Lesson 1

Simple Log Functions

Non-Calculator

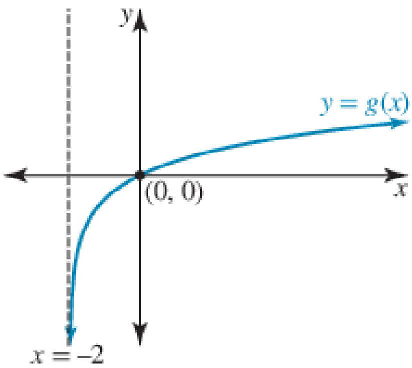
b. If $z = \log_3(x)$, find the following in terms of z.

i. $2x$

ii. $\log_x(27)$

13. If $\log_4(p) = x$ and $\log_4(q) = y$, show that $\log_4\left(\frac{64q^2}{p^3\sqrt{q}}\right) = 3 - 3x + \frac{3y}{2}$.

17. The graph shown has the rule $g(x) = \log_e(x - h) + k$, where h and k are constants.



a. State the value of h.

b. Show that $k = -\log_e(2)$.

c. Hence, rewrite the rule in the form $g(x) = \log_e\left(\frac{x - h}{c}\right)$, where c is a constant.

1 3 marks, 4.5 minutes

[Question 7 from VCE Mathematical Methods (CAS) Examination 1, 2012]

Solve the equation $2\log_e(x+2) - \log_e(x) = \log_e(2x+1)$, where $x > 0$, for x.

Calculator

13. **WE14** The diameter of a tree trunk increases according to the formula $D = A \times 10^{0.04t}$, where D cm is the diameter of the trunk t years after it is first measured and A cm is the diameter of the trunk when it is first measured.

   a. Write an equation for D in terms of t if the trunk had a diameter of 20 cm when it was first measured.

   b. When will the diameter be 25 cm?

   c. After how many years will the diameter be greater than 30 cm?

Deriving 'e'

Non-Calculator

b. $g(x) = e^{x^3+3x-2}$

c. $h(x) = 3e^{4x^2-7x}$

d. $y = -5e^{1-2x-3x^2}$

16. Determine the derivative of the function $f(x) = \dfrac{e^{3x}+2}{e^x}$ and hence find:

   a. $f'(1)$ in exact form

   b. $\{x : f'(x) = 0\}$

Calculator

15. The mass, m g, of a radioactive isotope remaining in a sample t hours after observations began is given by the rule $m(t) = ae^{-kt}$. Initially there are 4 grams of the isotope. After 6 hours, the mass of the isotope has decreased to 2.8 g.

   a. Evaluate the values of a and k. Give your answers correct to 3 decimal places where necessary.

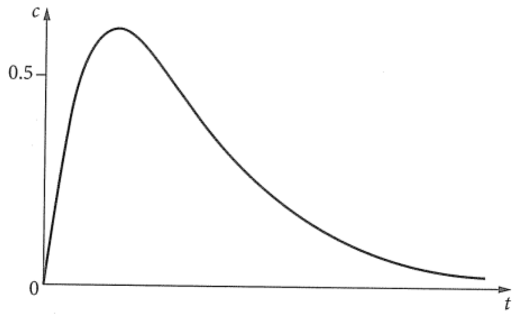
   b. Calculate the rate of decay of the isotope as a function of t.

   c. Calculate the rate of decay after 6 hours. Give your answer correct to 2 decimal places.

2 8 marks, 12 minutes

[Question 3 a–c from Section 2 VCE Mathematical Methods (CAS) Examination 2, 2014, illustrations redrawn]

In a controlled experiment, Juan took some medicine at 8 pm. The concentration of medicine in his blood was then measured at regular intervals. The concentration of medicine in Juan's blood is modelled by the function $c(t) = \frac{5}{2}te^{-\frac{3t}{2}}, t \geq 0$, where c is the concentration of medicine in his blood, in milligrams per litre, t hours after 8 pm. Part of the graph of the function c is shown below.



(a) What was the maximum value of the concentration of medicine in Juan's blood, in milligrams per litre, correct to two decimal places? 1 mark (1.5 min)

Deriving ln(x)

3 1 mark, 1.5 minutes

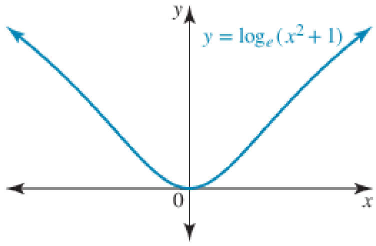
[Question 14 from VCE Mathematical Methods Examination 1, 2001]

Determine the derivative of $\log_e(2x)$ with respect to x.

6. Differentiate the following with respect to x.
   a. $y = \log_e(3x^4)$
   b. $y = \log_e(x^2 + 3)$
   c. $y = \log_e(x^2 + 4x)$

15. If $y = \log_e(x + 5)$, determine the equation of the tangent to the curve at the point where $x = e - 5$.

20. The graph of the function $y = \log_e(x^2 + 1)$ is shown.



a. Differentiate the function with respect to x.

b. Points A and B lie on the curve with x values of 2 and -2 respectively. Show that the point of intersection, T, of the tangents at A and B lies on the y-axis.

Deriving Trigonometric Functions

1. **WE10** Determine the derivative of each of the following functions.
   a. $y = \sin(8x)$
   b. $y = \sin(-6x)$

2. Differentiate each of the following.
   a. $y = \cos(3x)$
   b. $y = \cos(-2x)$

c. $y = \sin(e^x)$

5. Determine the derivative of each of the following.
   a. $y = \cos(x^2 - 4x + 3)$
   b. $y = \sin(10 - 5x + x^2)$