

Methods Lesson 6

QUESTION 15 (4 marks)

Solve the following equations.

a) $4e^x = 100$

b) $2 \log_4 x - \log_4(x-1) = 1$

QUESTION 12 (5 marks)

An object is moving in a straight line from a fixed point. The object is at the origin initially.

The acceleration a (in m s^{-2}) of the object is given by

$$a(t) = \pi \cos(\pi t) \quad t \geq 0, \text{ where } t \text{ is time in seconds.}$$

The velocity at $t = 1$ is 0.5 m s^{-1}

a) Determine the initial acceleration.

[1 mark]

b) Determine the initial velocity.

[2 marks]

c) Determine the displacement after one second.

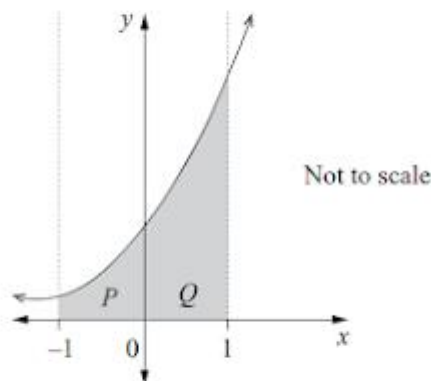
[2 marks]

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QUESTION 18 (4 marks)

The graph of $y = f(x)$, where $f(x)$ is the quadratic function $f(x) = ax^2 + bx + 4$, is shown.

Two regions of the area between the graph of $y = f(x)$ and the x -axis are shaded.



Region P has an area of $\frac{13}{6}$ units² and Region Q has an area of $\frac{43}{6}$ units².

Determine the values of a and b .

- 26 a** Differentiate $\sin^3(2x)$ with respect to x .
b Write $\cos^3(2x)$ as $(1 - \sin^2(2x)) \cos(2x)$ and hence find an anti-derivative of $\cos^3(2x)$.
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QUESTION 11 (5 marks)

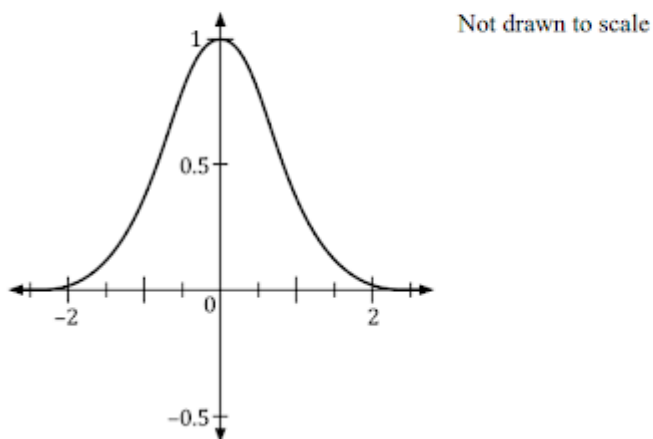
Determine:

- a) $\frac{d}{dx}(3 \ln(5x))$ [1 mark]
b) $\frac{d}{dx}(x^3 \cos(x^4 + 1))$ (You do not need to simplify.) [3 marks]
c) $\int 3e^{4x} dx$ [1 mark]
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QUESTION 18 (6 marks)

Determine the area of the largest rectangle that has one side on the x -axis and two vertices on the curve $y = e^{-x^2}$. Express your answer in simplest form. The curve is sketched below.



Question 12

For $f(x) = x \sin(5x)$, it is given that $f'(x) = 5x \cos(5x) + \sin(5x)$.

Use this to determine an anti-derivative of $x \cos(5x)$.

Question 10 CF (3 marks)

The area of the region bounded by the y -axis, the x -axis, the curve $y = e^{\frac{x}{2}}$ and the line $x = a$ is 4. Find the value of a .

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Question 1 SF (3 marks)

Let $f(x) = Ae^{kx} + 3$. It is known that the points $(0, 6)$ and $(5, 50)$ lie on the curve of $y = f(x)$. Find the exact value of A and the value of k correct to two decimal places.

Question 6 SF (2.5 marks)

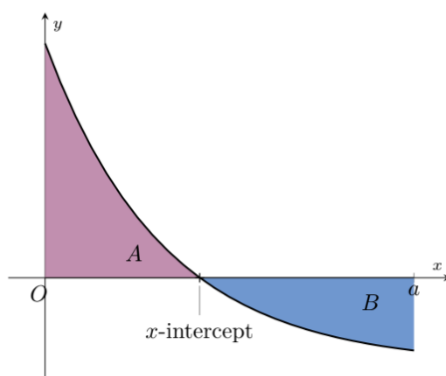
Calculate the value(s) of x for which $f(x) = e^{(x+2)^2}$ and $g(x) = 3 \sin\left(\frac{x}{2}\right)$ have the same gradient for $-2 \leq x \leq 2$.

Question 9 CF (5 marks)

Calculate the value/s of x , correct to three decimal places, for which $f(x) = \ln(2 - 4x) - \frac{x^2}{2}$ and $g(x) = -2 \cos(3x)$ have the same gradient for $-\pi \leq x \leq \pi$.

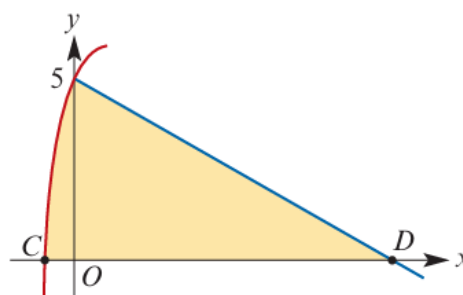
Question 11 CF (6 marks)

A part of the graph of $y = e^{(2-\frac{x}{2})} - 2$ is shown. Region A is bounded by the axes and the graph of $y = e^{(2-\frac{x}{2})} - 2$. Region B is bounded by the x -axis, the graph of $y = e^{(2-\frac{x}{2})} - 2$ and the line $x = a$. Find the value of a , correct to 2 decimal places, such that the area of region A is equal to the area of region B.



- 33** Let $f(x) = 6 - e^{-2x}$. The diagram shows part of the graph of f and also shows the normal to the graph of f at the point $(0, 5)$.

- Find the coordinates of points C and D .
- Find the area of the shaded region.



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