Methods Lesson 2

Warm up

9. A curve has a gradient function $f'(x) = 4\cos(2x) + ke^x$, where k is a constant, and a stationary point at $(0, -1)$. Calculate:
   a. the value of k
   b. the equation of the curve $f(x)$

3. Determine the equation of the curve $f(x)$ given that:
   a. $f'(x) = (x+4)^3$ and the curve passes through $(-2, 5)$
   b. $f'(x) = 8(1-2x)^{-5}$ and $f(1) = 3$
   c. $f'(x) = (x+5)^{-1}$ and the curve passes through $(-4, 2)$

Recognition

18. **WE15** Differentiate $x\sin(x)$ and hence determine an antiderivative of $x\cos(x)$.
19. Differentiate $x\ln(x)$ and hence determine an antiderivative of $\ln(x)$.
20. Differentiate $y = 2xe^{3x}$ and hence determine an antiderivative of xe^{3x}.

14. **WE13** Given that $y = \sqrt{x^2+1}$, determine $\frac{dy}{dx}$ and hence determine the antiderivative of $\frac{5x}{\sqrt{x^2+1}}$.

15. If $y = (5x^2+2x-1)^4$, determine $\frac{dy}{dx}$ and hence determine an antiderivative of $16(5x+1)(5x^2+2x-1)^3$.

17. If $y = \ln(\cos(x))$, determine:
    a. $\frac{dy}{dx}$
    b. $\int \tan(x)dx$

Kinematics with antidifferentiation

22. A particle moves in a straight line so that its velocity, in metres per second, can be defined by the equation $v = 3t^2 + 6t, t \geq 0$. Determine:
    a. the displacement of the particle, x metres, as a function of t, if it is known that the particle was initially 2 metres to the left of the origin
    b. the position of the particle after 5 seconds.
23. Determine the displacement of a particle that starts from the origin and has a velocity defined by:
    a. $v = e^{(3t-1)}$
    b. $v = -\sin(2t+3)$

25. A particle starting at the origin moves in a straight line with a velocity of $\frac{12}{(t-1)^2} + 6$ metres per second after t seconds.
    a. Determine the rule relating the position of the particle, x metres, to t.
    b. Determine the position of the particle after 3 seconds.

Worded problems

13. If $f'(x) = a\sin(mx) - be^{nx}$ and $f(x) = \cos(2x) - 2e^{-2x} + 3$, calculate the exact constants a, b, m and n.

4. If a curve has a stationary point at $(1, 5)$ and a gradient of $8x + k$ where k is a constant, determine:
   a. the value of k
   b. the value of y when $x = -2$.

15. A curve has a gradient function $f'(x) = \dfrac{k}{2x+3}$, where $k \in R$. It is known that the function has a gradient of 2 when $x = 1$.
   a. Determine the value of k.
   b. Hence, determine the general rule for the function $f(x)$.

17. A particle attached to a spring moves up and down in a straight line so that at time t seconds its velocity, v metres per second, is given by

$$v = 3\pi \sin\left(\frac{\pi t}{8}\right), t \geq 0.$$

Initially the particle is stationary and at the origin.
   a. Determine the rule relating the position of the particle, x centimetres, to t.
   b. What is the maximum displacement of the particle?
   c. Where is the particle, relative to the stationary position, after 4 seconds?

(If you can do this then you've mastered antidifferentiation)

Two rowers are in a race, The leader is 88m away from the finish line, travelling at a constant velocity of 8m/s. A challenger is 12m behind the leader and is travelling at a velocity of 7.5m/s and accelerating at 0.5m/s^2. Who will win the race?

Definite Integrals

Evaluate the following definite integrals.

a. $$\int_0^3 (3x^2 + 4x - 1)\,dx$$

b. $$\int_1^2 \frac{4}{(2x+1)^3}\,dx$$