OLIGOPOLY
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1 Cournot and Stackelberg Oligopoly Models

Problem 1. Cournot Equilibrium
There are only two firms in the industry. The cost function of firm A is $c_1(y_1) = 0.5y_1$ and the cost function of firm B is $c_2(y_2) = 1.5y_2$. The demand is $p(y) = 2 - y$, where $y = y_1 + y_2$. Two firms choose y_i simultaneously.

1. Find the Cournot equilibrium.

2. Find the Cournot equilibrium, when $c_1(y_1) = y_1^2$ and $c_2(y_2) = 2y_2^2$.

Problem 2. Stackelberg Equilibrium
There are only two firms in the industry. The cost function of each firm is $c_i(y_i) = 2y_i$, where $i \in \{1, 2\}$. The demand function is $y = 8 - p$. Suppose that the first firm (leader) decides how much to produce, and then the second firm (follower) makes a decision about the output, considering the choice of the first firm as given.

1. Find the equilibrium outputs of each firm.

2. Show that the isoprofit curve of the first firm refers to the reaction curve of the second firm in equilibrium. Illustrate graphically.

3. Compare the results with the case where two firms make decision simultaneously (Cournot equilibrium).
   Illustrate the equilibrium outputs on the same graph (from (2.)).

4. Suppose that $c_2(y_2) = 6y_2$. Find the Stackelberg equilibrium.

Problem 3. The License Fee
There are N firms in the industry producing a homogeneous product. The output of the i-th firm is y_i, and the industry output equals to y, $y = \sum_{i=1}^{N} y_i$. The inverse function of demand for products of the firms has the form $p = 100 - y$. Each firm carries variable costs $VC(y_i) = y_i^2$ in the production of goods, and also buys a license at a price F (the license must be purchased only for non-zero output).

1. Assume that the number of the firms in the industry is small enough (so that each of the N firms receives a positive profit). Find the output levels and profit for each firm in the Cournot equilibrium.

2. Assume that the license fee is the only barrier to entry. Find the equilibrium number of N^* firms in the industry in the long-run if initially each firm independently decides whether to entry or not, then the Cournot competition begins. Discuss the relationship of N^* to the value of F.

Problem 4. Duopoly or Monopoly
Consider an industry where two firms are involved in Cournot competition. Assume that technologies of both firms are characterized with constant marginal costs c_j, where $c_1 > c_2$. The inverse demand function in the industry is $p(y) = a - by$, where $a > b > 0$, $a > c_1$.

1. Under which condition only one firm will produce the goods in the equilibrium? Which firm will it be? Find the equilibrium output of this firm.

2. Suppose that the condition from (1.) does not hold. Find the equilibrium in this case.

3. How the equilibrium outputs and profit of each firm, which we've found in (2.), will change if marginal costs change.

4. Show that for any decreasing demand function and any increasing costs function in a case when marginal costs for every volume of production of the first firm are bigger than costs of the second one, the first firm produces less than second one in the equilibrium.

Problem 5. Teddy bears

Assume that demand for teddy bears is described as an equation $p(y) = 100 - y$. Only three firms can produce these bears. Firms have zero fixed costs and variable costs are different for every firm. Give an example of firm costs if only two firms produce bears.

Problem 6. Tax t

Consider an industry with N firms producing identical goods which are involved in Cournot competition. Assume that all the firms have the same constant marginal costs $c > 0$. The inverse demand function is defined by this formula $p(y) = a - by$, where $a > c$, $b > 0$. Suppose the industry introduces a tax t on the production of each unit.

1. How the equilibrium price of the products will change once the tax is imposed?

2. How the equilibrium price of the products will change once the tax is imposed given the fact there is a monopoly on the market?

3. Compare the results obtained in (1.) and (2.): in which case imposition of the tax will lead to the greater change in the price of products?

Problem 7. Oil Market

Countries A and B are the only oil producers on the world market. Interaction between them takes place only ones. Every country can choose one of three possible volumes of outputs (L, M or H). Payoff matrix is the following:

	L	M	H
L	16; 16	14; 21	10; 20
M	21; 14	15; 15	9; 12
H	20; 10	12; 9	5; 5

1. Suppose that countries choose the level of output simultaneously. Find the Nash equilibrium. Is it effective?

2. Consider the following order of interaction. Country A pronounces, how much oil it's going to produce. After that country B makes an announcement. Find the SPNE.

Problem 8. Valenki in Paris

Ivan and Maria sell Valenki in Paris. They form a Cournot duopoly: $p = a - y_I - y_M$, where y_I and y_M are the volumes of Ivan's and Maria's supplies. Before the competition every player can conduct a marketing campaign: Ivan and Maria simultaneously choose the volume of advertising a_I and a_M. Advertising influence on the demand is $a = a_0 + a_I + a_M$, where a_0 is constant. Advertising costs equal to a_I^2 and a_M^2 respectively. Player observes a volume of advertising chosen by his opponent before the Cournot competition stage. Find the SPNE.

Problem 9. Stackelberg Equilibrium with Three Firms

Consider the model of Stakelberg oligopoly where 3 firms consequentially choose a level of output. Every firm takes the output of the previous firms as given and it is aware of its influence on the output of the subsequent firms. Demand function is $y = 1 - P$, firms do not have variable costs.

1. Find the equilibrium outputs of the firms in this game.

2. Compare total output from (1.) with total output in a Cournot model with the same firms and the same demand.

3. Show for a random N that in these conditions the output of each firm depends only on its number and does not depend on the number of followers of this firm.

2   Price Competition

Problem 10. Bertrand Equilibrium

Consider an industry with N firms involved in Bertrand competition with constant marginal costs c_i for a firm I (no constant costs) which choose prices of products p_i simultaneously. The demand on the market is determined by an equation $P = a - y$, $a > c$ and money are infinitely dividable.

1. Let $N = 2$, $c_1 = c_2$. Find the equilibrium outcome and show that all the other outcomes cannot be realized in an equilibrium.

2. Let $N = 3$, $c_1 = c_2 = c_3$. Find all the equilibrium and generalize this result for every $N > 2$.

3. Answer the question in (1) for $c_1 < c_2$.

4. (On your own) Let $\varepsilon > 0$ be the smallest finite unit of money (for example, 1 cent). Will your answers in $(1.) - (3.)$ change?

5. (On your own) Imagine that in (1.) one of the firms before the interaction made a following announcement: *If the other firm sets a smaller price than mine, I will compensate the difference in price to all the consumers who come to me to buy a product.* Find the equilibrium prices.

Problem 11. Imperfect Substitutes in Bertrand Competition

On the market of two imperfect substitutes the demand looks like $y_1 = 64 - 4P_1 + 2P_2$ for the first product and $y_2 = 50 - P_2 + P_1$ for the second one. Besides, it is known that marginal costs of production for the first product are 5, and 4 for the second one.

1. Find the equilibrium if the first firm sets the price for its drink first and then the second one, knowing the price of the first firm, announce its price. Illustrate this graphically.

2. Which option of competition is more preferable for each company?

3. What prices will maximize the total welfare of firms?

Problem 12. Corrupt Officials

Vasya and Petya are the corrupt authorities. Vasya works in Sanitary and Epidemiological Service and Petya works in Fire Safety Authority. Every person who wants to start business has to get permission from both of the authorities. Demand for every document is specified by the same functions $y = D(P)$, where $D'(\cdot) < 0$ and $P = p_1 + p_2$ (p_1, p_2 are bribes for Vasya and Petya correspondingly). Each authority independently sets the size of a bribe, which he takes, maximizing his corrupt profit. Having known about it, the mayor of the city decided to fire Vasya and Petya, to unite two departments and ask Masha to distribute the documents. Unfortunately, Masha is corrupted either,

1. Show that despite that equilibrium bribes will decrease after the union of departments. If you cannot solve it generally, choose any demand function satisfying the conditions and prove the fact for it.

2. Come up with an economic interpretation of the result. Specifically, explain why equilibrium prices under a monopoly change not in an ordinary way.

3   Accommodation Model

Problem 13. Old good Jim and Jack
There are two cities in the country and a straight road between them. In every city exactly a third of the population lives and the rest of people are uniformly distributed along the road. Jim Beam and Jack Daniels are the only holders of the licences on the sale of alcoholic drinks in the country. This market is strictly regulated: government sets a fixed price p for a bottle of any alcohol drink. Every producer has fixed marginal costs c, $(c < p)$. Firstly, Jim and Jack independently decide (maximising each his own profit) where to place shops (in one of the cities or along the road) and then every citizen makes a decision which shop to choose for purchase (the closer the shop the better). If shops are situated in one point, consumers are equally divided between them.

1. Find the Nash equilibrium location of shops.

2. (On your own) Is this location effective?

Problem 14. Watch out for Big Bad Wolf
In the dark-dark forest there is an alley with length L. From every point of the alley the tree grows, under every tree a hare sits, every hare wants to eats one carrot (the trees are distributed with a density 1). Carrots are sold only in two shops which are also situated on this alley (the first one sells the carrot for price p_A and is situated on the distance of a from the begging of the alley, the second one sells it for price p_B and is located on the distance b from the end of the alley). Marginal costs of production of one carrot equal to 0 for shops. Big Bad Wolf walks around the alley, so hares are afraid to get to the shops themselves. Safe delivery of hares to the shop and back is accomplished by Ded Mazay who takes t rubles for one unit of distance from the tree under which a hare sits to the shop (he takes money only for delivery to the shop, the return is free). Every hare goes to the shop where his expenses (carrot price plus shipping charges) are minimal.

1. Assume all the shops simultaneously and independently choose p_A and p_B maximizing their profit. Find their equilibrium values and profit of every shop.

2. Assume shops can choose not only prices but their location along the alley before that. Prove that in this case there is no equilibrium.

3. Let $L = 100$, $a = 40$, $b = 34$, $t = 1$. The second shop had hired Red Hat who secured from Big Bad Wolf a half of the alley where this shop is located. In other words, movement along this half of the alley is safe and free for hares. Red Hat does her job for a small fixed payment. Find the equilibrium in the case when shops simultaneously choose prices.

4   Price Leadership

Problem 15. Price Leadership
Price Leadership. Consider an industry where two firms with costs functions $c_1(y_1) = cy_1$, $c > 0$, and $c_2(y_2) = y_2^2/2$ produce identical goods. Firstly, one firm (the leader) sets the price, then second

one (the follower) chooses output which maximizes its profit. Demand function is $D(p) = a - bp$, $a, b > 0$; $a/b \gg 0$.

1. Find the equilibrium and depict it graphically.

2. (On your own) Answer the question in (a) in case firms switch (the order changes).

Problem 16. Implicit collusion in repeated Cournot oligopoly

   In Canadian province British Columbia there is a perfectly competitive oil market. Oil is procured by 1000 firms whose costs function are the same and equal to $MC = y_i + 5$, where y_i is the output of the firm i. Produced oil is sold right in this province and demand is determined by an equation $y = 70000 - 2000p$, where p is price per barrel and y is measured in thousands of barrels. One of the American oil company got a license to procure light tight oil. Marginal costs of its production are 15 dollars per barrel. Consider the model of price leadership where an American firm sets the oil price first, and other 1000 firms are price takers. What is the equilibrium in this case? Compare total consumer surpluses on the market of oil consumption in the first (perfectly competitive) and the second cases.

5   Collusion

Problem 17. Collusion in Cournot Competition

   There are three firms in the industry producing identical goods. Output of the i-th firm is y_i, $y = y_1 + y_2 + y_3$. Demand function for this product is $p(y) = 100 - y$. Costs function for every firm looks like $TC(y_i) = 10y_i + 2$

1. Find the output and profit for every firm in the Cournot equilibrium.

2. Firms 2 and 3 merged into a new company (let's call it firm 4). Show that in the new Cournot equilibrium the profit of firm 4 is less than total profit of firms 2 and 3 from (a). Suggest an economic interpretation of this fact: why profit can decrease under consistent actions of firms?

3. Assume that simultaneously with merger firms can make an announcement what output firm 4 will produce and under no event they could deviate from this level. Firm 1 chooses output given this announcement. Will profit of the firm 4 increase in comparison with its profit in (b) and with total profit of firms 2 and 3 in (a). Suggest an economic interpretation of the results.

Problem 18. Endless Repetition of Bertrand's Duopoly

   Consider an infinitely repeated Bertrand model of duopolistic competition where firms which use strategies of returning to the Nash equilibrium, discount their future gains in according to the discount factor $\delta = 2/3$ Inverse function of aggregated demand is $P = 10 - y$. Assume that in the first period marginal costs of firms equal to 2, and beginning from the second one they equal to 4. Can the prices, which is equaled to monopoly prices in corresponding period, be supported as SPNE? How will your answer change if costs in all periods except for the first one equal to 8 instead of 4?

Problem 19. Collusion in Cournot Competition. Advanced

   Consider an industry with three firms involved in Cournot competition. All the firms have equal constant marginal costs c. Aggregated demand for products looks like $y = a - p$ where $a > c$. Firms 1 and 2 are planning to merge, resulting in costs of the merged firm to be equal to c_4 where $0 < c_4 \leq c$. Decision about the merger will be made iff (if and only if) profit of the merged firm exceeds the sum of profits of firms 1 and 2 before the merger.

1. Why this merger may not be? Suggest an economic explanation of this fact: why profit can decrease when firms begin to act in a coherent manner.

2. Find the equilibrium outcome and profit of every firm before the merger, output in the industry and price of products.

3. Suppose that merger has happened. Find equilibrium in this Cournot duopoly. Under which condition equilibrium output of firm 3 will be 0? Is it possible that in an equilibrium output of the merged firm (firm 4) will be 0?

4. Show that once $c_4 = c$ the merger does not happen.

5. Assume that simultaneously with the merger firms 1 and 2 can make an announcement what output firm 4 will produce and under no condition they can deviate from this level. Firm 3 chooses output given this announcement. Which values of c_4 will let merger happen under these conditions? Will it happen under $c_4 = c$?

Problem 20. Implicit collusion in repeated Cournot oligopoly

Consider an infinitely repeated Cournot oligopoly with N firms. The demand function is $p = 1 - y$. Firms have zero costs and common discount factor $\delta \in (0, 1)$. Under which restrictions imposed on parameters cooperative behavior where firms together produce monopolistic output (volumes of firms sales are equal) can be supported as SPNE in trigger strategies: firms begin with cooperative behavior described above and forever switch to Cournot equilibrium if at least one of them deviates.

Problem 21. Antitrust Authorities

Consider an industry with inverse demand function $P = 1 - y$, where N firms with equal constant marginal costs $c_i = 0.4$ are involved in Cournot competition. All the firms come to the antitrust authorities to ask for permission to merge into one monopoly. According to expert's review, because of the merger by means of synergetic effect marginal costs will decrease to $c = 0.4(1 - x)$. Find all the values of $x \in (0, 1)$ under which social welfare will rise as a result of the merger or prove that there are no such values.

Problem 22. Cournot Competition Again

Consider an infinitely repeated model of Cournot duopolistic competition with a discount factor $\delta \in (0, 1)$. Both firms have equal constant marginal costs $c > 0$. Demand function is $p = \theta - \gamma q$, $\theta > c$, $\gamma > 0$. Assume that firms strategies are the following: every firm produces a half of monopolistic output $(y_1, y_2) = (y^m/2, y^m/2)$ while no one deviates from this output and switch to the Nash equilibrium in a static Cournot model otherwise. Under which conditions firms will keep implicit agreement producing a half of monopolistic output in every period?

Problem 23. Administration Mistake

Having suspected of the collusion on the market of product A in a small city T, municipal government decided to publish the price of this product in every shop (the good thing is there is only 5 shops) in the evening newspaper. Administration representatives reasoned this way: now when all the information about price is available to everyone firms will be ashamed to take part in a collusion and prices will go down. However, despite expectations, one day after the publication the price for product A in all shops not only did not decrease but even increase. What is the most probable mistake of administration office?