

Homework 2

Logs

7)

$$a) \log_3 x = 5$$

$$x = 3^5 \\ = 243$$

$$b) \log_3 (x-2) - \log_3 (5-x) = 2$$

$$\log_3 \left(\frac{x-2}{5-x} \right) = 2$$

$$\frac{x-2}{5-x} = 3^2$$

$$x-2 = 3^2 (5-x)$$

$$= 9(5-x)$$

$$= 45 - 9x$$

$$10x = 47$$

$$x = \frac{47}{10}$$

$$2b) \log_2 (\sqrt{x}) = \log_2 x^{\frac{1}{2}}$$

$$= \frac{1}{2} \log_2 x$$

$$e) \log_2 \sqrt{\frac{x^4}{y^2}} = \log_2 \frac{x^2}{y}$$

Derivatives

$$2a) y = e^{6x-2}$$
$$y' = 6e^{6x-2}$$

$$b) y = e^{8-6x}$$
$$y' = -6e^{8-6x}$$

$$s) f(x) = 2(e^x + 1)$$
$$= 2e^x + 2$$
$$f'(x) = 2e^x$$

$$b) f(x) = 3e^{2x}(e^x + 1)$$
$$f(x) = 3e^{3x} + 3e^{2x}$$
$$f'(x) = 9e^{3x} + 6e^{2x}$$

$$4a) y = \log_e(2x+5)$$
$$y' = \frac{2}{2x+5}$$

$$b) y = \log_e(6x+1)$$
$$y' = \frac{6}{6x+1}$$

$$e) y = \log_e(x^3 + 2x^2 - 7x)$$
$$y' = \frac{3x^2 + 4x - 7}{x^3 + 2x^2 - 7x}$$

$$f) y = \log_e(x^2 - 2x^3 + x^4)$$
$$y' = \frac{2x - 6x^2 + 4x^3}{x^2 - 2x^3 + x^4}$$

$$c) y = \ln(\sqrt{x^2+2})$$
$$y' = \frac{1}{\sqrt{x^2+2}} \times \frac{x}{\sqrt{x^2+2}}$$
$$= \frac{x}{x^2+2}$$

$$u = \sqrt{x^2+2}$$
$$= (x^2+2)^{\frac{1}{2}}$$
$$u' = \frac{1}{2} \times 2x (x^2+2)^{-\frac{1}{2}}$$
$$= \frac{x}{(x^2+2)^{\frac{1}{2}}}$$
$$= \frac{x}{\sqrt{x^2+2}}$$

$$y = \ln u$$
$$y' = \frac{1}{u} \times u'$$

$$d) y = \ln(x+3)^{\frac{1}{4}}$$

$$u = (x+3)^{\frac{1}{4}}$$

$$u' = \frac{1}{4}(x+3)^{-\frac{3}{4}}$$

$$= \frac{1}{4(x+3)^{\frac{3}{4}}}$$

$$y' = \frac{1}{(x+3)^{\frac{1}{4}}} \times \frac{1}{4(x+3)^{\frac{3}{4}}}$$

$$= \frac{1}{4(x+3)}$$

$$2) y = \cos(3x)$$

$$y' = -3\sin(3x)$$

$$5) a) y = \cos(x^2 - 4x + 6)$$

$$y' = -(2x - 4)\sin(x^2 - 4x + 3)$$

$$3) y = \sin(2x+3)$$

$$y' = 2\cos(2x+3)$$

$$b) y = \sin(10 - 5x + x^2)$$

$$y' = (2x - 5)\cos(10 - 5x + x^2)$$

$$1) a) y = (5x-4)^3$$

$$y' = 3 \times 5(5x-4)^2$$

$$= 15(5x-4)^2$$

$$b) y = \sqrt{3x+1}$$

$$= (3x+1)^{\frac{1}{2}}$$

$$y' = \frac{1}{2} \times 3 \times (3x+1)^{-\frac{1}{2}}$$

$$y' = \frac{3}{2\sqrt{3x+1}}$$

$$1) a) f(x) = 3\cos(x^2-1)$$

$$f'(x) = -6x\sin(x^2-1)$$

$$b) f(x) = 5e^{3x^2-1}$$

$$f'(x) = 30xe^{3x^2-1}$$

1)

a) $f(x) = \sin(3x) \cos(3x)$

$$u = \sin(3x) \quad v = \cos(3x)$$

$$u' = 3\cos(3x) \quad v' = -3\sin(3x)$$

$$f'(x) = -3\sin^2(3x) + 3\cos^2(3x)$$

b) $x^2 e^{3x}$

$$u = x^2 \quad v = e^{3x}$$

$$u' = 2x \quad v' = 3e^{3x}$$

$$f'(x) = 3x^2 e^{3x} + 2x e^{3x}$$

Antidifferentiation

c) $\int (3x^2 + 5x - 8) dx = x^3 + \frac{5}{2}x^2 - 8x + C$

d) $\int (2x^3 + 3x^2 - 6x - 9) dx = \frac{1}{2}x^4 + x^3 - 3x^2 - 9x + C$

6)

a) $\int (x^4 - e^{-4x}) dx = \frac{1}{5}x^5 + \frac{1}{4}e^{-4x} + C$

b) $\int \left(\frac{1}{2}e^{2x} - \frac{2}{3}e^{-\frac{x}{2}} \right) dx = \frac{1}{4}e^{2x} + \frac{4}{3}e^{-\frac{x}{2}} + C$

3a) $\int \frac{1}{5x+6} dx = \frac{1}{5} \ln(5x+6) + C$

b) $\int \frac{3}{2x-5} dx = \frac{1}{2} \times 3 \ln(2x-5) + C$
 $= \frac{3}{2} \ln(2x-5) + C$

4)

a) $\int e^{4x} + \sin(2x) + x^3 dx = \frac{1}{4}e^{4x} - \frac{1}{2}\cos(2x) + \frac{1}{4}x^4 + C$

b) $\int 3x^2 - 2\cos(2x) + 6e^{3x} dx = x^3 - \sin(2x) + 2e^{3x} + C$

5a) $\int \sin(x) + \cos(x) dx = -\cos(x) + \sin(x) + C$

b) $\int \sin(2x) - \cos(x) dx = -\frac{1}{2}\cos(2x) - \sin(x) + C$