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Exchange economy, with exogenous dividned process

$$d\delta = \mu_\delta dt + \sigma_\delta dw$$

Bond and Stock process :

$$\begin{aligned} \frac{dB}{B} &= rdt \\ \frac{ds + \delta dt}{s} &= \mu dt + \sigma \partial w \end{aligned}$$

(r, μ, σ) to be determined endogenously in equilibrium.

Define:

- $\alpha \equiv$ amount in Bond
- $\theta \equiv$ amount in Stock
- $W \equiv \alpha + \theta =$ Wealth

$$\begin{aligned} dW + cdt &= (\alpha r + \theta\mu)dt + \theta\sigma d\vec{w} \\ U_i(\{c\}) &= E\left[\int_0^T e^{-\rho t} U_i(c_t)dt\right] \end{aligned}$$

Throughout assume $U_2(c_2) = Lc_2$

Endowments:
Agent 1 : Long 1 share stock, short β shares of bond
Agent 2 : Long β shares of bond

Equilibrium:
1) $w_i = \alpha_i + \theta_i = E_t[\int_0^T ds \frac{\zeta^i s}{\zeta^i t} c(s)]$ 2) Market clearing

(i) $c_1 + c_2 = \delta$

(ii) $\alpha_1 + \alpha_2 = 0$

(iii) $\theta_1 = s$

Unrestricted Case: - Dynamically complete markets
Introduce representative agent $U[c, \lambda] = E[\int_0^T e^{-\rho t} u[c_t, \lambda]dt]$ where λ is a constant and where $u[c, \lambda] = \max_{c_1+c_2=c} u_1(c_1) + \lambda u_2(c_2)$

In equal, $(c = \delta)$, so SDF follows

$$\begin{aligned} \zeta(t) &= e^{-\rho t} \frac{U_c \delta_{t,\lambda}}{U_c \delta_{0,\lambda}} \\ \frac{\partial \zeta}{\zeta} &= -rdt - K\partial w \\ K &= \frac{\mu - r}{\sigma} \\ -r = \frac{1}{dt} E\left[\frac{d\zeta}{\zeta}\right] &= \frac{1}{dt} E\left[-\rho dt + \frac{u_{cc}}{u_c} d\delta + \frac{1}{2}\frac{u_{ccc}}{u_c} d\delta^2\right] \end{aligned}$$

$$r = \rho + A\mu_\delta - \frac{1}{2}A\rho\sigma_\delta^2 \tag{12}$$

$$\begin{aligned}
\zeta_t S_t &= E_t\left[\int_t^T \zeta(u)\delta(u)du\right] \\
&\Rightarrow \zeta_t S_t + \int_t^T \zeta(u)\delta(u)du = \text{ P martingale} \\
&\Rightarrow 0 = E\left[\frac{d\zeta}{\zeta} + \frac{\partial s}{s} + \frac{d\zeta}{\zeta}\frac{\partial s}{s} + \frac{\delta}{s}dt\right] \\
&\Rightarrow E\left[\frac{ds + \delta dt}{s}\right] - rdt = -E\left[\frac{\partial\zeta}{\zeta}\frac{\partial s}{s}\right]
\end{aligned}$$

$$\mu - r = K\sigma \tag{13}$$

where, K is defined via, $-K = \frac{d\zeta}{\zeta}|_{stoch} = \frac{U_{cc}}{U_c}\sigma_\delta$

$$K = A\sigma_\delta \tag{14}$$