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1 Sierpinsky Fractal

The Sierpinsky Fractal is a famous fractal that is generated by recursively dividing a triangle into smaller triangles. The Sierpinsky Fractal is a self-similar fractal, which means that the fractal is made up of smaller copies of itself. The Sierpinsky Fractal is named after Wacław Sierpiński, a Polish mathematician who described the fractal in 1915.

The algorithm for generating the Sierpinsky Fractal is simple. We start from a random number (N.B.: the random point could be also out of the triangle), then we choose a random vertex of the triangle and move halfway towards that vertex. We repeat this process for a large number of iterations, and the result is the Sierpinsky Fractal.

An interesting property of the Sierpinsky Fractal is that it has a fractal dimension of 1.585, which means that it is a fractal that is between one and two dimensions. This makes the Sierpinsky Fractal a very interesting and unique fractal. Having a fractal dimension between one and two dimensions means that the Sierpinsky Fractal is a fractal that is more complex than a line but less complex than a plane.

An example of a pseudo-code to generate the Sierpinsky Fractal is shown below:

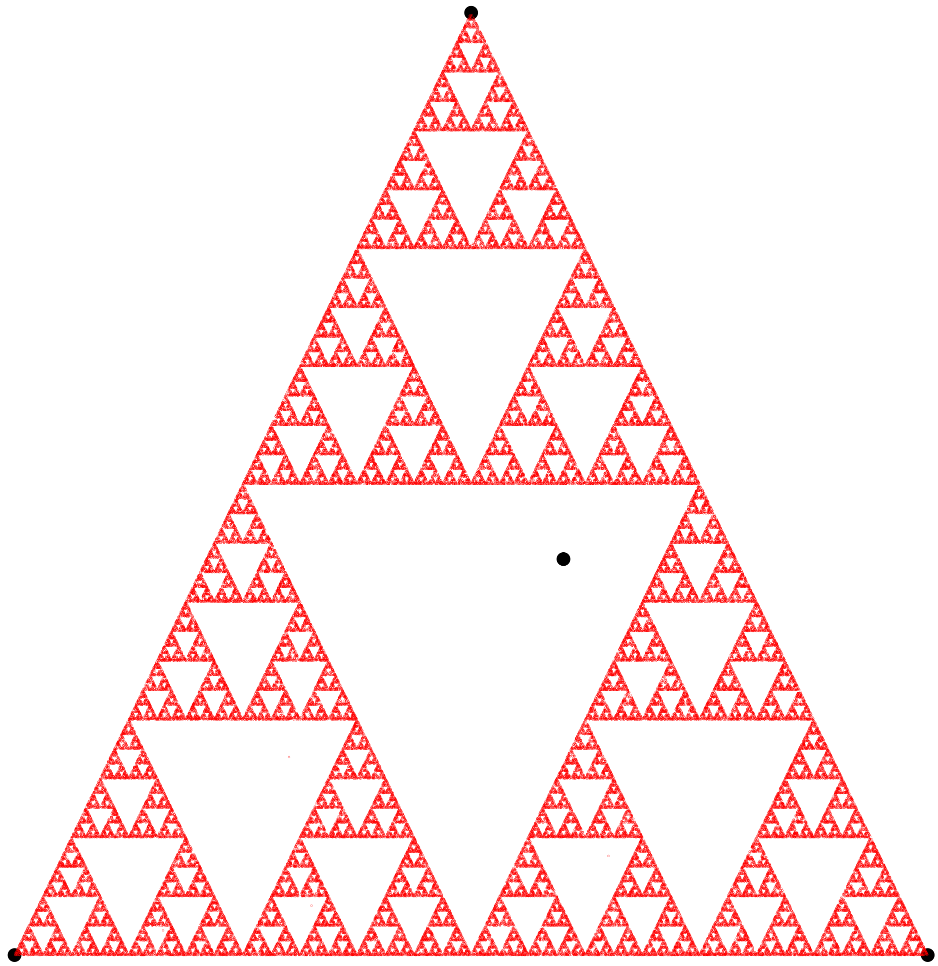
```
1. Choose a random point inside the triangle
2. For i = 1 to N:
3.     Choose a random vertex of the triangle
4.     Move halfway towards that vertex
5.     Plot the new point
```

Let's do it in Python!

[42]:
```python
import random
import matplotlib.pyplot as plt
import numpy as np
plt.rcParams['font.family'] = 'serif'
plt.rc('text', usetex=True)
```

[43]:
```python
Vertex_1 = [0, 0]
Vertex_2 = [1, 0]
Vertex_3 = [0.5, np.sqrt(3) / 2]

# initialize the plot
fig, ax = plt.subplots(figsize=(10, 10), dpi=1000)
```

```python
# remove the axis
ax.axis('off')

# axis limit
ax.set_xlim(-0.1, 1.1)
ax.set_ylim(-0.1, 0.9)

# draw point
ax.plot(Vertex_1[0], Vertex_1[1], 'ko')
ax.plot(Vertex_2[0], Vertex_2[1], 'ko')
ax.plot(Vertex_3[0], Vertex_3[1], 'ko')

# take a random point inside the triangle
x = random.random()
y = random.random()
# plot the first point
ax.plot(x, y, 'ko')

# choose a number of interation for the plot
n = 100000

for i in range(n):
    # choose randomly a vertexes
    chosen = random.choice([Vertex_1, Vertex_2, Vertex_3])

    # find the mid-point between the dot and the vertex
    midpoint_x = (x + chosen[0]) / 2
    midpoint_y = (y + chosen[1]) / 2

    # plot the dot
    ax.plot(midpoint_x, midpoint_y, 'ro', markersize=.1)

    # update the new point
    x = midpoint_x
    y = midpoint_y

# save the plot
plt.savefig('Sierpinski_triangle.png')
```



2 Some Curiosities about the Sierpinski Triangle

- **Origin of the Name**: The Sierpinski Triangle is named after the Polish mathematician Wacław Sierpiński, who described this figure in 1915.
- **Fractal Properties**: It is a classic example of a self-similar fractal, which means that it is composed of smaller copies of itself. Each sub-triangle is a scaled-down version of the whole triangle.
- **Fractal Dimension**: The fractal dimension of the Sierpinski Triangle is $log(3)/log(2) \approx 1.585$, which is a measure of its complexity.
- **Applications**: The Sierpinski Triangle has applications in various fields, including number theory, graph theory, and computer graphics. It is also used in electronic circuits to create

antennas with unique properties.

- **Iterative Construction**: It can be constructed iteratively starting from an equilateral triangle and removing the central triangle in each iteration, leaving three smaller triangles. This process is repeated infinitely.
- **Geometric Aspects**: Each iteration of the Sierpinski Triangle reduces the total area of the original triangle. After infinite iterations, the total area of the fractal tends to zero, but the perimeter becomes infinite.
- **Historical Curiosities**: Although the Sierpinski Triangle was formally described in the 20th century, similar structures were observed in art and architecture long before, such as in Islamic mosaics.
- **Relation to Other Fractals**: The Sierpinski Triangle is closely related to other fractals like the Sierpinski Carpet and the Sierpinski Curve, which share self-similar properties.

These curiosities show how fascinating and versatile the Sierpinski Triangle is, both from a mathematical and an applied perspective.