Lesson 2

Lesson 1 Review

QUESTION 6

If $\log_2 a = 5$ and $\log_2 b = 3$, then $a - b$ equals

(A) $5 - 3$

(B) 2^{5-3}

(C) $2^5 - 2^3$

(D) $5^2 - 3^2$

a. $y = \dfrac{\sin(x)}{x}$

b. $y = \dfrac{\sin(4x)}{\cos(2x)}$

a) $2\ln(x+1) - \ln(x+5) = \ln(x-1)$ where $x > 1$

16. Determine the equation of the tangent and the line perpendicular to the tangent to the curve $y = 3\cos(x)$ at the point where $x = \pi$.

Derivative Practice

QUESTION 13 (5 marks)

a) Determine the derivative of $f(x) = x^2(2\ln(x) - 1)$ and simplify.

11. A particle moves in a straight line so that at time t seconds its displacement, x metres, from a fixed origin O is given by $x(t) = t^3 - 6t^2 + 9t, t \geq 0$.

   a. How far is the particle from O after 2 seconds?

   b. What is the velocity of the particle after 2 seconds?

   c. After how many seconds does the particle reach the origin again, and what is its velocity at that time?

   d. What is the particle's acceleration when it reaches the origin again?

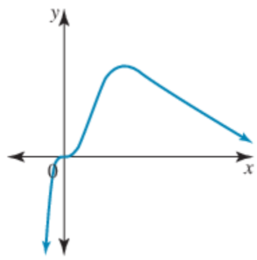
12. A particle moves in a straight line so that its displacement a point, O, at any time, t, is $x = \sqrt{3t^2 + 4}$.
   Determine:

   a. the velocity as a function of time

   b. the acceleration as a function of time

   c. the velocity and acceleration when $t = 2$.

10. The graph of the function $f: R \to R,\ f(x) = 3x^3e^{-2x}$ is shown. The derivative may be written as $f'(x) = ae^{-2x}(bx^2 + cx^3)$ where a, b and c are constants.

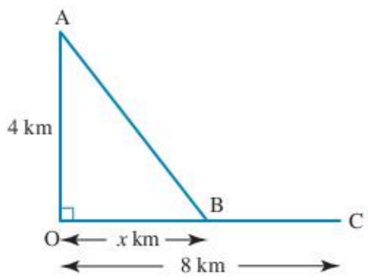


   a. Calculate the exact values of a, b and c.

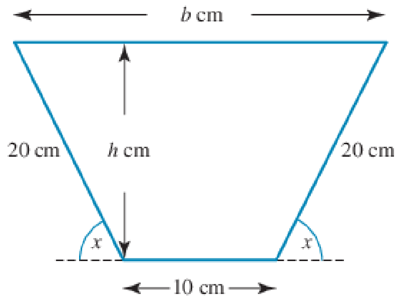
   b. Calculate the exact coordinates where $f'(x) = 0$.

   c. Determine the equation of the tangent to the curve at $x = 1$.

14. A rower is in a boat 4 km from the nearest point, O, on a straight beach. His destination is 8 km along the beach from O. If he is able to row at 5 km/h and walk at 8 km/h, what point on the beach should he row to in order to reach his destination in the least possible time? Give your answer correct to 1 decimal place.



13. Metal box guttering has to be formed on a common wall between two adjacent town houses. The cross section of the box guttering is shown.



For the most efficient elimination of rain water, this box guttering needs to have a maximum cross-sectional area within the given dimensions.

a. Determine an expression for h, the height of the trapezium, in terms of the angle x in radians, as shown.

b. Determine an expression for b, the base length of the trapezium, in terms of x.

c. Show that the cross-sectional area of the box guttering, A cm^2, is given by $A = 200\sin(x)(2\cos(x) + 1)$.

d. Determine, correct to 3 decimal places, the value of x that gives maximum cross-sectional area, and find this maximum area correct to the nearest cm^2.

QUESTION 15 (4 marks)

The graphs of $y = \ln x$ and $y = e^x$ are shown on the same axes below. The tangent to the curve $y = e^x$ at the point $(1, e)$ is also shown.

Determine the coordinates of the point on the graph of $y = \ln x$ where the gradient is parallel to the tangent shown.