

Mathematics Compulsory Part Paper 1

Solution	Marks	Remarks
<p>1. $k = \frac{3x-y}{y}$ $yk = 3x - y$ $y = \frac{3x}{k+1}$</p> <p>2. $\frac{(m^4 n^{-1})^3}{(m^{-2})^5}$ $= \frac{m^{12} n^{-3}}{m^{-10}}$ $= \frac{m^{22}}{n^3}$</p> <p>3. (a) $x^2 - 4xy + 3y^2 = (x-3y)(x-y)$ (b) $x^2 - 4xy + 3y^2 + 11x - 33y$ $= (x-3y)(x-y) + 11(x-3y)$ $= (x-3y+11)(x-y)$</p> <p>4. Let x, y be the number of regular tickets and concessionary tickets sold respectively $\begin{cases} x = 5y \\ 126x + 78y = 50976 \end{cases}$ $126x + 78\left(\frac{x}{5}\right) = 50976$ $\begin{cases} x = 360 \\ y = 72 \end{cases}$ The number of admission tickets sold that day $= 432$</p> <p>5. (a) $7(x-2) \leq \frac{11x+8}{3}$ and $6-x < 5$ $21(x-2) \leq 11x+8$ and $x > 1$ $x \leq 5$ and $x > 1$ $\therefore 1 < x \leq 5$ (b) 2, 3, 4, 5 are the only integers satisfying (a) So, there are 4 integers satisfying both inequalities in (a)</p>		

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6.	<p>(a) The coordinates of $A' = (-4, -3)$ The coordinates of $B' = (9, 9)$</p> <p>(b) Slope of AB $= \frac{-9 - 4}{9 - (-3)}$ $= -\frac{13}{12}$ Slope of A'B' $= \frac{-3 - 9}{-4 - 9}$ $= \frac{12}{13}$ Slope of AB \times Slope of A'B' $= -\frac{13}{12} \times \frac{12}{13}$ $= -1$ $\therefore AB \perp A'B'$</p>		
7.	<p>(a) $\frac{x}{360^\circ} = \frac{1}{9}$ $x = 40^\circ$</p> <p>(b) Let N be the number of students in the school $\frac{180}{N} = \frac{360^\circ - 90^\circ - 40^\circ - 158^\circ}{360^\circ}$ $N = 900$ The number of students in the school $= 900$</p>		
8.	<p>(a) $y = \frac{k}{\sqrt{x}}$ When $y = 81$, $x = 144$ $81 = \frac{k}{\sqrt{144}}$ $k = 972$ $y = \frac{972}{\sqrt{x}}$</p> <p>(b) Change in value of y $= \frac{972}{\sqrt{324}} - \frac{972}{\sqrt{144}}$ $= -27$</p>		

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9.	<p>(a) Let x mL be the actual capacity of a standard bottle</p> $200 - \frac{10}{2} \leq x < 200 + \frac{10}{2}$ $195 \leq x < 205$ <p>The least possible capacity is 195 mL</p> <p>(b) $23400 \leq 120x < 24600$ $23.4L \leq 120x < 24.6L$ Least total capacity = 23.4 L No, I don't agree the claim.</p>		
10.	<p>(a) $OP = OR$ (<i>given</i>) $PS = RS$ (<i>given</i>) $OS = OS$ (<i>common</i>) $\therefore \triangle OPS \cong \triangle ORS$ (<i>SSS</i>)</p> <p>(b) $\angle POQ = 10^\circ$ $\angle POQ = \angle QOR = 10^\circ$ $\angle POR = 20^\circ$ Area of the sector OPQR $= \frac{20^\circ}{360^\circ} \times \pi \times 6^2$ $= 2\pi \text{ cm}^2$</p>		
11.	<p>(a) $\frac{895 + 70 + a + 80 + b}{15} = 70$ $a + b = 5$ $80 + b - 61 = 22$ $b = 3$ $a = 2$ median = \$69 standard deviation $= \\$7.330302404$ $\approx \\$7.33$ (corr to 3 sig fig)</p>		

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<p>12. (a) Let $V_1 \text{ cm}^3$ and $V_2 \text{ cm}^3$ be the volume of smaller right pyramid and larger right pyramid respectively</p> $\frac{V_1}{V_2}$ $= \left(\sqrt{\frac{4}{9}} \right)^3$ $= \frac{8}{27}$ $\frac{V_1 + V_2}{V_2}$ $= (84)(20)$ $= 1680$ $= 1680 \times \frac{27}{27 + 8}$ $= 1296$ <p>The volume of the larger pyramid</p> $= 1296 \text{ cm}^3$ <p>(b) Volume of smaller pyramid = 384 cm^3</p> <p>Height of smaller pyramid</p> $= \frac{2}{3} \times 12$ $= 8$ $\frac{1}{3}(\text{Base Area})(8) = 384$ <p>Base Area = 144 cm^2</p> <p>Length of square in base</p> $= \sqrt{144}$ $= 12$ <p>Total surface area</p> $= \frac{1}{2}(12)\sqrt{\left(\frac{12}{2}\right)^2 + 8^2} \times 4 + 144$ $= 384 \text{ cm}^2$		

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<p>13. (a) Let the equation of C be $(x-4)^2 + (y+1)^2 = r$, where r is a real constant</p> <p>Since C passes through $(-6,5)$,</p> <p>So, we have $(-6-4)^2 + (5+1)^2 = r$ $r = 100$ The equation of C: $(x-4)^2 + (y+1)^2 = 100$</p> <p>(b) Radius of $C = 10$ GF $= \sqrt{(2-(-3))^2 + (-1-11)^2}$ $= 13$ > 10 F lies outside C</p> <p>(c) (i) F, G, H are collinear</p> <p>(ii) The equation of straight line which passes through F and H:</p> $\frac{y-11}{x+3} = \frac{11-(-1)}{-3-2}$ $y = -\frac{12}{5}x + \frac{19}{5}$ $12x + 5y - 19 = 0$		

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14.	<p>(a) $f(x)$</p> $= (3x+7)(2x^2+ax+4)+bx+c$ $= 6x^3 + (3a+14)x^2 + (12+7a+b)x + 28+c$ $\equiv 6x^3 - 13x^2 - 46x + 34$ $3a+14 = -13$ $a = -9$ <p>(b) (i) Since $g(x)$ is a quadratic polynomial, degree of quotient when $g(x)$ is divided by $2x^2+ax+4$ is 0</p> $g(x) = A(2x^2-9x+4)+bx+c, \text{ where } A \text{ is a constant}$ $f(x) - g(x)$ $= (3x+7)(2x^2+ax+4)+bx+c - (A(2x^2-9x+4)+bx+c)$ $= (3x+7-A)(2x^2+ax+4)$ $\therefore f(x) - g(x) \text{ is divisible by } 2x^2+ax+4$ <p>(b) (ii) $f(x) - g(x) = 0$</p> $(3x+7-A)(2x^2-9x+4) = 0$ $x = \frac{A-7}{3} \text{ or } x = 4 \text{ or } x = \frac{1}{2}$ <p>Since $\frac{1}{2}$ is not an integer, I don't agree the claim</p>		

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<p>15. $\begin{cases} 0 = a + \log_b 9 \\ 3 = a + \log_b 243 \end{cases}$ $3 = \log_b 243 - \log_b 9$ $3 = \log_b \frac{243}{9}$ $b^3 = 27$ $b = 3$ $a = -2$</p> <p>$y = -2 + \log_3 x$ $y + 2 = \log_3 x$ $x = 3^{y+2}$</p> <p>16. (a) The total volume of water imported $= 1.5 \times 10^7 + 0.9 \times 1.5 \times 10^7 + 0.9^2 \times 1.5 \times 10^7 + \dots + 0.9^{19} \times 1.5 \times 10^7$ $= 1.5 \times 10^7 \times (1 + 0.9 + 0.9^2 + \dots + 0.9^{19})$ $= 1.5 \times 10^7 \times \frac{1 - 0.9^{20}}{1 - 0.9}$ $= 1.31763501 \times 10^8$ $\approx 1.32 \times 10^8 \text{ m}^3$ (corr to 3 sig fig)</p> <p>(b) Since the water imported every year is positive, The total volume of water imported $< 1.5 \times 10^7 + 0.9 \times 1.5 \times 10^7 + 0.9^2 \times 1.5 \times 10^7 + \dots$ $= 1.5 \times 10^7 \times (1 + 0.9 + 0.9^2 + \dots)$ $= 1.5 \times 10^7 \times \frac{1}{1 - 0.9}$ $= 1.5 \times 10^8$ $< 1.6 \times 10^8$ No, I don't agree the claim.</p>		
<p>Suppose the total water imported can exceed $1.6 \times 10^8 \text{ m}^3$ at n^{th} year $1.5 \times 10^7 + 0.9 \times 1.5 \times 10^7 + 0.9^2 \times 1.5 \times 10^7 + \dots + 0.9^{n-1} \times 1.5 \times 10^7 > 1.6 \times 10^8$ $1.5 \times 10^7 \times (1 + 0.9 + 0.9^2 + \dots + 0.9^{n-1}) > 1.6 \times 10^8$ $\frac{1 - 0.9^n}{1 - 0.9} > \frac{32}{3}$ $0.9^n < -\frac{1}{15}$ which is impossible No, I don't agree the claim.</p>		

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17. (a)	Required probability $= \frac{C_4^4 C_1^{15}}{C_5^{19}}$		
	$= \left(\frac{4}{19}\right)\left(\frac{3}{18}\right)\left(\frac{2}{17}\right)\left(\frac{1}{16}\right)\left(\frac{15}{15}\right) C_4^5$		
	$= \frac{5}{3876}$		r.t. 0.00129
(b)	Required probability $= \frac{C_3^4 C_2^{15}}{C_5^{19}}$ $= \left(\frac{4}{19}\right)\left(\frac{3}{18}\right)\left(\frac{2}{17}\right)\left(\frac{15}{16}\right)\left(\frac{14}{15}\right) C_3^5$ $= \frac{35}{969}$		r.t. 0.0361
(c)	Required probability $= 1 - \frac{5}{3876} - \frac{35}{969}$		
	$= \left(\frac{15}{19}\right)\left(\frac{14}{18}\right)\left(\frac{13}{17}\right)\left(\frac{12}{16}\right)\left(\frac{11}{15}\right) C_0^5 + \left(\frac{4}{19}\right)\left(\frac{15}{18}\right)\left(\frac{14}{17}\right)\left(\frac{13}{16}\right)\left(\frac{12}{15}\right) C_1^5$ $+ \left(\frac{4}{19}\right)\left(\frac{3}{18}\right)\left(\frac{15}{17}\right)\left(\frac{14}{16}\right)\left(\frac{13}{15}\right) C_2^5$		
	$= \frac{3731}{3876}$		r.t. 0.963

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<p>18. (a) $\begin{cases} y = 19 \\ y = 2x^2 - 2kx + 2x - 3k + 8 \end{cases}$ $19 = 2x^2 - 2kx + 2x - 3k + 8$ $2x^2 - 2kx + 2x - 3k - 11 = 0$ Δ $= (-2k + 2)^2 - 4(2)(-3k - 11)$ $= 4k^2 + 16k + 92$ $= 4(k + 2)^2 + 76$ ≥ 76, for all real values of k $\therefore \Delta > 0$, for all real values of k L and Γ intersect at two distinct points.</p> <p>(b) (i) a, b are the roots of $2x^2 - 2kx + 2x - 3k - 11 = 0$ ab $= \frac{-3k - 11}{2}$ $= -\frac{3k + 11}{2}$ $a + b$ $= -\frac{-2k + 2}{2}$ $= k - 1$ $(a - b)^2$ $= a^2 + b^2 - 2ab$ $= (a + b)^2 - 4ab$ $= (k - 1)^2 - 4\left(-\frac{3k + 11}{2}\right)$ $= k^2 - 2k + 1 + 6k + 22$ $= k^2 + 4k + 23$</p> <p>(ii) AB $= \sqrt{(a - b)^2}$ $= \sqrt{k^2 + 4k + 23}$ $= \sqrt{(k + 2)^2 + 19}$ $\geq \sqrt{19} = 4.358898944$, for all real values of k No, it is impossible.</p>		

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19.	<p>(a) $\frac{AC}{\sin(180^\circ - 30^\circ - 42^\circ)} = \frac{24}{\sin 30^\circ}$ $AC = 45.65071278$ $AC \approx 45.7 \text{ cm}$ (corr to 3 sig fig) The length of AC is 45.7 cm</p> <p>(b) (i) $\frac{CF}{CF + AC} = \frac{2}{10}$ $CF = \frac{1}{4} AC$ $CF = \frac{1}{4} \times 45.65071278$ $CF = 11.4126782$ $CF \approx 11.4 \text{ cm}$ (corr to 3 sig fig)</p> <p>(ii) Area of $\triangle ABF = \frac{1}{2}(AB)(AF) \sin 30^\circ$ $AF = CF + AC = 57.06339098$ $AB^2 = AC^2 + BC^2 - 2(AC)(BC) \cos 42^\circ$ $AB^2 = (45.65071278)^2 + 24^2 - 2(24)(45.65071278) \cos 42^\circ$ $AB = 32.11826911$ Area of $\triangle ABF$ $= \frac{1}{2}(32.11826911)(57.06339098) \sin 30^\circ$ $= 458.1943369 \text{ cm}^2$ $\approx 458 \text{ cm}^2$ (corr to 3 sig fig)</p>		

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<p>(iii) $BF^2 = AF^2 + AB^2 - 2(AF)(AB)\cos 30^\circ$ $BF^2 = (57.06339098)^2 + (32.11826911)^2 - 2(57.06339098)(32.11826911)\cos 30^\circ$ $BF = 33.36690449$ Let h be the height of $\triangle ABF$ with base BF $\frac{1}{2}(h)(BF) = \text{Area of } \triangle ABF$ $\frac{1}{2}(h)(BF) = 458.1943369$ $\frac{1}{2}(h)(33.36690449) = 458.1943369$ $h = 27.46400026$ Let the inclination of the thin metal sheet ABC to horizontal ground be θ $\sin \theta = \frac{10}{h}$ $\sin \theta = \frac{10}{27.46400026}$ $\theta = 21.35300646^\circ$ The inclination of the thin metal sheet ABC to horizontal ground = 21.4°</p> <p>(iv) $BF = 33.36690449$ $BD = \sqrt{AB^2 - 10^2}$ $BD = 30.52184808$ $DF = \sqrt{CF^2 - 10^2}$ $DF = 56.18033989$ Let $s = \frac{BF + BD + DF}{2} = 60.03454623$ Area of $\triangle BDF$ $= \sqrt{s(s - BF)(s - BD)(s - DF)}$ $= 426.741482$ < 460 No, I don't agree the claim.</p>		