Digital Control Technologies and Architectures - 01PDCYP, 01PDCOV, 01PDCLP

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Laboratory practice 6

Objectives: MPC control design of CT systems using `MPCtools`, closed loop constrained discrete time quadratic optimal control through the receding horizon principle, review on stability and response computation of discrete time 1dof control systems.

Problem 1 (MPC design for output set-point tracking using `MPCtools`.)

Consider the mass-damper system introduced in Laboratory practice 4 described by the following state space representation in continuous time.

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t), \; x(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t)$$

Assume a sampling time $T_s = 0.05$ s and suppose that the system state can be measured. Design an MPC controller using `MPCtools` to meet the following requirements:

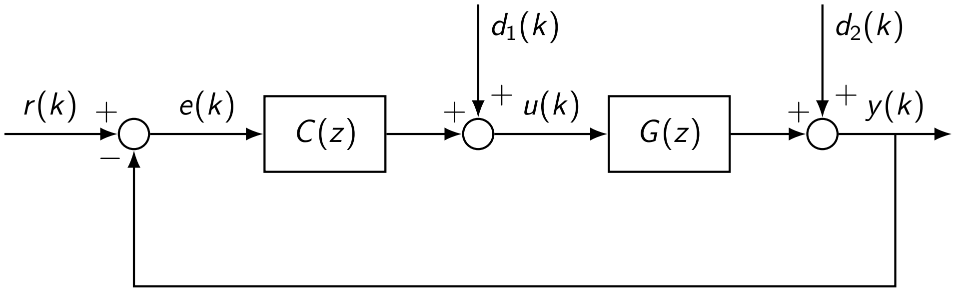
1. zero steady state tracking error ($|e_r^\infty| = 0$) in the presence of a unitary step reference;
2. $t_{s,1\%} \approx 2$ s with a tolerance of 5%;
3. $\max_t |u(t)| \leq 1$.

Hint: start with $Q = 1$ and $R = 1$ and a sufficiently high prediction horizon H_p; then tune the weights Q and or R to adjust the response speed; then evaluate, if needed, the effect of including an output constraint and further tuning of the weights Q and or R; finally, try to reduce the prediction horizon H_p, and, if possible, the control horizon H_c.

Compare the results with those obtained in Problem 2 of Laboratory practice 4, using the LQ approach with integral action.

Problem 2 (review on stability, response computation, steady state analysis of discrete time 1dof control systems)

Given the 1dof control architecture,



where:

$$C(z) = 0.01\frac{z-0.2}{z-0.8}, \ G(z) = \frac{z-1}{z^2-0.2z+0.02}, \ T_s = 1 \text{ s}$$

1. Study the stability properties.
2. Compute the analytical expression of the output response $y(k)$ when $r(k) = 2\,\varepsilon(k)$ and the other inputs are set to zero.
3. Compute, if possible, the steady state value y_∞ of $y(k)$ when $r(k) = 2\,\varepsilon(k)$ and the other inputs are set to zero. (Hint: use the final value theorem).
4. Compute, if possible, the steady state value y_∞ of $y(k)$ when $d_1(k) = \varepsilon(k)$ and the other inputs are set to zero. (Hint: use the final value theorem).

Provide motivations for the values obtained in points 3. and 4.

Solution

1. The given control system is stable.
2. $y(k) = (0.0236 \cdot 0.8024^k + 0.0425 \cdot 0.1321^k \cos(0.781k - 2.1596))\varepsilon(k)$.
3. $y_\infty = 0$.
4. $y_\infty = 0$.

Problem 3 (closed loop constrained discrete time quadratic optimal control through the receding horizon principle)

Given the LTI discrete time dynamical system,

$$x(k+1) = \begin{bmatrix} 0.9616 & 0.1878 \\ -0.3756 & 0.8677 \end{bmatrix} x(k) + \begin{bmatrix} 0.0192 \\ 0.1878 \end{bmatrix} u(k),\ x(0) = \begin{bmatrix} -0.3 \\ 0.4 \end{bmatrix},\ T_s = 0.2 \text{ s}.$$

Consider the feedback state control law obtained by applying the receding horizon principle (RH) to the following constrained finite optimization problem

$$\min_{U(k)} \sum_{i=0}^{H_p-1} \left(x^\top(k+i)\, Q\, x(k+i) + u^\top(k+i)\, R\, u(k+i) \right) + x^\top(k+H_p)\, S\, x(k+H_p)$$

s.t.

$|u(k|k+i)| \leq 0.5,\ i = 0, \ldots, H_p - 1;$

Given $H_p = 3$, $\quad Q = S = \begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix}$, $\quad R = 0.1$,

1. simulate the state response over a time interval of 8 s, i.e., 40 steps;
2. plot $x_1(k)$, $x_2(k)$, $u(k)$, $\|x(k)\|_2$;
3. evaluate the zero regulation time t_{reg} based on $\|x(k)\|_2$ and considering a tolerance of 10^{-4}.

Problem 4 (closed loop constrained discrete time quadratic optimal control through the receding horizon principle)

Repeat Problem 3 including the following state constraint

$$x(k|k+i) \leq x_{\max},\ i = 1, \ldots, H_p,\ x_{\max} \leq \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}.$$

Compare the results obtained in Problem 3 in terms of regulation time.