

## Methods Lesson 7

1. **WE1** Determine  $\frac{d^2y}{dx^2}$  for each of the following functions.

a.  $y = x^4 - 5x^3 + x^2 - 9$

b.  $y = x^3 - 4x^2$

c.  $y = 4 - x^2$

At  $x=0$  are they concave up or down?

7. a. If  $f(x) = 4 \log_e(2x - 3)$ , calculate  $f''(3)$ .

b. If  $f(x) = e^{x^2}$ , calculate  $f''(1)$ .

14. Consider the function  $y = e^x \sin(x)$ .

a. Show that the function has a stationary point at  $x = \frac{3\pi}{4}$ .

What is the concavity at that point? Is this a maximum or a minimum stationary point?

5. a. For the function  $y = x^3 + 2x^2 - 3x + 1$ , determine where the function is:

i. concave up

ii. concave down.

b. Hence, state the coordinates of the point of inflection.

10. Determine the point(s) of inflection of the function  $f(x) = \frac{1}{2}x^2 - 3x^4$ . Hence, state where the function is concave down.

12. Determine the value of  $k$  if the function  $f(x) = 2x^3 - kx^2 + 3x$  has a point of inflection when  $x = 3$ .

2. An object travelling in a straight line has its displacement (in metres) after  $t$  seconds given by  $x(t) = 2 \cos(3t - 1) + 3$ .

a. Determine the maximum and minimum displacement.

b. Determine when the velocity is first equal to 0.

c. How long after the object is first at rest is it next at rest?

d. Determine an expression for the acceleration.

6. **WE11** The sum of two positive numbers is 32. Determine the numbers if their product is a maximum.

7. The sum of two positive numbers is 8. Determine the numbers if the sum of the cube of one and the square of the other is a minimum.

11. A rectangular box with an open top is to be constructed from a rectangular sheet of cardboard measuring 16 cm by 10 cm. The box will be made by cutting equal squares of side length  $x$  cm out of the four corners and folding the flaps up.

a. Express the volume as a function of  $x$ .

b. Determine the dimensions of the box with greatest volume and give this maximum volume.