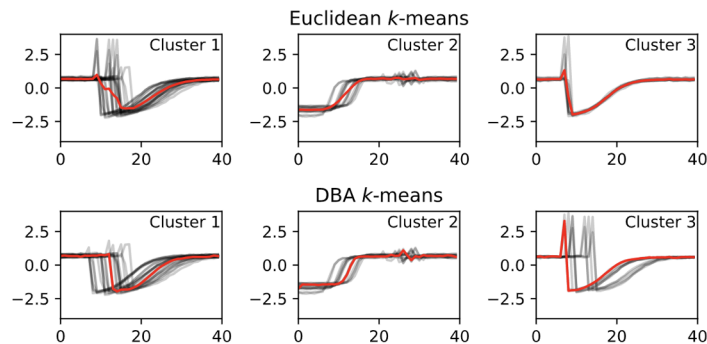

Time Series Clustering

K-means

k -means clustering for time series: Similarity function

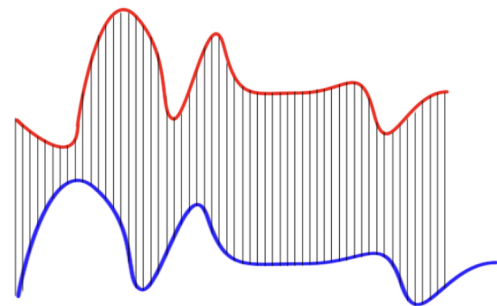
- Many variants of the algorithm are available, two of them:
 - Euclidean k -means
 - DBA- k -means (for DTW Barycenter Averaging)



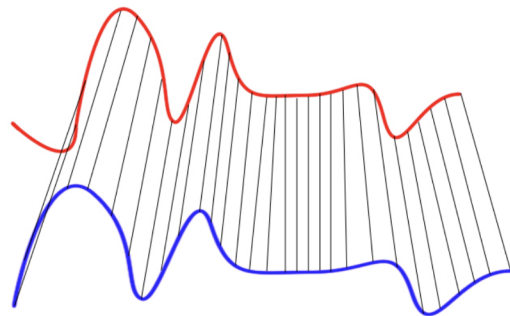
Training set series for a cluster (in black)
and centroid of the cluster (in red).

Dynamic Time Warping(DTW)

- Euclidean distance - not invariant to time shifts.
- Alternate similarity measure between time series, which may vary in speed.
- Based on the Levenshtein distance (edit distance)



Euclidean Matching



Dynamic Time Warping Matching

Similarity Functions

Let $A = \langle a_1, a_2, \dots, a_T \rangle$ and $B = \langle b_1, b_2, \dots, b_T \rangle$ be two sequences, and let δ be a distance between elements (or coordinates) of sequences.

- Euclidean Distance:
$$D(A,B) = \sqrt{\delta(a_1,b_1)^2 + \dots + \delta(a_T,b_T)^2}$$
- Dynamic time warping:
$$D(A_i,B_j) = \delta(a_i,b_j) + \min \left\{ \begin{array}{l} D(A_{i-1},B_{j-1}) \\ D(A_i,B_{j-1}) \\ D(A_{i-1},B_j) \end{array} \right\}$$
- Numerical Example [[YT video](#)]

k -means and Dynamic Time Warping

- Cluster centers (aka centroids) are computed with respect to DTW.
- Clusters gather **time series of similar shapes**
 - DTW deals with time shifts.
- Capable of retrieving **sensible average shape** of the temporal shifts in the cluster.

Further Readings

- <https://www.cs.unm.edu/~mueen/DTW.pdf>
- <https://dl.acm.org/doi/10.1145/2939672.2945383>