The Median Voter Theorem: Why Politicians Move to the Center

Jørgen Veisdal

In public choice economics, the median voter theorem states that

> A majority rule voting system will select the outcome most preferred by the median voter.[1]

In other words, the voter in the middle of the probability distribution picks the winner of the election. The prediction of the model, therefore, is what intuitively seems questionable (given today's politics), namely that:

> Candidates position themselves around the center.

The theorem rests on two core assumptions:

- Candidates and/or parties may be placed along a one-dimensional political spectrum; and
- Voters' preferences are single-peaked, meaning that voters have one alternative they prefer over the other.

History

The dynamics and predictions of the median voter theorem made their first appearance in economist Harold Hotelling's[2] legendary 1929 paper *Stability in Competition*,[3] in which Hotelling in passing notes that political candidates' platforms seem to converge during majoritarian elections. His paper regards the positioning of stores by two sellers along a line segment, in which buyers are uniformly distributed. The prediction of his model, now known simply as "Hotelling's law"[4] is that in many markets it is rational for producers to

make their products as similar as possible, the so-called "*principle of minimum differentiation*."

The formal analysis of the principle of Hotelling's law in majority voting systems was provided in a related 1948 paper entitled *On the Rationale of Group Decision-making* by economist Duncan Black.[5] Anthony Downs,[6] inspired by Adam Smith, further expanded on Black's work in his 1957 book *An Economic Theory of Political Action in a Democracy*.

A Simple Voting Model

The principle of the median voter theorem is successfully illustrated in a game theory lecture by Yale economist Ben Polak,[7] which is available through the Open Yale Courses website.[8] His very simple model goes as follows:

A Simple Voting Model (Polak 2007)

Two candidates choose their positions along a political spectrum from 1 to 10:

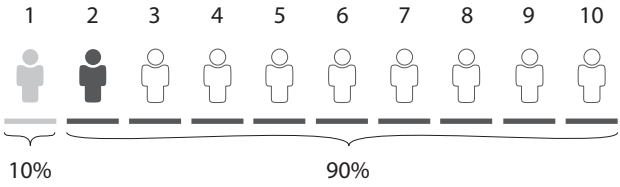
–	–	–	–	–	–	–	–	–	–
1	2	3	4	5	6	7	8	9	10

Imagine perhaps positions 1–3 being left-wing positions and positions 8–10 being right-wing positions. Assume that there are 10% of voters in each position (a uniform distribution of voters). Assume also that voters choose the closest candidate to their position. If there is a tie, votes are split 50%/50% between the two candidates.

The question for both candidates is where they should position themselves along the political spectrum 1–10 in order to maximize their shares of the vote.

First, as candidate A, let us assume that candidate B places himself or herself at the extreme left, in position 1. The question we need to ask ourselves is then whether position 2 is a better strategy for maximizing our share of the vote. Formally, we're asking if $u_A(2,1) > u_A(1,1)$, i.e., whether the utility we obtain from choosing position 2 is greater than that we obtain if we choose position 1, given our assumption that the other candidate chooses position 1. We know that each position

potentially holds 10% of the votes and if both choose the same position, we split the election. We—in other words—know that if both choose position 1, (1,1), both get 50% of the votes. If we as candidate A choose position 2, however, (2,1), candidate B gets the 10% of votes in position 1, but we get the remaining 90% of votes in positions 2–10 as the result of our assumption that voters choose their closest candidate:



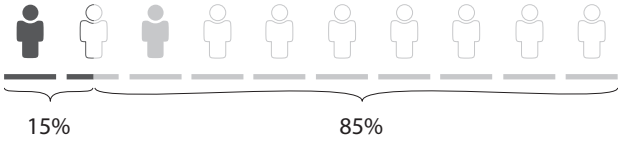
u(1,2): Voter shares if candidate A chooses position 1 when candidate B chooses position 2.

We can conclude that if candidate B chooses position 1, choosing position 2 is better for us as candidate A than choosing position 1 (90% vs. 50% of votes).

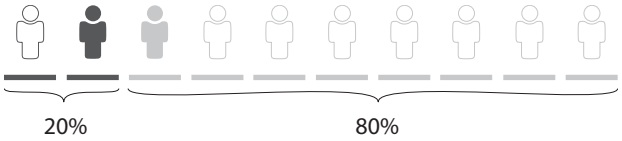
Next, again as candidate A, let us assume that candidate B places herself or himself at the second most extreme left, in position 2. Is choosing position 2 still strictly better for us than choosing position 1? Formally, we're asking whether $u_A(2,2) > u_A(1,2)$. Immediately, since both are choosing the same position (2,2), we know that our payoff for choosing position 2 is 50%, i.e., splitting the election. If we choose position 1 against position 2, we find ourselves in the inverse of the latter scenario (in the figure above) and can expect a payoff of 10%. We must therefore conclude that if candidate B chooses position 2, as candidate A we are still better off choosing position 2 (50% > 10% of votes), than choosing position 1.

Next, again as candidate A, let us assume that candidate B places himself or herself at the third most extreme left, in position 3. Is choosing position 2 still strictly better for us than choosing position 1? Formally, now we're asking whether $u_A(2,3) > u_A(1,3)$. In the first of these scenarios, we get all the voters in position 1 plus all the voters in position 2 (a total of 20%), while candidate B gets all the voters in positions 3–10 (80%). In the second scenario, we get all the voters in position 1 plus half the voters in position 2 (since we are equally far from position 2 as candidate B is, a total of 15%). Either way, we see that position 2

is still strictly better for us as candidate A than position 1 (20% > 15% of votes):

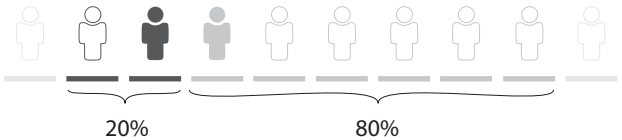


$u(1,3)$: Voter shares if candidate A chooses position 1 when candidate B chooses position 3.



$u(2,3)$: Voter shares if candidate A chooses position 2 when candidate B chooses position 3.

Continuing for all strategies against choosing position 1, we will find that position 2 always yields a higher payoff. Formally, we say that the strategy of choosing position 2 *strictly dominates* the strategy of choosing position 1, for both candidates, no matter what the other candidate chooses. We can hence eliminate position 1 as a viable strategy, since it is never better than position 2 for either candidate. By considerations of symmetry, we can do the same for position 10 vs. position 9:

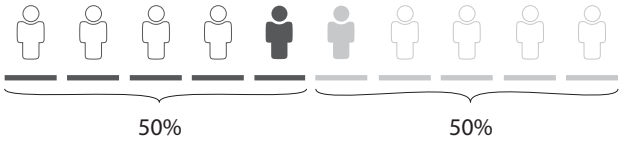


$u(2,3)$ in the reduced political spectrum.

Having eliminated positions 1 and 10 as viable strategies for maximizing our number of votes, we can next perform a similar analysis, but now with a reduced model consisting of positions 2–9. Unexpectedly, an analysis of position 2 vs. 3 (and 9 vs. 8) will yield the same result, and so we may delete strategies 2 and 9 as well, and so on. In the end, our model will have reduced to two viable positions, 5 and 6; let's call them "left and right."

$u(5,6) = u(6,5)$ in reduced political spectrum after eliminating dominated strategies.

An analysis of this reduced model will find that if we expect candidate B to position herself or himself on the right, it is rational for us to position ourselves on the left. If we expect candidate B to position himself or herself on the left, it is rational for us to position ourselves on the right:



$u(5,6) = u(6,5)$: Voter shares if candidate A chooses position 5 and candidate B chooses position 6.

Our finding, in other words, is the prediction of the median voter theorem, namely that:

Candidates position themselves around the center.

Clean as it looks, as with any model, our model depicting the essence of the median voter theorem is ripe with limitations, including (but not limited to) the following:

- Voters are in reality *not* evenly distributed (10% at each position);
- There are often more than two candidates in an election;
- Real candidates cannot simply "position" themselves—their positioning has to be believable; and
- Even in referendums, there is often more than one issue being voted on.

Notes

1. *Holcombe, Randall G. (2006). Public Sector Economics: The Role of Government in the American Economy. Pearson, London, p 155.* https://en.wikipedia.org/wiki/Median_voter_theorem.

2. https://en.wikipedia.org/wiki/Harold_Hotelling#Spatial_economics.

3. Hotelling, Harold. (1929). "Stability in Competition." *Economic Journal* 39, 153 (1929): 41–57.

4. https://en.wikipedia.org/wiki/Hotelling%27s_law.

5. https://en.wikipedia.org/wiki/Duncan_Black.

6. https://en.wikipedia.org/wiki/Anthony_Downs.

7. https://en.wikipedia.org/wiki/Ben_Polak.

8. https://oyc.yale.edu/economics/econ-159.