

Grade 11 Methods Lesson 2

(1)

Q8) Concepts: Circles
Completing the Square

$$x^2 + y^2 - 4x + 2y - 4 = 0$$

→ Need to get to form

$$(x - h)^2 + (y - k)^2 = r^2$$

(h, k) gives center

$$x^2 - 4x + y^2 + 2y = 4$$

$$x^2 - 4x + \underbrace{\left(-\frac{4}{2}\right)^2 - \left(\frac{4}{2}\right)^2}_{\text{Completing the square}} + y^2 + 2y + \underbrace{\left(\frac{2}{2}\right)^2 - \left(\frac{2}{2}\right)^2}_{\text{Completing the square}} = 4$$

$$\underbrace{x^2 - 4x + 4}_{\text{Factorise}} - 4 + \underbrace{y^2 + 2y + 1}_{\text{Factorise}} - 1 = 4$$

$$(x - 2)^2 - 4 + (y + 1)^2 - 1 = 4$$

$$(x - 2)^2 + (y + 1)^2 = 4 + 4 + 1 \\ = 9$$

$$(x - 2)^2 + (y + 1)^2 = 9$$

∴ center is $(2, -1)$

$$\therefore r = \sqrt{9} \\ = 3$$

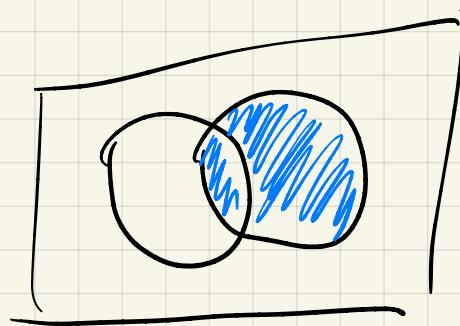
b) $A = \pi r^2$
 $= \pi \times 3^2 = 9\pi$

(2) Concept: Probabilities

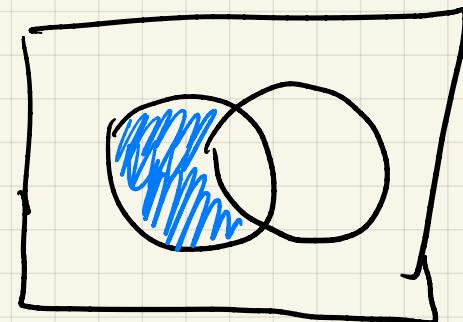
Q5)

a)

$$\text{i)} P(\text{soccer}) = \frac{25}{50} = \frac{1}{2}$$



$$\text{ii)} P(\text{Tennis} \cap \text{soccer}') = \frac{15}{50} = \frac{3}{10}$$



$$P(\text{Tennis} \cap \text{soccer}') = P(\text{Tennis}) \times P(\text{soccer}' | \text{Tennis})$$

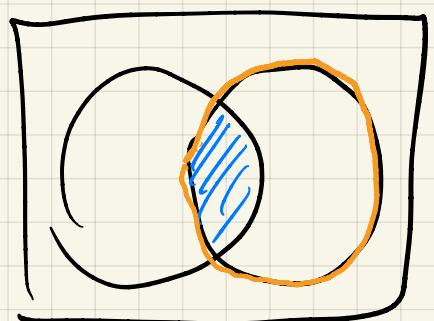
$$= \frac{20}{50} \times \frac{15}{20} \\ = \frac{15}{50} \\ = \frac{3}{10}$$

See how it
works with the
formulae

$$\text{b)} P(\text{Tennis} | \text{soccer}) = \frac{P(\text{Tennis} \cap \text{soccer})}{P(\text{soccer})}$$

$$= \frac{\left(\frac{5}{50}\right)}{\left(\frac{25}{50}\right)}$$

$$= \frac{50 \times 5}{25 \times 50} \\ = \frac{5}{25} = \frac{1}{5}$$



(3) CONCEPT: Factorisation and Cubics

1) $P(x) = x^3 + 5x^2 + 3x - 9$

Sub $x=1$

$$P(1) = 1 + 5 + 3 - 9$$

$$= 0$$

$\therefore (x-1)$ is a factor

$$P(x) = (x-1)(ax^2 + bx + c)$$

$$\begin{array}{r} \overline{x^2 + 6x + 9} \\ x-1 \sqrt{x^3 + 5x^2 + 3x - 9} \\ - \overline{x^3 - x^2} \\ \hline 6x^2 + 3x \\ - 6x^2 - 6x \\ \hline - 9x - 9 \\ - 9x - 9 \\ \hline 0 \end{array}$$

$$P(x) = (x-1)(x^2 + 6x + 9)$$

$$= (x-1)(x+3)^2$$

2) $P(x) = 6x^4 - 17x^3 - 11x^2 + 32x + 20$

If $(x-2)$ and $(x+1)$ is a factor

$\therefore (x-2) \times (x+1)$ is a factor

$$\begin{aligned} (x-2)(x+1) &= x^2 + x - 2x - 2 \\ &= x^2 - x - 2 \end{aligned}$$

$$\begin{array}{r} \overline{6x^2 - 11x - 10} \\ x^2 - x - 2 \sqrt{6x^4 - 17x^3 - 11x^2 + 32x + 20} \\ - \overline{6x^4 - 6x^3 - 12x^2} \\ \hline - 11x^3 + x^2 + 32x \\ - 11x^3 + 11x^2 + 22x \\ \hline - 10x^2 + 16x + 20 \\ - 10x^2 + 16x + 20 \\ \hline 0 \end{array}$$

$$\therefore P(x) = (6x^2 - 11x - 10)(x-2)(x+1)$$

↑

$$\begin{array}{r} \underline{-15 + 4} \\ \underline{-15 \times 4} \end{array} \quad \begin{array}{l} = -11 \\ = -60 \end{array}$$

$$\begin{aligned} b &= 6x^2 + 4x - 15x - 10 \\ &= 2x(3x+2) - 5(3x+2) \\ &= (2x-5)(3x+2) \end{aligned}$$

$$\therefore P(x) = (2x-5)(3x+2)(x-2)(x+1)$$

$$3) \quad P(x) = x^3 - ax^2 + bx - 3 \quad x^3 - 6x^2 - 2x - 3$$

$$\frac{x^3 - ax^2 + bx - 3}{x-1} = 2$$

$$\therefore P(1) = 2$$

$$\begin{aligned} P(1) &= 1 - a + b - 3 \\ &= -2 - a + b \end{aligned}$$

$$\frac{x^3 - ax^2 + bx - 3}{x+1} = -4$$

$$P(-1) = -4$$

$$\begin{aligned} P(-1) &= -1 - a - b - 3 \\ &= -a - b - 4 \end{aligned}$$

$$-2 - a + b = 2$$

$$-a - b = -4$$

$$b = 2 + 2 + a$$

$$-a - b = 0$$

$$\textcircled{1} \quad = 4 + a$$

$$-a = b$$

Sub in \textcircled{1}

$$-a = 4 + a$$

$$0 = 4 + 2a$$

$$2a = -4$$

$$a = -2$$

$$\begin{aligned} \therefore b &= 4 + (-2) \\ &= 2 \end{aligned}$$

(4) CONCEPT: PROBABILITY

(i) $P(A) = 0.7$, $P(B) = 0.3$, $P(A \cup B) = 0.8$

a) $P(A \cap B)$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\begin{aligned} \therefore P(A \cap B) &= P(A) + P(B) - P(A \cup B) \\ &= 0.7 + 0.3 - 0.8 \\ &= 0.2 \end{aligned}$$

b) $P(A|B) = \frac{P(A \cap B)}{P(B)}$

$$\begin{aligned} &= \frac{0.2}{0.3} \\ &= \frac{2}{3} \end{aligned}$$

c) $P(B|A) = \frac{P(B \cap A)}{P(A)}$

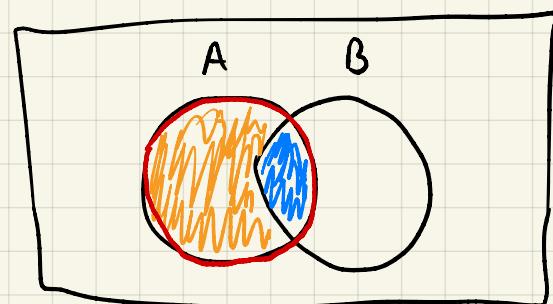
$$\begin{aligned} &= \frac{0.2}{0.7} \\ &= \frac{2}{7} \end{aligned}$$

d) $P(A|B') = \frac{P(A \cap B')}{P(B')}$

$$P(B') = 1 - P(B)$$

$$\begin{aligned} &= 1 - 0.3 \\ &= 0.7 \end{aligned}$$

$$P(A|B') = \frac{0.2}{0.7} = \frac{2}{7}$$



$$A \cap B' = P(A) - P(A \cap B)$$

$$= 0.7 - 0.2$$

$$= 0.5$$

$$11) P(A) = 0.6, P(B) = 0.5, P(A \cup B) = 0.8$$

$$\text{a)} P(A \cap B) = P(A) + P(B) - P(A \cup B)$$
$$= 0.3$$

$$\text{b)} P(A|B) = \frac{P(A \cap B)}{P(B)}$$
$$= \frac{3}{5}$$

$$\text{c)} P(B|A) = \frac{3}{6} = \frac{1}{2}$$

$$\text{d)} P(A|B') = \frac{P(A \cap B')}{P(B')}$$

$$P(A \cap B') = P(A) - P(A \cap B)$$
$$= 0.6 - 0.3$$
$$= 0.3$$

$$= \frac{0.3}{0.5}$$
$$= \frac{3}{5}$$

$$12) P(A) = 0.6, P(B) = 0.7, P(A \cap B) = 0.4$$

$$\text{a)} P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= 0.6 + 0.7 - 0.4$$
$$= 0.9$$

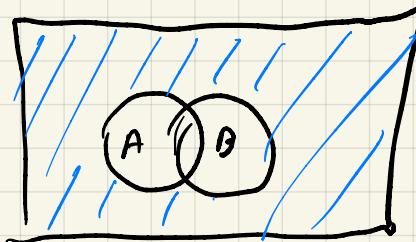
$$\text{b)} P(A|B) = \frac{P(A \cap B)}{P(B)}$$
$$= \frac{0.4}{0.7} = \frac{4}{7}$$

$$\text{c)} P(B|A') = \frac{P(B \cap A')}{P(A')} \quad P(B \cap A') = P(B) - P(A \cap B)$$
$$= 0.7 - 0.4$$
$$= 0.3$$
$$P(A') = 0.4$$

$$P(B|A') = \frac{0.3}{0.4}$$
$$= \frac{3}{4}$$

$$d) P(A' \mid B') = \frac{P(A' \cap B')}{P(B')}$$

$$P(A' \cap B') \rightarrow$$



$$P(B') = 1 - 0.7$$

$$= 0.3$$

$$P(A' \cap B') + P(A \cup B) = 1$$

$$\begin{aligned} P(A' \cap B') &= 1 - P(A \cup B) \\ &= 1 - 0.9 \\ &= 0.1 \end{aligned}$$

$$\begin{aligned} P(A' \mid B') &= \frac{0.1}{0.3} \\ &= \frac{1}{3} \end{aligned}$$

(5) 15) $R(x) = 6(2x^2 + 10x + 3)$
 $C(x) = x(6x^2 - x + 1)$

a) $R(x) : 2$

$C(x) : 3$

$$\begin{aligned} b) R(1) &= 6(2 \cdot 1^2 + 10 \cdot 1 + 3) & C(1) &= 1(6 \cdot 1^2 - 1 + 1) \\ &= 6 \times 15 & &= 1 \times 6 \\ &= 90 & &= 6 \end{aligned}$$

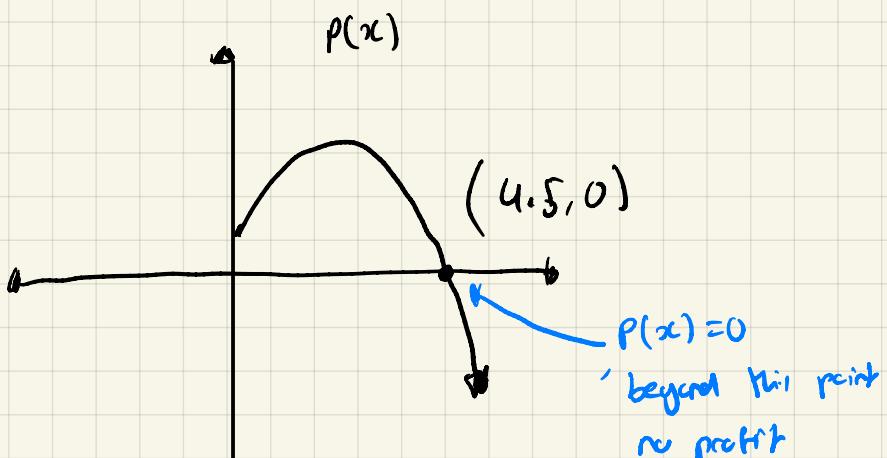
$$\text{Profit} = \text{Revenue} - \text{Cost}$$

$$= 90 - 6$$

$$= 84$$

$$\begin{aligned} c) P(x) &= R(x) - C(x) \\ &= 6(2x^2 + 10x + 3) - x(6x^2 - x + 1) \\ &= 12x^2 + 60x + 18 - 6x^3 + x^2 - x \\ &= -6x^3 + 13x^2 + 59x + 18 \end{aligned}$$

e) Using Graphics

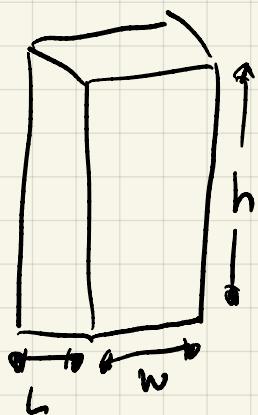


4500 units no profit.

4501 units is loss

$$d = 4501$$

⑥



$$\text{Surface area} : 2hw + 2Lh + 2Lw$$

$$SA = 2hx + 2(2x)h + 2(2x)x$$

$$= 2hx + 4xh + 4x^2$$

$$= 6hx + 4x^2$$

$$\text{Total area} = 48m^2$$

$$48 = 6hx + 4x^2$$

$$= 2x(3h + 2x)$$

$$\frac{24}{x} = 3h + 2x$$

$$3h = \frac{24}{x} - 2x$$

$$h = \frac{8}{x} - \frac{2}{3}x$$

a) Volume = $L \times w \times h$

$$= x \times 2x \times \left(\frac{8}{x} - \frac{2}{3}x\right)$$

$$= 2x^2 \left(\frac{8}{x} - \frac{2}{3}x\right)$$

$$V(x) = 16x - \frac{4}{3}x^3$$

b) Using Graphics Calculator
plotting $V(x)$

Largest volume at $x=2$

$$c) V(2) = 16 \times 2 - \frac{4}{3} \times 2^3$$

$$= 32 - \frac{4}{3} \times 8$$

$$\approx 21.3 \text{ m}^3$$