Methods Lesson 4

3)

a) $\int_0^{\frac{\pi}{2}} \sin(x)\,dx = \left[-\cos(x)\right]_0^{\frac{\pi}{2}}$

$= \left(-\cos\left(\frac{\pi}{2}\right)\right) - \left(-\cos(0)\right)$

$= 0 - -1$

$= 1$

b) $\int_{\frac{\pi}{2}}^{\pi} 3\sin(4x)\,dx = \frac{3}{4}\left[-\cos(4x)\right]_{\frac{\pi}{2}}^{\pi}$

$= -\frac{3}{4}\left(\left(-\cos(4\pi)\right) - \left(-\cos\left(\frac{\pi}{2}\right)\right)\right)$

$= -\frac{3}{4}(-1-0)$

$= \frac{3}{4}$

c) $\int_0^{\pi} 5\sin\left(\frac{x}{4}\right)dx = \left[-20\cos\left(\frac{x}{4}\right)\right]_0^{\pi}$

$= -20\left(\cos\left(\frac{\pi}{4}\right) - \cos\left(\frac{0}{4}\right)\right)$

$= -20\left(\frac{1}{\sqrt{2}} - 1\right)$

$= \frac{-20}{\sqrt{2}} + 20$

$= 20 - \frac{20}{\sqrt{2}}$

$= 20 - \frac{20\sqrt{2}}{2}$

$= 20 - 10\sqrt{2}$

7)

a) $\int_2^5 3m(x)\,dx = 3\int_2^5 m(x)\,dx$

$= 3 \times 7$

$= 21$

b) $\int_2^5 (2m(x)-1)\,dx = \int_2^5 2m(x)\,dx - \int_2^5 1\,dx$

$$= 2\int_2^5 m(x)dx - \int_2^5 1dx$$

$$= 2\times 7 - \int_2^5 1dx$$

$$= 14 - [x]_2^5$$

$$= 14 - (5-2)$$

$$= 14 - 3$$

$$= 11$$

5)

a) $$\int_0^3 (3x^2 - 2x + 3)dx = [x^3 - x^2 + 3x]_0^3$$

$$= 27 - 9 + 9 - 0$$

$$= 27$$

b) $$\int_1^2 \frac{2x^3 + 3x^2}{x} dx = \int_1^2 2x^2 + 3x \, dx$$

$$= \left[\frac{2x^3}{3} + \frac{3x^2}{2}\right]_1^2$$

$$= \left(\frac{2\times 2^3}{3} + \frac{3\times 2^2}{2}\right) - \left(\frac{2\times 1^2}{3} + \frac{3\times 1^2}{2}\right)$$

$$= \left(\frac{16}{3} + 6\right) - \left(\frac{2}{3} + \frac{3}{2}\right)$$

$$= \frac{14}{3} + 6 + \frac{3}{2}$$

$$= \frac{28}{6} + \frac{9}{6} + 6$$

$$= \frac{37}{6} + 6$$

$$= 12 + \frac{1}{6}$$

$$\approx 12.16$$

20) $y = 2\sin(x)$

a) $\int_0^{\frac{\pi}{2}} 2\sin(x)\,dx = -2\left[\cos(x)\right]_0^{\frac{\pi}{2}}$

$= -2\left(\cos\left(\frac{\pi}{2}\right) - \cos(0)\right)$

$= -2(-1)$

$= 2$

b) $2 \times \frac{\pi}{2} = \pi$

$\pi - 2 = \frac{1}{2}$ shaded region ($\because$ symmetry)

Shaded region $= 2\pi - 4$

14) a) $\int_0^1 3x^3\,dx = \left[\frac{3x^4}{4}\right]_0^1$

$= \frac{3}{4}$

b) $3 \times 1 = 3$

$3 - \frac{3}{4} = 2.25 \text{ units}^2$

5)

a) y-int

$x = 0$

$y = (-1)(-9)$

$= 9$

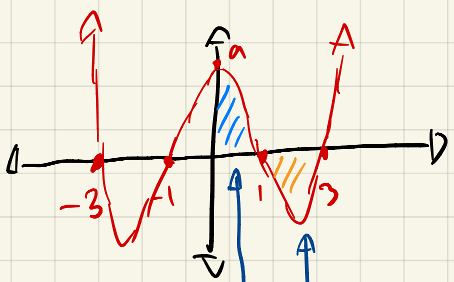
x-int

$y = (x^2-1)(x^2-9)$

$= (x+1)(x-1)(x+3)(x-3)$

$x = -1, 1, -3, 3$

↑ all intercepts



Add them seperately

then ×2

$\because$ Symmetrical

using a minus sign so that $\int_1^3 y\,dx$ is positive (because its below x-axis)

b) $\int_0^1 (x^2-1)(x^2-9)\,dx - \int_1^3 (x^2-1)(x^2-9)\,dx$

Expand this

$\int_0^1 (x^4 - 9x^2 - x^2 + 9)\,dx \ldots$

$= \int_0^1 (x^4 - 10x^2 + 9)\,dx - \int_1^3 (x^4 - 10x^2 + 9)\,dx$

$$\int_0^1 (x^4 - 10x^2 + 9)\,dx = \left[\frac{x^5}{5} - \frac{10x^3}{3} + 9x\right]_0^1$$

$$= \frac{1}{5} - \frac{10}{3} + 9$$

$$= \frac{3}{15} - \frac{50}{15} + 9$$

$$= -\frac{47}{15} + 9$$

$$= \frac{-47}{15} + \frac{135}{15}$$

$$= \frac{88}{15}$$

$$\int_1^3 (x^4 - 10x^2 + 9) = \left[\frac{x^5}{5} - \frac{10x^3}{3} + 9x\right]_1^3$$

$$= \frac{3^5}{5} - \frac{10\times 3^3}{3} + 9\times 3 - \frac{88}{15}$$

$$= \frac{243}{5} - \frac{270}{3} + 27 - \frac{88}{15}$$

$$= \frac{243}{5} - 90 + 27 - \frac{88}{15}$$

$$= \frac{243}{5} - 63 - \frac{88}{15}$$

$$= \frac{729}{15} - \frac{88}{15} - 63$$

$$= \frac{641}{15} - 63$$

$$= 42 - 63 + \frac{11}{15}$$

$$= -21 + \frac{11}{15} \quad \Big\} \int_1^3 f(x)\,dx$$

$$\underset{\int_0^1 f(x)\,dx}{\frac{88}{15}} - \left(-21 + \frac{11}{15}\right) = 21 + \frac{77}{15}$$

$$\times 2 = 42 + \frac{154}{15}$$

$$= 42 + 10 + \frac{4}{15}$$

$$= 52 + \frac{4}{15} \text{ units}^2$$

×2 because of symmetry

Find the Area

a)

$$\int_{-2}^{0} x^3 \, dx = \left[\frac{x^4}{4} \right]_{-2}^{0}$$

$$= 0 - \left(\frac{(-2)^4}{4} \right)$$

$$= -\left(\frac{16}{4} \right)$$

$$= -4$$

∵ under x-axis

∴ take absolute value

area = 4 units^2

e)

$$\int_{-1}^{1} x^3 + 2x^2 - x - 2 \, dx = \left[\frac{x^4}{4} + \frac{2x^3}{3} - \frac{x^2}{2} - 2x \right]_{-1}^{1}$$

$$= \left(\frac{1}{4} + \frac{2}{3} - \frac{1}{2} - 2 \right)$$

$$- \left(\frac{1}{4} - \frac{2}{3} - \frac{1}{2} + 2 \right)$$

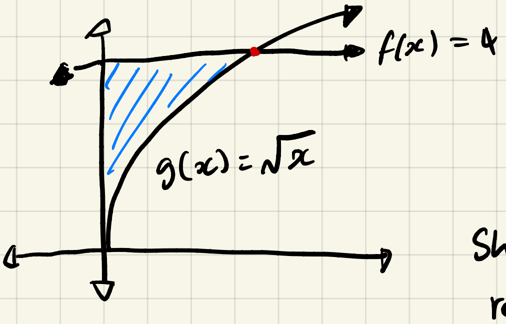
$$= \frac{4}{6} - 4$$

$$= \frac{4}{6} - \frac{24}{6}$$

$$= -\frac{20}{6}$$

∵ under curve

∴ Area = $\frac{20}{6}$ units^2



let a be the intersection point between the curves

Shaded region $= \int_0^a (f(x) - g(x))\,dx$

intersection point

$f(x) = g(x)$

$4 = \sqrt{x}$

$16 = x$

$a = 16$

$-\int x^{\frac{1}{2}} = \frac{2x^{\frac{3}{2}}}{3}$

$$\text{Area} = \int_0^{16} 4 - \sqrt{x}\,dx$$

$$= \left[4x - \frac{2\sqrt{x}^3}{3}\right]_0^{16}$$

$$= 4\times16 - \frac{2\sqrt{16}^3}{3}$$

$$= 64 - \frac{2\times4^3}{3}$$

$$= 64 - \frac{128}{3}$$

$$= 64 - (42 + \tfrac{2}{3})$$

$$= 22 - \tfrac{2}{3}$$

$$= 21\tfrac{1}{3} \text{ units}^2$$