If G is a finite soluble group with $|G| = p_1^{e_1} p_2^{e_2} \dots p_r^{e_r}$, the primes $p_1, p_2, \dots, p_r$ being pairwise distinct and $e_1, \dots, e_r$ being positive integers, then the order of $\operatorname{Out} G$ divides the number $\prod_{i=1}^r m_i p_i^{d_i(e_i-d_i)}$, where d_i is the minimum number of generators of a Sylow p_i-subgroup of G and $m_i = |\mathrm{GL}(d_i, p_i)|$.

Since G is finite soluble, a result of P. Hall guarantees the existence of a Sylow basis for G. Pick one, say $\mathscr{S} = \{P_1, P_2, \dots, P_r\}$. So for all $i, j \in \{1, 2, \dots, r\}$, P_i is a Sylow p_i-subgroup of G and they permute: $P_i P_j = P_j P_i$. Consider the set of those elements $(\theta_1, \dots, \theta_r)$ of $L = \operatorname{Aut}(P_1) \times \operatorname{Aut}(P_2) \times \dots \times \operatorname{Aut}(P_r)$ for which there exists an automorphism $\beta \in \operatorname{Aut} G$ such that $P_i^\beta = P_i$ and $\beta|_{P_i} = \theta_i$. Because $\mathscr{S}$ is a Sylow basis, there exists at most one such automorphism: every element of G can be expressed in one, and only one, way as a product of an element of P_1, by an element of P_2, and so on. Clearly, these elements form a subgroup H of L. For an element $g \in G$, we shall denote by the same symbol the element of $\operatorname{Inn} G$ induced by conjugation by g. If $N(\mathscr{S})$ is the normalizer of $\mathscr{S}$, that is, $N(\mathscr{S}) = \bigcap_{i=1}^r N_G(P_i)$, we have a homomorphism $N(\mathscr{S}) \to H$ given by $x \mapsto \bigl(x|_{P_1}, x|_{P_2}, \dots, x|_{P_r}\bigr)$; let its image be $K \le H$. Now choose a right transversal $\{\gamma_1, \gamma_2, \dots, \gamma_s\}$ for $\operatorname{Inn} G$ in $\operatorname{Aut} G$. We have $\mathscr{S}^{\gamma_j}$ a Sylow basis for G, hence, by the same result of P. Hall, there is an element of G conjugating each member of $\mathscr{S}$ into its counterpart in $\mathscr{S}^{\gamma_j}$. For each $j \in \{1, 2, \dots, s\}$, choose a $g_j \in G$ such that $P_1^{\gamma_j} = P_1^{g_j{}^{-1}}, P_2^{\gamma_j} = P_2^{g_j{}^{-1}}, \dots, P_r^{\gamma_j} = P_r^{g_j{}^{-1}}$, so that $\gamma_j g_j \in \operatorname{Aut} G$ leaves invariant each subgroup in $\mathscr{S}$. We thus have $u_j := \bigl((\gamma_j g_j)|_{P_1}, (\gamma_j g_j)|_{P_2}, \dots, (\gamma_j g_j)|_{P_r}\bigr) \in H$. Let the space of right cosets of K in H be $K \backslash H$. Define a function $\alpha : \operatorname{Out} G \to K \backslash H$ by $\gamma_j^\alpha = \overline{u_j}$. If $\overline{u_j} = \overline{u_k}$, then $\gamma_j g_j = a \gamma_k g_k$ for some $a \in N(\mathscr{S})$, so $\gamma_j \gamma_k^{-1} = a g_k^{\gamma_k^{-1}} \bigl(g_j^{-1}\bigr)^{\gamma_k^{-1}} \in \operatorname{Inn} G$, hence $\gamma_k = \gamma_j$, so α is one-to-one. Now let $(\theta_1, \dots, \theta_r) \in H$ and let $\delta \in \operatorname{Aut} G$ be the automorphism restricting to θ_i in P_i. Write $\delta = \gamma_\ell w = (\gamma_\ell g_\ell)(g_\ell^{-1} w)$ for some $w \in \operatorname{Inn} G$ and some ℓ. (Recall that $\operatorname{Inn} G \lhd \operatorname{Aut} G$) We have $g_\ell^{-1} w \in N(\mathscr{S})$, so $(g_\ell^{-1} w)^{(\gamma_\ell g_\ell)^{-1}} \in N(\mathscr{S})$, thus $\delta = b(\gamma_\ell g_\ell)$ for some $b \in N(\mathscr{S})$, so $(\theta_1, \dots, \theta_r) = \bigl(b|_{P_1}, b|_{P_2}, \dots, b|_{P_r}\bigr) u_\ell$, therefore $\overline{y}$ lies in the image of α for every $y \in H$, i.e., α is onto. Conclusion: $|\operatorname{Out} G| = [H : K]$. Therefore, $|\operatorname{Out} G|$ divides $|L| = \prod_{i=1}^r |\operatorname{Aut} P_i|$. By a well-known result of Burnside, if P is a finite p-group with order p^m, p prime, then $C_{\operatorname{Aut} P}(P/\operatorname{Frat} P)$, which coincides with $\operatorname{Ker}(\operatorname{Aut} P \to \operatorname{Aut}(P/\operatorname{Frat} P))$, has order dividing $p^{r(m-r)}$, where $r = \dim_{\mathrm{GF}(p)}(P/\operatorname{Frat} P)$, the minimum number of generators of P. Since $\operatorname{Aut}(P/\operatorname{Frat} P) \cong \mathrm{GL}(r, p)$, it follows that $|\operatorname{Aut} P|$ divides $|\mathrm{GL}(r, p)| p^{r(m-r)}$.