Analytical Rigid Body Reconstruction after Micro-Doppler Removal

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***Abstract*— Recently, an L-statistics based method for a successful micro-Doppler effects removal and reconstruction of the rigid body in ISAR/SAR has been proposed. However, during the procedure of the micro-Doppler removal, a part of the short-time Fourier transform samples that correspond to the rigid body is eliminated, as well. Consequently, in the Fourier transform of a rigid body, reconstructed based on the remaining short-time Fourier transform samples, one highly concentrated peak is obtained, as in the original Fourier transform, with a number of undesired low-concentrated components, being spread around this peak. In this paper, we propose a procedure for analytical reconstruction of the removed short-time Fourier transform samples that correspond to the rigid body. The proposed procedure leads to a highly concentrated Fourier transform (ISAR/SAR image) obtained by summing over time the resulting short-time Fourier transform filled up with the estimation of the missing rigid body related samples.**

Keywords*— *Radar imaging; micro-Doppler; L-statistics; short-time Fourier transform

I. INTRODUCTION

Micro-Doppler (m-D) is an effect that appears as a consequence of the existence of fast moving reflectors and can severely degrade radar image readability and detection of rigid body reflectors. Thus, the problem of eliminating m-D effect has attracted significant research attention [1]-[6]. Recently, a very simple and efficient method for the m-D effect elimination has been proposed in [7]. This method is based on the L- statistics. In order to remove the m-D effect, the short-time Fourier transform (STFT), within the coherent integration time (CIT), is calculated first. Then, the fact that the rotating reflectors and rigid bodies have different representations in the time-frequency (TF) plane is used. Namely, the m-D part of the signal has highly variable frequency content, and it covers a wide range of frequencies, but only for a couple of samples in time. Rigid body related part of the signal, on the other hand, has constant or almost constant frequency content along time. This information is used to eliminate the m-D part of the signal. If the STFT of the original signal is sorted along time, the m-D part covers wide range of frequencies but only for a few strongest values. Therefore, if we eliminate a number of the strongest STFT values, for each frequency, we can completely remove the m-D effect [7], without significantly degrading rigid body part of the signal. By summing the remaining STFT samples over time, the Fourier transform (FT) of the rigid body is obtained. This method has very simple implementation, fast realization and gives better results than other existing complex procedures, [7],[13].

However, for frequencies that correspond to rigid body, the samples omitted during the process of the m-D removal are part of the rigid body STFT representation. Consequently, the concentration of the FT, reconstructed by summing the m-D free STFT, will have lower concentration than the original m-D free FT of the rigid body, [7]. The original m-D free FT of the rigid body can be obtained, in the absence of the m-D, by summing over time all the STFT samples. Missing STFT samples of the rigid body after m-D removal induce residual smearing around the peak in the reconstructed FT. Thus, estimation of the missing rigid body samples would improve its concentration in the FT based radar image.

Due to the high number of the missing rigid body related STFT samples, a genetic algorithm for their estimation is proposed in [8]. Amplitudes at each frequency are reconstructed as median amplitude of the remaining STFT samples at the same frequency. The phases are estimated using genetic algorithm, where the first generation is taken as randomly generated possible values of phases. The fitness function is defined in a way that forces improvements in each new generation, [8]. The best individual in the last generation of the genetic algorithm represents the final phases' estimation of the missing STFT samples. This approach successfully estimates the missing STFT samples and results in the FT reconstructed as if the m-D was not present at the first place. Moreover, it is not based on any restriction, thus it will work in any radar scenario. The main drawback of this approach is its high computational demand (on Intel i3 3.2GHz it takes 33s for this estimation [8]). However, this would be a small price if we do not have any knowledge regarding the missing STFT samples that should be recovered, except their positions. Namely, in this case, the alternative would be standard optimisation techniques, which involve the multidimensional search procedure that is computationally more extensive than the genetic algorithm proposed in [8].

In this paper we propose a new method for the phases' estimation of the missing rigid body related STFT samples. Phases are recovered using knowledge about analytical form of the rigid body related STFT. The rigid body is assumed to be complex sinusoid, as it is usually done in the literature, [7], [9], [10]. At each frequency, amplitudes of the missing rigid body related STFT samples are estimated as the median of the STFT amplitudes remained after the m-D removal, as in [8]. By the proposed procedure, missing STFT samples are recovered in less than a second, while the FT is reconstructed with even higher concentration than in [8]. In addition, for the FT reconstruction, we use only those frequencies that correspond to the rigid body. In this manner, the m-D effect and noise are almost completely eliminated. The proposed procedure gives better results than the genetic algorithm proposed in [8] when the assumptions about the rigid body signal hold. However, in the case of very close rigid body components with the STFTs overlapped in the TF plane, the genetic algorithm outperforms the proposed algorithm. Namely, in this case, our analytical form of rigid body related STFT samples' phases cannot be used, since very close components interact with each other in the TF plane. The genetic algorithm proposed in [8], on the other hand, does not use any assumption related to the form of the STFT samples' phases.

Due to the fast realization of the procedure proposed in this paper, it could be used as the first step after the m-D removal. In the most of the practical cases it will produce good results, and further processing will not be needed, while in the case of close components or when the rigid body form does not satisfy our assumption we should proceed, with the genetic algorithm proposed in [8].

The paper is organized in four sections. In Section 2, the radar signal model, the procedure for the rigid body separation (based on the L-statistics) and the missing STFT samples recovering procedure are presented. Performance of the proposed reconstruction method is illustrated through the simulations in Section 3, while the conclusion is given in Section 4.

II. Micro-Doppler Removal Based on the L-Statistics and STFT Reconstruction

Consider continuous wave (CW) radar that transmits signal toward a target. The received signal, reflected from a target, is delayed with respect to the transmitted signal for $t_d=2d(t)/c$, where $d(t)$ is the target distance from the radar and c is the speed of light. At the reception this signal is demodulated to the baseband, with possible distance compensation and other preprocessing operations.

In order to analyze the influence of cross-range non-stationarities in the radar imaging, we will consider only the Doppler part in the received signal of a point target, as it is usually done in the radar literature [7], [9]:

$$s(t)=\sigma\exp(j2d(t)\omega_0/c)\,, \tag{1}$$

where σ is the reflection coefficient of the target, while ω_0 is the radar operating frequency. A component of the radar signal that corresponds to the rigid body can be modeled as a complex sinusoid, while the component related to fast rotating reflector can be modeled as a sinusoidally modulated signal, [7]. Radar signal that corresponds to K rigid bodies and P fast-rotating reflectors can be then analytically expressed as in [1]:

$$s(m)\cong\sum_{i=1}^{K}\sigma_{Bi}e^{jy_{Bi}m}+\sum_{i=1}^{P}\sigma_{Ri}e^{jA_{Ri}\sin(\omega_{Ri}m+\varphi_i)}\,, \tag{2}$$

where σ_{Bi} and σ_{Ri} represent reflection coefficients of a rigid body and a fast-rotating component, respectively, y_{Bi} is proportional to the distance of the reflector from the center of rotation, ω_{Ri} represents rotation frequency, and $m=0,...,M-1$. M is number of emitted pulses.

Frequency content of the m-D related part of the signal varies over time, while the rigid body part has almost constant frequency content. Thus, in order to represent this multicomponent signal we have to use TF representation. The STFT is the simplest TF representation:

$$STFT(m,k)=\sum_{i=0}^{M-1}s(i)w(i-m)e^{-j2\pi ik/M}\,, \tag{3}$$

where $w(i)$ represents window function, used to localize frequency content of the signal. In our simulations we used Hann(ing) window of length M_w, zero-padded to M. Zerro-padding is performed in order to obtain the same frequency grid as it would be in the FT calculated with M samples, and which we would like to reconstruct from the STFT samples, after m-D removal.

When all off the STFT samples are available, the FT can be accurately reconstructed by summing them over time:

$$\begin{aligned}\sum_{m=0}^{M-1}STFT(m,k)&=\sum_{i=0}^{M-1}s(i)\left[\sum_{m=0}^{M-1}w(i-m)\right]e^{-j2\pi ik/M}=\\&=\sum_{i=0}^{M-1}s(i)w_1(i)e^{-j2\pi ik/M}=S_{w_1}(k).\end{aligned} \tag{4}$$

However, in the presence of the m-D we will not have all the STFT samples available. In the presence of the m-D, we did not use the FT at the first place, since the m-D, as a non-stationary signal, covers wide range of frequencies masking the rigid body and making its detection from the FT almost impossible. On the other hand, the fact that the STFT part related to the rigid body is located at same frequencies during the entire CIT, while the m-D related part covers a wide range of frequencies, but only for the small time intervals, can be used for their separation in the STFT, before the reconstruction of the FT is performed. Then, the FT reconstructed by (4), using the STFT part related to the rigid body only, is the m-D free [7].

In order to illustrate this procedure, we may use signal (2), with $K=1$ and $P=1$. The STFT representation of this signal is given in Fig. 1.(a). It confirms that the rigid body has constant frequency content. The frequency content of the fast-rotating component changes over time. Then we can see what happens when we sort the STFT over time, Fig. 1.(b). The rigid body is stationary so sorting does not significantly change its distribution. On the other hand, the sorted m-D related part of the STFT have non-zero values over wide range of frequencies, but only for a few samples in time, i.e. in a small region with the strongest values. Therefore, if we remove these strongest STFT samples for each frequency, we will eliminate the most or all of the m-D effect. Then we can sum the remaining STFT samples over time to reconstruct the FT of the rigid body (Fig. 1.(d))

$$S_L(k) = \sum_{m \in D_k} STFT(m,k) \ , \qquad (5)$$

where, for each k, D_k is a subset of $\{0,1,2,...,M-1\}$ with M_Q elements. If we compare the reconstructed FT to the original one, given in Fig. 1.(c), it is obvious that the detectability of the rigid body is significantly improved. In almost every of our examples in [7] and [8] we removed 50% of the STFT samples in order to eliminate the m-D, so we will do the same in this paper.

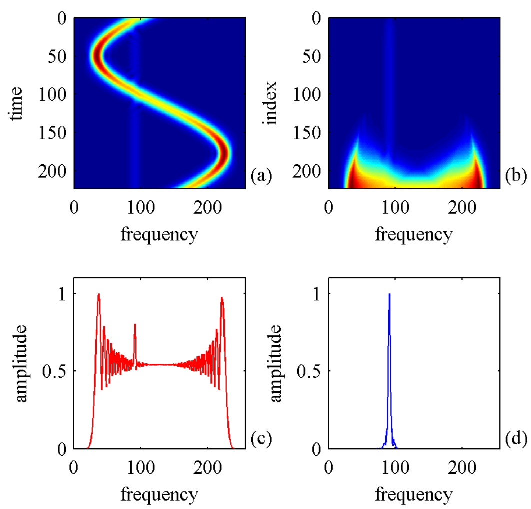

Fig. 1. One rigid body reflector and fast-rotating reflector. (a) STFT. (b) Sorted STFT. (c) Original FT. (d) FT after rigid body elimination.

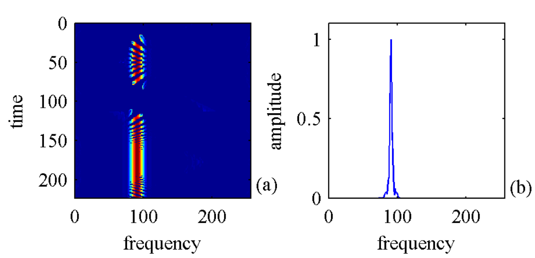

Fig. 2. Rigid body after the m-D removal. (a) STFT with 50% missing samples. (b) FT after reconstruction.

Using the procedure proposed in [7], by removing 50% of the strongest STFT samples, the m-D free FT of the rigid body is reconstructed. However, this procedure removes half of the rigid body related STFT samples, as well. The remaining STFT samples are presented in Fig. 2.(a). The FT reconstructed by using only these 50% remaining rigid body related samples in (5) have undesired side lobes, spreaded around the strong peak, Fig. 2.(b).

In [8] we proposed a genetic algorithm for the estimation of the rigid body related STFT samples removed during the process of the m-D elimination. The fact that all of the STFT samples at fixed frequency are with the same amplitudes is used in its implementation. The amplitudes for each frequency are estimated as the median amplitude of the remaining STFT samples at the same frequency. On the other hand, only the STFT samples at the frequency $k=k_0$ are in phase, and for their estimation we may use the median. Samples at the frequencies ($k=k_0+l,\ l\neq 0$) are with different phases. Possible values of these phases are randomly selected from the range $[0,2\pi)$, and binary coded [11], [12]. They form initial population in the genetic algorithm proposed in [8]. Then, the genetic algorithm is used to improve these randomly selected phases and to reconstruct the missing STFT samples. The genetic algorithm gives very good estimation of the missing STFT samples [8]. The corresponding FT (5) is highly concentrated and looks almost as if the m-D effect was not present at the first place and as if all of the STFT samples were used for its reconstruction. Moreover, it is not based on any assumption related to the form of the phases that should be estimated. Hence, the genetic algorithm performs well in any radar scenario. Its drawback is calculation burden.

In this paper, we propose a new highly computationally efficient method for reconstruction of the missing rigid body related STFT samples. We use additional information obtained by analysing the STFT of the rigid body only. If we denote the discrete FT (DFT) as the $W(k)$ and if we consider the STFT of the rigid body only we get two cases, [7]:

1. Case for $k=k_0$, corresponding to the position of the rigid body point: At this frequency, all terms in the STFT sum in (4) are the same and equal to $W(0)$.

2. Case for $k=k_0+l$, where $l\neq 0$: The STFT samples at the fixed frequency are of the form $x_l(m)=W(l)\exp(j2\pi ml/M)$, $m=0,...,M-1$, for a given l.

If we analyse more carefully these two cases, we can analytically calculate the missing, unknown STFT samples of rigid body. Amplitudes of the missing STFT samples at any frequency are estimated as the median of the remaining samples at the same frequency. Phases of the samples at frequencies near k_0 $(k=k_0+l,\ l\neq 0)$ are calculated as $2\pi ml/M$, $m=0,...,M-1$ (case 2). Note that for $k=k_0$, l equals zero (case 1).

Number of frequencies l on the both sides of the rigid body's frequency k_0 which need to be estimated equals to the number of the FT samples of the Hann(ing) window with width M_w zero-padded to M, [8],

$$|l| \leq 1 + 2(M / M_w - 1) . \quad (6)$$

This is actually width of the rigid body in the STFT. For every time instant the rigid body related STFT is the FT of the used window (Hann) centered at the frequency that corresponds to the rigid body position [8]. Therefore, in order to recover the rigid body related missing STFT samples, we need to recover the STFT samples at these frequencies only. Non-zero values at the other frequencies are due to residual m-D effect or noise and should be set to zero.

In order to better remove the m-D effect and noise in the rigid body region, we also remove and then re-estimate the STFT samples at that region if their amplitudes deviates more than 10% from the median value of the remaining STFT samples at the same frequency. Moreover, we calculate phases of all the STFT values in the rigid body related region in order to correct their values in the case that noise has altered them. All of this is feasible because the calculation process is extremely fast.

The proposed procedure is done very quickly, and the reconstructed FT is obtained much faster than in the case of using genetic algorithm, [8], while it is with the same or even higher concentration. Both procedures were tested using 3.2GHz Intel i3 processor. Average duration of the genetic algorithm was *33s*, while this newly proposed procedure was always executed in less than one second.

The proposed procedure gives better results than the genetic algorithm proposed in [8], in the most of the radar scenarios encountered in practice. However, in the case of very close rigid body components, with the STFTs overlapped in the TF plane, the genetic algorithm outperforms it. In this case, analytical form of phases for rigid body related STFT samples does not hold. The genetic algorithm, on the other hand, does not use any assumption related to the form of the STFT samples' phases. Therefore, it will successfully estimate the missing STFT samples and obtain sharp peaks in the FT, even in this case. However, due to the fast realization of the proposed procedure (less than second), it could be used as the first step after m-D removal. In the most of the practical cases it produces good results, and no further processing is needed. In the case of close components we should proceed with the genetic algorithm proposed in [8], but we need only additional second to do that.

III. SIMULATIONS

Example 1: Consider radar return of one stationary reflector and one fast-rotating reflector modeled as:

$$s(m) = \exp(-j0.3\pi m) + 15\exp(j96\cos(2\pi m / 256)) , \quad (7)$$

in the presence of complex valued, white Gaussian noise with variance $\sigma^2=4.5$. $M=256$ samples are used, while the window width is $M_w=M/8$. The STFT of the original signal and its FT are presented in Fig. 3.(a), (b), respectively. In order to reconstruct the FT of the rigid body, 50% of the smallest STFT values are summed over time. The STFT samples that were summed and the corresponding reconstructed FT are presented in Fig. 3.(c), (d). Even in the presence of very strong noise and with *15* times higher reflection coefficient of the rotating reflector than the reflection coefficient of the rigid body this procedure provides good results. However, due to the missing STFT samples in (5), the concentration of the reconstructed FT is severely lowered.

Therefore, the proposed method for analytical estimation of the missing STFT samples related to the rigid body is applied in order to improve concentration of the reconstructed rigid body's FT and to discard noise, thus improving readability of the rigid body. The reconstructed STFT and the corresponding FT obtained by summing it along time are illustrated in Fig. 3. (e), (f).

For the final representation we only use the STFT columns that were reconstructed by the proposed algorithm ($k=k_0+l$, $l\neq 0$). The STFT samples at the other columns (frequencies) are consequence of noise or residual m-D and are set to zero. In this manner we almost completely remove noise, Fig. 3. (f). Concentration of the FT is almost the same as it would have been if the m-D and noise were not present in the first place, so that all the STFT samples could be used.

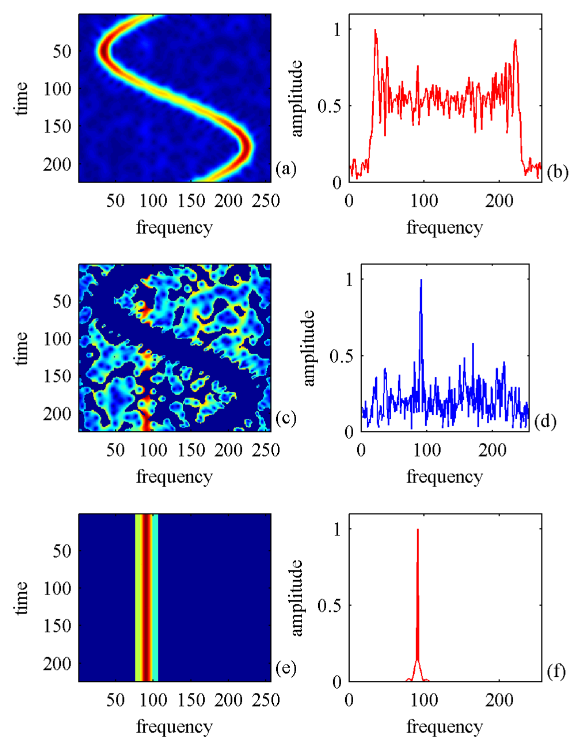

Fig. 3. One rigid body reflector and one fast-rotating reflector. (a) Original STFT. (b) Original FT. (c) STFT after m-D removal. (d) Reconstructed FT after m-D removal. (e) STFT of the rigid body after reconstruction with the proposed algorithm. (f) Reconstructed FT of the rigid body after reconstruction with the proposed algorithm.

Example 2: In this example proposed method is tested using the signal:

$$s(m) = \sum_{i=1}^{K} \exp(jy_{Bi}m + j\phi_i) + 5\exp(j96\cos(2\pi m / 256)) \quad (8)$$

which has $K=5$ rigid body points and one fast-rotating point with y_{Bi}=[1.7π, 1.8π, 1.9π, 2π, 2.1π], Φ_i=[π/3, 0, -π/6, π/3, 0], for $i=1,\ldots,5$, $M=256$ and $M_w=M/4$. In this example higher value of M_w is used, in order to achieve higher resolution in the initial STFT of five closely positioned rigid bodies. The STFT and the FT of this signal are presented in Fig. 4.(a), (b). By eliminating 50% of the strongest STFT values and by summing the remaining ones over time we remove the m-D from the analysed signal, Fig. 4.(c), (d).

The proposed method for the STFT reconstruction can be also used in the case of more than one rigid body. In this case, the proposed method should be simply applied once for each rigid body.

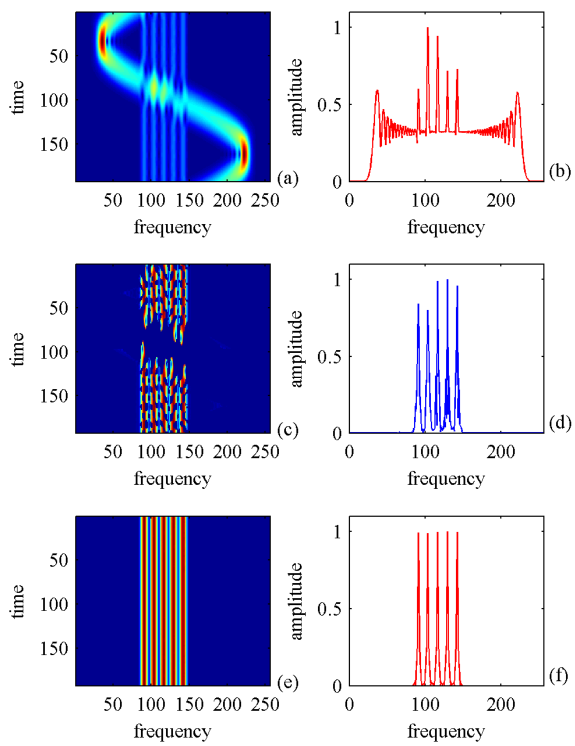

Fig. 4. Five rigid body reflectors and one fast-rotating reflector. (a) Original STFT. (b) Original FT (c) STFT after m-D removal. (d) Reconstructed FT after m-D removal. (e) STFT of the rigid body after reconstruction with the proposed algorithm. (f) Reconstructed FT of the rigid body after reconstruction with the proposed algorithm.

In order to find the positions and number of the rigid bodies we use procedure similar to the one proposed in [10]. At the beginning of this procedure we assume that there is at least one rigid body reflector and we set a threshold ε'' to 2% of the energy of the m-D free signal. This is reasonable assumption since the analysed signal correspond to a radar signal at a range cell, while there are plenty well established thresholds for detection of range cells that contains a rigid body, [7], [10]. The position of the rigid body is determined as a maximum value in the m-D free FT and detected rigid body is removed by setting zeros at the maximum value and the surrounding l values (6). Then, the remaining energy is calculated. While this energy is higher than the threshold ε'' the number of rigid bodies is incremented and the peak-peeling procedure is repeated. Using this algorithm for rigid body counting and by applying, for each detected rigid body, the proposed method for analytical reconstruction of the missing STFT samples, we get five highly concentrated peaks in the reconstructed FT. These five peaks represent five easily detectable rigid bodies, Fig. 4.(f). Final result (Fig. 4.(f)) is even better than the result obtained in [8], and it is obtained in a much shorter time.

In the case of very close components, the STFTs of different rigid body reflectors are overlapped in the TF plane and mutually interacted. Consequently, the assumption related to the form of their STFT samples' phases does not hold, and the genetic algorithm outperforms the proposed procedure.

IV. Conclusion

The micro-Doppler effect can severely decrease readability of rigid bodies in radar images. Elimination of the m-D effect is very important in order to obtain focused radar image. By removing, for each frequency, several highest STFT values the m-D effect is almost completely removed. But, in this way a part of the rigid body is also removed resulting in the lower concentrated FT of the rigid body, reconstructed by summing the remaining STFT samples. In this paper we propose a very fast method for the estimation of missing rigid body related STFT samples which improves the concentration of the reconstructed FT. Concentration of the FT, reconstructed by summing the STFT samples remaining after the m-D removal and filled up with the estimation of the missing STFT samples, is almost the same as it would have been if the m-D and noise were not present in the first place. The proposed procedure achieves high concentration of the reconstructed FT even in the presence of very strong fast moving component and high complex noise. Comparing to the recently proposed genetic algorithm for the STFT reconstruction, [8], this procedure provides almost the same, or in the presence of noise even better results, while it is much faster. However, it is based on the assumption of analytical form of the rigid body related STFT's phases and the genetic algorithm outperforms it in the case of very close rigid body reflectors with the STFTs overlapped in the TF plane. Nevertheless, the proposed procedure finishes in less than a second thus it should be always used as the first step after the m-D removal. In the most of the radar scenarios it produces highly concentrated reconstructed FT of rigid body reflectors and no additional processing is needed. In the case of close components the genetic algorithm should be used afterwards since it is not overloaded with the assumption of the analytical form of the rigid body related STFT's phases. However, we would add less than a second to its execution time.

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