# Digital Control Technologies and Architectures - 01PDCYP, 01PDCOV, 01PDCLP

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## Laboratory practice 4

*Objectives*: digital LQ control of CT LTI systems.

### Problem 1

The continuous time state space representation below describes, for suitable value of the physical parameters, a mass-damper system. The state components $x_1$ and $x_2$ are the position and the speed of the mass, respectively. The input $u$ is the force applied to the mass while the output $y$ of the system output is the position of the mass.

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$
$$y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t)$$

Assume a sampling time $T_s = 0.05$ s and design, if possible, a digital infinite horizon LQ controller through the optimization of the following cost function

$$J = \sum_{i=0}^{\infty} x^\top(k+i)Qx(k+i) + u^\top(k+i)Ru(k+i),$$

to obtain zero regulation of the state of the system according to the Euclidean norm (within a tolerance of $10^{-5}$) in about 5 s with a tolerance of 2% starting with the initial condition $x(0) = \begin{bmatrix} 0.8 & 0 \end{bmatrix}^\top$.

### Problem 2

Consider the mass-damper system introduced in Problem 1.

1. Design, if possible, a digital infinite horizon LQ controller with integral action to impose zero steady state tracking error in the presence of a step reference input and a settling time $t_{s,1\%} \approx 2$ s with a tolerance of 2% starting with the initial condition $x(0) = \begin{bmatrix} 0 & 0 \end{bmatrix}^\top$. (Hint: start from the design obtained in Problem 1, then tune the weight of the integral state to meet the settling time requirement; what is the effect of increasing such a weight too much? )

2. Modify the design performed at the previous point so that $|u(t)| \leq 1, \forall t$ when $r(t) = \varepsilon(t)$. Is it still possible to obtain the same performance in terms of settling time $t_{s,1\%} \approx 2$?
   (Hint : start from the design obtained at the previous point, then tune the input weight until you meet the magnitude condition for the control input).