Assignment 1: Registration of Fluorescein Angiography Retinal Images

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Computer Science and Engineering Program
CS419 - Digital Image and Analysis
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1. Padding and Calculating the Affine Transformation Matrix

1.1 Padding the Images

Given two images, a reference image I_{ref} and a test image I_{test}, with dimensions $(H_{\text{ref}}, W_{\text{ref}})$ and $(H_{\text{test}}, W_{\text{test}})$, we aim to pad the smaller image to match the dimensions of the larger image. The target dimensions are defined as:

$$H_{\text{target}} = \max(H_{\text{ref}}, H_{\text{test}})$$

$$W_{\text{target}} = \max(W_{\text{ref}}, W_{\text{test}})$$

For the reference image I_{ref}, the padding required on each side is computed as follows:

$$\begin{aligned}
\text{pad}_{\text{top}}^{\text{ref}} &= \left\lfloor \frac{H_{\text{target}} - H_{\text{ref}}}{2} \right\rfloor \\
\text{pad}_{\text{bottom}}^{\text{ref}} &= H_{\text{target}} - H_{\text{ref}} - \text{pad}_{\text{top}}^{\text{ref}} \\
\text{pad}_{\text{left}}^{\text{ref}} &= \left\lfloor \frac{W_{\text{target}} - W_{\text{ref}}}{2} \right\rfloor \\
\text{pad}_{\text{right}}^{\text{ref}} &= W_{\text{target}} - W_{\text{ref}} - \text{pad}_{\text{left}}^{\text{ref}}
\end{aligned}$$

Similarly, for the test image I_{test}:

$$
\begin{aligned}
\text{pad}_{\text{top}}^{\text{test}} &= \left\lfloor \frac{H_{\text{target}} - H_{\text{test}}}{2} \right\rfloor \\
\text{pad}_{\text{bottom}}^{\text{test}} &= H_{\text{target}} - H_{\text{test}} - \text{pad}_{\text{top}}^{\text{test}} \\
\text{pad}_{\text{left}}^{\text{test}} &= \left\lfloor \frac{W_{\text{target}} - W_{\text{test}}}{2} \right\rfloor \\
\text{pad}_{\text{right}}^{\text{test}} &= W_{\text{target}} - W_{\text{test}} - \text{pad}_{\text{left}}^{\text{test}}
\end{aligned}
$$

After computing these padding values, both images will have dimensions $(H_{\text{target}}, W_{\text{target}})$ for further processing.

1.2 Calculating the Affine Transformation Matrix

The affine transformation between corresponding points in the reference image I_{ref} and the test image I_{test} can be expressed as follows:

$$
\begin{pmatrix} x_i' \\ y_i' \\ 1 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & t_x \\ a_{21} & a_{22} & t_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix}
$$

Where:

- $a_{11}, a_{12}, a_{21}, a_{22}$ are the coefficients of the affine transformation matrix T.
- t_x, t_y are the translation components.

Rearranging the transformation leads to the following linear equations:

1. $x_i' = a_{11}x_i + a_{12}y_i + t_x$
2. $y_i' = a_{21}x_i + a_{22}y_i + t_y$

These equations can be represented in matrix form:

$$
A \cdot \begin{pmatrix} a_{11} \\ a_{12} \\ t_x \\ a_{21} \\ a_{22} \\ t_y \end{pmatrix} = \begin{pmatrix} x_1' \\ y_1' \\ x_2' \\ y_2' \\ \vdots \\ x_n' \\ y_n' \end{pmatrix}
$$

Where the matrix A is constructed as follows:

$$A = \begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_1 & y_1 & 1 \\ x_2 & y_2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_2 & y_2 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_n & y_n & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_n & y_n & 1 \end{bmatrix}$$

Example Corresponding Points

The following pairs of corresponding points are provided:

Reference Points (x_i, y_i)	Test Points (x'_i, y'_i)
$(388, 822)$	$(388, 822)$
$(357, 1849)$	$(357, 1849)$
$(1153, 2487)$	$(1153, 2487)$
$(1843, 400)$	$(1843, 400)$
$(2358, 2000)$	$(2358, 2000)$
$(2556, 1198)$	$(2556, 1198)$
$(1949, 1194)$	$(1949, 1194)$
$(670, 1192)$	$(670, 1192)$
$(1944, 1460)$	$(1944, 1460)$
$(1328, 1784)$	$(1328, 1784)$

Table 1: Corresponding Points for Affine Transformation

This results in the matrix A:

$$
A = \begin{bmatrix}
388 & 822 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 388 & 822 & 1 \\
357 & 1849 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 357 & 1849 & 1 \\
1153 & 2487 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1153 & 2487 & 1 \\
1843 & 400 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1843 & 400 & 1 \\
2358 & 2000 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 2358 & 2000 & 1 \\
2556 & 1198 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 2556 & 1198 & 1 \\
1949 & 1194 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1949 & 1194 & 1 \\
670 & 1192 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 670 & 1192 & 1 \\
1944 & 1460 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1944 & 1460 & 1 \\
1328 & 1784 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1328 & 1784 & 1
\end{bmatrix}
$$

2. Applying the Affine Transformation Matrix

To register the test image to the reference image, we utilize the affine transformation matrix obtained from the corresponding control points. The function `registered_image` computes this transformation and applies it to the reference image.

2.1 Affine Transformation Calculation

In this function, we first compute the affine transformation matrix using the `affTransMat` function. The transformation matrix captures the necessary scaling, rotation, and translation needed to align the test image with the reference image.

2.2 Application of the Transformation

Next, we apply the affine transformation to the reference image using the `warpAffine` function. This process involves the following steps:

1. Compute the affine transformation matrix T from control points. 2. Use `warpAffine` to apply T to the reference image.

This alignment ensures that the test image is accurately registered according to the control points defined previously.

3. Measuring the MSE for Method Accuracy

To evaluate the accuracy of our affine transformation method, we need to assess how closely the registered image approximates the reference image. This involves two key mathematical steps:

1. Inverse Transformation: Transforming the registered image back into the recovered image using the inverse of the transformation matrix. 2. Mean Squared Error (MSE) Calculation: Quantifying the difference between the recovered image and the reference image.

3.1 Inverse Transformation

The first step is to apply the inverse transformation matrix T^{-1} to the registered image. Given the affine transformation matrix T derived from the control points, we can compute its inverse. The transformation can be represented in homogeneous coordinates, allowing us to express points in a 2D space as:

$$\mathbf{x}' = T \cdot \mathbf{x}$$

To recover the original image, we must perform the following operation:

$$\mathbf{x} = T^{-1} \cdot \mathbf{x}'$$

where $\mathbf{x}$ is a point in the reference image and $\mathbf{x}'$ is the corresponding point in the registered image. The application of T^{-1} effectively reverses the affine transformation, aligning the registered image back to its original position relative to the reference image.

3.2 Mean Squared Error Calculation

Next, we compute the Mean Squared Error (MSE) to quantify the difference between the recovered image and the reference image. The MSE is defined mathematically as follows:

$$\text{MSE} = \frac{1}{N} \sum_{i=1}^{N} (I_A(i) - I_B(i))^2$$

where I_A and I_B represent the pixel values of the recovered and reference images, respectively, and N is the total number of pixels. This equation sums the squared differences of pixel values across all corresponding pixels in the two images, providing an aggregate measure of the error.

A lower MSE value indicates better alignment and accuracy of the transformation method, as it signifies that the recovered image closely resembles the reference image. This quantitative assessment allows us to validate the effectiveness of the affine transformation process in registering the images.

Lastly, in this assignment the MSE values of 1.14697 and 1.27500 indicate successful image registration, reflecting minimal pixel differences. Both values suggest effective alignment of registered images with their references, validating the transformation method.