Homework Lesson 7

Q1

8. Surface Area of Cylinder

$A = 2\pi r h + 2\pi r^2$

a) $200 = 2\pi r h + 2\pi r^2$

$= 2\pi r(h + r)$

$h + r = \frac{100}{\pi r}$

$h = \frac{100}{\pi r} - r$

$V = \pi r^2 \times h$

$= \pi r^2 \times \left(\frac{100}{\pi r} - r\right)$

$= \frac{100\pi r^2}{\pi r} - \pi r^3$

$= 100r - \pi r^3$

b) $V(r) = 100r - \pi r^3$

c) Max volume at $V'(r) = 0$

$V'(r) = 100 - 3\pi r^2$

$0 = 100 - 3\pi r^2$

$100 = 3\pi r^2$

$r = \sqrt{\frac{100}{3\pi}} = 10\sqrt{\frac{1}{3\pi}}$

$V''(r) = -6\pi r$

$V''\left(\sqrt{\frac{100}{3\pi}}\right) < 0 \therefore$ maximum

diameter $= 2r$

$= 2\sqrt{\frac{100}{3\pi}}$

$= 2\sqrt{100 \times \frac{1}{3\pi}}$

$= 20\sqrt{\frac{1}{3\pi}}$

height $= \frac{100}{\pi r} - r$

$= \frac{100}{\pi \times 10\sqrt{\frac{1}{3\pi}}} - 10\sqrt{\frac{1}{3\pi}}$

$= \frac{100\sqrt{\frac{1}{3\pi}}}{\pi \times 10 \times \frac{1}{3\pi}} - 10\sqrt{\frac{1}{3\pi}}$

$$= \frac{300\pi\sqrt{\frac{1}{3\pi}}}{10\pi} - 10\sqrt{\frac{1}{3\pi}}$$

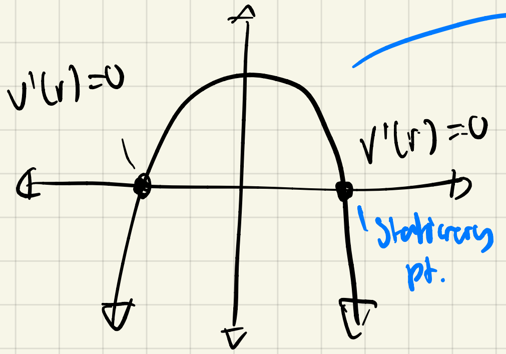
$$= 30\sqrt{\frac{1}{3\pi}} - 10\sqrt{\frac{1}{3\pi}}$$

$$= 20\sqrt{\frac{1}{3\pi}}$$

$\therefore$ diameter = height at max volume

d) $V'(r) = -3\pi r^2 + 100$

$\therefore$ $V'(r)$ graph looks like



Only one stationary point at $V'(r)=0$
$\therefore$ either end point has to be minimum

radius can't be negative so disregard left side

$V(2) = 200 - 8\pi = 8(25 - \pi)$
$V(4) = 400 - 64\pi = 8(50 - 8\pi)$

$V(2)$ is smaller
$\therefore$ $V(2)$ is minimum

(Probably need calculator for this Q)

Q10)

a) $L = r\theta$

$\therefore \; P = L + 2r$

$8 = L + 2r$

$= r\theta + 2r$

$r\theta = 8 - 2r$

$\theta = \frac{8}{r} - 2$

b) $A_{sector} = \frac{1}{2}r^2\theta$

$= \frac{1}{2}r^2 \times \left(\frac{8}{r} - 2\right)$

$= \frac{4r^2}{r} - r^2$

$= 4r - r^2$

$A(r) = 4r - r^2$

c) $A'(r) = 0$, r is max

$A'(r) = 4 - 2r$

$0 = 4 - 2r$

$2r = 4$

$r = 2$

$\theta = \frac{8}{r} - 2$

$= \frac{8}{2} - 2$

$= 4 - 2$

$= 2$ radians

$A''(r) = -2$

$\therefore r = 2$ is a maximum