6.1: The heavy pill.

Take i pills from the ith bottle, measure the weight w. Then the $\frac{w-20}{0.1}$ th bottle is the heavier one.

6.2: Basketball.

Probability of winning Game 1: p
Probability of winning Game 2: $C_3^2 p^2(1-p) + p^3 = 3p^2(1-p) + p^3 = 3p^2 - 2p^3$
When $p > 3p^2 - 2p^3$, $0 < p < 0.5$, choose Game 1. When $p < 3p^2 - 2p^3$, $0.5 < p < 1$, choose Game 2. When $p = 0.5$ or $p = 0$ —— $p = 1$, both games are OK.

6.3: Dominos.

It's impossible. Label the block in first row first colmn black. Label all block adjacent to black block white. If we remove the diagnoally opposite corner, there will be 30 black blocks and 32 white blocks left. Each domino covers a black block and a white block. So 31 dominos will cover 31 black blocks and 31 white blocks. It is contradict to that there is 30 black blocks and 32 white blocks.

6.4: Ants on a triangle.

Every ant has two directions to walk. So, there are 2^3 kinds of results, only two of which will not lead to collision. So, the probability of collision is $\frac{6}{8} = 0.75$.

For n-vertex polygon, the probability is $\frac{2^n-2}{2^n}$

6.5: Jugs of water.

Fill the five-quart jug. Use the water in five-quart jug to fill the three-quart jug. So there will be 2 quart water left in the five-quart jug. Clear the three-quart jug and move the 2 quart water to the three-quart jug. Fill the five-quart jug again and use this water to fill the three-quart jug. There will be 4 quart water left in the five-quart jug.

6.6: Blue-eyed island.

If there is only one blue-eyed person, in the first day, he will find that all of other people are not blue-eyed, then he will know that he is the only one who has blue eyes. He will get on the plane.

If there are two blue-eyed person. For the two blue-eyed person, they will find that only one blue-eyed person. If that guy doesn't leave in the first day, he will know he should leave.

...

So, it takes n days to let all blue-eyed people leave, where n is the number of the blue-eyed people on the island.

6.7: The apocalypse.

For one family, the expected number of children is:

$$\sum_{1}^{\infty} i \times (1 - \frac{1}{2}) \times (\frac{1}{2})^{i-1} = \frac{1}{1 - \frac{1}{2}} = 2$$

Every family has a girl, so the expected number of girl is 1. Then the expected number of boy is 1.

The gender ratio of new generation will be 1 : 1

6.8: The egg drop problem.

The first egg drops from the 10th, 20th, 30th, ..., 100th floor. Suppose it breaks at the nth floor, then the second egg drops from (n-9)th, (n-8)th, ..., nth floor. At worst case, it takes 20 drops.

Further more, if we have n eggs to drop, the worst case takes $n\lceil \sqrt[n]{100} \rceil$ drops.

6.9: 100 Lockers.

The ith locker will be toggled at the jth pass, where j is a factor of i. So, locker i is open if and only if it has odd number of factors. Suppose:

$$i = \sum_{1}^{n} a_i^{k_i}$$

where a_i are prime numbers, k_i a positive integers.
The number of facotrs of i is:

$$\prod_{1}^{n} k_i + 1$$

So, locker i is open if and only if k_i are even. Then i is a squre. So, locker 1, 4, 9, 16, 25, 36, 49, 64, 81, 100 will be open.

6.10: Poison

Bottle i drops on trips k if $i \,\&\, 2^{k-1} \neq 0$. After 7 days, suppose the set of trips turn positive is $S = \{s_i\}$, the

$$\sum_{1}^{|S|} 2^{s_i - 1}$$

bottle is poisoned.