Suggested References

Next slides refer also to material and topics discussed and presented in the following references:

- ISTAT: https://www.istat.it/it/archivio/238228
- Leontief, W. (Ed.) (1986). Input-output economics. Oxford University Press.
- OECD: https://data.oecd.org/
- Ten Raa, T. (2006). The economics of input-output analysis. Cambridge University Press.
- United Nations, Department of Economic and Social Affairs (2018). Handbook on Supply and Use Tables and Input Output-Tables with Extensions and Applications

Input-Output (I-O) Tables

"*In practical terms, the economic system to which input-output analysis is applied may be as large as a nation or even the entire world economy, or as small as the economy of a metropolitan areas or even a single enterprise. In all instances the approach is essentially the same*" (Leontief, 1986).

Input-Output (I-O) Tables

I-O table represents interlinkages between economic industries at a given time

- There are several types of tables (symmetric, rectangular, ..)
- They can be represented on different levels of aggregation
- Simplistic HPs (e.g. productivity capacity, technology, ..)
- Can represent a domestic, inter-regional or even cross-countries perimeter
- Presence of accounting peculiarities across national systems
- ..

Input-Output (I-O) Tables

- IOTs describe the **flow of goods and services** between all sectors of an economy over a period of time
- IOTs provide information on **production structures** and may be arranged to cover all inputs which are used in production: **intermediate inputs, labour, capital and land**
- Production processes are always **interdependent** and form a system of products produced using also products of other sectors
- With **globalization**, there is **more competition** and **more interdependent production processes**, a greater division of labour and more diversity and **complexity of products**.

Leontief vs. Stone

Leontief (m=n)

- Squared tables referring to flows of goods and services defined according to industry outputs (industry × industry) or according to product outputs (product × product)
- Rows indicate the industry suppliers of inputs for the production of column industries

Stone (m≠n)

- Asymmetric tables in which resources/inputs (supply) are matched to the corresponding uses
- Rows typically refer to inputs/products/services (also secondary products), columns to economic industries

Example: Symmetric Matrix (in quantities)

from\to	Sector 1	Sector 2	Sector 3	Total
Sector 1	5	10	200	**215**
Sector 2	15	25	20	**60**
Sector 3	5	10	5	**20**

Rows:

- Sector 1 produces 215 units, of which 5 consumed by itself, 10 by Sector 2 and 200 by Sector 3

Columns:

- Sector 1 uses 5 input units from Sector 1, 15 from Sector 2 and 5 from Sector 3

Example: Symmetric Matrix (in value)

from\to	Sector 1	Sector 2	Sector 3	Total
Sector 1	15	5	20	**40**
Sector 2	10	20	2	**32**
Sector 3	15	7	25	**47**

Rows:

- Sector 1 transfers goods for 40€, of which 15€ to itself, 5€ to Sector 2 and 20€ to Sector 3

Columns:

- Sector 1 acquires 40€ of goods, of which 15€ from itself, 10€ from Sector 2 and 15€ from Sector 3

Symmetric Matrix

Industries clear their positions at each time, meaning that they receive enough inputs for their production

$$(I)\begin{cases} q_{11}+q_{12}+..+q_{1n}=q_1 \\ q_{21}+q_{22}+..+q_{2n}=q_2 \\ .. \\ .. \\ q_{n1}+q_{n2}+..+q_{nn}=q_n \end{cases}$$

$$(II)\begin{cases} q_{11}p_1+q_{21}p_2+..+q_{n1}p_n=q_1p_1 \\ q_{12}p_1+q_{22}p_2+..+q_{n2}p_n=q_2p_2 \\ .. \\ .. \\ q_{1n}p_1+q_{2n}p_2+..+q_{nn}p_n=q_np_n \end{cases}$$

Matrix of Technical Coefficients (**A**)

from\to	Sector 1	Sector 2	Sector 3
Sector 1	a_{11}	a_{12}	a_{13}
Sector 2	a_{21}	a_{22}	a_{23}
Sector 3	a_{31}	a_{32}	a_{33}

$$a_{ij} = \frac{q_{ij} p_{ij}}{q_j p_j}$$

Value portion of inputs from industry *i* required to produce one unit of value of output *j* given the available technologies

Matrix A

$$\begin{cases} a_{11}X_1 + a_{12}X_2 + a_{13}X_3 + Y_1 = X_1 \\ a_{21}X_1 + a_{22}X_2 + a_{23}X_3 + Y_2 = X_2 \\ a_{31}X_1 + a_{32}X_2 + a_{33}X_3 + Y_3 = X_3 \end{cases}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$$

An increase of demand in one sector will determine also an increase of inputs required from the other sectors in the system, which themselves also generate additional demand, ..

Matrix A

$$\begin{cases} a_{11}X_1 + a_{12}X_2 + a_{13}X_3 + Y_1 = X_1 \\ a_{21}X_1 + a_{22}X_2 + a_{23}X_3 + Y_2 = X_2 \\ a_{31}X_1 + a_{32}X_2 + a_{33}X_3 + Y_3 = X_3 \end{cases}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$$

What is **Y**?

Leontief Model

$$\begin{cases} X_m + J_m = \sum_n K_{mn} + Y_m, \forall m \\ K_{mn} = a_{mn} X_n, \forall m \end{cases}$$

X: Total output

J: Imports

K_{mn}: Intermediate demand

Y: Final demand (households consumption, Gov.t expenditures, investments, Δ inventories, exports)

Leontief Model

$$\begin{cases} X_m + J_m = \sum_n K_{mn} + Y_m, \forall m \\ K_{mn} = a_{mn} X_n, \forall m \end{cases}$$

$$\Downarrow$$

$$X_m + J_m = \sum_n a_{mn} X_n + Y_m, \forall m$$

$$\mathbf{X + J = AX + Y}$$
$$\mathbf{X - AX = Y - J}$$
$$\mathbf{X(I-A) = Y - J}$$
$$\mathbf{X = (I-A)^{-1} (Y-J)}$$

$$(I - A)^{-1} \cong (I + A) + (A^2 + A^3 + A^4 + .. + A^n)$$

I Identity matrix
(I-A) Leontief matrix
$(I-A)^{-1}$ Inverse Leontief matrix

Leontief Model

$$\begin{cases} X_m + J_m = \sum_n K_{mn} + Y_m, \forall m \\ K_{mn} = a_{mn} X_n, \forall m \end{cases}$$

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$$\mathbf{X + J = AX + Y}$$
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$$\mathbf{X = (I-A)^{-1} (Y-J)}$$

$$(I - A)^{-1} \cong (I + A) + (A^2 + A^3 + A^4 + .. + A^n)$$

I Identity matrix
(I-A) Leontief matrix
$(I-A)^{-1}$ Inverse Leontief matrix

The Leontief inverse $(I - A)^{-1}$ reflects the direct and indirect requirements for intermediate consumption and one unit of output for final uses

I-O Tables for Economic Applications

- Medium-Long term Structural analysis: identification of most relevant industries, impact of value added, interdependencies import/export, ..
- Short-term analysis: impulse in a specific industry, exogeneous shocks, ..
- Evaluation of a Keynesian public expenditure programme to offset a recession or unemployment
- ..

Example

HP: Increase of final demand of 1000€ in Sector 1

From/to	Sector 1	Sector 2	Sector 3	Final Demand	Total
Sector 1	400	10	0	5000	**5410**
Sector 2	210	200	100	70	**580**
Sector 3	300	50	20	50	**420**
Value Added	4500	320	300		
Total Output	**5410**	**580**	**420**		

Example

HP: Increase of final demand of 1000€ in Sector 1

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Find I-A

Example

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Sector 1	400	10	0	5000	**5410**
Sector 2	210	200	100	70	**580**
Sector 3	300	50	20	50	**420**
Value Added	4500	320	300		
Total Output	**5410**	**580**	**420**		

Find I-A

From/To	Sector 1	Sector 2	Sector 3
Sector 1	0.93	- 0.02	-
Sector 2	- 0.04	0.66	- 0.24
Sector 3	- 0.06	- 0.09	0.95

Example

HP: Increase of final demand of 1000€ in Sector 1

From/to	Sector 1	Sector 2	Sector 3	Final Demand	Total
Sector 1	400	10	0	5000	**5410**
Sector 2	210	200	100	70	**580**
Sector 3	300	50	20	50	**420**
Value Added	4500	320	300		
Total Output	**5410**	**580**	**420**		

Find $(I-A)^{-1}$

Example

HP: Increase of final demand of 1000€ in Sector 1

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Sector 1	400	10	0	5000	**5410**
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Sector 3	300	50	20	50	**420**
Value Added	4500	320	300		
Total Output	**5410**	**580**	**420**		

Find $(I-A)^{-1}$

From/To	Sector 1	Sector 2	Sector 3
Sector 1	1.08	0.03	0.01
Sector 2	0.09	1.58	0.40
Sector 3	0.07	0.14	1.09

Example

HP: Increase of final demand of 1000€ in Sector 1

From/to	Sector 1	Sector 2	Sector 3	Final Demand	Total
Sector 1	400	10	0	5000	**5410**
Sector 2	210	200	100	70	**580**
Sector 3	300	50	20	50	**420**
Value Added	4500	320	300		
Total Output	**5410**	**580**	**420**		

$X = (I-A)^{-1} Y$		From/To	Sector 1	Sector 2	Sector 3				
		Sector 1	1.08	0.03	0.01		**6000**		6,491.51
		Sector 2	0.09	1.58	0.40	.	70	=	669.92
		Sector 3	0.07	0.14	1.09		50		491.11

Δx = 1,242.54 € = 1000€ +242.54€

What is this extra impact?

Example

HP: 100€ increase of exports of Sector 2

From/to	Sector 1	Sector 2	Sector 3	Final Demand	Exports	Total
Sector 1	500	200	50	4000	700	**5450**
Sector 2	325	128	240	480	20	**1193**
Sector 3	100	5	800	114	200	**1219**
Value Added	4455	835	89			
Total Output	**5380**	**1168**	**1179**			
Imports	70	25	40			

Find I-A

Example

HP: 100€ increase of exports of Sector 2

From/to	Sector 1	Sector 2	Sector 3	Final Demand	Exports	Total
Sector 1	500	200	50	4000	700	**5450**
Sector 2	325	128	240	480	20	**1193**
Sector 3	100	5	800	114	200	**1219**
Value Added	4455	835	89			
Total Output	**5380**	**1168**	**1179**			
Imports	70	25	40			

Find I-A

From/To	Sector 1	Sector 2	Sector 3
Sector 1	0.91	- 0.17	- 0.04
Sector 2	- 0.06	0.89	- 0.20
Sector 3	- 0.02	- 0.00	0.32

Example

HP: 100€ increase of exports of Sector 2

From/to	Sector 1	Sector 2	Sector 3	Final Demand	Exports	Total
Sector 1	500	200	50	4000	700	**5450**
Sector 2	325	128	240	480	20	**1193**
Sector 3	100	5	800	114	200	**1219**
Value Added	4455	835	89			
Total Output	**5380**	**1168**	**1179**			
Imports	70	25	40			

Find $(I-A)^{-1}$

From/To	Sector 1	Sector 2	Sector 3
Sector 1	1.12	0.22	0.29
Sector 2	0.09	1.14	0.74
Sector 3	0.07	0.03	3.14

Example

Y

HP: 100€ increase of exports of Sector 2

From/to	Sector 1	Sector 2	Sector 3	Final Demand	Exports	Total
Sector 1	500	200	50	4000	700	**5450**
Sector 2	325	128	240	480	20	**1193**
Sector 3	100	5	800	114	200	**1219**
Value Added	4455	835	89			
Total Output	**5380**	**1168**	**1179**			
Imports	70	25	40			

J

$X = (I-A)^{-1}(Y-J)$		From/To	Sector 1	Sector 2	Sector 3					
		Sector 1	1.12	0.22	0.29			4630		5,401.73
		Sector 2	0.09	1.14	0.74		.	**575**	=	1,282.42
		Sector 3	0.07	0.03	3.14			274		1,181.78

$\Delta x =$ **138.93 €**

Example: Supply Table

Sector/Industry

Product	Agriculture	Manufacturing	Construction	Transport	Real Estate
Agriculture	**4245**	0	0	10	0
Manufacturing	63	**8704**	350	12	2
Construction	0	156	**15480**	245	54
Transport	0	546	455	**5454**	54
Real Estate	0	44	48	45	**484**
Total	**4308**	**9450**	**16333**	**5766**	**594**

Example: Supply Table

Product	Sector/Industry: Agriculture	Manufacturing	Construction	Transport	Real Estate
Agriculture	**4245**	0	0	10	0
Manufacturing	63	**8704**	350	12	2
Construction	0	156	**15480**	245	54
Transport	0	546	455	**5454**	54
Real Estate	0	44	48	45	**484**
Total	**4308**	**9450**	**16333**	**5766**	**594**

In the main diagonal there are all products for which there is an identity between product and economic sector; off-diagonal: ‘’secondary products’’

Example: Supply Table

	Agriculture	Manufacturing	Construction	Transport	Real Estate	imports CIF	**Total (at basic prices)**
Agriculture	**4245**	0	0	10	0	450	**4705**
Manufacturing	63	**8704**	350	12	2	4546	**13677**
Construction	0	156	**15480**	245	54	254	**16189**
Transport	0	546	455	**5454**	54	45	**6554**
Real Estate	0	44	48	45	**484**	0	**621**
Total	**4308**	**9450**	**16333**	**5766**	**594**	**5295**	**41746**

CIF: Cost, insurance, and freight is an international shipping agreement. It represents the charges paid by a seller to cover the costs, insurance, and freight of a buyer's order.

Example: Supply Table

	Agriculture	Manufacturing	Construction	Transport	Real Estate	imports CIF	margins and net taxes	**Total (at purchaser prices)**
Agriculture	**4245**	0	0	10	0	450	23540	**28245**
Manufacturing	63	**8704**	350	12	2	4546	4587	**18264**
Construction	0	156	**15480**	245	54	254	-428	**15761**
Transport	0	546	455	**5454**	54	45	-5000	**1554**
Real Estate	0	44	48	45	**484**	0	-156	**465**
Total	**4308**	**9450**	**16333**	**5766**	**594**	**5295**	**22543**	**64289**

Transport margins and net taxes

Example: Uses Table

	Agriculture	Manufacturing	Construction	Transport	Real Estate	Households consumption	Gvt. Expenditures	Investments	Exports	**Total (at purchaser prices)**
Agriculture	224	1465	0	5	0	4000	450	18889	3212	**28245**
Manufacturing	450	12000	776	55	20	354	4347	141	121	**18264**
Construction	35	12	2000	202	65	881	254	11081	1231	**15761**
Transport	10	54	30	527	54	321	45	462	51	**1554**
Real Estate	5	10	29	35	254	132	0	0	0	**465**
Total	**724**	**13541**	**2835**	**824**	**393**	**5688**	**5096**	**30573**	**4615**	**64289**

What does this portion represent?

Example: Uses Table

	Agriculture	Manufacturing	Construction	Transport	Real Estate	Households consumption	Gvt. Expenditures	Investments	Exports	**Total (at purchaser prices)**
Agriculture	224	1465	0	5	0	4000	450	18889	3212	**28245**
Manufacturing	450	12000	776	55	20	354	4347	141	121	**18264**
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Total	**724**	**13541**	**2835**	**824**	**393**	**5688**	**5096**	**30573**	**4615**	**64289**

And this portion?

Example: Uses Table

	Agriculture	Manufacturing	Construction	Transport	Real Estate	Households consumption	Gvt. Expenditures	Investments	Exports	**Total (at purchaser prices)**
Agriculture	224	1465	0	5	0	4000	450	18889	3212	**28245**
Manufacturing	450	12000	776	55	20	354	4347	141	121	**18264**
Construction	35	12	2000	202	65	881	254	11081	1231	**15761**
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Real Estate	5	10	29	35	254	132	0	0	0	**465**
Total	**724**	**13541**	**2835**	**824**	**393**	**5688**	**5096**	**30573**	**4615**	**64289**

For example: 120.000€ is the value of product ‘Manufacturing’ needed to produce the overall production of goods and services of sector ‘Manufacturing’, thus considering both primary and secondary productions.

The difference between Uses table and symmetric table increases when many relevant secondary productions are present

Example: Uses Table

	Agriculture	Manufacturing	Construction	Transport	Real Estate	Households consumption	Gvt. Expenditures	Investments	Exports	**Total**
Agriculture	200	722	0	53	0	2915	50	15800	3800	**23540**
Manufacturing	27	100	190	25	100	100	3810	135	100	**4587**
Construction	-20	-3	-80	-10	-4	-140	-21	-100	-50	**-428**
Transport	-80	-10	-90	-55	-21	-650	-220	-3852	-22	**-5000**
Real Estate	-1	-2	-4	-2	-14	-79	0	-24	-30	**-156**
Total	**126**	**807**	**16**	**11**	**61**	**2146**	**3619**	**11959**	**3798**	**22543**

In this table there are reported transport margins and net taxes → it allows to switch from basic prices to purchasers prices

Example: Uses Table

	Agriculture	Manufacturing	Construction	Transport	Real Estate
Agriculture	224				0
Manufacturing	450	12000	776	55	20
Construction	35	12	2000	202	65
Transport	10	54	30	527	54
Real Estate	5	10	29	35	254
Total	**724**	**13541**	**2835**	**824**	**393**

	Agriculture	Manufacturing	Construction	Transport	Real Estate
Agriculture	24	743	0	-48	0
Manufacturing	423	11900	586	30	-80
Construction	55	15	2080	212	69
Transport	90	64	120	582	75
Real Estate	6	12	33	37	268
Total (at basic prices)	**598**	**12734**	**2819**	**813**	**332**
Margins & Net Taxes	126	807	16	11	61
Total (at purchasers' prices)	**724**	**13541**	**2835**	**824**	**393**

	Agriculture	Manufacturing	Construction	Transport	Real Estate
Agriculture	200	722	0	53	0
Manufacturing	27	100	190	25	100
Construction	-20	-3	-80	-10	
Transport	-80	-10	-90		-21
Real Estate	-1	-2	-4	-2	-14
Total	**126**	**807**	**16**	**11**	**61**

Example: Uses Table

	Agriculture	Manufacturing	Construction	Transport	Real Estate	Households consumption	Gvt. Expenditures	Investments	Exports	**Total**
Agriculture	24	743	0	-48	0	1085	400	3089	-588	**4705**
Manufacturing	423	11900	586	30	-80	254	537	6	21	**13677**
Construction	55	15	2080	212	69	1021	275	11181	1281	**16189**
Transport	90	64	120	582	75	971	265	4314	73	**6554**
Real Estate	6	12	33	37	268	211	0	24	30	**621**
Total (at basic prices)	**598**	**12734**	**2819**	**813**	**332**	**3542**	**1477**	**18614**	**817**	**41746**
Margins & Net Taxes	126	807	16	11	61	2146	3619	11959	3798	22543
Total (at purchaser prices)	**724**	**13541**	**2835**	**824**	**393**	**5688**	**5096**	**30573**	**4615**	**64289**

Exercise

Link (OECD Use at purchasers' prices): https://stats.oecd.org/Index.aspx?DataSetCode=SNA_TABLE40
(National Accounts->Supply and Use Table->40.Use at purchaser' prices)

Flow: Total
Country: Austria
Year: 2015
Unit: Euro, Millions

Ex.1: In which (intermediate) sectors the 'Products of agriculture, hunting and related services' are *used* more?
Ex.2: In which (intermediate) sectors the 'Food, beverages and tobacco products' are *used* more?

(If you prefer you may use different data sources. This is just an example of possible data.)

Multipliers

- Direct effects: increase in the industry where the impulse is provided
- Indirect effects: increase due to the structure of the interdependencies in the system and the configuration of the supply chain within/across industries
- Induced effects: increase due to the impact of additional employment and salaries

Examples ($L = (I-A)^{-1}$, L^* is L extended to households):

- Output multiplier (Type I, direct and indirect effects due to unit variations in the demand of *j*): $\sum_i L_{ij}$
- Output multiplier (Type II, direct, indirect and induced effects due to unit variations in the demand of *j*): $\sum_i L^*_{ij}$

Output Multiplier (type I)

The Output multiplier of a certain industry is defined as the total value of production in all industries of the economy that is necessary for all phases of production to produce one unit of product for final use in that industry

	Agriculture	Manufacturing	Construction	Transport	Real Estate	Households consumption
Agriculture	45	20	1	0	0	100
Manufacturing	24	120	60	32	20	20
Construction	15	36	125	55	84	84
Transport	2	62	54	95	10	10
Real Estate	5	10	5	2	23	23
Compensation of employees	80	12	207	40	50	
Consumption of fixed assets	325	555	600	357	180	
Total	**496**	**815**	**1052**	**581**	**367**	**237**

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	Agriculture	Manufacturing	Construction	Transport	Real Estate	Households consumption
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Manufacturing	24	120	60	32	20	20
Construction	15	36	125	55	84	84
Transport	2	62	54	95	10	10
Real Estate	5	10	5	2	23	23
Compensation of employees	80	12	207	40	50	
Consumption of fixed assets	325	555	600	357	180	
Total	**496**	**815**	**1052**	**581**	**367**	**237**

Matrix ***A***

	Agriculture	Manufacturing	Construction	Transport	Real Estate
Agriculture	0.09	0.02	0.00	-	-
Manufacturing	0.05	0.15	0.06	0.06	0.05
Construction	0.03	0.04	0.12	0.09	0.23
Transport	0.00	0.08	0.05	0.16	0.03
Real Estate	0.01	0.01	0.00	0.00	0.06

Output Multiplier (type I)

The Output multiplier of a certain industry is defined as the total value of production in all industries of the economy that is necessary for all phases of production to produce one unit of product for final use in that industry

	Agriculture	Manufacturing	Construction	Transport	Real Estate	Households consumption
Agriculture	45	20	1	0	0	100
Manufacturing	24	120	60	32	20	20
Construction	15	36	125	55	84	84
Transport	2	62	54	95	10	10
Real Estate	5	10	5	2	23	23
Compensation of employees	80	12	207	40	50	
Consumption of fixed assets	325	555	600	357	180	
Total	**496**	**815**	**1052**	**581**	**367**	**237**

Matrix ***A***

	Agriculture	Manufacturing	Construction	Transport	Real Estate
Agriculture	0.09	0.02	0.00	-	-
Manufacturing	0.05	0.15	0.06	0.06	0.05
Construction	0.03	0.04	0.12	0.09	0.23
Transport	0.00	0.08	0.05	0.16	0.03
Real Estate	0.01	0.01	0.00	0.00	0.06

Matrix $(I-A)^{-1}$

	Agriculture	Manufacturing	Construction	Transport	Real Estate
Agriculture	1.10	0.03	0.00	0.00	0.00
Manufacturing	0.07	1.19	0.08	0.09	0.09
Construction	0.05	0.08	1.15	0.14	0.29
Transport	0.01	0.11	0.08	1.21	0.06
Real Estate	0.01	0.02	0.01	0.01	1.07
Total	**1.24**	**1.43**	**1.32**	**1.45**	**1.51**

Output Multiplier (type II)

The Output multiplier of a certain industry is defined as the total value of production in all industries of the economy that is necessary for all phases of production to produce one unit of product for final use in that industry

	Agriculture	Manufacturing	Construction	Transport	Real Estate	Households consumption
Agriculture	45	20	1	0	0	100
Manufacturing	24	120	60	32	20	20
Construction	15	36	125	55	84	84
Transport	2	62	54	95	10	10
Real Estate	5	10	5	2	23	23
Compensation of employees	80	12	207	40	50	
Consumption of fixed assets	325	555	600	357	180	
Total	**496**	**815**	**1052**	**581**	**367**	**237**

Matrix ***A****

	Agriculture	Manufacturing	Construction	Transport	Real Estate	Households consumption
Agriculture	0.09	0.02	0.00	-	-	0.42
Manufacturing	0.05	0.15	0.06	0.06	0.05	0.08
Construction	0.03	0.04	0.12	0.09	0.23	0.35
Transport	0.00	0.08	0.05	0.16	0.03	0.04
Real Estate	0.01	0.01	0.00	0.00	0.06	0.10
Compensation of employees	0.16	0.01	0.20	0.07	0.14	-

Output Multiplier (type II)

The Output multiplier of a certain industry is defined as the total value of production in all industries of the economy that is necessary for all phases of production to produce one unit of product for final use in that industry

	Agriculture	Manufacturing	Construction	Transport	Real Estate	Households consumption
Agriculture	45	20	1	0	0	100
Manufacturing	24	120	60	32	20	20
Construction	15	36	125	55	84	84
Transport	2	62	54	95	10	10
Real Estate	5	10	5	2	23	23
Compensation of employees	80	12	207	40	50	
Consumption of fixed assets	325	555	600	357	180	
Total	**496**	**815**	**1052**	**581**	**367**	**237**

Matrix ***A****

	Agriculture	Manufacturing	Construction	Transport	Real Estate	Households consumption
Agriculture	0.09	0.02	0.00	-	-	0.42
Manufacturing	0.05	0.15	0.06	0.06	0.05	0.08
Construction	0.03	0.04	0.12	0.09	0.23	0.35
Transport	0.00	0.08	0.05	0.16	0.03	0.04
Real Estate	0.01	0.01	0.00	0.00	0.06	0.10
Compensation of employees	0.16	0.01	0.20	0.07	0.14	-

Matrix $(I-A^*)^{-1}$

	Agriculture	Manufacturing	Construction	Transport	Real Estate	Households consumption
Agriculture	1.212	0.060	0.140	0.068	0.124	0.581
Manufacturing	0.108	1.198	0.132	0.112	0.136	0.211
Construction	0.156	0.105	1.285	0.202	0.410	0.579
Transport	0.038	0.119	0.107	1.226	0.087	0.125
Real Estate	0.040	0.023	0.040	0.022	1.099	0.141
Compensation of employees	0.236	0.059	0.290	0.140	0.258	1.238
Total	**1.79**	**1.57**	**1.99**	**1.77**	**2.11**	**2.87**

Exercise

Link: https://stats.oecd.org/Index.aspx?DataSetCode=IOTS_2021
(Industry and Services -> STAN -> Input Output Database -> IOTs 2021: Input-Output Tables)

Variable: TTL Total
Country: ITA Italy
Time: 2018

Ex.1: If the Italian Gvt would gave an impulse to the economy stimulating final demand, which economic sector would generate the greatest direct and indirect effects?

(If you prefer you may use different data sources. This is just an example of possible data.)

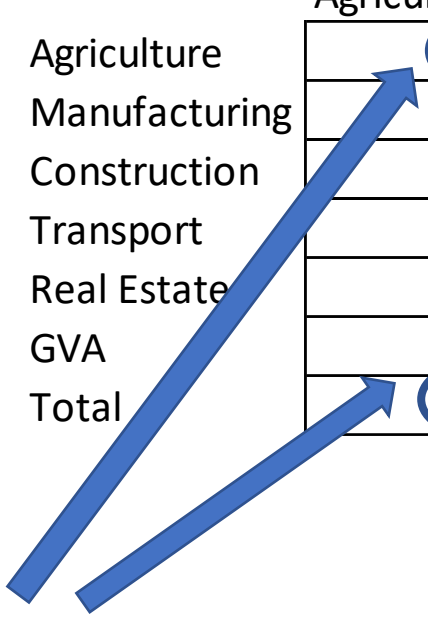
Exercise

	Agriculture	Manufacturing	Construction	Transport	Real Estate
Agriculture	45	20	1	0	0
Manufacturing	24	120	60	32	20
Construction	15	36	125	55	84
Transport	2	62	54	95	10
Real Estate	5	10	5	2	23
GVA	250	28	420	232	160
Total	**341**	**276**	**665**	**416**	**297**

Ex.1: Given this simplistic IOT, compute the matrix of technical coefficients (***A***).

Exercise

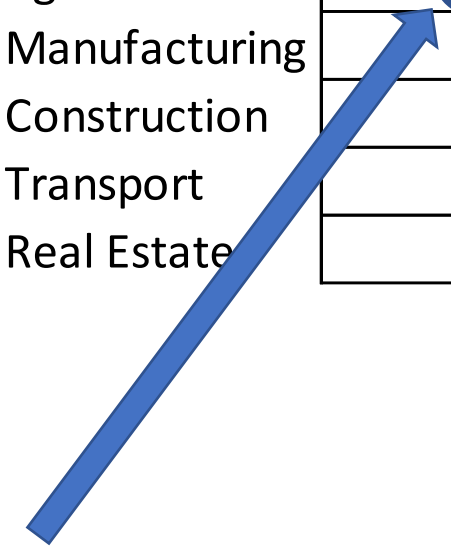
	Agriculture	Manufacturing	Construction	Transport	Real Estate
Agriculture	45	20	1	0	0
Manufacturing	24	120	60	32	20
Construction	15	36	125	55	84
Transport	2	62	54	95	10
Real Estate	5	10	5	2	23
GVA	250	28	420	232	160
Total	**341**	**276**	**665**	**416**	**297**



$a_{11} = 45/341 = 0.13$

Exercise

	Agriculture	Manufacturing	Construction	Transport	Real Estate
Agriculture	0.13	0.07	0.00	-	-
Manufacturing	0.07	0.43	0.09	0.08	0.07
Construction	0.04	0.13	0.19	0.13	0.28
Transport	0.01	0.22	0.08	0.23	0.03
Real Estate	0.01	0.04	0.01	0.00	0.08



Technical coefficient

Exercise

	Agriculture	Manufacturing	Construction	Transport	Real Estate
Agriculture	0.13	0.07	0.00	-	-
Manufacturing	0.07	0.43	0.09	0.08	0.07
Construction	0.04	0.13	0.19	0.13	0.28
Transport	0.01	0.22	0.08	0.23	0.03
Real Estate	0.01	0.04	0.01	0.00	0.08

Ex.2: Given this matrix of technical coefficient (***A***), compute the Leontief matrix (***L***).

Exercise

Identity Matrix (*I*)

	Agriculture	Manufacturing	Construction	Transport	Real Estate
Agriculture	1	0	0	0	0
Manufacturing	0	1	0	0	0
Construction	0	0	1	0	0
Transport	0	0	0	1	0
Real Estate	0	0	0	0	1

Technical Coefficients Matrix (*A*)

	Agriculture	Manufacturing	Construction	Transport	Real Estate
Agriculture	0.13	0.07	0.00	-	-
Manufacturing	0.07	0.43	0.09	0.08	0.07
Construction	0.04	0.13	0.19	0.13	0.28
Transport	0.01	0.22	0.08	0.23	0.03
Real Estate	0.01	0.04	0.01	0.00	0.08

I-A

Exercise

	Agriculture	Manufacturing	Construction	Transport	Real Estate
Agriculture	0.87	-0.07	0.00	0.00	0.00
Manufacturing	-0.07	0.57	-0.09	-0.08	-0.07
Construction	-0.04	-0.13	0.81	-0.13	-0.28
Transport	-0.01	-0.22	-0.08	0.77	-0.03
Real Estate	-0.01	-0.04	-0.01	0.00	0.92

Leontief matrix (***L = I-A***)

Exercise

	Agriculture	Manufacturing	Construction	Transport	Real Estate
Agriculture	0.87	-0.07	0.00	0.00	0.00
Manufacturing	-0.07	0.57	-0.09	-0.08	-0.07
Construction	-0.04	-0.13	0.81	-0.13	-0.28
Transport	-0.01	-0.22	-0.08	0.77	-0.03
Real Estate	-0.01	-0.04	-0.01	0.00	0.92

Ex. Given the Leontief matrix, compute the inverse Leontief matrix.

Exercise

	Agriculture	Manufacturing	Construction	Transport	Real Estate
Agriculture	1.17	0.16	0.02	0.02	0.02
Manufacturing	0.18	1.96	0.24	0.24	0.23
Construction	0.11	0.45	1.31	0.27	0.45
Transport	0.07	0.62	0.21	1.40	0.16
Real Estate	0.03	0.09	0.02	0.02	1.10

Inverse Leontief matrix $\boldsymbol{(I-A)^{-1}}$

Exercise

	Agriculture	Manufacturing	Construction	Transport	Real Estate
Agriculture	1.17	0.16	0.02	0.02	0.02
Manufacturing	0.18	1.96	0.24	0.24	0.23
Construction	0.11	0.45	1.31	0.27	0.45
Transport	0.07	0.62	0.21	1.40	0.16
Real Estate	0.03	0.09	0.02	0.02	1.10

$$(I - A)^{-1} \cong (I + A) + (A^2 + A^3 + A^4 + .. + A^n)$$

- the unit matrix (I) on the diagonal: one unit of the product for final uses
- matrix A represents the direct input requirements
- A^2 until A^n represent the indirect requirements for intermediate consumption in the previous stages of production

Exercise

	Agriculture	Manufacturing	Construction	Transport	Real Estate
Agriculture	1.17	0.16	0.02	0.02	0.02
Manufacturing	0.18	1.96	0.24	0.24	0.23
Construction	0.11	0.45	1.31	0.27	0.45
Transport	0.07	0.62	0.21	1.40	0.16
Real Estate	0.03	0.09	0.02	0.02	1.10
Total	**1.56**	**3.29**	**1.81**	**1.95**	**1.95**

What is the interpretation of these values?

Exercise

	Agriculture	Manufacturing	Construction	Transport	Real Estate
Agriculture	1.17	0.16	0.02	0.02	0.02
Manufacturing	0.18	1.96	0.24	0.24	0.23
Construction	0.11	0.45	1.31	0.27	0.45
Transport	0.07	0.62	0.21	1.40	0.16
Real Estate	0.03	0.09	0.02	0.02	1.10
Total	**1.56**	**3.29**	**1.81**	**1.95**	**1.95**

What is the interpretation of these values?

For each column, they can be seen as output multipliers reflecting the cumulative output of the economy generated by one additional unit of final uses of a certain product (of a specific column)

For example: 'Agriculture'=1.56 means that if final uses for these products were to increase by 1.0 million, a cumulative output of about 1.56 million would be generated in that economy

Exercise

Link: https://stats.oecd.org/Index.aspx?DataSetCode=IOTS_2021

Variable: TTL Total
Country: ITA Italy
Time: 2018, 2006, 1995

Ex.2: has the Italian IOT structure evolved in time?
Ex.3: which are the top 3 sectors that increased more their output multipliers from 1995 to 2018?
Ex.4: which are the bottom 3 sectors that increased less (reduced) their output multipliers from 1995 to 2018?

(If you prefer you may use different data sources. This is just an example of possible data.)

Exercise

Link: https://stats.oecd.org/Index.aspx?DataSetCode=IOTS_2021

Variable: TTL Total
Country: ITA Italy, DEU Germany, LUX Luxembourg
Time: 2018

Ex.5: Do the three economies share similar IOT structures?

(If you prefer you may use different data sources. This is just an example of possible data.)