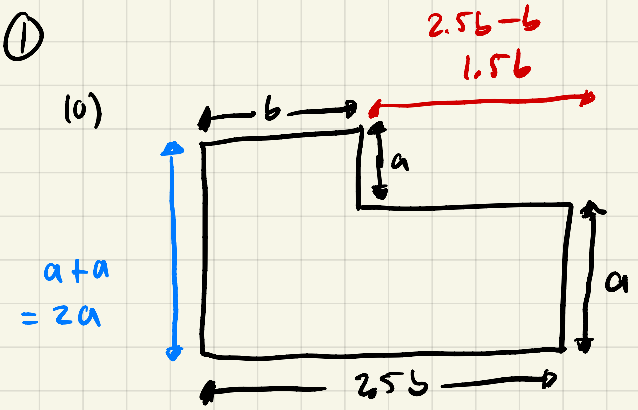
Methods Lesson 8

1

(10)



Perimeter $= 96m$

Perimeter $= 2a + b + a + 1.5b + a + 2.5b$

$= 4a + 5b$

$96 = 4a + 5b$

$4a = 96 - 5b$

$a = 24 - \frac{5}{4}b$

a) Area $= ab + a \times 2.5b$

$= ab + 2.5ab$

$= 3.5ab$

$A(b) = 3.5b \times (24 - \frac{5}{4}b)$

$= \frac{7}{2}b \times (24 - \frac{5}{4}b)$

$= 84b - \frac{35}{8}b^2$

$A'(b) = 84 - \frac{35}{4}b \longrightarrow A''(b) = -\frac{35}{4}$ $\therefore$ maximum

max at $A'(b) = 0$

$0 = 84 - \frac{35}{4}b$

$\frac{35}{4}b = 84$

$35b = 336$

$b = \frac{336}{35}$

≈ 9.6

$a = 24 - \frac{5}{4} \times 9.6$

$= 12$

b) $A(9.6) = 403.2m^2$

2

16) $V=\frac{2}{3}t^2(15-t)$

a) $V(10)=\frac{2}{3}\times 10^2(15-10)$

$=\frac{2}{3}\times 100(5)$

$=\frac{1000}{3}$

$\simeq 333.333\text{mL}$

b) $V'(t)=uv'+vu'$

$u=\frac{2}{3}t^2 \quad v=15-t$

$u'=\frac{4}{3}t \quad v'=-1$

$V'(t)=-\frac{2}{3}t^2+\frac{4}{3}t(15-t)$

c) $V'(3)=-\frac{2}{3}\times 3^2+\frac{4}{3}\times 3(15-3)$

$=-\frac{2}{3}\times 9+4\times(12)$

$=-6+48$

$=42\text{mL/s}$

d) 'Rate of flow' the greatest → Trick question

∴ when is $V'(t)$ the greatest

$V'(t)=-\frac{2}{3}t^2+\frac{4}{3}t(15-t)$

$=-\frac{2}{3}t^2+20t-\frac{4}{3}t^2$

$=20t-2t^2$

$V'(t)$ greatest when $V''(t)=0$

$V''(t)=20-4t$

$0=20-4t$

$t=5$ →

$V'''(t)=-4$

$V'''(5)=-4$

∴ maximum

$V'(5)=20\times 5-2\times 5^2$

$=100-50$

$=50\text{mL/s}$

Q3

17) $A(t) = 10t^2 - 4t^3$ → # of Algae

$A'(t) = 20t - 12t^2$ → Rate of change in Algae

To find maximum rate of change $A''(t) = 0$

$A''(t) = 20 - 24t$

$0 = 20 - 24t$

$24t = 20$

$t = \frac{20}{24}$

$= \frac{5}{6}$

$\frac{5}{6}$ hours = 50 minutes

∴ rate of max absorption occurs at 10:50am

4

a) $y = x^3 - x^2 - 16x + 16$

y-int

$x = 0, \ y = 16$

x-int

$y = 0$

$0 = x^3 - x^2 - 16x + 16$

sub $x = 1$

$= 1 - 1 - 16 + 16$

$= 0$

∴ $(x-1)$ is a factor

$$
\begin{array}{r|l}
 & x^2 + 0x - 16 \\
\hline
x-1 & x^3 - x^2 - 16x + 16 \\
 & -\ x^3 - x^2 \\
\hline
 & 0 - 16x \\
 & -\ 0 \\
\hline
 & -16x + 16 \\
 & -16x + 16 \\
\hline
 & 0
\end{array}
$$

∴ $y = (x-1)(x^2 - 16)$ ← $a^2 - b^2 = (a+b)(a-b)$

$= (x-1)(x+4)(x-4)$

y-int = 1, −4, 4

Stationary points → $y' = 0$

$y' = 3x^2 - 2x - 16$

$= 3x^2 + 6x - 8x - 16$

$= 3x(x+2) - 8(x+2)$

$= (3x-8)(x+2)$

Stationary points at $x = \frac{8}{3}$, $x = -2$

$y\left(\frac{8}{3}\right) = \left(\frac{8}{3}\right)^3 - \left(\frac{8}{3}\right)^2 - 16\left(\frac{8}{3}\right) + 16$

$= -14\frac{22}{27}$

$y(-2) = (-2)^3 - (-2)^2 - 16 \times -2 + 16$

$= 36$

Nature of stationary points

$y'' = 6x - 2$

$y''\left(\frac{8}{3}\right) = 6 \times \frac{8}{3} - 2$

$= 16 - 2$

$= 14$

positive ∴ local minimum

$y''(-2) = 6 \times -2 - 2$

$= -14$

negative ∴ local maximum

Point of inflection

$y''(x) = 0$

$0 = 6x - 2$

$2 = 6x$

$x = \frac{1}{3}$

$y''(1) = 6 - 2$

$= +ve$

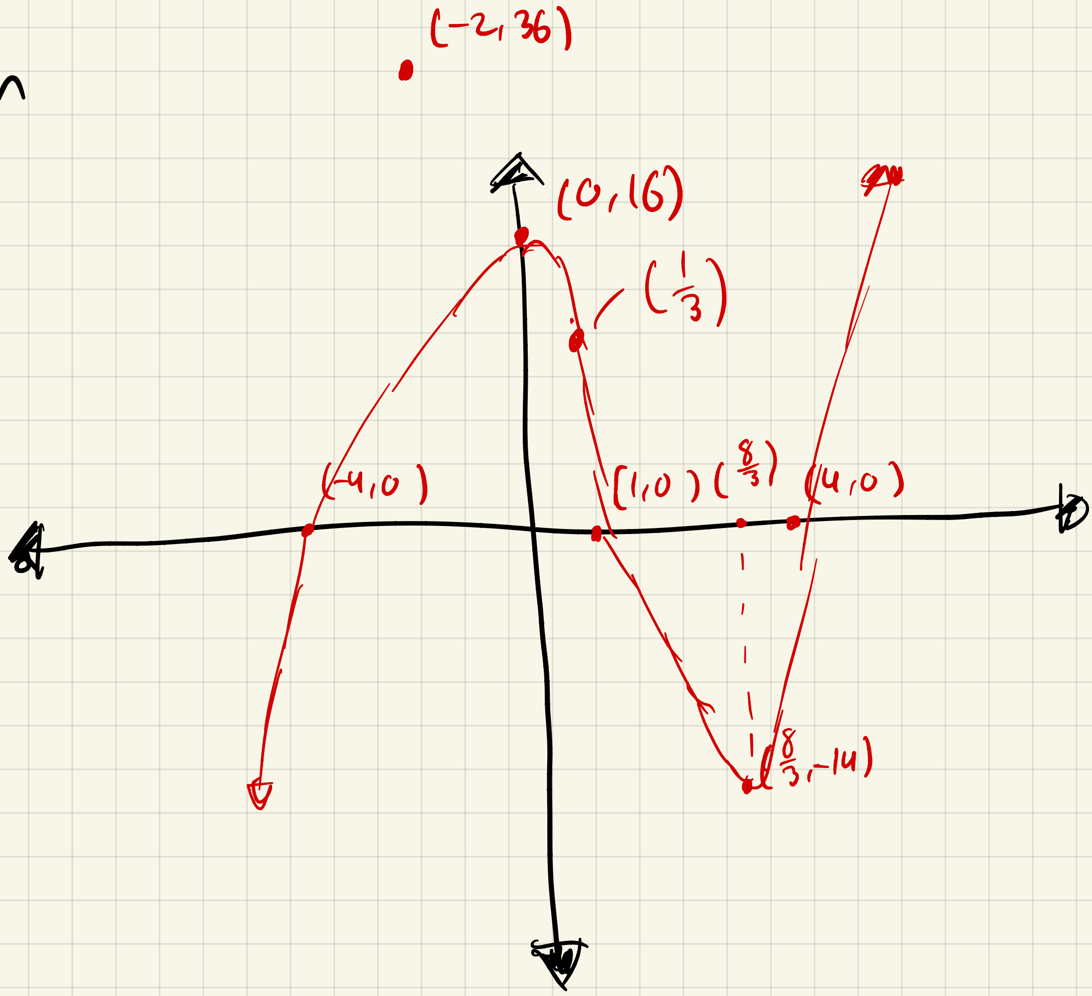
$y''(0) = 0 - 2$

$= -ve$

∴ $\frac{d^2y}{dx^2}$

0	$\frac{1}{3}$	1
-ve	0	+ve
concave down	pt. of inflection	concave up

Sketch



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Volume $50cm^3$

40cents / $100cm^2$ in surface Area → Price

a) Surface Area of cylinder $= 2\pi rh + 2\pi r^2$

Volume $= \pi r^2 \times h$

$50 = \pi r^2 h$

$h = \dfrac{50}{\pi r^2}$

$\therefore A(r) = 2\pi r \times \left(\dfrac{50}{\pi r^2}\right) + 2\pi r^2$

$= \dfrac{100\pi r}{\pi r^2} + 2\pi r^2$

$= \dfrac{100}{r} + 2\pi r^2$

b) minimum at $A'(r) = 0$

$$A'(r) = -\frac{100}{r^2} + 4\pi r \longrightarrow A''(r) = \frac{200}{r^3} + 4\pi$$

$$0 = -\frac{100}{r^2} + 4\pi r$$

$$\frac{100}{r^2} = 4\pi r$$

$$100 = 4\pi r^3$$

$$\pi r^3 = 25$$

$$r = \sqrt[3]{\frac{25}{\pi}}$$

$$A''\left(\sqrt[3]{\frac{25}{\pi}}\right) = \frac{200}{\left(\sqrt[3]{\frac{25}{\pi}}\right)^3} + 4\pi$$

$= +ve$

$\therefore$ minimum

$\approx 1.996\text{cm}$

$\approx 2\text{cm}$

c) $A(2) = \frac{100}{2} + 2\pi \times 2^2$

$= 50 + 8\pi$

$= 75.13\text{cm}^2$

d) $40c / 100\text{cm}^2$

$10\,000 \text{ cars} \times 75.13 \text{ cm}^2 = 751327 \text{cm}^2$

$40c / 100\text{cm}^2$

$751327 \times \frac{0.4}{100} = \$3005$

nearest \$20

$= \$3000$

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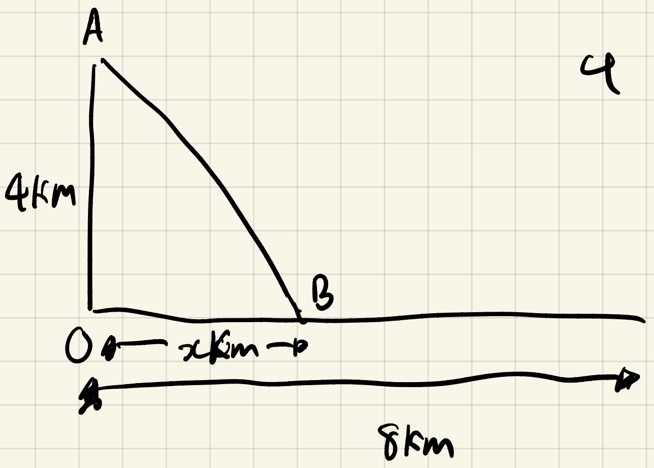

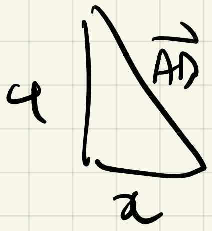

$4^2 + x^2 = AD^2$

$AD = \sqrt{4^2 + x^2}$

Total distance walking $= (8 - x)$ km

total distance rowing $= \overrightarrow{AD}$

$= \sqrt{4^2 + x^2}$

$= \sqrt{16 + x^2}$

let $T(x) =$ total time

$$\therefore T(x) = \frac{\text{distance to row}}{\text{rowing speed}} + \frac{\text{distance to walk}}{\text{walking speed}}$$

$$= \frac{km}{\frac{km}{s}} + \frac{km}{\frac{km}{s}}$$

$= s + s \rightarrow$ units of time

$$= \frac{\sqrt{16 + x^2}}{5} + \frac{8 - x}{8}$$

$$= \frac{(16 + x^2)^{\frac{1}{2}}}{5} + \frac{8}{8} - \frac{x}{8}$$

Graph on GCS to find minimum

$x = 3.203$

No need to find minimum time

$$T'(x) = \frac{2x(16 + x^2)^{-\frac{1}{2}}}{2 \times 5} - \frac{1}{8}$$

$$= \frac{x}{5\sqrt{16 + x^2}} - \frac{1}{8}$$

using GCS to find

$T'(x) = 0$

$x = 3.203$ km