

Tema de Casa 1

I. Se considera sistemul liniar neted (SLN) având realizarea de stare

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -a^2bc & -[a^2(b+c)+2abc] & -[a^2+2a(b+c)+bc] & -(2a+b+c) \end{bmatrix}; B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix};$$

$$x_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ (-1)^d \cdot e \end{bmatrix}; C = \begin{bmatrix} bd & b+c & 1 & 0 \\ ec & e & 0 & 0 \end{bmatrix}; ss^T = [x_1 \ x_2 \ x_3 \ x_4 \ y_1 \ y_2]$$

Sa se determine(numai) $ss(b)$, daca x_0 este cel mentionat, iar

$$u(t) = [b \cdot \sin 3(t-c) + d \cdot \cos 3(t-c)] \cdot 1(t-c)$$

II. Fie sistemul liniar discret (SLD), exprimat intrare-iesire, prin ecuatia cu diferente

$$y(t+4) + (b-2a) \cdot y(t+3) + (a^2-2ab) \cdot y(t+2) + a^2b \cdot y(t+1) = c \cdot u(t+1) + 5 \cdot u(t) \quad (1)$$

a) Sa se determine functia de transfer a sistemului (conditii initiale nule)

b) Sa se determine, utilizand transformata Z, raspunsul sistemului

(reprezentat ca in (1)), daca $u(t) = 1(t-1); y(0)=c, y(1)=-e, y(2)=-c, y(3)=d$ (2)

c) Sa se determine realizari(standard) de stare(inclusiv x_0) pentru sistemul (1)

III. Sa se analizeze stabilitatea interna si externa a SLN (utilizand criteriul Hurwitz)

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & ce(a^2+b^2)^2 \\ 1 & 0 & 0 & 0 & 0 & (c-e)(a^2+b^2)^2 \\ 0 & 1 & 0 & 0 & 0 & 2ce(b^2-a^2)-(a^2+b^2)^2 \\ 0 & 0 & 1 & 0 & 0 & 2(c-e)(b^2-a^2) \\ 0 & 0 & 0 & 1 & 0 & ce-2(b^2-a^2) \\ 0 & 0 & 0 & 0 & 1 & c-e \end{bmatrix};$$

$$B = \begin{bmatrix} -c(a^2+b^2) & c(a^2+b^2) \\ a^2+b^2+2ac-c(a^2+b^2) & -a^2-b^2-2ac-c(a^2+b^2) \\ a^2+b^2+2ac-2a-c & a^2+b^2+2ac+2a+c \\ 1-2a-c & -1-2a-c \\ 1 & 1 \\ 0 & 0 \end{bmatrix}; C = [0 \ 0 \ 0 \ 0 \ 0 \ 1]$$

n = numarul de ordine

n = 8

a = n mod 5 + 1

a = 4

b = n mod 6 + 1

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b = 3

c = n mod 4 + 1

c = 1

d = ultima cifra a grupei

d = 5

e = n mod 5 + 1

e = 4

Enunt personalizat:

I. Se considera sistemul liniar neted (SLN) avand realizarea de stare

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -48 & -88 & -51 & -12 \end{bmatrix}; B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}; x_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -4 \end{bmatrix};$$

$$C = \begin{bmatrix} 15 & 8 & 1 & 0 \\ 4 & 4 & 0 & 0 \end{bmatrix}; ss^T = [x_1 \ x_2 \ x_3 \ x_4 \ y_1 \ y_2]$$

Sa se determine(numai) ss(3), daca x_0 este cel mentionat, iar

$$u(t) = [3 \cdot \sin 3(t-1) + 5 \cdot \cos 3(t-1)] \cdot 1(t-1)$$

II. Fie sistemul liniar discret (SLD), exprimat intrare-iesire, prin ecuatie cu diferente

$$y(t+4) + (3-2 \cdot 4) \cdot y(t+3) + (4^2 - 2 \cdot 4 \cdot 3) \cdot y(t+2) + 4^2 \cdot 3 \cdot y(t+1) = 1 \cdot u(t+1) + 5 \cdot u(t)$$

$$y(t+4) - 5 \cdot y(t+3) - 8 \cdot y(t+2) + 48 \cdot y(t+1) = u(t+1) + 5 \cdot u(t) \quad (1)$$

a) Sa se determine functia de transfer a sistemului (conditii initiale nule)

b) Sa se determine, utilizand transformata Z, raspunsul sistemului

(reprezentat ca in (1)), daca $u(t) = 1(t-1); y(0)=1, y(1)=-4, y(2)=-1, y(3)=5$ (2)

c) Sa se determine realizari(standard) de stare(inclusiv x_0) pentru sistemul (1)

III. Sa se analizeze stabilitatea interna si externa a SLN(utilizand criteriul Hurwitz)

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 2500 \\ 1 & 0 & 0 & 0 & 0 & -1875 \\ 0 & 1 & 0 & 0 & 0 & -681 \\ 0 & 0 & 1 & 0 & 0 & 42 \\ 0 & 0 & 0 & 1 & 0 & 18 \\ 0 & 0 & 0 & 0 & 1 & -3 \end{bmatrix}; B = \begin{bmatrix} -25 & 25 \\ 8 & -58 \\ 24 & 42 \\ -8 & -10 \\ 1 & 1 \\ 0 & 0 \end{bmatrix}; C = [0 \ 0 \ 0 \ 0 \ 0 \ 1]$$

Rezolvare :

I.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -48 & -88 & -51 & -12 \end{bmatrix}; \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}; \quad x_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -4 \end{bmatrix}$$

$$C = \begin{bmatrix} 15 & 8 & 1 & 0 \\ 4 & 4 & 0 & 0 \end{bmatrix};$$

$$u(t) = [3 \cdot \sin 3(t-1) + 5 \cdot \cos 3(t-1)] \cdot 1(t-1)$$

$$u(t) = [3 \cdot \sin 3(t-1)] \cdot 1(t-1) + [5 \cdot \cos 3(t-1)] \cdot 1(t-1)$$

$$u(s) = e^{-s} \cdot 3 \frac{3}{s^2+9} + e^{-s} \cdot 5 \frac{s}{s^2+9}$$

$$u(s) = \frac{9+5s}{e^s(s^2+9)}$$

$$x_0 + B \cdot u(s) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{9+5s}{e^s(s^2+9)} - 4 \end{bmatrix}$$

$$(sI-A) = \begin{bmatrix} s & -1 & 0 & 0 \\ 0 & s & -1 & 0 \\ 0 & 0 & s & -1 \\ 48 & 88 & 51 & s+12 \end{bmatrix}$$

$$\chi(s) = \det(sI-A)$$

$$\chi(s) = (-1)^{1+1} \cdot s \cdot \begin{vmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 88 & 51 & s+12 \end{vmatrix} + (-1)^{1+2} \cdot (-1) \cdot \begin{vmatrix} 0 & -1 & 0 \\ 0 & s & -1 \\ 48 & 51 & s+12 \end{vmatrix}$$

$$\chi(s) = s^3(s+12) + 88s + 51s^2 + 48$$

$$\chi(s) = s^4 + 12s^3 + 51s^2 + 88s + 48$$

$$\chi(s) = (s+1)(s+3)(s+4)^2$$

$$(sI-A)^T = \begin{bmatrix} s & 0 & 0 & 48 \\ -1 & s & 0 & 88 \\ 0 & -1 & s & 51 \\ 0 & 0 & -1 & s+12 \end{bmatrix}$$

$$x(s) = \frac{1}{\chi(s)}(sI-A)^* \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{9+5s}{e^s(s^2+9)} - 4 \end{bmatrix}$$

$$x(s) = \frac{1}{(s+1)(s+3)(s+4)^2} \begin{bmatrix} ? & ? & ? & e_{14}^* \\ ? & ? & ? & e_{24}^* \\ ? & ? & ? & e_{34}^* \\ ? & ? & ? & e_{44}^* \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{9+5s}{e^s(s^2+9)} - 4 \end{bmatrix}$$

$$e_{14}^* = (-1)^{1+4} \begin{vmatrix} -1 & s & 0 \\ 0 & -1 & s \\ 0 & 0 & -1 \end{vmatrix} = 1$$

$$e_{24}^* = (-1)^{2+4} \begin{vmatrix} s & 0 & 0 \\ 0 & -1 & s \\ 0 & 0 & -1 \end{vmatrix} = s$$

$$e_{34}^* = (-1)^{3+4} \begin{vmatrix} s & 0 & 0 \\ -1 & s & 0 \\ 0 & 0 & -1 \end{vmatrix} = s^2$$

$$e_{44}^* = (-1)^{4+4} \begin{vmatrix} s & 0 & 0 \\ -1 & s & 0 \\ 0 & -1 & s \end{vmatrix} = s^3$$

$$x(s) = \begin{bmatrix} \frac{9+5s}{e^s(s^2+9)(s+1)(s+3)(s+4)^2} - \frac{4}{(s+1)(s+3)(s+4)^2} \\ \frac{s(9+5s)}{e^s(s^2+9)(s+1)(s+3)(s+4)^2} - \frac{4s}{(s+1)(s+3)(s+4)^2} \\ \frac{s^2(9+5s)}{e^s(s^2+9)(s+1)(s+3)(s+4)^2} - \frac{4s^2}{(s+1)(s+3)(s+4)^2} \\ \frac{s^3(9+5s)}{e^s(s^2+9)(s+1)(s+3)(s+4)^2} - \frac{4s^3}{(s+1)(s+3)(s+4)^2} \end{bmatrix}$$

$$x_3(s) = \frac{e^{-s} s^2 (9+5s)}{(s^2+9)(s+1)(s+3)(s+4)^2} - \frac{4s^2}{(s+1)(s+3)(s+4)^2}$$

$$\frac{9s^2+5s^3}{(s^2+9)(s+1)(s+3)(s+4)^2} = \frac{A}{s+1} + \frac{Bs+C}{s^2+9} + \frac{D}{s+3} + \frac{E}{s+4} + \frac{F}{(s+4)^2}$$

$$9s^2+5s^3 = A(s^2+9)(s+3)(s+4)^2 + (Bs+C)(s+1)(s+3)(s+4)^2 + D(s+1)(s^2+9)(s+4)^2 + E(s+1)(s^2+9)(s+3)(s+4) + F(s+1)(s^2+9)(s+3)$$

$$9s^2+5s^3 = A(s^5+11s^4+49s^3+147s^2+360s+432) + B(s^5+12s^4+51s^3+88s^2+48s) + C(s^4+12s^3+51s^2+88s+48) + D(s^5+9s^4+33s^3+97s^2+216s+144) + E(s^5+8s^4+28s^3+84s^2+171s+108) + F(s^4+4s^3+12s^2+36s+27)$$

$$s^5: A+B+D+E=0$$

$$s^4: 11A+12B+C+9D+8E+F=0$$

$$s^3: 49A+51B+12C+33D+28E+4F=5$$

$$s^2: 147A+88B+51C+97D+84E+12F=9$$

$$s^1: 360A+48B+88C+216D+171E+36F=0$$

$$s^0: 432A+48C+144D+108E+27F=0$$

$$A = \frac{1}{45}; B = \frac{147}{1250}; C = \frac{387}{1250}; D = \frac{3}{2}; E = \frac{-9224}{5625}; F = \frac{-176}{75}$$

$$\frac{4s^2}{(s+1)(s+3)(s+4)^2} = \frac{A}{s+3} + \frac{B}{s+4} + \frac{C}{(s+4)^2} + \frac{D}{s+1}$$

$$4s^2 = A(s+4)^2(s+1) + B(s+3)(s+4)(s+1) + C(s+3)(s+1) + D(s+3)(s+4)^2$$

$$s^3: A+B+D=0$$

$$s^2: 9A+8B+C+11D=4$$

$$s^1: 24A+19B+4C+40D=0$$

$$s^0: 16A+12B+3C+48D=0$$

$$A = -18; B = \frac{160}{9}; C = \frac{64}{3}; D = \frac{2}{9}$$

$$x_3(s) = e^{-s} \left(\frac{1}{45(s+1)} + \frac{147s}{1250(s^2+9)} + \frac{387}{1250(s^2+9)} + \frac{3}{2(s+3)} - \frac{9224}{5625(s+2)} - \frac{176}{75(s+4)^2} \right) - \left(\frac{-18}{s+3} + \frac{160}{9(s+4)} + \frac{64}{3(s+4)^2} + \frac{2}{9(s+1)} \right)$$

$$x_3(t) = \left(\frac{1}{45} \cdot e^{-1(t-1)} + \frac{147}{1250} \cdot \cos 3(t-1) + \frac{129}{1250} \cdot \sin 3(t-1) + \frac{3}{2} \cdot e^{-3(t-1)} - \frac{9224}{5625} \cdot e^{-2(t-1)} - \frac{176}{75} \cdot (t-1) e^{-4(t-1)} \right)$$

$$1(t-1) - \left(-18e^{-3t} + \frac{160}{9} \cdot e^{-4t} + \frac{64}{3} \cdot t \cdot e^{-4} + \frac{2}{9} \cdot e^{-t} \right) \cdot 1(t)$$

II.

a)

$$y(t+4)-5\cdot y(t+3)-8\cdot y(t+2)+48\cdot y(t+1)=u(t+1)+5\cdot u(t)$$

$$z^4 y(z)-[z^4 y(0)+z^3 y(1)+z^2 y(2)+z\cdot y(3)]-5\{z^3 y(z)-[z^3 y(0)+z^2 y(1)+z\cdot y(2)]\}-8\{z^2 y(z)-[z^2 y(0)+z\cdot y(1)]\}+48\{z\cdot y(z)-[z\cdot y(0)]\}=z\cdot u(z)-z\cdot u(0)+5\cdot u(z)$$

$$y(0)=y(1)=y(2)=y(3)=u(0)=0 \quad (\text{conditii initiale nule})$$

$$\Rightarrow (z^4-5\cdot z^3-8\cdot z^2+48\cdot z)\cdot y(z)=(z+5)\cdot u(z)$$

$$y(z)=H(z)\cdot u(z)$$

$$\Rightarrow H(z)=\frac{z+5}{z^4-5\cdot z^3-8\cdot z^2+48\cdot z}$$

b)

$$u(t)=1(t-1); y(0)=1; y(1)=-4; y(2)=-1; y(3)=5$$

$$z^4 y(z)-(z^4-4\cdot z^3-z^2+5\cdot z)-5\cdot [z^3\cdot y(z)-(z^3-4\cdot z^2-z)]-8\cdot [z^2\cdot y(z)-(z^2-4\cdot z)]+48\cdot [z\cdot y(z)-z]=Z\{1(t)\}+5\cdot Z\{1(t-1)\}=\frac{z+5}{z-1}$$

$$(z^4-5\cdot z^3-8\cdot z^2+48\cdot z)\cdot y(z)-(z^4-9\cdot z^3+11\cdot z^2+90\cdot z)=\frac{z+5}{z-1}$$

$$(z^4-5\cdot z^3-8\cdot z^2+48\cdot z)\cdot y(z)=\frac{z^5-10\cdot z^4+20\cdot z^3+79\cdot z^2-89\cdot z+5}{z-1}$$

$$\Rightarrow y(z)=\frac{z^5-10\cdot z^4+20\cdot z^3+79\cdot z^2-89\cdot z+5}{z^5-6\cdot z^4-3\cdot z^3+56\cdot z^2-48\cdot z}$$

$$\frac{y(z)}{z}=\frac{z^5-10\cdot z^4+20\cdot z^3+79\cdot z^2-89\cdot z+5}{z\cdot (z^5-6\cdot z^4-3\cdot z^3+56\cdot z^2-48\cdot z)}$$

$$\frac{y(z)}{z}=\frac{A}{z^2}+\frac{B}{z-1}+\frac{C}{z}+\frac{D}{z+3}+\frac{E}{z-4}+\frac{F}{(z-4)^2}$$

$$A(z^4-6z^3-3z^2+56z-48)+B(z^5-5z^4-8z^3+48z^2)+C(z^5-64z^4-3z^3+56z^2-48z)+D(z^5-9z^4+24z^3-16z^2)+E(z^5-2z^4-11z^3+12z^2)+F(z^4+2z^3-3z^2)=z^5-10\cdot z^4+20\cdot z^3+79\cdot z^2-89\cdot z+5$$

$$z^5: B+C+D+E=1$$

$$z^4: A-5B-6C-9D-2E+2F=-10$$

$$z^3: -6A-8B-3C+24D-11E+2F=20$$

$$z^2: 48B+56C-16D+12E-3F=79$$

$$z^1: 56A-48C=-89$$

$$z^0: -48A=5$$

$$\Rightarrow A=\frac{-5}{48}; B=\frac{1}{6}; C=\frac{499}{288}; D=\frac{305}{882}; E=\frac{-5857}{4704}; F=\frac{219}{112}$$

$$y(z) = \frac{-5}{48} \cdot z^{-1} + \frac{1}{6} \frac{z}{z-1} + \frac{499}{288} + \frac{305}{882} \frac{z}{z+3} - \frac{5857}{4704} \frac{z}{z-4} + \frac{219}{112} \frac{z}{(z-4)^2}$$

$$y(t) = \left[\frac{-5}{48} u_0(t-1) + \frac{1}{6} + \frac{499}{288} u_0(t) + \frac{305}{882} (-3)^t - \frac{5857}{4704} 4^t + \frac{219}{112} 4^{t-1} \cdot t \right] \cdot 1(t)$$

c)

III.

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 2500 \\ 1 & 0 & 0 & 0 & 0 & -1875 \\ 0 & 1 & 0 & 0 & 0 & -681 \\ 0 & 0 & 1 & 0 & 0 & 42 \\ 0 & 0 & 0 & 1 & 0 & 18 \\ 0 & 0 & 0 & 0 & 1 & -3 \end{bmatrix}; B = \begin{bmatrix} -25 & 25 \\ 8 & -58 \\ 24 & 42 \\ -8 & -10 \\ 1 & 1 \\ 0 & 0 \end{bmatrix}; C = [0 \ 0 \ 0 \ 0 \ 0 \ 1]$$

a) stabilitatea internă

$\chi(s) = -2500 + 1875 \cdot s + 681 \cdot s^2 - 42 \cdot s^3 - 18 \cdot s^4 + 3 \cdot s^5 + s^6$ (Realizare standard observabilă)

$$\chi(s) = s^6 + 3 \cdot s^5 - 18 \cdot s^4 - 42 \cdot s^3 + 681 \cdot s^2 + 1875 \cdot s - 2500$$

$$H = \begin{bmatrix} 3 & -42 & 1875 & 0 & 0 & 0 \\ 1 & -18 & 681 & -2500 & 0 & 0 \\ 0 & 3 & -42 & 1875 & 0 & 0 \\ 0 & 1 & -18 & 681 & -2500 & 0 \\ 0 & 0 & 3 & -42 & 1875 & 0 \\ 0 & 0 & 1 & -18 & 681 & -2500 \end{bmatrix}$$

$$H_1 = 3 > 0$$

$$H_2 = 3 \cdot (-18) - 1 \cdot (-42) = -12 > 0 \Rightarrow \text{SLN nu e stabil intern}$$

b) stabilitatea externă

$$T(s) = C \cdot (sI - A)^{-1} \cdot B = \frac{\bar{R}(s)}{\chi(s)} = \frac{R(s)}{p(s)}$$

$$T(s) = [1 \ s \ s^2 \ s^3 \ s^4 \ s^5] \cdot B \frac{1}{\chi(s)}$$

$$T(s) = [1 \ s \ s^2 \ s^3 \ s^4 \ s^5] \cdot \begin{bmatrix} -25 & 25 \\ 8 & -58 \\ 24 & 42 \\ -8 & -10 \\ 1 & 1 \\ 0 & 0 \end{bmatrix} \frac{1}{s^6 + 3 \cdot s^5 - 18 \cdot s^4 - 42 \cdot s^3 + 681 \cdot s^2 + 1875 \cdot s - 2500}$$

$$T(s) = \frac{[-25:25] \cdot 1 + [8:-58] \cdot s + [24:42] \cdot s^2 + [-8:-10] \cdot s^3 + [1:1] \cdot s^4 + [0:0] \cdot s^5}{s^6 + 3 \cdot s^5 - 18 \cdot s^4 - 42 \cdot s^3 + 681 \cdot s^2 + 1875 \cdot s - 2500}$$

$$T(s) = \frac{[s^4 - 8s^3 + 24s^2 + 8s - 25 : s^4 - 10s^3 + 42s^2 - 58s + 25]}{s^6 + 3 \cdot s^5 - 18 \cdot s^4 - 42 \cdot s^3 + 681 \cdot s^2 + 1875 \cdot s - 2500} = \frac{[r_{11}^- : r_{12}^-]}{\chi(s)}$$

$$\chi(s) = (s-1)(s+4)(s^2-8s+25)(s^2+8s+25)$$

$$r_{11}^- = (s-1)(s+1)(s^2-8s+25)$$

$$r_{12}^- = (s-1)^2(s^2-8s+25)$$

$$\begin{array}{ccc} \frac{s^4 - 8s^3 + 24s^2 + 8s - 25}{s^4 - 10s^3 + 42s^2 - 58s + 25} & \rightarrow & \frac{s^4 - 8s^3 + 24s^2 + 8s - 25}{-2s^3 + 18s^2 - 64s + 50} \\ s^6 + 3s^5 - 18s^4 - 42s^3 + 681s^2 + 1875s - 2500 & & 0 \end{array}$$

$$\rightarrow \begin{array}{ccc} \frac{s^3 - 9s^2 + 32s - 25}{s^4 - 8s^3 + 24s^2 + 8s - 25} & \rightarrow & \frac{s^3 - 9s^2 + 32s - 25}{0} \\ & & 0 \end{array}$$

$$\begin{array}{l} \frac{s^6 + 3s^5 - 18s^4 - 42s^3 + 681s^2 + 1875s - 2500}{-s^6 + 8s^5 - 24s^4 - 8s^3 + 25s^2} \\ \hline 11s^5 - 42s^4 - 50s^3 + 706s^2 + 1875s - 2500 \\ \hline -11s^5 + 88s^4 - 264s^3 - 88s^2 + 275s \\ \hline 46s^4 - 314s^3 + 618s^2 + 2150s - 2500 \\ \hline -46s^4 + 368s^3 - 1104s^2 - 368s - 1150 \\ \hline 54s^3 - 486s^2 + 1782s - 1350 \\ \hline 54(s^3 - 9s^2 + 33s - 25) \end{array} \quad \div \frac{s^4 - 8s^3 + 24s^2 + 8s - 25}{s^2 + 11s + 64}$$

$$\begin{array}{l} \frac{s^4 - 8s^3 + 24s^2 + 8s - 25}{-s^4 + 9s^3 - 33s^2 + 25s} \\ \hline s^3 - 9s^2 + 33s - 25 \\ \hline -s^3 + 9s^2 - 33s + 25 \\ \hline 0 \end{array} \quad \div \frac{s^3 - 9s^2 + 33s - 25}{s+1}$$

$$\begin{array}{r}
s^6 + 3s^5 - 18s^4 - 42s^3 + 681s^2 + 1875s - 2500 \\
\underline{-s^6 + 9s^5 - 33s^4 + 25s^3} \\
12s^5 - 51s^4 - 17s^3 + 681s^2 + 1875s - 2500 \\
\underline{-12s^5 + 108s^4 - 396s^3 + 3000s^2} \\
57s^4 - 413s^3 + 981s^2 + 1875s - 2500 \\
\underline{-57s^4 + 513s^3 - 1881s^2 + 1425s} \\
100s^3 - 900s^2 + 3300s - 2500 \\
\underline{-100s^3 + 800s^2 - 3300s + 2500} \\
0
\end{array}
\quad \div \quad \frac{s^3 - 9s^2 + 33s - 25}{s^3 + 12s^2 + 57s + 100}$$

$$p(s) = \frac{\chi(s)}{\text{cmmdc}\{\chi(s), r_{11}^-, r_{12}^-\}} = s^3 + 12s^2 + 57s + 100$$

$$H = \begin{bmatrix} 12 & 100 & 0 \\ 1 & 57 & 0 \\ 0 & 12 & 100 \end{bmatrix} \quad \begin{array}{l} H_1 = 12 > 0 \\ H_2 = 12 \cdot 57 - 100 = 674 > 0 \end{array}$$

$$H_3 = 12 \cdot 57 \cdot 100 - 100 \cdot 100 = 100 \cdot H_2 > 0$$

\Rightarrow SLN e strict stabil extern