Methods Lesson 2

9)

a) $f'(x) = 4\cos(2x) + ke^{x}$

Stationary point $(0,-1)$

↑

gradient $=0$ at $(0,-1)$

$f'(x) =$ gradient

$f'(x) = 0$ at $(0,-1)$

$f'(0) = 0$

$0 = 4\cos(2\times 0) + ke^{0}$

$= 4\cos(0) + k$

$= 4 + k$

$-4 = k$

b) $f'(x) = 4\cos(2x) - 4e^{x}$

$f(x) = \int f'(x)$

$= 2\sin(2x) - 4e^{x} + c$

point $(0,-1)$

$f(0) = -1$

$-1 = 2\sin(2\times 0) - 4e^{0} + c$

$= 0 - 4 + c$

$3 = c$

$f(x) = 2\sin(2x) - 4e^{x} + 3$

3)

a) $f'(x) = (x+4)^3$

$f(x) = \dfrac{(x+4)^4}{4} + c$

point $(-2,5)$

$f(-2) = 5$

$5 = \dfrac{(-2+4)^4}{4} + c$

$= \dfrac{2^4}{4} + c$

$= 4 + c$

$1 = c$

$f(x) = \dfrac{(x+4)^4}{4} + 1$

b) $f'(x) = 8(1-2x)^{-5}$

$f(x) = \dfrac{8(1-2x)^{-4}}{-4 \times -2} + c$

$= (1-2x)^{-4} + c$

$f(1) = 3$

$3 = (1-2\times 1)^{-4} + c$

$= \dfrac{1}{(1-2)^4} + c$

$= 1 + c$

$c = 2$

c) $f'(x) = (x+5)^{-1}$

$f(x) = \ln(x+5) + c$

point $(-4,2)$

$f(-4) = 2$

$2 = \ln(-4+5) + c$

$= \ln(1) + c$

$= c$

$f(x) = \ln(x+5) + 2$

2024 Methods Lesson 2

18) $y = x\sin(x)$

$u = x \quad v = \sin(x)$

$u' = 1 \quad v' = \cos(x)$

$$\frac{dy}{dx} = x\cos(x) + \sin(x)$$

$$\therefore \int x\cos(x) + \sin(x)\,dx = x\sin(x)$$

$$\int x\cos(x)\,dx + \int \sin(x)\,dx = x\sin(x)$$

$$\int x\cos(x)\,dx = x\sin(x) - \int \sin(x)\,dx$$

$$= x\sin(x) - (-\cos(x))$$

$$= x\sin(x) + \cos(x) + C$$

19. $y = x\ln(x)$

$u = x \quad v = \ln(x)$

$u' = 1 \quad v' = \frac{1}{x}$

$$\frac{dy}{dx} = 1 + \ln(x)$$

$$\therefore \int 1 + \ln(x)\,dx = x\ln(x)$$

$$\int 1\,dx + \int \ln(x)\,dx = x\ln(x)$$

$$\int \ln(x)\,dx = x\ln(x) - \int 1\,dx$$

$$\int \ln(x)\,dx = x\ln(x) - x + C$$

2024 Methods lesson 2

26)

$y = 2xe^{3x}$

$u = 2x \quad v = e^{3x}$

$u' = 2 \quad v' = 3e^{3x}$

$\dfrac{dy}{dx} = 6xe^{3x} + 2e^{3x}$

$\therefore \int 6xe^{3x} + 2e^{3x} = 2xe^{3x}$

$6\int xe^{3x} = 2xe^{3x} - \int 2e^{3x}$

$= 2xe^{3x} - \frac{2}{3}e^{3x} + c$

$\int xe^{3x} = \frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x} + c$

14) $y = \sqrt{x^2+1}$

$= (x^2+1)^{\frac{1}{2}}$

$\dfrac{dy}{dx} = \dfrac{2x}{2}(x^2+1)^{-\frac{1}{2}} = \dfrac{x}{\sqrt{x^2+1}}$

$\int \dfrac{5x}{\sqrt{x^2+1}}dx = 5\int \dfrac{x}{\sqrt{x^2+1}}dx$

$= 5\sqrt{x^2+1} + c$

$\cancel{15)\ y = (5x^2+2x-1)^4}$

$\cancel{\dfrac{dy}{dx} = 4(10x+2)(5x^2+2x-1)^3 = 8(5x+1)(5x^2+2x-1)^3}$

$\cancel{\int 16(5x^2+2x-1)^3 dx = 4(5x^2+2x-1)^4 + c}$

$\cancel{\int 16(5x+1)(5x^2+2x-1)^3} = 2$

$\cancel{= 2 \times \int 8(5x+1)(5x^2+2x-1)^3 = 2(5x^2+2x-1)^4}$

15) $y = (5x^2 + 2x - 1)^4$

$$\frac{dy}{dx} = 4(10x - 2)(5x^2 + 2x - 1)^3$$

$$\int 4(10x - 2)(5x^2 + 2x - 1)^3 = (5x^2 + 2x - 1)^4 + C$$

factor out 2 from $(10x - 2)$
$\rightarrow 2(5x - 1)$

$$\int 4 \times 2 \times (5x - 1)(5x^2 + 2x - 1)^3 = (5x^2 + 2x - 1)^4 + c$$

$$\int 8(5x - 1)(5x^2 + 2x - 1)^3 = (5x^2 + 2x - 1)^4 + c$$

$\times 2$ on both sides to go from $8 \to 16$

$$\int 16(5x - 1)(5x^2 + 2x - 1)^3 = 2(5x^2 + 2x - 1)^4 + c$$

o Rearrange
o Multiply/divide until you see the integral you need

22)

a) particle initially 2m to left
$\therefore \; x(0) = -2$

$$\int v(t) = x(t)$$

$$\int 3t^2 + 6t = t^3 + 2t^2 + c$$

$$-2 = 0 + 6 + c$$

$$c = -2$$

$$x(t) = t^3 + 2t^2 - 2$$

b) $x(5) = 125 + 50 - 2$
$= 173\text{m}$

23) particle starts at origin
$\therefore \; x(0) = 0$

a) $\int e^{(3t-1)} = \dfrac{e^{3t-1}}{3} + c$

$$0 = \frac{e^{3 \times 0 - 1}}{3} + c$$

$$0 = \frac{e^{-1}}{3} + c$$

$$= \frac{1}{3e} + c$$

$$c = -\frac{1}{3e}$$

$$x(t) = \frac{e^{(3t-1)}}{3} - \frac{1}{3e}$$

b) $v = -\sin(2t+3)$

$$x(t) = \int -\sin(2t+3)\,dt$$

$$= \frac{\cos(2t+3)}{2} + c$$

$x(0) = 0$

$$0 = \frac{\cos(3)}{2} + c$$

$$c = \frac{-\cos(3)}{2}$$

$$x(t) = \frac{\cos(2t+3)}{2} - \frac{\cos(3)}{2}$$

25) $\frac{12}{(t-1)^2} + 6 = v(t)$

$x(0)=0$ ∵ It starts from origin

$$\int 12(t-1)^{-2} + 6\,dt = \frac{-12}{(t-1)} + 6t + c$$

$$0 = \frac{-12}{-1} + 0 + c$$

$$= 12 + c$$

$$c = -12$$

$$\therefore \quad x(t) = \frac{-12}{(t-1)} + 6t - 12$$

$\therefore x(t) = -24\cos\left(\frac{\pi t}{8}\right) + 24$

b) Max displacement when $\cos\left(\frac{\pi t}{8}\right) = -1$

$\cos(\theta) = -1$ when $\theta = \pi$

$\frac{\pi t}{8} = \pi$

$\pi t = 8\pi$

$t = 8$ seconds

c) $x(4) = -24\cos\left(\frac{4\pi}{8}\right) + 24$

$= 0 + 24$

$= 24$ meters

Mastery Question

Leader - 88m away
Challenger - Leader +12m away
} From finish

let x = displacement from finish line

$\therefore$ at $x(t) = 0$, t = time taken to complete race

Leader

- initial displacement is -88m from finish
- velocity = 8m/s

$\therefore$ $\int 8\,dt = 8t + c$

$x(0) = -88$

$-88 = 8 \times 0 + c$
$= c$

$c = -88$

$\therefore x(t) = 8t - 88$

$0 = 8t - 88$

$88 = 8t$

$t = 11$ seconds to finish for the leader

Challenger

initially $-100m$ from finish

initial velocity $- 7.5m/s$ → $v(0)=7.5$

acceleration - constant $0.5m/s^2$

$$\int a(t) = v(t)$$

$$\int 0.5\,dt = 0.5t + c$$

$$v(0) = 7.5$$

$$c = 7.5$$

$$v(t) = 0.5t + 7.5$$

$x(0) = -100$ $\therefore$ $c = -100$

$$x(t) = \int 0.5t + 7.5\,dt$$

$$= \frac{0.5t^2}{2} + 7.5t + c$$

$$x(t) = 0.25t^2 + 7.5t - 100$$

$0 = 0.25t^2 + 7.5t - 100$ ← $x=0$ is finish line, solve for t

$= t^2 + 30 - 400$ ← multiply all by 4 to make factorization easier

$= (t-10)(t+20)$

$t = 10$ seconds ← time cannot $= -20$

$\therefore$ Challenger will win the race as it takes him 10 seconds rather than 11 seconds definite integrals

a) $\int_0^3 (3x^2+4x-1)\,dx = \left[x^3+2x^2-x\right]_0^3$

$= (3^3+2\times3^2-3)-(0^3+2\times0^2-0)$

$= 27+18-3$

$= 42$

b) $\int_1^2 \frac{4}{(2x+1)^3}\,dx = \int_1^2 4(2x+1)^{-3}\,dx$ $\longrightarrow$ $\int 4(2x+1)^{-3}dx = \frac{4(2x+1)^{-2}}{-2\times2}$

$= (2x+1)^{-2}$

divide by new power

divide by derivative

$= \left[-(2x+1)^{-2}\right]_1^2$

$= \left[-\frac{1}{(2x+1)^2}\right]_1^2$

$= -\left(\left(\frac{1}{(2\times2+1)^2}\right)-\left(\frac{1}{(2\times1+1)^2}\right)\right)$

$= -\left(\frac{1}{25}-\frac{1}{9}\right)$

$= -\left(\frac{9}{225}-\frac{25}{225}\right)$

$= \frac{16}{225}$