Methods Lesson 5

Applications of integration

A particle starting from rest accelerates according to the rule $a = 3t(2 - t)$.

a. Determine a relationship between the velocity of the particle, v metres/second, and the time, t seconds.

b. Determine the displacement of the particle after 4 seconds.

c. Sketch the graph of velocity versus time for the first 4 seconds of the motion.

d. Calculate the distance travelled by the particle in the first 4 seconds.

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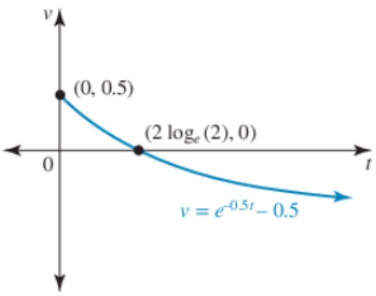
14. A particle moves in a straight line. At time t seconds its velocity, v metres per second, is defined by the rule $v = e^{-0.5t} - 0.5$, $t \geq 0$. The graph of the motion is shown.

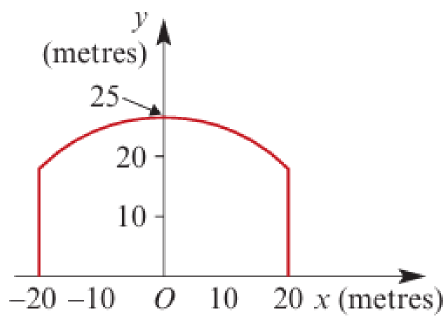


a. Determine the acceleration of the particle, $a\ \mathrm{m/s^2}$, in terms of t.

b. Determine the displacement of the particle, x m, if $x = 0$ when $t = 0$.

c. Determine the displacement of the particle after 4 seconds.

15 The roof of an exhibition hall has the shape of the function $f(x) = 25 - 0.02x^2$, $x \in [-20, 20]$. The hall is 80 metres long. A cross-section of the hall is shown in the figure. An air-conditioning company wishes to find the volume of the hall so that a suitable system may be installed. Find this volume.



QUESTION 17 (4 marks)

A snail is travelling along a straight path from point A. The snail's velocity (cm min^{-1}) is modelled by $v(t) = 1.4\ln\left(1 + t^2\right)$, where t is time (in minutes) for $0 \leq t \leq 15$.

An ant passes point A 12 minutes after the snail and follows the snail's path. The ant moves with a constant acceleration of 2 cm min^{-2} and passes the snail at $t = 15$ minutes.

Determine the ant's velocity at point A.

Exam preparation

1 Solve each of the following equations for x:

a $\ln x + \ln 25 = \ln(x^3)$ **b** $3 \times 3^{2x} + 3^x - 10 = 0$

c $\log_{16}(3x - 1) = \log_4(3x) + \log_4\left(\frac{1}{2}\right)$

5 Evaluate:

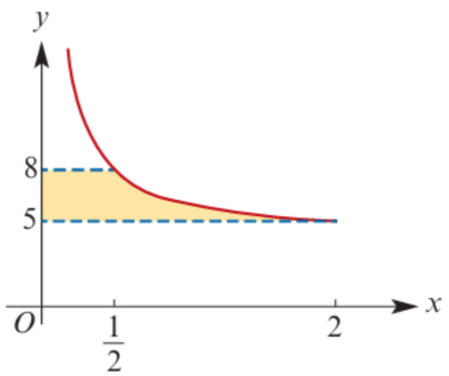
a $\int_0^1 \frac{2}{5x+5}\,dx$ **b** $\int_1^4 2x + \frac{2}{x}\,dx$

4 Solve the equation $\log_2(7x^2 + 8x + 3) = \log_2(x^2) + 1$.

8 The diagram shows the graph of the function

$$f(x) = 4 + \frac{2}{x}, \quad 0 < x \leq 2$$

and the lines $y = 5$ and $y = 8$. Find the area of the shaded region.



Question 4 (6 marks)

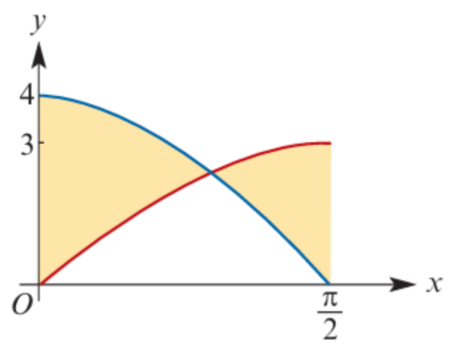
A particle moves along the x-axis with position at time t given by $x(t) = e^t \sin(t)$ for $0 \leq t \leq 2\pi$. The rate of change of position with respect to time is called velocity. Determine all possible values of t when the particle is at rest.

1 The diagram shows the graphs of $y = 3\sin x$ and $y = 4\cos x$ for $0 \leq x \leq \frac{\pi}{2}$.

a The graphs intersect at the point $P(a, b)$. Determine the value of:

i $\tan a$ **ii** $\sin a$ **iii** $\cos a$

b Determine the total area of the shaded regions.



Question 5 (6 marks)

Determine the coordinates of the stationary point of the function $f(x)=\frac{\ln(2x)}{x}, x>0$.

Question 6 (5 marks)

The spread of a flu in a certain school is modelled by the equation $P(t)=\frac{100}{1+e^{b-t}}, t\geq 0$ where $P(t)$ is the total number of students infected after t days.

Given that the number of students infected after day 3 is 50, determine the rate the flu is spreading at this time.

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QUESTION 16 (5 marks)

A handbag designer finds that the number of handbags she sells in a week is given by the equation, $n = 92e^{-0.01p}$ where n is the number sold and p is the price ($) per handbag she sells.

Calculate the number of handbags she should aim to sell in a week to obtain maximum revenue from sales.

Question 11 (7 marks)

Terry is an avid skier. The equation that best models his favourite run is given by $H=1.8e^{-x}+0.43$ where H is the height in kilometres above sea level and x represents the cross-sectional width of the run in kilometres.

The run terminates at a place that is 1 km above sea level. The cover of snow on the mountain is 2 m (assume this cover is constant across the entire run). If the run is 300 m wide, calculate the volume of snow on the run.

Question 10 (8 marks)

Determine the line $y=mx$ that divides the area under the curve $y=2x(2-x)$ over $[0,2]$ into two regions of equal area.

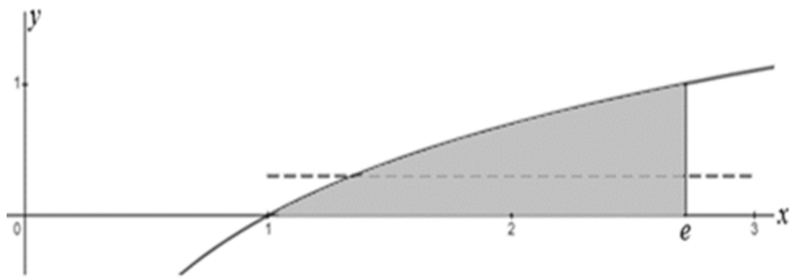
Justify all procedures and decisions by explaining mathematical reasoning.

Evaluate the reasonableness of your solution.

QUESTION 20 (7 marks)

a) Show that $\frac{d}{dx}(x\ln(x) - x) = \ln(x)$. *[1 mark]*

b) The area between the curve $y = \ln(x)$, the x-axis and the line $x = e$ is shaded below. *[5 marks]*



Determine the value of c such that the line $y = c$ cuts this area in half. Give your answer correct to three decimal places.

Question 9 (4 marks)

In the figure below, $f(x)$ and $g(x)$ intersect at A and B.

If $f(x) = 2e^x - 3x$ and $g(x) = -3e^{-x} + x^2 + 3$, find the area of the region bounded by $f(x)$ and $g(x)$.