

Methods Lesson 3

$$1) \quad y = \ln(3x^2 + 7)$$

$$\frac{dy}{dx} = \frac{1}{3x^2 + 7} + 6x$$

$$= \frac{6x}{3x^2 + 7} \quad \therefore \int \frac{6x}{3x^2 + 7} dx = \ln(3x^2 + 7) + C$$

$$\int \frac{6x}{3x^2 + 7} dx = 6 \int \frac{x}{3x^2 + 7} dx$$

∴

$$6 \int \frac{x}{3x^2 + 7} dx = \ln(3x^2 + 7)$$

$$\int \frac{x}{3x^2 + 7} dx = \frac{\ln(3x^2 + 7)}{6} + C$$

(6')

$$y = \ln(3x^2 + 4)$$

$$\frac{dy}{dx} = \frac{6x}{3x^2 + 4}$$

$$6 \int \frac{x}{3x^2 + 4} dx = \ln(3x^2 + 4) + C$$

$$\int \frac{x}{3x^2 + 4} dx = \frac{\ln(3x^2 + 4)}{6} + C$$

$$16) f'(x) = \cos(2x) - \sin(2x)$$

$$\begin{aligned}f(x) &= \int \cos(2x) - \sin(2x) \\&= \frac{\sin(2x)}{2} + \frac{\cos(2x)}{2} + C\end{aligned}$$

Point $(\pi, 2)$

$$f(\pi) = 2$$

$$\begin{aligned}2 &= \frac{\sin(2\pi)}{2} + \frac{\cos(2\pi)}{2} + C \\&= 0 + \frac{1}{2} + C\end{aligned}$$

$$\frac{3}{2} = C$$

$$\underline{f(x) = \frac{\sin(2x)}{2} + \frac{\cos(2x)}{2} + \frac{3}{2}}$$

5.

Need to expand the brackets

$$\begin{aligned}\int \frac{(2x+5)^2}{x} dx &= \int \frac{4x^2 + 20x + 25}{x} dx \\&= \int 4x + 20 + \frac{25}{x} dx \\&= \underline{x^2 + 20x + 25 \ln(x) + C}\end{aligned}$$

$$\begin{aligned}6) \int \frac{(3x+2)^2}{x^2} dx &= \int \frac{9x^2 + 12x + 4}{x^2} dx \\&= \int 9 + \frac{12}{x^2} + \frac{4}{x^2} dx \\&= \underline{9x + 12 \ln(x) - \frac{4}{x} + C}\end{aligned}$$

$$\begin{aligned}
 5a) \int (e^{2x+1} - 4)^2 dx &= \int e^{2x+1} e^{2x+1} - 8e^{2x+1} + 16 dx \\
 &= \int e^{4x+2} - 8e^{2x+1} + 16 dx \\
 &= \frac{e^{4x+2}}{4} - 4e^{2x+1} + 16x + C
 \end{aligned}$$

10.

$$\begin{aligned}
 \frac{dy}{dx} &= x \left(1 - \frac{1}{x}\right)^2 \\
 &= x \left(1 - \frac{2}{x} + \frac{1}{x^2}\right) \\
 &= x - 2 + \frac{1}{x}
 \end{aligned}$$

$$\begin{aligned}
 y &= \int x - 2 + \frac{1}{x} dx \\
 &= \frac{x^2}{2} - 2x + \ln(x) + C
 \end{aligned}$$

$$16) \frac{d}{dx} \ln(x^2 + 3) = \frac{2x}{x^2 + 3}$$

$$\int \frac{2x}{(x^2 + 3)} = \ln(x^2 + 3) + C$$

x6 x6

$$6 \int \frac{2x}{(x^2 + 3)} dx = 6 \ln(x^2 + 3) + C$$

$$\int \frac{12x}{(x^2 + 3)} dx = 6 \ln(x^2 + 3) + C$$

$$\begin{aligned}
 11. \frac{d}{dx} \frac{\cos(x)}{\sin(x)} &= \frac{-\sin^2(x) - \cos^2(x)}{\sin^2(x)} \\
 u &= \cos(x) \quad v = \sin(x) \\
 u' &= -\sin(x) \quad v' = \cos(x)
 \end{aligned}$$

$$\sin^2(x) + \cos^2(x) = 1 \quad \text{A Pythagorean identity}$$

$$\cos^2(x) = 1 - \sin^2(x)$$

$$\begin{aligned} \therefore \frac{-\sin^2(x) - \cos^2(x)}{\sin^2(x)} &= \frac{-\sin^2(x) - (1 - \sin^2(x))}{\sin^2(x)} \\ &= -\sin^2(x) - 1 + \sin^2(x) \\ &= \underline{\underline{-1}} \end{aligned}$$

$$\therefore \int \frac{-1}{\sin^2(x)} dx = \frac{\cos(x)}{\sin(x)}$$

$$\int \frac{1}{\sin^2(x)} dx = -\frac{\cos(x)}{\sin(x)} \quad \text{or } -\frac{1}{\tan(x)} \quad \text{or } -\arctan(x)$$

12)

$$-3 + \frac{4}{3-2x} = \frac{-3(3-2x)}{(3-2x)} + \frac{4}{(3-2x)}$$

Multiply top
and bottom
by $(3-2x)$

> Do this so the denominator
can be the same

> so you can add them
together

$$= \frac{-9+6x}{(3-2x)} + \frac{4}{(3-2x)}$$

$$= \frac{-9+6x+4}{(3-2x)}$$

$$= \frac{6x-5}{(3-2x)}$$

Prove by polynomial long
division

$$\begin{array}{r} -3 \\ \hline -2x+3 \sqrt{6x-5} \\ -6x-9 \\ \hline 0+4 \end{array}$$

$$\therefore \int \frac{6x-5}{3-2x} dx = \int -3 + \frac{4}{(3-2x)} dx$$

$$= -3x + \frac{4 \ln(3-2x)}{-2} + C$$

$$= -3x - 2 \ln(3-2x) + C$$

$$\begin{aligned} \therefore \frac{6x-5}{3-2x} &= -3 + \frac{4}{-2x+3} \\ &= -3 + \frac{4}{(3-2x)} \end{aligned}$$

$$6) f'(x) = 3x^2 - 8x + 3$$

PASSES THE ORIGIN

\therefore point $(0,0)$
exists

$$\int 3x^2 - 8x + 3 \, dx = x^3 - 4x^2 + 3x + C$$

$$f(0) = 0$$

$$0 = 0^3 - 4 \times 0^2 + 3 \times 0 + C$$

$$C = 0$$

$$\therefore f(x) = x^3 - 4x^2 + 3x$$

x intercepts

$$f(x) = x(x^2 - 4x + 3) \quad \text{Factor } x \text{ out}$$

$$= x(x+4)(x-1) \quad \text{Solve the quadratic}$$

$$\therefore x = 0, -4, 1 \quad \text{find } x\text{-values via null factor}$$

axis intercepts $(0, f(0)), (-4, f(-4)), (1, f(1))$

Solve for the
 $f(x)$, I didn't
do them here
but this should
be easy

$$6) \text{ acceleration} = -10 \text{ m/s}^2$$

$$\text{velocity initially} = 25 \text{ m/s} \rightarrow v(0) = 25$$

$$\therefore a(t) = -10$$

$$x(t) = \int v(t) dt$$

$$v(t) = -10t + C$$

$$= \int -10t + 25 dt$$

$$v(0) = 25$$

$$= -5t^2 + 25t + C$$

$$25 = -10 \times 0 + C$$

at time of firing, displacement = 0

$$C = 25$$

$$x(0) = 0$$

$$\therefore C = 0$$

a) $v(t) = -10t + 25$

b) $x(t) = -5t^2 + 25t$

Max height found at
stationary point

max height at
 $t = 2.5$

time to return
($x=0$)

$$\therefore \text{at } v(t) = 0$$

$$0 = -10t + 25$$

$$10t = 25$$

c) $t = 2.5$ seconds

To confirm it is a maximum, take
 $\frac{dx^2}{dt^2}$ and observe

its value

$$\therefore a(t) = \frac{d x^2}{dt^2}$$

$$= -10$$

\therefore -ve \therefore maximum

$$\therefore x(2.5) = -5x(2.5)^2 + 25x2.5$$

$$= -5 \times \frac{25}{4} + 25 \times \frac{5}{2}$$

$$= -\frac{125}{4} + \frac{125}{2}$$

$$= -\frac{125}{4} + \frac{250}{4}$$

$$= \frac{125}{4}$$

a) $= 31.25 \text{ m}$

$$1) \int_0^1 x^2 dx = \left[\frac{x^3}{3} \right]_0^1$$

$$= \left(\frac{1^3}{3} \right) - \left(\frac{0^3}{3} \right)$$

$$= \frac{1}{3}$$

$$\int_0^3 x^3 dx = \left[\frac{x^4}{4} \right]_0^3$$

$$= \left(\frac{3^4}{4} \right) - \left(\frac{0^4}{4} \right)$$

$$= \frac{81}{4}$$

$$\approx 20.25$$

$$c) \int_0^\pi 5 \sin\left(\frac{x}{a}\right) dx = \left[-20 \cos\left(\frac{x}{a}\right) \right]_0^\pi$$

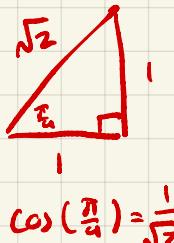
$$= \left(-20 \cos\left(\frac{\pi}{a}\right) \right) - \left(-20 \cos\left(\frac{0}{a}\right) \right)$$

$$= \left(-20 \times \frac{1}{\sqrt{2}} \right) - (-20)$$

$$= \frac{-20}{\sqrt{2}} + 20$$

$$= \frac{-20\sqrt{2}}{2} + 20$$

$$= -10\sqrt{2} + 20$$



11)

$$\int_0^a e^{\frac{x}{2}} dx = 4$$

↓

evaluate this
first

$$= \left[2e^{\frac{x}{2}} \right]_0^a = (2e^{\frac{a}{2}}) - (2e^{\frac{0}{2}}) \\ = 2e^{\frac{a}{2}} - 2$$

$$2e^{\frac{a}{2}} - 2 = 4$$

$$2e^{\frac{a}{2}} = 6$$

$$e^{\frac{a}{2}} = 3$$

$$\ln e^{\frac{a}{2}} = \ln 3$$

$$\frac{a}{2} = \ln 3$$

$$\underline{a = 2\ln 3 \quad \text{or} \quad \ln 9}$$

12)

$$\int_0^a e^{-2x} dx = \left[-\frac{e^{-2x}}{2} \right]_0^a$$

To make things easier,
you can factor out
constants in the expression

$$\left[-\frac{e^{-2x}}{2} \right]_0^a = -\frac{1}{2} \left[e^{-2x} \right]_0^a$$

$$-\frac{1}{2} (e^{-2a} - e^0) = -\frac{1}{2} (e^{-2a} - 1)$$

$$= -\frac{e^{-2a}}{2} + \frac{1}{2}$$

$$-\frac{e^{-2a}}{2} + \frac{1}{2} = \frac{1}{2} \left(1 - \frac{1}{e^{2a}} \right)$$

$$-e^{-2a} + 1 = 1 - \frac{1}{e^{2a}}$$

$$-e^{-2a} = -\frac{1}{e^{2a}}$$

$$e^{-2a} = e^{-8}$$

$$\underline{a = 4}$$

$$3) y = x + 1$$

bounds $x=0, x=2$

$$\begin{aligned}\int_0^2 x+1 \, dx &= \left[\frac{x^2}{2} + x \right]_0^2 \\ &= 2+2 \\ &= 4 \text{ units}^2\end{aligned}$$

$$4) y = x^2$$

bounds $x=0, x=3$

$$\begin{aligned}\int_0^3 x^2 \, dx &= \left[\frac{x^3}{3} \right]_0^3 \\ &= \frac{27}{3} \\ &= 9 \text{ units}^2\end{aligned}$$