In[20]:=
```
NN = 1241143;
k = 8 × 9 × 5 × 7 × 11 × 13
```

Out[21]= 360360

In[22]:=
```
Mod[2^k - 1, NN]
```

Out[22]= 861525

In[23]:=
```
GCD[Mod[2^k - 1, NN], NN]
```

Out[23]= 547

In[24]:=
```
FactorInteger[NN]
```

Out[24]= {{547, 1}, {2269, 1}}

In[25]:=
```
FactorInteger[546]
```

Out[25]= {{2, 1}, {3, 1}, {7, 1}, {13, 1}}

In[121]:=
```
l = 100;
p = RandomPrime[{2^l, 2^(l+1)}];
q = RandomPrime[{2^l, 2^(l+1)}];
NN = p q
FactorInteger[p - 1]
FactorInteger[q - 1]
```

Out[124]= 3 658 257 361 478 318 230 350 717 195 121 690 412 201 036 551 628 760 139 948 459

Out[125]= {{2, 4}, {3, 1}, {7, 1}, {23, 1}, {37, 1},
{47, 1}, {83, 1}, {1 844 791 708 796 414 711 521, 1}}

Out[126]= {{2, 1}, {3, 4}, {397, 1}, {443, 1}, {16 123 099, 1}, {19 990 561, 1}, {193 597 337, 1}}

In[34]:=
```
k = 8 × 9 × 5 × 7 × 11;
GCD[Mod[2^k - 1, NN], NN]
p
q
```

Out[35]= 199

Out[36]= 199

Out[37]= 131

In[127]:=
```
L[n_, u_, v_] := Exp[v Log[n]^u Log[Log[n]]^(1-u)]
```

In[131]:=
```
L[nn, 0, v]
```

Out[131]= Log[nn]^v

In[132]:=
```
L[nn, 1, v]
```

Out[132]= nn^v

Quadratic Sieve

In[136]:= $QS[n\_] := L\left[n, \frac{1}{2}, 1\right]$

Number Field Sieve (NFS/GNFS)

In[135]:= $NFS[n\_] := L\left[n, \frac{1}{3}, \left(\frac{64}{9}\right)^{\frac{1}{3}}\right]$

In[146]:= `DiscretePlot[NFS[10^n] / QS[10^n], {n, 10, 200}] // N // MatrixForm`

Out[146]//MatrixForm=