I.

$$A = \begin{vmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -16 & -40 & -33 & -10 \end{vmatrix} \qquad B = \begin{vmatrix} 0 \\ 0 \\ 0 \\ 1 \end{vmatrix} \qquad X_0 = \begin{vmatrix} 0 \\ 0 \\ 0 \\ 1 \end{vmatrix}$$

$$C = \begin{vmatrix} 8 & 6 & 1 & 0 \\ 4 & 1 & 0 & 0 \end{vmatrix}$$

$$u(t) = [\ 4^*\sin3(t-4) + 2^*\cos3(t-4)\ ] * 1(t-4)$$

$$u(s) = e^{-4s} *(\ 4^* \frac{3}{s^2+9} + 2^* \frac{s}{s^2+9}) = e^{-4s} * \frac{s+12}{s^2+9}$$

$$X_0 + B^* u(s) = \begin{vmatrix} 0 \\ 0 \\ 0 \\ 1 \end{vmatrix} + \begin{vmatrix} 0 \\ 0 \\ 0 \\ u(s) \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ 0 \\ 1+u(s) \end{vmatrix}$$

$$X(s) = \frac{1}{\mathcal{X}(s)} * (sI-A)^* * [X_0 + B^*u(s)]$$

$$Y(s) = \frac{1}{\mathcal{X}(s)} * C * (sI-A)^* * [X_0 + B^*u(s)]$$

$$\mathcal{X}(s) = \det(sI-A) = s^4 + 10s^3 + 33s^2 + 40s + 16 = (s+1)^2 * (s+4)^2$$

$$(sI-A)^t = \begin{vmatrix} s & 0 & 0 & 16 \\ -1 & s & 0 & 40 \\ 0 & -1 & s & 33 \\ 0 & 0 & -1 & s+10 \end{vmatrix} \qquad (sI-A)^* = \begin{vmatrix} ? & ? & ? & -1 \\ ? & ? & ? & s \\ ? & ? & ? & s^2 \\ ? & ? & ? & s^3 \end{vmatrix}$$

II.

$$y(t+4) + 2*y(t+3) - 7*y(t+2) + 4*y(t+1) = 4*u(t+1) + 2*u(t)$$

a) $y(z) = H(z) * u(z) \Rightarrow H(z) = \dfrac{y(z)}{u(z)}$

$$y(z)*z^4 + 2*y(z)*z^3 - 7*y(z)*z^2 + 4*y(z)*z = 4*u(z)*z + 2*u(z)$$

$$H(z) = \frac{4z+2}{z^4 + 2z^3 - 7z^2 + 4z} = \frac{4z^{-3} + 2z^{-4}}{1 + 2z^{-1} - 7z^{-2} + 4z^{-3}}$$

III.

$$A = \begin{vmatrix} 0 & 0 & 0 & 0 & 0 & 1156 \\ 1 & 0 & 0 & 0 & 0 & 867 \\ 0 & 1 & 0 & 0 & 0 & -169 \\ 0 & 0 & 1 & 0 & 0 & 90 \\ 0 & 0 & 0 & 1 & 0 & -26 \\ 0 & 0 & 0 & 0 & 1 & 3 \end{vmatrix} \qquad B = \begin{vmatrix} -68 & 68 \\ -43 & -93 \\ 19 & 31 \\ -5 & -7 \\ 1 & 1 \\ 0 & 0 \end{vmatrix}$$

$$C = \begin{vmatrix} 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$

$$\mathcal{X}(s) = \det(sI-A) = s^6 -3s^5 + 26s^4 -90s^3 + 169s^2 - 867s -1156$$

$$H = \begin{vmatrix} -3 & -90 & -867 & 0 & 0 & 0 \\ 1 & 26 & 169 & -1156 & 0 & 0 \\ 0 & -3 & -90 & -867 & 0 & 0 \\ 0 & 1 & 26 & 169 & -1156 & 0 \\ 0 & 0 & -3 & -90 & -867 & 0 \\ 0 & 0 & 1 & 26 & 169 & -1156 \end{vmatrix}$$

$H_1 = -3 < 0$

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$$T(s) = C^*(sI-A)^{-1}{}^*B = \frac{R(s)}{p(s)}$$

$$T(s) = \frac{[\; s^4 -5s^3 +19s^2 -43s -68 \qquad s^4 -7s^3 +31s^2 -93s +68 \;]}{s^6 -3s^5 + 26s^4 -90s^3 + 169s^2 - 867s -1156}$$

Aplicam algoritmul lui Euclid pt aflarea cmmdc

$s^4 -5s^3 +19s^2 -43s -68$ <-- *pivot*

$s^4 -7s^3 +31s^2 -93s +68$

$s^6 -3s^5 + 26s^4 -90s^3 + 169s^2 - 867s -1156$

III.

$$\begin{array}{r|l} s^4-7s^3+31s^2-93s+68 & s^4-5s^3+19s^2-43s-68 \\ \cline{2-2} s^4-5s^3+19s^2-43s-68 & s \\ \cline{1-1} -2s^3+12s^2-50s+136 & \end{array}$$

$$\begin{array}{r|l} s^6-3s^5+26s^4-90s^3+169s^2-867s-1156 & s^4-5s^3+19s^2-43s-68 \\ \cline{2-2} s^6-5s^5+19s^4-43s^3-68s^2 & s^2+2s \\ \cline{1-1} 2s^5+7s^4-47s^3+237s^2-867s-1156 & \\ 2s^5-5s^4+19s^3-43s^2-68s & \\ \cline{1-1} 17s^4-85s^3+323s^2-731s-1156 & \end{array}$$

Algoritmul Euclid pt:

$-2s^3+12s^2-50s+136$ <-- *pivot*

$s^4-5s^3+19s^2-43s-68$

$17s^4-85s^3+323s^2-731s-1156$

$$\begin{array}{r|l} s^4-5s^3+19s^2-43s-68 & -2s^3+12s^2-50s+136 \\ \cline{2-2} s^4-6s^3+25s^2-68s & -\frac{1}{2}s-\frac{1}{2} \\ \cline{1-1} s^3-6s^2+25s-68 & \\ s^3-6s^2+25s-68 & \\ \cline{1-1} 0 & \end{array}$$

Deci cmmdc $= -2s^3+12s^2-50s+136 \;/: (-1/2)$

$= s^3-6s^2+25s-68$

III.

$$T(s) = \frac{[\, s+1 \qquad s-1 \,]}{s^3 + 3s^2 + 19s + 17} = \frac{R(s)}{p(s)}$$

$$H = \begin{vmatrix} 3 & 17 & 0 \\ 1 & 19 & 0 \\ 0 & 3 & 17 \end{vmatrix}$$

$H_1 = 3 > 0$

$H_2 = 3*19 - 17 > 0$

$H_3 = 3*19*17 - 17^2 > 0$

Deci SLN este stabil extern