Digital Control Technologies and Architectures - 01PDCYP, 01PDCOV, 01PDCLP
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Laboratory practice 5

Objectives: finite horizon unconstrained/constrained discrete time quadratic optimal control. Receding Horizon principle.

Problem 1

Given the LTI discrete time dynamical system and the generic time instant k.

$$x(k+1) = \begin{bmatrix} 0.3 & 1.5 \\ 0.5 & -0.4 \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k), \; x(0) = \begin{bmatrix} 20 \\ 20 \end{bmatrix}, \; T_s = 0.1 \text{ s.}$$

Consider the unconstrained finite horizon quadratic optimal control problem described by the cost function:

$$\min_{U(k)} J(x(k), U(k)) =$$

$$= \min_{U(k)} \sum_{i=0}^{H_p-1} \left(x^\top(k+i)Qx(k+i) + u^\top(k+i)Ru(k+i) \right) + x^\top(k+H_p)Sx(k+H_p)$$

$$U(k) = \begin{bmatrix} u(k) & u(k+1) & \cdots & u(k+H_p-1) \end{bmatrix}$$

Given $H_p = 3$, $Q = S = \begin{bmatrix} 20 & 0 \\ 0 & 5 \end{bmatrix}$, $R = 5$, compute the optimal control sequence: $U^*(k) =$

$\begin{bmatrix} u^*(k) \\ u^*(k+1) \\ u^*(k+2) \end{bmatrix}$ and the corresponding state ahead predictions $X^*(k) = \begin{bmatrix} x(k) \\ x^*(k+1) \\ x^*(k+2) \\ x^*(k+3) \end{bmatrix}$

Problem 2

Repeat Problem 1 in the presence of the following input saturation constraint.

$$|u(k)| \leq 13, k = 0, \ldots, H_p - 1$$

Problem 3

Repeat Problem 2 by applying the Receding Horizon procedure over a time interval of 2 s, i.e., 20 steps.
Then

1. plot the time course of $x_1(k)$, $x_2(k)$, $\|x(k)\|_2$ and $u(k)$;

2. evaluate the zero regulation time considering a tolerance of 10^{-4}.