1 Introduction

A manifold M with periodic end X is a Riemann manifold of the form

$$Z_\infty = Z \cup W_0 \cup W_1 \cup W_2 \cup \cdots ,$$

where W_k are isometric copies of the fundamental segment W obtained by cutting X open along an oriented connected submanifold Y and Z is a smooth compact manifold with boundary Y.

The completion of the de Rham complex of M in the L^2 norm using over the end a Riemann measure dx lifted from that on X is not Fredholm. To rectify this, we will use L^2_δ norms, which are the L^2 norms on M with respect to the measure $e^{\delta f(x)}\,dx$ over the end. Here δ is a real number and $f : \tilde{X} \to \mathbb{R}$ is a smooth function such that $f(\tau(x)) = f(x) + 1$ with respect to the covering translation $\tau : \tilde{X} \to \tilde{X}$. We shall denote the L^2_δ completion of the de Rham complex on M by $\Omega^*_\delta(M)$.

Theorem 1.1. *Let M be a smooth Riemannian manifold manifold with a periodic end modeled on $\tilde{X}$, and suppose that $H_*(M;\mathbb{C})$ is finite-dimensional. Then $\Omega^*_\delta(M)$ is Fredholm for all but finitely many δ of the form $\delta = \ln|\lambda|$, where λ is a root of the characteristic polynomial of $\tau_* : H_*(\tilde{X};\mathbb{C}) \to H_*(\tilde{X};\mathbb{C})$.*

Given a manifold M as in the above theorem, the complex $\Omega^*_\delta(M)$ has a well-defined index $\mathrm{ind}_\delta(M)$. It is known that $\mathrm{ind}_\delta(M)$ is an even or odd function of δ according to whether $\dim M = n$ is even or odd, and that $\mathrm{ind}_\delta(M) = (-1)^n \xi(M)$ for sufficiently large $\delta > 0$.

Theorem 1.2. *Let M be as in Theorem 1.1. Then $\mathrm{ind}_\delta(M)$ is a piecewise constant function of δ whose only jumps occur at $\delta = \ln|\lambda|$, where λ is a root of the characteristic polynomial $A_k(t)$ of $\tau_* : H_k(\tilde{X};\mathbb{C}) \to H_k(\tilde{X};\mathbb{C})$ for some $k \in [0 : n-1]$. Every such λ contributes $(-1)^{k+1}$ times its multiplicity as a root of $A_k(t)$ to the jump.*