

## Geodesy using the Swedish permanent GPS network: Effects of snow accumulation on estimates of site positions

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**Abstract.** We have observed variations at the several centimeter level in estimates of the vertical coordinate of site position. The estimates are obtained from our analysis of data acquired from the Swedish permanent Global Positioning System (GPS) network. The observed variations are strongly correlated with changes in the indirectly inferred accumulation of snow, which we assume collects on the radomes and pillars; the GPS sites could not be observed directly due to their remoteness. Numerical simulations which assume a simple geometry for the snow cover are used to study the effects of snow accumulation on GPS phase observables and hence on estimates of the vertical coordinate of site position obtained from these observables. Our results indicate that the variations in the vertical coordinate of site position can be fully explained by reasonable accumulations of snow which retard the GPS signals and enhance signal scattering effects.

### Introduction

The 21 sites of the Swedish permanent Global Positioning System (GPS) network, or SWEPOS [Johansson *et al.*, 1996], are located at latitudes ranging from 56° to about 68° north, that is in a region where snow falls are common during autumn, winter, and spring. Signal propagation delay during snow storms has been investigated by, e.g., Tranquilla and Al-Rizzo [1993, 1994], who demonstrated that due to the localized nature of many snow storms differential effects may cause systematic variations at the centimeter level in estimates of the vertical coordinate of site position. Systematic variations introduced by snow storms may, however, if short-lived (minutes to hours), be reduced to a high degree by data averaging. A potentially more serious effect of heavy snow precipitation is the accumulation of snow on the top of the GPS antenna and on its surroundings, such as on the top of the GPS pillar or, when present, on the radome covering the antenna.

This accumulation may last for days, weeks, or months. Webb *et al.* [1995] reported variations on the order of 0.4 m in estimates of the vertical coordinate of site position. The variations were apparently correlated with the accumulation of snow over the antenna.

In this paper we investigate the effects of snow accumulation on estimates of the vertical coordinate of site position based on data from SWEPOS. Data from this network has been used since August 1993 to measure the three-dimensional rates associated with post-glacial rebound. These rates are expected to be less than ~10 mm/yr and ~3 mm/yr in the vertical and horizontal coordinates, respectively [e.g., Mitrovica *et al.*, 1994]. Thus, the existence of unmodeled systematic effects at the centimeter level is clearly a serious problem. To assess and model the effects due to snow accumulation, we investigate and quantify variations in the vertical coordinate of site position using simulations based on two models. These models include effects of signal scattering and the excess propagation delay due to accumulated snow. In Elósegui *et al.* [1995] and Jaldehag *et al.* [1995] the effects of signal scattering (i.e., reflections and diffractions) on estimates of (relative) site positions were studied. It was shown that structures associated with the mounting of the GPS antennas to the pillars and with the pillars themselves, caused differential phase errors due to signal scattering. Here we demonstrate that signal scattering from accumulated snow in combination with signal propagation delay effects can cause similar errors.

### Observed Snow Effects

In results of daily estimates of site positions from SWEPOS data, systematic variations in the vertical coordinate were found to be strongly correlated with changes in local weather conditions related to snow precipitation in combination with air temperature changes. As we illustrate below, the effects are due to snow, in one form or another, accumulating both on the conical radome covering the GPS antenna and on the top of the 3-m high concrete pillar, shown in Figure 1. The effects are only evident during winter periods, and at a few sites of the network.

Since many of the sites of SWEPOS are extremely remote, we cannot observe them directly, and we there-

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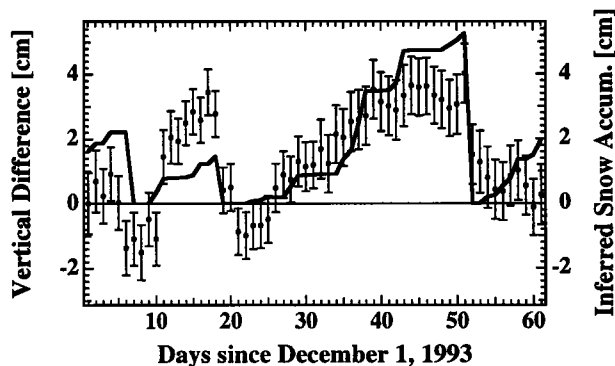
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**Figure 1.** Photograph of the top of a 3-m high GPS concrete pillar, used at SWEPOS sites, including the radome that covers the antenna.

fore have no direct measure of snow accumulation. We thus seek a qualitative model of the time-dependence of the depth of snow on the pillar and on the radome. This model should indicate periods of snow build-up, and periods of melting. Our simple model is based on recorded snow precipitation and air temperature. The depth is inferred by equating it to a running sum of the precipitation. The measured accumulated precipitation is reset to zero whenever the maximum daily temperature exceeds  $0^{\circ}\text{C}$ . Both the precipitation and the temperature are recorded at nearby ( $\sim 50\text{ km}$  distant) meteorological stations operated by the Swedish Meteorological and Hydrological Institute. By comparing the model to the time series of height estimates for a given receiver, we can infer whether snow accumulation might be the cause of the variations in estimated height. However, we do not expect this model to be able to predict the exact height or distribution of snow on the pillars or on the radomes.

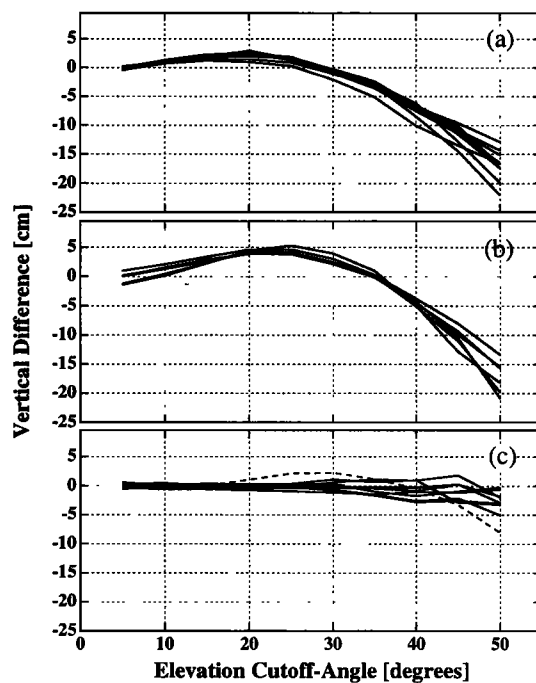
Figure 2 shows an example of the daily estimates of the vertical coordinate of position of the Sveg site (located at a latitude of  $62.0^{\circ}$  north) from December 1, 1993, to January 30, 1994, obtained from a network solution of all sites within SWEPOS [Johansson *et al.*,



**Figure 2.** Daily estimates (relative to a nominal value) of the vertical coordinate of position of the Sveg site (points with error bars, left scale). The error bars are the statistical standard deviation for these estimates based on a propagated phase measurement noise of  $10\text{ mm}$ . Inferred depth of accumulated snow (solid line, right scale).

1996] with an elevation cutoff-angle (the minimum elevation angle of the data used in the solution) of  $15^{\circ}$ . (All GPS results presented in this paper are processed using the GIPSY software package [Webb and Zumberge, 1993].) In the same figure, we have plotted the inferred depth of accumulated snow, calculated as described above. The inferred accumulation curve closely mimics the estimated vertical variations. Our hypothesis is that as more and more snow covers the top of the pillar and the radome, the error in the estimates of the vertical coordinate increase due to increased scattering from, and increased electrical path length through, the snow layer. As soon as the maximum temperature exceeds  $0^{\circ}\text{C}$  the snow melts and/or slides off. The fact that the estimated vertical coordinate does not vary on days without precipitation (unless the maximum temperature exceeds  $0^{\circ}\text{C}$ ) indicates that scattering and/or increased electrical path length due to accumulation, and not the precipitation as such, are the main sources of error.

Systematic biases can also be seen in estimates of the vertical coordinate when plotted as a function of elevation cutoff-angle. Figure 3 shows estimates of vertical coordinate of position of the Sveg site in January 1994 (excluding January 19, 20, 30, and 31 for reasons that are not relevant to the results of this paper) plotted as a function of elevation cutoff-angle. These estimates were obtained from daily single baseline solutions, including Sundsvall as a fixed site (located at a latitude of  $62.2^{\circ}$  north). Sundsvall was chosen because the elevation cutoff-angle tests and time series for this site did



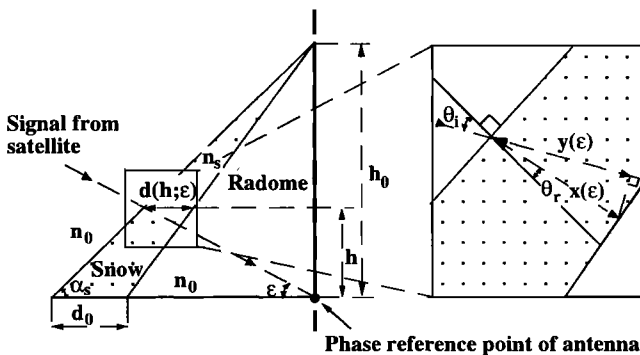
**Figure 3.** Estimates of the vertical coordinate of position (relative to the solution obtained using  $5^{\circ}$  elevation cutoff-angle on January 1) of the Sveg site as a function of elevation cutoff-angle. The results are divided into three time blocks: (a) January 1–12; (b) January 13–18; and (c) January 21–29. In (c) the dashed line corresponds to January 21 (see text).

not indicate the presence of an elevation-angle dependent error. This lack of problem is consistent, under our hypothesis, with the very low inferred snow accumulation for Sundsvall determined using our qualitative model described above. On the basis of the elevation cutoff-angle tests, we have divided these curves into three time blocks: January 1–12, January 13–18, and January 21–29 (day numbers 32–43, 44–49, 52–60, respectively in Figure 2). Clear systematic dependence on elevation cutoff-angle can be seen in Figure 3 for the first two time blocks. The difference (about 2 cm at 20° elevation cutoff-angle) between results from these two periods, is probably associated with a 10 mm snowfall precipitation on January 12 (day number 43 in Figure 2). All days in the third block exhibit a non-systematic behavior with elevation cutoff-angle. This normal behavior may be explained by all the snow (or a major part of it) melting and sliding off from the pillar and the radome on January 21 (dashed line in Figure 3c and day number 52 in Figure 2), which may also explain the unusual systematic behavior with elevation cutoff-angle on January 21 itself. The inferred accumulated snow in the third time block is, in spite of the normal behavior, not zero. It thus appears that our simple qualitative model for the prediction of snow accumulation may not be universally valid. There are many obvious reasons, however, why snow need not accumulate on the radome or on the pillar. For example, if the snow is dry and the winds high it might simply be blown off.

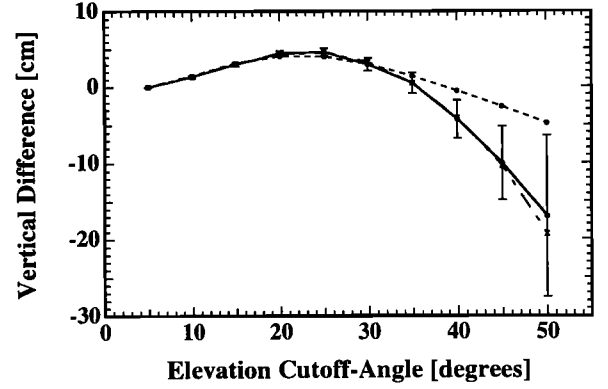
## Simulation of Snow Accumulation Effects

We have considered three models for the effects of snow accumulation on the GPS carrier phase observable: (model 1) signal scattering off an infinitely large horizontal plane layer of snow with an average reflection coefficient  $\alpha$  and located a distance  $H$  below the phase reference point of the antenna; (model 2) an excess electrical path length due to snow accumulated with a given distribution above the antenna; (model 3) a combination of model 1 and model 2.

In model 1, the model for the top of the pillar, and therefore that of the accumulated snow, is that of an infinitely large horizontal plane. This model was successfully used by [Elóseguí *et al.*, 1995]. In model 2, the



**Figure 4.** Diagram of the geometry of the signal excess path length due to a layer of snow distributed over the radome (model 2).



**Figure 5.** Estimates of the vertical coordinate of position of the Sveg site as a function of elevation cutoff-angle relative to the 5° estimate (solid line). The estimates and the error bars are as explained in the text. The dashed line (model 1) and the dashed-dot-dashed line (model 3) shows the results from our simulations for the two examples described in the text and in Table 1.

excess electrical path length  $\delta\phi$ , that the GPS signal will expose when propagating through a layer of snow as compared to propagating through free space, can be estimated using Snell's law of refraction:

$$\delta\phi(\epsilon; n, d(\epsilon)) = n_s x(\epsilon) - n_0 y(\epsilon) \quad (1)$$

where (see Figure 4)  $\epsilon$  is the satellite elevation angle,  $n_s$  (assumed real) the index of refraction of the accumulated snow,  $n_0 = 1$  the index of refraction in free space,  $x(\epsilon)$  the length of the refracted (true) signal path,  $y(\epsilon)$  the length of the unrefracted (straight-line) signal path,  $n_s x(\epsilon) = d(\epsilon)/\cos\theta_i$ , the electrical path length through the layer of snow, and  $n_0 y(\epsilon) = x(\epsilon)\cos(\theta_i - \theta_r)$  the electrical path length in free space;  $d(\epsilon)$  is the thickness distribution of the snow layer over the radome, and  $\theta_i$  and  $\theta_r$  are the angles of incidence and refraction, respectively. The phase in (1) is taken to be in units of length. The index of refraction of snow depends on its state of compactness, wetness, etc. It can vary between slightly over 1 for dry and moist snow, to about 1.7 for wet snow, and to about 5 for watery snow [e.g., Tranquilla and Al-Rizzo, 1994].

In our model, we further assume that the thickness of the snow layer over the radome vary linearly with the vertical distance  $h$  from the base to the top of the radome as

$$d(h) = d_0(1 - h/h_0) \quad (2)$$

where  $d_0$  is the thickness of the snow layer at the base of the radome, and  $h_0$  is the total height of the radome. This gives a thickness distribution of snow over the radome as a function of elevation angle as

$$d(\epsilon) = d_0 \left( 1 - \frac{\tan\epsilon}{\tan\alpha_s + \tan\epsilon} \right) \quad (3)$$

where  $\alpha_s$  and  $d(\epsilon)$  are pictured in Figure 4. In Figure 4 it is seen that  $d(h)$  and  $d(\epsilon)$  are equal, and are the horizontal thickness of the snow at the specific elevation angle. The difference between the horizontal thickness and the thickness following the actual ray-path is at the most a few millimeters, so we have chosen the thickness

distribution according to (3) for simplicity. Choosing the thickness distribution following the actual ray-path would only change the estimated parameters insignificantly regarding the uncertainty of the estimated parameters and the uncertainty in the overall snow distribution model. We also assume that differential effects due to the excess signal path delay through the radome are canceled out as identical radomes are employed at all sites of SWEPOS.

To quantify the effect of snow accumulation in the vicinity of the antenna on GPS estimates of the vertical coordinate of site position, we have used a simplified analysis based on least-squares inversions [Elósegui *et al.*, 1995]. We compared variations in estimates of the vertical coordinate from real data with those from simulations based on these models. Figure 5 shows differences in the estimated vertical coordinate of the Sveg site as a function of elevation cutoff-angle. Each estimate is shown relative to the solution using a  $5^\circ$  elevation cutoff-angle. The estimated values at each indicated elevation cutoff-angle are the arithmetic mean of the estimates obtained from the solutions on January 13–18, 1994, shown in Figure 3b (the second time block as defined above). The error bars are the 1-sigma statistical standard deviations of the differences between the indicated solution and the  $5^\circ$  solution [Davis *et al.*, 1985]. Figure 5 also shows the results from our simulations using model 1 and model 3. To perform the simulations we used a realistic satellite configuration at the Sveg site on January 14 and then carried out the least-squares inversion. For model 3, we assumed a thickness distribution over the radome, as described in (2) and (3), of wet snow with  $n_s = 1.7$ . Least-squares estimates of the parameters  $H$ ,  $\alpha$ , and, when present,  $d_0$  as obtained from fits between the observed variations and the simulated variations, are summarized in Table 1 along with normalized  $\chi^2$  values of the fits. As deduced from the  $\chi^2$  values and Figure 5, a closer fit is obtained using model 3, i.e., the systematic variations in the vertical coordinate with elevation cutoff-angle are not completely explained by scattering effects only. Intentionally we show no results from the test with model 2 only, as these tests resulted in unrealistic (negative) estimates of the parameter  $d_0$ . This indicates, also, that the systematic variations are due to a combination of scattering and signal propagation effects. The  $\chi^2$  values in Table 1 indicate that the apriori uncertainties adopted for the carrier phase observable, 10 mm, might be too large. An apriori uncertainty of about 4 mm would produce a  $\chi^2$  of about 1 for the fit using model 3 in Table 1.

## Discussion and Future Work

One way to reduce accumulation of snow on the pillar and the radome is to use a more suitable radome. The

**Table 1.** Results from Simulation of Snow Accumulation on Pillar and Radome

model	$H$ [cm]	$\alpha$ [%]	$d_0$ [cm]	$\chi^2$
1	$10.8 \pm 0.1$	$7.7 \pm 0.3$	–	1.52
3	$10.1 \pm 0.2$	$9.3 \pm 0.6$	$5.1 \pm 1.7$	0.15

type of radome used on SWEPOS pillars up to December 1995 covered the antenna but not the top of the pillar. The diameter of the pillar ( $\sim 70$  cm) exceeded well the diameter of the base of the radome ( $\sim 40$  cm) making snow accumulation possible. We are therefore testing a new type of radome which covers the complete pillar top, and is steeper in order for the snow to slide off. The new radome is now employed at all sites of SWEPOS except at the Onsala site. Results of daily estimates of site positions obtained using the new radome will be reported elsewhere.

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