II.

a)

$$y(t+4)-5\cdot y(t+3)-8\cdot y(t+2)+48\cdot y(t+1)=u(t+1)+5\cdot u(t)$$

$$z^4 y(z)-[z^4 y(0)+z^3 y(1)+z^2 y(2)+z\cdot y(3)]-5\{z^3 y(z)-[z^3 y(0)+z^2 y(1)+z\cdot y(2)]\}$$
$$-8\{z^2 y(z)-[z^2 y(0)+z\cdot y(1)]\}+48\{z\cdot y(z)-[z\cdot y(0)]\}=z\cdot u(z)-z\cdot u(0)+5\cdot u(z)$$

$y(0)=y(1)=y(2)=y(3)=u(0)=0$ (conditii initiale nule)

$$\Rightarrow(z^4-5\cdot z^3-8\cdot z^2+48\cdot z)\cdot y(z)=(z+5)\cdot u(z)$$
$$y(z)=H(z)\cdot u(z)$$

$$\Rightarrow H(z)=\frac{z+5}{z^4-5\cdot z^3-8\cdot z^2+48\cdot z}$$

b)

$$u(t)=1(t-1);\ y(0)=1;\ y(1)=-4;\ y(2)=-1;\ y(3)=5$$

$$z^4 y(z)-(z^4-4\cdot z^3-z^2+5\cdot z)-5\cdot[z^3\cdot y(z)-(z^3-4\cdot z^2-z)]-8\cdot[z^2\cdot y(z)-(z^2-4\cdot z)]+48\cdot[z\cdot y(z)-z]$$
$$=Z\{1(t)\}+5\cdot Z\{1(t-1)\}=\frac{z+5}{z-1}$$
$$(z^4-5\cdot z^3-8\cdot z^2+48\cdot z)\cdot y(z)-(z^4-9\cdot z^3+11\cdot z^2+90\cdot z)=\frac{z+5}{z-1}$$
$$(z^4-5\cdot z^3-8\cdot z^2+48\cdot z)\cdot y(z)=\frac{z^5-10\cdot z^4+2o\cdot z^3+79\cdot z^2-89\cdot z+5}{z-1}$$
$$\Rightarrow y(z)=\frac{z^5-10\cdot z^4+20\cdot z^3+79\cdot z^2-89\cdot z+5}{z^5-6\cdot z^4-3\cdot z^3+56\cdot z^2-48\cdot z}$$

$$\frac{y(z)}{z}=\frac{z^5-10\cdot z^4+20\cdot z^3+79\cdot z^2-89\cdot z+5}{z\cdot(z^5-6\cdot z^4-3\cdot z^3+56\cdot z^2-48\cdot z)}$$
$$\frac{y(z)}{z}=\frac{A}{z^2}+\frac{B}{z-1}+\frac{C}{z}+\frac{D}{z+3}+\frac{E}{z-4}+\frac{F}{(z-4)^2}$$
$$A(z^4-6z^3-3z^2+56z-48)+B(z^5-5z^4-8z^3+48z^2)+C(z^5-64z^4-3z^3+56z^2-48z)$$
$$+D(z^5-9z^4+24z^3-16z^2)+E(z^5-2z^4-11z^3+12z^2)+F(z^4+2z^3-3z^2)=z^5-10\cdot z^4+20\cdot z^3$$
$$+79\cdot z^2-89\cdot z+5$$

$z^5: B+C+D+E=1$

$z^4: A-5B-6C-9D-2E+2F=-10$

$z^3: -6A-8B-3C+24D-11E+2F=20$

$z^2: 48B+56C-16D+12E-3F=79$

$z^1: 56A-48C=-89$

$z^0: -48A=5$

$$\Rightarrow A=\frac{-5}{48};\ B=\frac{1}{6};\ C=\frac{499}{288};\ D=\frac{305}{882};\ E=\frac{-5857}{4704};\ F=\frac{219}{112}$$

$$y(z)=\frac{-5}{48}\cdot z^{-1}+\frac{1}{6}\frac{z}{z-1}+\frac{499}{288}+\frac{305}{882}\frac{z}{z+3}-\frac{5857}{4704}\frac{z}{z-4}+\frac{219}{112}\frac{z}{(z-4)^2}$$

$$y(t)=[\frac{1}{6}+\frac{305}{882}(-3)^t-\frac{5857}{4704}4^t+\frac{219}{112}4^{t-1}\cdot t]\cdot 1(t)-\frac{5}{48}u_0(t-1)+\frac{499}{288}u_0(t)$$

c)

$$y(z)=T(z)\cdot u(z)$$

$$H(z)=\frac{z+5}{z^4-5\cdot z^3-8\cdot z^2+48\cdot z}$$

$$T(z)=H(z)$$

Realizarea standard controlabila :

$$A=\begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & -48 & 8 & 5\end{bmatrix};\qquad B=\begin{bmatrix}0\\0\\0\\1\end{bmatrix};\qquad C=\begin{bmatrix}5 & 1 & 0 & 0\end{bmatrix};$$

$$x_0=O$$

Realizarea standard observabila :

$$\bar{A}=\begin{bmatrix}0 & 0 & 0 & 0\\ 1 & 0 & 0 & -48\\ 0 & 1 & 0 & 8\\ 0 & 0 & 1 & 5\end{bmatrix};\qquad \bar{B}=\begin{bmatrix}5\\1\\0\\0\end{bmatrix};\qquad \bar{C}=\begin{bmatrix}0 & 0 & 0 & 1\end{bmatrix};$$

$$x_0=O$$