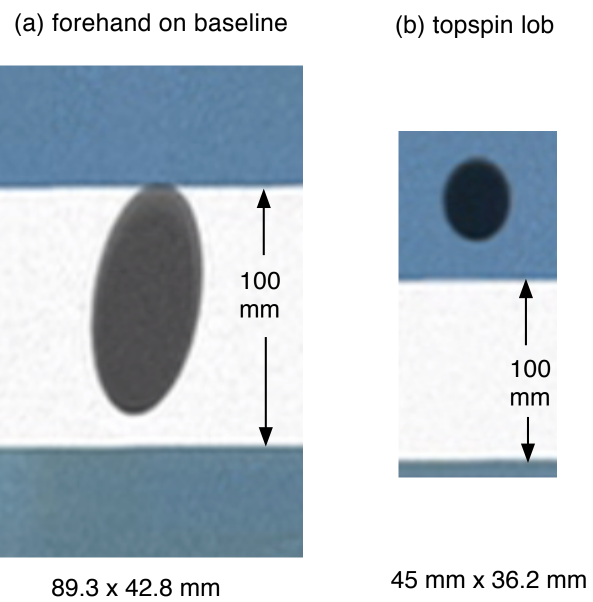

FIG. 9: Hawk-Eye data showing footprints of two shots that landed on and near the 100 mm wide baseline. The length and width of each footprint, shown below each shot, was scaled from the line width.

The results shown in Fig. 9 are those for relatively low speed shots landing on or near the baseline. As a result, the length of the footprint is less than that for a fast serve, being 89.3 mm and 45 mm respectively for the two shots in Fig. 9. The short length of the topspin lob shot is not surprising, but the 36.2 mm width is surprising given that the ball falls onto the court from a large height. If the ball falls from a height of say 5 m, then it will land on the court at a vertical speed of about 10 m/s. According to the results in Fig. 5, the footprint should then be about 44 mm wide for the Wilson ball used in the Brisbane tournament, and even larger if the ball falls from a height greater than 5 m.

6 Discussion

Many factors can affect the accuracy of a line calling system, as described by Capel-Davies and Miller [15]. They include visual interference by players and their racquets, shadows, expansion of the court during the day etc. The main factor described in this paper is the rapid initial expansion of the contact area at the bottom of the ball. The experimental results are consistent with the simple ball model shown in Fig. 1, although it would be of interest to consider the predictions of more sophisticated ball models such as those described by Carre et al [8], Maw et al [13], Goodwill et al [16], Hubbard and Stronge [17] and Bridge [18]. A more extensive set of data would also assist in evaluating the significance of the problem, for example by measuring the expansion rate as functions of ball speed, incident angle, ball spin, ball type, court type and coefficient of friction between the ball and the court surface. Such studies may have already been undertaken for the Hawk-Eye system, but if they have they have not been published.

Uneveness of the court surface is another factor that could affect a line calling system. The problem was encountered in the present experiment due to slight curvature in the blackboard used for the oblique angle experiments. The problem was solved by nailing the board to a flat table top so that the blackboard surface itself was accurately flat and horizontal. The original problem was that one section of the board was about 1 mm higher than other parts of the board. A ball incident at say 15° to the horizontal will impact the board 3.7 mm earlier if the board is raised by 1 mm. The stated accuracy of the Hawk-Eye system [2] is about 3.6 mm, implying that no part of a line on the court surface can be 1 mm higher or lower than any other part. That degree of flatness is unlikely to be observed on a grass or a clay court, and is also unlikely to be achieved on a hard court, given that 1 mm deep puddles are commonly observed on a rain-affected court. Furthermore, drainage of most courts is usually achieved by sloping the court downwards from the centre to the sidelines. The ITF recommends a slope of 1 in 100 for acrylic courts [19], meaning that the sidelines can be 55 mm lower than the centre line. It can be presumed that Hawk-Eye makes allowances for such large differences in the height of the court surface, although it would be more difficult to make allowances for localised 1 mm variations in the height.

In the oblique impact experiments, the video camera was zoomed in to enlarge the image of the ball so that the coordinates of the bottom of the ball could be measured as accurately as possible. The full width of the video screen captured an image that was 500 mm wide. At 300 frames/s, the recorded image size was 512 x 384 pixels. At 600 frames/s, the recorded image size was 432 x 192 pixels. Consequently, the measurement accuracy of any given coordinate was at best 1 pixel or 1.16 mm when recording at 600 frames/s. By fitting 4 or 5 data points with a linear or quadratic fit, random pixel errors in each coordinate were effectively averaged out or reduced. However, if each point was systematically in error by one pixel, then the latter error would not be reduced by curve fitting. The overall measurement accuracy was estimated as about ± 1 mm when determining the initial contact point of the bottom of the ball on the blackboard.

The same type of analysis applied to Hawk-Eye suggests that there could be a relatively large error in determining the coordinates of the bottom of the ball. Each Hawk-Eye camera views approximately half the length of the court or about 12 m [2]. If each camera operated in Full HD mode, with 1920 pixels horizontally, then an error of one pixel would give a measurement accuracy of 6 mm. If the ball is brighter on top than on the bottom, due to sunlight, or if the ball emerges from a shadow into sunlight, there might be a systematic error of one or two pixels. Around ten cameras are used to triangulate and average the coordinates, but there is clearly a technical difficulty in ensuring that the average error in the initial contact point on the court is less than 6 mm.

According to the ITF web site [3], the Hawk-Eye system was approved by the ITF as a result of testing against high speed video operating at 2000 frames/sec, using balls projected at speeds between 30 and 50 m/s. The results in the present paper point to three possible deficiencies in the testing protocol. One is that a frame rate of 10,000 frames/s or higher should preferably be used to determine the distance from the back edge of the footprint to the initial contact point. As explained earlier, the distance between the back edge and the initial contact point is a maximum during the first 0.1 ms of the impact for a high speed ball, meaning that such an effect may not be detected if the frame rate is less than 10,000 frames/s. Alternatively, physical contact with the line could be measured with a system such as that described by Fisher [5]. A second problem is that the width of the footprint does not appear to be part of the testing protocol. The third problem is that conditions relevant to a topspin lob should also be tested to ensure that the assumed footprint is physically realistic.

7 Conclusions

As far as the author is aware, the results presented in this paper represent the first published measurements of the footprint of a obliquely bouncing ball. The initial objective was to measure the rate of expansion of the contact region at the bottom of the ball, given that it has potential consequences for the measurement accuracy of the Hawk-Eye line calling system. It is not known whether the effect has been considered in the design of the system or whether it is included in estimates of the footprint presented by Hawk-Eye. The effect was not measured and apparently not detected when the system was originally approved by the ITF, given that the system was originally tested against a video camera operating at 2000 frames/s. Expansion of the contact area of the ball is most rapid in the first 0.1 ms of the bounce. After 0.1 ms, the back edge of the footprint for a fast serve is typically about 5 mm behind the initial contact point of the ball on the court. At lower ball speeds, such as those encountered in a topspin lob, the back edge of the footprint will be about 22 mm behind the initial contact point if the ball is incident vertically at about 10 m/s on the court.

Measurements of the footprint highlighted several other potential problems with the Hawk-Eye system. One is that the back edge of the footprint of a tennis ball is not well defined, since the edge of the ball itself is not well defined. Fibres extending out from the ball contact the court surface about 10 mm before the ball begins to compress. The effect was clearly detected in the measured footprints. From a practical point of view, the effect can probably be ignored since it is unlikely to be detected by players or line referees. Nevertheless, there is no requirement in the rules of tennis that the effect should be ignored. If any part of the ball touches a line, then the ball is conventionally assumed to be "in".

It was also found that local variations in the height of the contact surface, even as small as 1 mm, can alter the initial contact point of a ball on the court by up to 3 or 4 mm when the ball is incident at a small angle on the court, as it is in a fast serve. Pixel resolution also represents a potential limitation to the accuracy of the Hawk-Eye system. In the present experiment, the video camera used to view the ball was zoomed in to improve the accuracy of the measured ball coordinates. In the Hawk-Eye system, the cameras view a much wider image of the court. An approximate estimate of pixel resolution in that case suggests that pixel resolution might limit the measured accuracy of the ball coordinates to about 6 mm.

Additional results obtained in the present experiment are relevant to the problem of modelling the impact of a ball. Measurements of the maximum contact diameter of a ball provide a useful measure of the one-sided compression of a ball during the impact. In previous studies, the compression of a ball during a rapid impact has been estimated from measurements of the force on the ball and the resulting displacement of its centre of mass. The shape of a ball footprint is easily measured and would provide a useful test of ball impact models. In addition, the footprint provides two-dimensional information on the grip-slip process that occurs when a ball is incident at a sufficiently large angle to grip the impact surface.

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