Methods lesson 1.5

18a)

$$\frac{x-12}{x+3} = \frac{(x+3)-15}{(x+3)}$$

$$= 1 - \frac{15}{x+3}$$

Partial fractions

b)

$$\frac{4x+7}{2x+1} = \frac{2(2x+1)+5}{2x+1}$$

$$= 2 + \frac{5}{(x+1)}$$

19) a)

$$\begin{array}{r} 2x^2 - x + 7 \\ x-2 \overline{)\,2x^3 - 5x^2 + 8x + 6} \\ -2x^3 - 4x^2 \\ \hline -x^2 + 8x \\ -x^2 + x \\ \hline 7x + 6 \\ -7x - 14 \\ \hline 20 \end{array}$$

Remainder $= 20$

b)

$$\begin{array}{r} -\frac{1}{2}x^2 - \frac{1}{4}x - \frac{1}{8} \\ -2x+1 \overline{)\,x^3 + 0x^2 + 0x + 10} \\ -x^3 - \frac{1}{2}x^2 \\ \hline \frac{1}{2}x^2 \\ -\frac{1}{2}x^2 - \frac{1}{4}x \\ \hline \frac{1}{4}x + 10 \\ -\frac{1}{4}x - \frac{1}{8} \\ \hline 9\frac{7}{8} \rightarrow \frac{81}{8} \end{array}$$

$$= -\frac{1}{2}x^2 - \frac{1}{4}x - \frac{1}{8} + \frac{81}{8(1-2x)}$$

6a)

$Q(2) = 0$ if

$x-2$ is remainder

$Q(2) = 4\times 2^4 + 4\times 2^3 - 25\times 2^2 - 2 + 6$

$= 4\times 16 + 4\times 8 - 25\times 4 - 2 + 6$

$= 64 + 32 - 100 - 2 + 6$

$= 0$

6b) $P(x) = 3x^3 + ax^2 + bx - 2$

$P(1) = 0$

$0 = 3 + a + b - 2$

$= 1 + a + b$

$P(-1) = -22$

$= -3 + a - b - 2$

$-22 = -3 + a - b - 2$

$= -5 + a - b$

Simultaneous Equations

$-5 + a - b = -22$

$-\ \ 1 + a + b = 0$

$-6 - 2b = -22$

$-2b = -16$

$b = 8$

$0 = 1 + a + b$

$= 1 + a + 8$

$a = -9$

$\therefore P(x) = 3x^2 - 9x^2 + 8x - 2$

13)

a) $P(x) = x^3 + 5x^2 + 2x - 8$

Sub values in like 1, -1

$P(1) = 1 + 5 + 2 - 8$

$= 0$

$\therefore (x-1)$ is a factor

Equating coefficients

$x^3 + 5x^2 + 2x - 8 = (x-1)(ax^2 + bx + c)$

$ax^2 \times x = x^3 \quad \therefore a = 1$

$ax^2 \times -1 + bx \times x = 5x^2$

$-x^2 + bx^2 = 5x^2$

$-1 + b = 5 \quad \therefore b = 6$

$bx \times -1 + cx = 2x$

$-b + c = 2$

$-6 + c = 2$

$c = 8$

$\therefore P(x) = (x-1)(x^2 + 6x + 8)$

$= (x-1)(x+4)(x+2)$

12a) $P(x) = x^3 - x^2 - 10x - 8$

$P(4) = 0$

$\therefore (x-4)(ax^2 + bx + c)$

$a = 1$

$ax^2 \times -4 + bx^2 = -x^2$

$-4 + b = -1$

$b = 3$

$cx - 4bx = -10x$

$c - 12 = -10$

$c = 2$

$P(x) = (x-4)(x^2 + 3x + 2) = (x-4)(x+2)(x+1)$

13b) $x^3 + 10x^2 + 31x + 30$

Sub $x = -2$

$-8 + 40 - 62 + 30 = 0$

$\therefore$ $x+2$ is a factor

$\therefore$

$$x+2 \overline{)\, x^3 + 10x^2 + 31x + 30}$$

$x^2 + 8x + 15$

$-(x^3 + 2x^2)$

$8x^2 + 31x$

$-(8x^2 + 16x)$

$15x + 30$

$-(15x + 30)$

0

$(x+2)(x^2+8x+15)$

$(x+2)(x+5)(x+3)$ → fully factorised

c) $2x^3 - 13x^2 + 13x + 10$

Sub $x = 2$

$2 \times 8 - 52 + 26 + 10$

$16 - 52 + 26 + 10 = 0$

$\therefore (x-2)$ is factor

$$x-2 \overline{)\, 2x^3 - 13x^2 + 13x + 10}$$

$2x^2 - 9x - 5$

$-(2x^3 - 4x^2)$ ↓

$-9x^2 + 13x$

$-(-9x^2 + 18x)$

$-5x + 10$

$-5x + 10$

$(x-2)(2x^2 - 9x - 5)$

$-10 \times 1 = -10$

$-10 + 1 = -9$

$(x-2)(2x^2 - 10x + x - 5)$

$(x-2)(2x(x-5) + (x-5))$ ← there is imaginary '1' here

$(x-2)(2x+1)(x-5)$

12a)

$P(x) = x^3 - x^2 - 10x - 8$

$\because (x-4)$ is factor

$\therefore$

$$
\begin{array}{r|l}
 & x^2 + 3x + 2 \\
\hline
x - 4 & x^3 - x^2 - 10x - 8 \\
 & -x^3 - 4x^2 \\
 & \quad 3x^2 - 10x \\
 & \quad -3x^2 - 12x \\
 & \quad\quad 2x - 8 \\
 & \quad\quad -2x - 8 \\
 & \quad\quad\quad 0
\end{array}
$$

$P(x) = (x-4)(x^2+3x+2)$

$= (x-4)(x+1)(x+2)$

12b)

$P(x) = 3x^3 + 40x^2 + 44x + 12$

$(x+12)$ is factor

$\therefore$

$$
\begin{array}{r|l}
 & 3x^2 + 4x + 1 \\
\hline
x + 12 & 3x^3 + 40x^2 + 44x + 12 \\
 & -3x^3 + 36x^2 \\
 & \quad 4x^2 + 44x \\
 & \quad -4x^2 + 48x \\
 & \quad\quad x + 12
\end{array}
$$

$\therefore P(x) = (x+12)(3x^2+4x+1)$

$\underline{3} \times \underline{1} = 3$

$\underline{3} + \underline{1} = 4$

$P(x) = (x+12)(3x^2 + 3x + x + 1)$

$= (x+12)(3x(x+1) + (x+1))$

$= (x+12)(3x+1)(x+1)$

10 a) $p(x) = 4x^3 + kx^2 - 10x - 4$

$p(1) = 15$

$15 = 4 + k - 10 - 4$

$k = 25$

b) $Q(-2) = -5$

$-5 = 4a + 2a + 7$

$-4a = 36$

$a = -9$

10c)

$P(2) = 3P(-1)$

$P(2) = 8 - 24 + 2n + 2$

$= -14 + 2n$

$P(-1) = -1 - 6 - n + 2$

$= -5 - n$

$3P(-1) = -15 - 3n$

$\therefore -14 + 2n = -15 - 3n$

$1 + 5n = 0$

$5n = -1$

$n = -\frac{1}{5}$

10d) $Q(x) = -x^2 + bx + c$

$Q(0) = 5 \quad c = 5 \quad \because Q(0) = -0^2 + 5(0) + c$

$5 = c$

$Q(5) = 0$

$Q(x) = -x^2 + bx + 5$

$0 = -25 + 5b + 5$

$20 = 5b$

$b = 4$

7) a) $x = -4, 3, -5$

b) $x = 7, -\frac{5}{3}, 9$

c) $x = -1, 6, 8$

d) $2x^3 + 7x^2 - 9 = 0$

Sub $x = 1$

$2 + 7 - 9 = 0$

$\therefore x - 1$ is a factor

$$x-1 \overline{)\,2x^3 + 7x^2 + 0x - 9}$$

$2x^3 + 7x^2 + 0x - 9$

$-2x^3 - 2x^2$

$9x^2 + 0x$

$-9x^2 - 9x$

$9x - 9$

$-9x - 9$

0

$\therefore (x-1)(2x^2 + 9x + 9) = 0$

$\frac{6}{2} \times \frac{3}{1} = 18$

$\frac{6}{2} + \frac{3}{1} = 9$

$(x-1)(2x^2 + 6x + 3x + 9)$

$(x-1)(2x(x+3) + 3(x+3))$

$(x-1)(2x+3)(x+3)$

$x = 1, -\frac{3}{2}, -3$

Polynomial Long division

3) $x^3 - 9x^2 + 15x + 25$

at $x = 5$, $p(x) = 0$

$\therefore (x-5)$ is factor

$$x-5 \overline{)\,x^3 - 9x^2 + 15x + 25}$$

$x^2 - 4x - 5$

$x^3 - 5x^2$

$-4x^2 + 15x$

$-4x^2 + 20x$

$-5x + 25$

$-5x + 25$

0

$\therefore p(x) = (x-5)(x^2 - 4x - 5)$

$= (x-5)(x-5)(x+1)$

3 solutions