Game theory

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All problems marked in orange wouldn't be discussed at the seminar.

1 Normal-form game

Problem 1. Classical Examples

Find all the Nash equilibria (in pure and mixed strategies).

1. Prisoner's Dilemma
(Gifts of Santa Claus)

	D	C
D	1; 1	3; 0
C	0; 3	2; 2

2. Chicken out
(Volleyball Game)

	L	G
L	0; 0	-2; 1
G	1; -2	-4; -4

3. Battle of the Sexes
(So which wheel went down?)

	L	R
L	2; 3	0; 0
R	1; 1	3; 2

4. Controller vs Free Rider
(Matching Pennies)

	I	II
I	1; -1	-1; 1
II	-1; 1	1; -1

Problem 2. Are you Hermione Granger?

The lecturer of the course announced that students' grade depends on their relative success exclusively, in other words, on the rating position of the student. It is a common knowledge that top 40 % of students will get the 10, 40% more — 7, and the rest of students — 5. Formalize the game. Find the Nash equilibria. Which outcomes can be realized in reality?

Problem 3. Rock-Paper-Scissors

Find all the Nash equilibria in the Rock-Paper-Scissors game.

	R	P	S
R	0; 0	-1; $+1$	$+1$; -1
P	$+1$; -1	0; 0	-1; $+1$
S	-1; $+1$	$+1$; -1	0; 0

Find the payoffs of each player in the equilibrium.

Problem 4. A Beautiful Mind

On Friday evening two men, John and Nielson, were sitting in the bar when 3 girls came in. Both of them prefer the blonde, however they realize that:

- if they both come to the blonde, she won't be able to make a choice and will deny both;
- if they come to the brunettes after the blonde, girls will deny them, because they do not want to be the second choice;
- it is better to be with a brunette than without a girl at all.

Guys were blinded with the girls' beauty and couldn't agree on who goes to whom. Formalize the game. Find the Nash equilibria. Which outcomes can be realized in reality?

Problem 5. A Penalty

The idea and data of this task are borrowed from Palacios-Huerta, "Professionals play minimax".

Consider the game "Penalty". Participants of the game are a forward (f) and a goalkeeper (g). The forward must decide which side to strike: a habitual one (the left one for right-handers or the right one for left-handers) or an unusual one (vice versa). At the same time the goalkeeper decides which side to jump. The movement to a habitual side is called the strategy R, and to an unusual one is called the strategy L. The payoff in the game is the probability of scoring (forwards) or nonscoring (for the goalkeeper). The payoff matrix obtained from the real data is presented in the table.

	L_g	R_g
L_f	$58.30; 41; 70$	$94.97; \ \ 5.03$
R_f	$92.91; \ \ 7.09$	$69.92; 30.08$

1. Is there an equilibrium in pure strategies?
2. Find all the Nash equilibria.
3. Compare the result with the real data: $\pi(L_g) = 42.31$; $\pi(L_f) = 39.98$.

Problem 6. Dominant Strategies

Two players place an object on the plane which means to select its coordinates (x, y). Player 1 is at $(0, 0)$, and player 2 is at $(1, 1)$. Player 1 selects the x coordinate and Player 2 selects the y coordinate. Everyone strives for the object to be as close as possible to it. Write down the game in a normal form. Show that each player has a strictly dominant strategy in this game.

Problem 7. Keynesian Beauty Contest

The Professor invites 50 students to play the following game. All the students simultaneously and in secret for each other write an integer from 0 to 100 on the sheet of paper. Then the professor calculates the average M and gives the student who wrote the nearest number to $0.5M$ \$50.

1. Prove that choosing 70 is the dominated strategy.
2. One student wrote the number 10. All the rest have found it out. What numbers should other students call in this case?
3. Find Nash equilibria (hint: use deletion of dominated strategies).

2 Extensive-form Game

Problem 8. From Extensive to Normal-Form

Transform the games from the task 2 to an extensive-form game, where the first player goes first, and the second player goes second. Then transform these games to a normal-form game and find the NE and SPNE.

Problem 9. Repeated Game Alice and Bob play in a repeated game with the following payoffs:

Alice \ Bob	D	C
D	$2; 1$	$4; 0$
C	$1, 5$	$3, 4$

Find the values of the discount factor δ in which the following strategies are Nash equilibria:

1. Always Cooperate
2. Always Defect
3. Grim Trigger
4. Win–Stay, Lose–Switch

Problem 10. Predation

There is the only firm A on the market which gets the profit of 2 dollars. The firm B is thinking whether to enter the market or not. If it doesn't enter, the firm A still gets 2 dollars and the firm B gets nothing. If it does enter, the firm A can either let the firm B be on the market and then divide the profit equally, or force it to go from the market with the price war which will lead to the profit of A being equal to $(-\$1)$ and the profit of B – $(-\$3)$.

Draw the tree of the game. Find all the NE and SPNE. Notice, that not all the NE are SPNE, why so?

Let there be 5 periods in the game now and the firm B can try to enter the market only in the first period. If A lets it do that, then B exists for the whole 5 periods on the market, if not – it suffers losses in the first period and goes away for good. Find NE and SPNE which depend only on the discount factor δ.

Problem 11. Commitment*

Terrorist Bob (player T) kidnapped the victim Alice and demands a ransom r. Alice's parents (player P) can not pay more than v. After P makes a decision to pay the ransom, Bob can either release Alice (costs from this action equal to f) or kill her (costs equal to c). Imagine the game in a normal and extensive form, find all the NE and SPNE depending on the parameters. Are you consistent with the results of known facts about abductions and ransom in reality? If not, what can be modified in the model to restore compliance?

*Author: Danil Fedorovykh