Cox/Huang N assets

$$
\begin{aligned}
\frac{ds_i}{s_i} &= \mu_i dt + \sum_{j=1}^{n} \sigma_{ij} dZ_j \\
\frac{\partial B}{B} &= \gamma dt \\
\text{All Zero divident scc... satisfy } 0 &= E\left[\frac{\partial \wedge}{\wedge} + \frac{\partial X}{X} + \frac{\partial \wedge}{\wedge}\frac{\partial X}{X}\right] \\
\text{Notationally specify } \frac{\partial \wedge}{\wedge} &= -\alpha dt - \sum_{j=1}^{n} p_j dZ_j \\
\text{Bond}: 0 &= -\alpha + \gamma \Rightarrow \alpha = \gamma \\
s_i : 0 &= -\gamma + \mu_i - \left(\sum_{m=1}^{n} p_m dZ_m\right)\left(\sum_j \sigma_j dZ_j\right) \\
\Rightarrow \mu_i &= \gamma + \sum_j \sigma_{ij} p_j \\
\Rightarrow \sigma\beta_i &= (\mu - \gamma 1) \\
\Rightarrow \beta &= \sigma^{-1}\left[\mu - \gamma 1\right] \\
\Rightarrow \frac{\partial \wedge}{\wedge} &= -\gamma dt - \left[\mu^T - \gamma 1^T\right]\sigma^{-1} dZ \\
\text{Check}: 0 &= -\gamma + \mu_i + \left(\sum_{jm} \left(u_j - \gamma\right)\sigma_{jm}^{-1} dZ_m\right)\left(\sum_k \sigma_{ik} dZ_k\right) \\
&= -\gamma + \mu_i + \sum_{jm} \left(\mu_j - \gamma\right)\sigma_{jm}^{-1}\sigma_{mi} \\
&= -\gamma + \mu_i + \sum_{j} \left(\mu_j - \gamma\right) 1_{(j=i)}
\end{aligned}
$$

$\max E\left[\frac{w^{1-\gamma}}{1-\gamma}\right]$ s.t., $\wedge_* \, w_* = E_*\left[\wedge_T w_T\right]$

$$
\begin{aligned}
L &= \int dw_T D(w_T) \frac{W(w_T)^{1-\gamma}}{1-\gamma} + \beta\left[\wedge_* w_* - \int \partial w_T D(w_*) \wedge (w_T) W(w_T)\right. \\
\frac{\partial Z}{\partial W(w_T)} : 0 &= D(w_T^{\gamma}) W(w')^{-\gamma} - \beta D(w'_T) \wedge (w'_T) \\
\Rightarrow W(w_T)^{-\gamma} &= \beta \wedge (w_T) \forall w_T \epsilon \Omega_T \\
\Rightarrow W(w_T) &= \beta^{-\frac{1}{\gamma}} \wedge^{-\frac{1}{\gamma}(w)}
\end{aligned}
$$

to solve for β plug back into constraint:

$$
\begin{aligned}
\wedge_* w_* &= E_*\left[\wedge_T \beta^{-\frac{1}{\gamma}} \wedge^{-\frac{1}{\gamma}}\right] \\
&= \beta^{-\frac{1}{\gamma}} E_*\left[\wedge_T^{-\frac{\gamma-1}{\gamma}}\right] \\
Y &= Z \wedge \\
dY &= \frac{\partial \wedge}{\wedge} - \frac{1}{x}\left(\frac{\partial \wedge}{\wedge}\right)^2 \\
&= \gamma d * - \left(\mu - \gamma 1\right)^T \sigma^{-1} dZ -
\end{aligned}
$$

$$
\begin{aligned}
& \frac{1}{2}\left(\sum_{ij}\left(\mu_i-\gamma 1_i\right)\sigma_{ij}^{-1}dZ_j\right)\left(\sum_{mn}\left(\mu_n-1\right)\sigma_{mn}^{-1}dZ_n\right) \\
\text{last term} &= \sum_{ijm}\left(\mu_i-\gamma\right)\sigma_{ij}^{-1}\left(\mu_m-\gamma\right)\sigma_{mj}^{-1} \\
\text{Define : } \sum_j \sigma_{ij}^{-1}\sigma_{mi}^{-1} &\equiv \sum_{ij}^{-1} \\
\text{last term} &= \left(\mu-\gamma 1\right)^T \sum^{-1}\left(\mu-\gamma 1\right) \\
\wedge(T) &= \wedge(*)e^{\left\{\left(-\gamma-\frac{1}{2}(\mu-\gamma 1)^T\sum^{-1}(\mu-\gamma 1)\right)(T-*)-(\mu\gamma 1)^T\sigma^{-1}[Z_T-Z_*]\right\}} \\
\text{For convenience, define vector } \sigma^{-1}(\mu-\gamma 1) &\equiv \theta \\
\wedge(w_T) &= \wedge(w_*)e^{(-\gamma-\frac{1}{2}\theta^T\theta)(T-*)-\theta^T(Z(w_T)-Z(w_*))} \\
\wedge_* w_* &= \beta^{-\frac{1}{\gamma}}\left[\wedge_* e^{-\left(\gamma+\frac{1}{2}\theta^\gamma\theta\right)(\gamma-*)}\right]^{\frac{\gamma-1}{\gamma}} E_*\left[e^{-\frac{1-\gamma}{\gamma}\theta^T\Delta Z}\right] \\
w_* &= \beta^{-\frac{1}{\gamma}}\wedge_*^{-\frac{1}{\gamma}} e^{\left(\frac{1-\gamma}{\gamma}\right)\left(\gamma+\frac{1}{2}\theta^T\theta\right)(T-*)} \\
& e^{\frac{1}{\gamma}\left(\frac{1-\gamma}{\gamma}\right)^2\theta^T\theta(T-*)} \\
\beta^{-\frac{1}{\gamma}} &= w_*\wedge_*^{\frac{1}{\gamma}} e^{\left(\frac{\gamma-1}{\gamma}\right)\gamma(T-*)}e^{\frac{1}{2}\theta^T\theta(T-*)\left(\frac{\gamma-1}{\gamma}\right)\left(1-\left(\frac{\gamma-1}{\gamma}\right)\right)} \\
&= w_*\wedge_*^{\frac{1}{\gamma}} e^{\left(\frac{\gamma-1}{\gamma}\right)\gamma(T-*)}e^{\frac{1}{2}\theta^T\theta(T-*)\left(\frac{\gamma-1}{\gamma^2}\right)} \\
\text{Plugging back for } \beta^{-\frac{1}{\gamma}} & \\
W(w_T) &= \beta^{-\frac{1}{\gamma}}\wedge^{-\frac{1}{\gamma}}(w_T) \\
&= w_*\left[\wedge_*^{\frac{1}{\gamma}}e^{\left(\frac{\gamma-1}{\gamma}\right)\gamma(T-*)}e^{\frac{1}{2}\theta^T\theta(T-*)\left(\frac{\gamma-1}{\gamma^2}\right)}\right] \\
& \left[\wedge_*^{-\frac{1}{\gamma}}e^{\frac{1}{\gamma}\left(\gamma+\frac{\theta^T\theta}{2}\right)(T-*)+\frac{1}{\gamma}\theta^T\Delta Z}\right] \\
&= w_* e^{\gamma(T-*)\left(1-\frac{1}{\gamma}+\frac{1}{\gamma}\right)}e^{\frac{1}{2}\theta^T\theta(T-*)\left(\frac{1}{\gamma^2}\right)(\gamma-1+\gamma)}e^{\frac{1}{\gamma}\theta^T\Delta Z_{(T-*)}} \\
&= w_* e^{\gamma(T-*)}e^{\frac{1}{2}\theta^T\theta(T-*)\left(\frac{2\gamma-1}{\gamma^2}\right)}e^{\frac{1}{\gamma}\theta^T\Delta Z_{(T-*)}} \\
\text{Solve at some intermediate date} & \\
(s*<s<T) & \\
\wedge_s w_s &= E_s\left[\wedge_T w_T\right] \\
&= \wedge_* e^{-\left(1+\frac{1}{2}\theta^T\theta\right)(T-*)}w_* e^{\gamma(T-*)}e^{\frac{1}{2}\theta^T\theta(T-*)\left(\frac{2\gamma-1}{\gamma^2}\right)} \\
& E_s\left[e^{-\theta^T[(Z_T-Z_s)+(Z_s-Z_*)]\left[1-\frac{1}{\gamma}\right]}\right] \\
&= \wedge_* w_* e^{\frac{1}{2\gamma^2}\theta^T\theta(T-*)\left(2\gamma-1-\gamma^2\right)}e^{-\left(1-\frac{1}{\gamma}\right)\theta^T(Z_s-Z_*)}e^{\frac{1}{2\gamma^2}\theta^T\theta\left(1-\frac{1}{\gamma}\right)^2(T-s)} \\
&= \wedge_* w_* e^{\frac{1}{2\gamma^2}\theta^T\theta(T-s)\left(2\gamma-1-\gamma^2+(\gamma-1)^2\right)}e^{-\frac{1}{2\gamma^2}\theta^T\theta(s-*)(\gamma-1)^2} \\
& e^{\frac{1-\gamma}{\gamma}\theta^T\theta(Z_T-Z_*)} \\
&= \wedge_* w_* e^{-\frac{1}{2}\left(\frac{\gamma-1}{\gamma}\right)^2\theta^T\theta(s-*)}e^{-\frac{\gamma-1}{\gamma}\theta^T(Z_s-Z_*)} \\
d\left(\wedge_s w_s\right) &= -\wedge_s w_s\left(\frac{\gamma-1}{\gamma}\right)\theta^T dZ_s \rightarrow \text{ p-martingale} \\
\text{Separately,} & \\
d\left(\wedge_s w_s\right) &= -\wedge_s w_s\left(\frac{\partial\wedge_s}{\wedge_s}+\frac{\partial w_s}{w_s}+\frac{\partial\wedge_s}{\wedge_s}\frac{\partial w_s}{s_s}\right)
\end{aligned}
$$

We know the different terms vanish. To check use,

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\begin{aligned}
dw &= \sum_i n_{s_i} ds_i + n_B dB \\
\text{Define : } \frac{n_{s_i} s_i}{w} &= X_i \\
\frac{n_B B}{w} &= 1 - \sum X_i \\
\frac{dw}{w} &= \sum_i X_i \frac{\partial s_i}{s_i} + \left(1 - \sum_i X_i\right) \gamma \partial * \\
&= \sum_i X_i \left(\mu_i \partial * + \sum_i \sigma_{ij} dZ_j\right) + \left(1 - \sum_i X_i\right) \gamma \partial * \\
d\left(\wedge_s w_s\right) &= \wedge_s w_s \left\{ \left[-\gamma ds - \left(\mu^T - \gamma 1^T\right) \sigma^{-1} dZ\right] \right. \\
&\quad \left. + \left[\gamma ds + X^T \left(\mu - \gamma 1\right) d * + X^T \sigma dZ\right] + \left[\left(\mu^T - \gamma 1^T\right) X d*\right] \right\} \\
&= - \wedge_s w_s \left[\left(\mu^T - \gamma 1^\gamma\right) \sigma^{-1} dZ - X^T \sigma dZ\right] \\
&= - \wedge_s w_s \left(\frac{\gamma - 1}{\gamma}\right) \theta^T dZ \\
\Rightarrow X^T \sigma &= -\left(\frac{\gamma - 1}{\gamma}\right) \theta^T + \theta^T \\
&= \frac{1}{\gamma} \theta^T \\
\Rightarrow X^T &= \frac{1}{\gamma} \theta^T \sigma^{-1}
\end{aligned}
$$