Methods lesson 3

1) $y = \ln(3x^2+7)$

$$\frac{dy}{dx} = \frac{1}{3x^2+7} + 6x$$

$$= \frac{6x}{3x^2+7} \quad \therefore \int \frac{6x}{3x^2+7}\,dx = \ln(3x^2+7) + C$$

$$\int \frac{6x}{3x^2+7}\,dx = 6\int \frac{x}{3x^2+7}\,dx$$

$\therefore$

$$6\int \frac{x}{3x^2+7}\,dx = \ln(3x^2+7)$$

$$\int \frac{x}{3x^2+7}\,dx = \frac{\ln(3x^2+7)}{6} + C$$

16)
$y = \ln(3x^2+4)$

$$\frac{dy}{dx} = \frac{6x}{3x^2+4}$$

$$6\int \frac{x}{(3x^2+4}\,dx = \ln(3x^2+4) + C$$

$$\int \frac{x}{(3x^2+4)}\,dx = \frac{\ln(3x^2+4)}{6} + C$$

10) $f'(x) = \cos(2x) - \sin(2x)$

$$f(x) = \int \cos(2x) - \sin(2x)$$

$$= \frac{\sin(2x)}{2} + \frac{\cos(2x)}{2} + C$$

Point $(\pi, 2)$

$f(\pi) = 2$

$$2 = \frac{\sin(2\pi)}{2} + \frac{\cos(2\pi)}{2} + C$$

$$= 0 + \frac{1}{2} + C$$

$$\frac{3}{2} = C$$

$$f(x) = \frac{\sin(2x)}{2} + \frac{\cos(2x)}{2} + \frac{3}{2}$$

5.

Need to expand the brackets

$$\int \frac{(2x+5)^2}{x}\,dx = \int \frac{4x^2+20x+25}{x}\,dx$$

$$= \int 4x + 20 + \frac{25}{x}\,dx$$

$$= x^2 + 20x + 25\ln(x) + C$$

6) $$\int \frac{(3x+2)^2}{x^2}\,dx = \int \frac{9x^2+12x+4}{x^2}\,dx$$

$$= \int 9 + \frac{12}{x} + \frac{4}{x^2}\,dx$$

$$= 9x + 12\ln(x) - \frac{4}{x} + C$$

5a) $\int (e^{2x+1}-4)^2\,dx = \int e^{2x+1} e^{2x+1} - 8e^{2x+1} + 16\,dx$

$$= \int e^{4x+2} - 8e^{2x+1} + 16\,dx$$

$$= \frac{e^{4x+2}}{4} - 4e^{2x+1} + 16x + C$$

10.

$$\frac{dy}{dx} = x\left(1-\frac{1}{x}\right)^2$$

$$= x\left(1-\frac{2}{x}+\frac{1}{x^2}\right)$$

$$= x - 2 + \frac{1}{x}$$

$$y = \int x - 2 + \frac{1}{x}\,dx$$

$$= \frac{x^2}{2} - 2x + \ln(x) + C$$

16)

$$\frac{d}{dx}\ln(x^2+3) = \frac{2x}{x^2+3}$$

$$\int \frac{2x}{(x^2+3)} = \ln(x^2+3) + c$$

$\times 6$ $\times 6$

$$6\int \frac{2x}{(x^2+3)}\,dx = 6\ln(x^2+3) + c$$

$$\int \frac{12x}{(x^2+3)}\,dx = 6\ln(x^2+3) + C$$

11. $\frac{d}{dx}\frac{\cos(x)}{\sin(x)} = \frac{-\sin^2(x) - \cos^2(x)}{\sin^2(x)}$

$u = \cos(x)$ $v = \sin(x)$

$u' = -\sin(x)$ $v' = \cos(x)$

$\sin^2(x) + \cos^2(x) = 1$ ← Pythagorean identity

$\cos^2(x) = 1 - \sin^2(x)$

$$\therefore \frac{-\sin^2(x) - \cos^2(x)}{\sin^2(x)} = \frac{-\sin^2(x) - (1 - \sin^2(x))}{\sin^2(x)}$$

$$= -\sin^2(x) - 1 + \sin^2(x)$$

$$= \frac{-1}{\sin^2(x)}$$

$$\therefore \int \frac{-1}{\sin^2(x)}\,dx = \frac{\cos(x)}{\sin(x)}$$

$$\int \frac{1}{\sin^2(x)}\,dx = -\frac{\cos(x)}{\sin(x)} \quad \text{or} \quad -\frac{1}{\tan(x)} \quad \text{or} \quad -\operatorname{arctan}(x)$$

12)

$$-3 + \frac{4}{3-2x} = \frac{-3(3-2x)}{(3-2x)} + \frac{4}{(3-2x)}$$

multiply top and bottom by $(3-2x)$

> Do this so the denominator can be the same

> so you can add them together

$$= \frac{-9+6x}{(3-2x)} + \frac{4}{(3-2x)}$$

$$= \frac{-9+6x+4}{(3-2x)}$$

$$= \frac{6x-5}{(3-2x)}$$

Proof by polynomial long division

$$\begin{array}{r|l} & -3 \\ -2x+3 & 6x-5 \\ & -6x-9 \\ \hline & 0+4 \end{array}$$

r 4 $\therefore \dfrac{6x-5}{3-2x} = -3 + \dfrac{4}{-2x+3}$

$= -3 + \dfrac{4}{(3-2x)}$

$$\therefore \int \frac{6x-5}{3-2x}\,dx = \int -3 + \frac{4}{(3-2x)}\,dx$$

$$= -3x + \frac{4\ln(3-2x)}{-2} + c$$

$$= -3x - 2\ln(3-2x) + c$$

6) $f'(x) = 3x^2 - 8x + 3$

PASSES THE ORIGIN

$\therefore$ point $(0,0)$ exists

$$\int 3x^2 - 8x + 3 \, dx = x^3 - 4x^2 + 3x + C$$

$f(0) = 0$

$0 = 0^3 - 4 \times 0^2 + 3 \times 0 + C$

$C = 0$

$\therefore f(x) = x^3 - 4x^2 + 3x$

x intercepts

$f(x) = x(x^2 - 4x + 3)$ Factor x out

$= x(x-3)(x-1)$ Solve the Quadratic

$\therefore x = 0, 3, 1$ find x-values via null factor

$(0,0), (3,0), (1,0)$

6) acceleration $= -10$m/s

velocity initially $= 25$m/s $\rightarrow v(0) = 25$

$\therefore$

$a(t) = -10$

$v(t) = -10t + C$

$v(0) = 25$

$25 = -10 \times 0 + C$

$C = 25$

a) $v(t) = -10t + 25$

$x(t) = \int v(t) \, dt$

$= \int -10t + 25 \, dt$

$= -5t^2 + 25t + C$

at time of firing, displacement $= 0$

$x(0) = 0$

$\therefore C = 0$

b) $x(t) = -5t^2 + 25t$

max height found at stationary point

$\therefore$ at $v(t)=0$

$0 = -10t + 25$

$10t = 25$

c) $t = 2.5$ seconds

To confirm it is a maximum, take $\frac{dx^2}{d^2t}$ and observe it's value

$\therefore a(t) = \frac{dx^2}{d^2t}$

$= -10$

$\therefore$ -ve $\therefore$ maximum

max height at $t=2.5$

$\therefore x(2.5) = -5\times(2.5)^2 + 25\times 2.5$

$= -5\times\frac{25}{4} + 25\times\frac{5}{2}$

$= -\frac{125}{4} + \frac{125}{2}$

$= -\frac{125}{4} + \frac{250}{4}$

$= \frac{125}{4}$

d) $= 31.25m$

time to return $(x=0)$

$0 = -5t^2 + 25t$

$= -5t(t-5)$

$t = 5$ seconds

1) $\int_0^1 x^2 dx = \left[\frac{x^3}{3}\right]_0^1$

$= \left(\frac{1^3}{2}\right) - \left(\frac{0}{2}\right)$

$= \frac{1}{2}$

$\int_0^3 x^3 dx = \left[\frac{x^4}{4}\right]_0^3$

$= \left(\frac{3^4}{4}\right) - \left(\frac{0^4}{4}\right)$

$= \frac{81}{4}$

$\doteq 20.25$

c) $\int_0^{\pi} 5\sin\left(\frac{x}{4}\right)dx = \left[-20\cos\left(\frac{x}{4}\right)\right]_0^{\pi}$

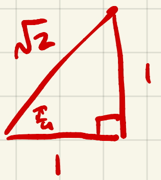
$= \left(-20\cos\left(\frac{\pi}{4}\right)\right) - \left(-20\cos\left(\frac{0}{4}\right)\right)$

$= \left(-20\times\frac{1}{\sqrt{2}}\right) - (-20)$

$= \frac{-20}{\sqrt{2}} + 20$

$= \frac{-20\sqrt{2}}{2} + 20$

$= -10\sqrt{2} + 20$



$\cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

11)

$$\int_0^a e^{\frac{x}{2}}\,dx = 4$$

evaluate this first

$$= \left[2e^{\frac{x}{2}}\right]_0^a = \left(2e^{\frac{a}{2}}\right) - \left(2e^{\frac{0}{2}}\right)$$

$$= 2e^{\frac{a}{2}} - 2$$

$$2e^{\frac{a}{2}} - 2 = 4$$

$$2e^{\frac{a}{2}} = 6$$

$$e^{\frac{a}{2}} = 3$$

$$\ln e^{\frac{a}{2}} = \ln 3$$

$$\frac{a}{2} = \ln 3$$

$$a = 2\ln 3 \quad \text{or} \quad \ln 9$$

12)

$$\int_0^a e^{-2x}\,dx = \left[-\frac{e^{-2x}}{2}\right]_0^a$$

To make things easier, you can factor out constants in the expression

$$\left[-\frac{e^{-2x}}{2}\right]_0^a = -\frac{1}{2}\left[e^{-2x}\right]_0^a$$

$$-\frac{1}{2}\left(e^{-2a} - e^0\right) = -\frac{1}{2}\left(e^{-2a} - 1\right)$$

$$= \frac{-e^{-2a}}{2} + \frac{1}{2}$$

$$\frac{-e^{-2a}}{2} + \frac{1}{2} = \frac{1}{2}\left(1 - \frac{1}{e^8}\right)$$

$$-e^{-2a} + 1 = 1 - \frac{1}{e^8}$$

$$-e^{-2a} = -\frac{1}{e^8}$$

$$e^{-2a} = e^{-8}$$

$$a = 4$$

3) $y = x+1$

bounds $x=0$, $x=2$

$$\int_0^2 x+1 \, dx = \left[\frac{x^2}{2} + x \right]_0^2$$

$$= 2+2$$

$$= 4 \text{ units}^2$$

4) $y = x^2$

bounds $x=0$, $x=3$

$$\int_0^3 x^2 dx = \left[\frac{x^3}{3} \right]_0^3$$

$$= \frac{27}{3}$$

$$= 9 \text{ units}^2$$