Introduction to Glue Semantics

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Course overview

1. Basic theory
2. Applications
3. Crossover – seting up the computational tools
4. Computational algorithms
5. Constraining Glue

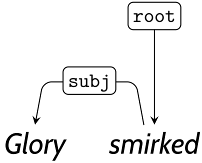
Goals

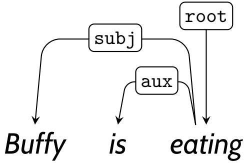
1. To familiarise you with the key concepts and tools of Glue Semantics.
2. To make you aware of the complexities that arise when we try to implement theory computationally.
3. To give you some hands-on experience of such computational implementation.
4. To make you think critically about certain default assumptions regarding the connection between syntax and semantics.

Exercise 1: Thinking with dependencies (1/3)

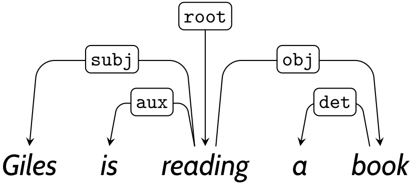
spaCy/displaCy

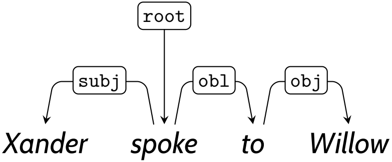
https://demos.explosion.ai/displacy

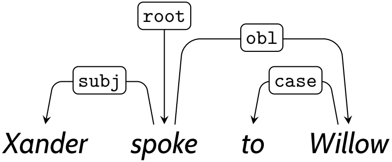
1. 

2. 

Exercise 1. Thinking with dependencies (2/3)

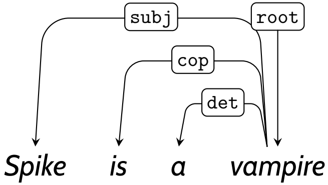
3


4


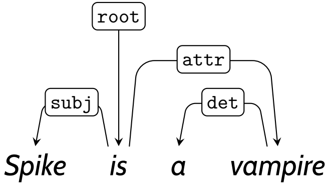
OR


Exercise 1. Thinking with dependencies (3/3)

5



OR



Exercise 2: Meaning constructors (1/2)

1. a. *hates* $\rightsquigarrow \lambda y.\lambda x.\textbf{hates}(x, y) : E(\bullet\ \texttt{obj}) \multimap [E(\bullet\ \texttt{subj}) \multimap T(\bullet)]$
   b. *sleeps* $\rightsquigarrow \lambda x.\textbf{sleeps}(x) : E(\bullet\ \texttt{subj}) \multimap T(\bullet)$
   c. *Giles* $\rightsquigarrow \textbf{giles} : E(\bullet)$
   d. *book* $\rightsquigarrow \lambda x.\textbf{book}(x) : E(\bullet) \multimap T(\bullet)$
   e. *sister* $\rightsquigarrow \lambda y.\lambda x.\textbf{sister}(x, y) : E(\bullet\ \texttt{poss}) \multimap [E(\bullet) \multimap T(\bullet)]$
   f. *everyone* $\rightsquigarrow \lambda P.\forall x.\textbf{person}(x) \rightarrow P(x) : \forall\alpha.[E(\bullet) \multimap T(\alpha)] \multimap T(\alpha)$

Exercise 2: Meaning constructors (2/2)

❷

$$\dfrac{\dfrac{\begin{array}{c}\textbf{[\textit{hates}]}\\ \lambda y.\lambda x.\textbf{hates}(x,y) : E(a) \multimap [E(s) \multimap T(h)]\end{array} \quad \begin{array}{c}\textbf{[\textit{Angel}]}\\ \textbf{angel} : E(a)\end{array}}{\lambda x.\textbf{hates}(x,\textbf{angel}) : E(s) \multimap T(h)} \quad \begin{array}{c}\textbf{[\textit{Spike}]}\\ \textbf{spike} : E(s)\end{array}}{\textbf{hates}(\textbf{spike},\textbf{angel}) : T(h)}$$

Exercise 3: Practising with Glue proofs (1/3)

1.

$$\dfrac{\begin{array}{ll}\textbf{[}\textbf{\textit{laughed}}\textbf{]} & \textbf{[}\textbf{\textit{Buffy}}\textbf{]} \\ \lambda x.\mathbf{laughed}(x) : E(b) \multimap T(l) & \mathbf{buffy} : E(b)\end{array}}{\mathbf{laughed}(\mathbf{buffy}) : T(l)} \multimap_{\mathcal{E}}$$

Exercise 3: Practising with Glue proofs (2/3)

❷

$$\cfrac{\cfrac{\begin{array}{l}\textbf{[}\textbf{\textit{scolded}}\textbf{]}\\ \lambda y.\lambda x.\textbf{scolded}(x,y) : E(b) \multimap [E(g) \multimap T(s)]\end{array} \quad \begin{array}{l}\textbf{[}\textbf{\textit{Buffy}}\textbf{]}\\ \textbf{buffy} : E(b)\end{array}}{\lambda x.\textbf{scolded}(x,\textbf{buffy}) : E(g) \multimap T(s)} \quad \begin{array}{l}\textbf{[}\textbf{\textit{Giles}}\textbf{]}\\ \textbf{giles} : E(g)\end{array}}{\textbf{scolded}(\textbf{giles},\textbf{buffy}) : T(s)}$$

Exercise 3: Practising with Glue proofs (3/3)

2

$$
\dfrac{\dfrac{\begin{array}{l}\textbf{[}\textbf{\textit{is}}\textbf{]}\\ \lambda P.\lambda x.P(x):\\ [E(v) \multimap T(v)] \multimap E(s) \multimap T(v)\end{array} \qquad \begin{array}{l}\textbf{[}\textbf{\textit{vampire}}\textbf{]}\\ \textbf{vampire}:\\ E(v) \multimap T(v)\end{array}}{\begin{array}{l}\lambda x.\textbf{vampire}(x):\\ E(s) \multimap T(v)\end{array}} \qquad \begin{array}{l}\textbf{[}\textbf{\textit{Spike}}\textbf{]}\\ \textbf{spike}:\\ E(s)\end{array}}{\textbf{vampire}(\textbf{spike}) : T(v)}
$$

Exercise 4: Quantificational determiners (1/10)

1. *every* $\rightsquigarrow$
   $\lambda P.\lambda Q.\forall x.P(x) \rightarrow Q(x)$
   $[E(\hat{\bullet}) \multimap T(\hat{\bullet})] \multimap \forall\alpha.[E(\bullet) \multimap T(\alpha)] \multimap T(\alpha)$

   – We need a way to refer to a node's mother (here we use '$\hat{\bullet}$').

2.

$$\dfrac{\dfrac{\dfrac{\begin{array}{c}[\textbf{\textit{every}}]\\ \lambda P.\lambda Q.\forall x.P(x) \rightarrow Q(x) :\\ [E(v) \multimap T(v)] \multimap \forall\alpha.[[E(v) \multimap T(\alpha)] \multimap \alpha]\end{array} \quad \begin{array}{c}[\textbf{\textit{vampire}}]\\ \lambda x.\textbf{vampire}(x) :\\ E(v) \multimap T(v)\end{array}}{\begin{array}{c}\lambda Q.\forall x.\textbf{vampire}(x) \rightarrow P(x) :\\ \forall\alpha.[E(v) \multimap T(\alpha)] \multimap T(\alpha)\end{array}}}{\begin{array}{c}\lambda Q.\forall x.\textbf{vampire}(x) \rightarrow Q(x) :\\ [E(s) \multimap T(s)] \multimap T(s)\end{array}} \forall_{\mathcal{E}} \quad \begin{array}{c}[\textbf{\textit{snarled}}]\\ \lambda x.\textbf{snarled}(x) :\\ E(v) \multimap T(s)\end{array}}{\forall x.\textbf{vampire}(x) \rightarrow \textbf{snarled}(x) : T(s)}$$

Exercise 4: Quantificational determiners (2/10)

❸ MEANING CONSTRUCTORS:

$$\begin{array}{rcl}
\textit{every} & \rightsquigarrow & \lambda P.\lambda Q.\forall x.P(x) \rightarrow Q(x) : \\
& & [E(v) \multimap T(v)] \multimap \forall \alpha.[[E(v) \multimap T(\alpha)] \multimap T(\alpha)] \\
\textit{vampire} & \rightsquigarrow & \lambda x.\textbf{vampire}(x) : \\
& & E(v) \multimap T(v) \\
\textit{killed} & \rightsquigarrow & \lambda y.\lambda x.\textbf{killed}(x, y) : \\
& & E(w) \multimap [E(v) \multimap T(k)] \\
\textit{a} & \rightsquigarrow & \lambda P.\lambda Q.\exists x.P(x) \wedge Q(x) : \\
& & [E(w) \multimap T(w)] \multimap \forall \beta.[[E(w) \multimap T(\beta)] \multimap T(\beta)] \\
\textit{werewolf} & \rightsquigarrow & \lambda x.\textbf{werewolf}(x) : \\
& & E(w) \multimap T(w)
\end{array}$$

Exercise 4: Quantificational determiners (3/10)

❸ **[*every*], [*vampire*]** $\vdash$ $\lambda Q.\forall x.\textbf{vampire}(x) \rightarrow Q(x)$:
$\forall \alpha.[E(v) \multimap T(\alpha)] \multimap T(\alpha)$

[*a*], [*werewolf*] $\vdash$ $\lambda Q.\exists x.\textbf{werewolf}(x) \wedge Q(x)$:
$\forall \beta.[E(w) \multimap T(\beta)] \multimap T(\beta)$

Exercise 4: Quantificational determiners (4/10)

3

$$
\dfrac{
\dfrac{
\dfrac{
\dfrac{\begin{array}{l}\textbf{[\textit{killed}]}\\ \lambda y.\lambda x.\mathbf{killed}(x,y) : \\ E(w) \multimap [E(v) \multimap T(k)]\end{array} \qquad \left[\begin{array}{l} y : \\ E(w)\end{array}\right]^{1}}
{\begin{array}{l}\lambda x.\mathbf{killed}(x,y) : \\ E(v)\multimap T(k)\end{array}}
\qquad
\begin{array}{l}\textbf{[\textit{every vampire}]}\\ \lambda Q.\forall x.\mathbf{vampire}(x)\rightarrow Q(x) : \\ [E(v)\multimap T(k)]\multimap T(k)\end{array}
}{\begin{array}{l}\forall x.\mathbf{vampire}(x)\rightarrow\mathbf{killed}(x,y) : \\ T(k)\end{array}}
}{\begin{array}{l}\lambda y.\forall x.\mathbf{vampire}(x)\rightarrow\mathbf{killed}(x,y) : \\ E(w)\multimap T(k)\end{array}}
\multimap_{\mathcal{I},1}
\qquad
\begin{array}{l}\textbf{[\textit{a werewolf}]}\\ \lambda Q.\exists y.\mathbf{werewolf}(y)\wedge Q(y) : \\ [E(w)\multimap T(k)]\multimap T(k)\end{array}
}{\exists y.\mathbf{werewolf}(y)\wedge\forall x.\mathbf{vampire}(x)\rightarrow\mathbf{killed}(x,y) : T(k)}
$$

Exercise 4: Quantificational determiners (5/10)

3

$$
\dfrac{
  \dfrac{
    \dfrac{
      \dfrac{
        \dfrac{
          \begin{array}{l}\textbf{[}\textbf{\textit{killed}}\textbf{]}\\ \lambda y.\lambda x.\mathbf{killed}(x,y) :\\ E(w) \multimap [E(v) \multimap T(k)]\end{array}
          \quad
          \left[\begin{array}{l} y :\\ E(w)\end{array}\right]^{1}
        }{
          \begin{array}{l}\lambda x.\mathbf{killed}(x,y) :\\ E(v) \multimap T(k)\end{array}
        }
        \quad
        \left[\begin{array}{l} x :\\ E(v)\end{array}\right]^{2}
      }{
        \begin{array}{l}\mathbf{killed}(x,y) :\\ T(k)\end{array}
      }
      \multimap_{\mathcal{I},1}
    }{
      \begin{array}{l}\lambda y.\mathbf{killed}(x,y) :\\ E(w) \multimap T(k)\end{array}
    }
    \quad
    \begin{array}{l}\textbf{[}\textbf{\textit{a werewolf}}\textbf{]}\\ \lambda Q.\exists y.\mathbf{werewolf}(y) \wedge Q(y) :\\ [E(w) \multimap T(k)] \multimap T(k)\end{array}
  }{
    \begin{array}{l}\exists y.\mathbf{werewolf}(y) \wedge \mathbf{killed}(x,y) :\\ T(k)\end{array}
  }
  \multimap_{\mathcal{I},2}
}{
  \begin{array}{l}\lambda x.\exists y.\mathbf{werewolf}(y) \wedge \mathbf{killed}(x,y) :\\ E(v) \multimap T(k)\end{array}
}
\quad
\begin{array}{l}\textbf{[}\textbf{\textit{every vampire}}\textbf{]}\\ \lambda Q.\forall x.\mathbf{vampire}(x) \rightarrow Q(x) :\\ [E(v) \multimap T(k)] \multimap T(k)\end{array}
$$

$$
\overline{\forall x.\mathbf{vampire}(x) \rightarrow \exists y.\mathbf{werewolf}(y) \wedge \mathbf{killed}(x,y) : T(k)}
$$

Exercise 4: Quantificational determiners (6/10)

❹ MEANING CONSTRUCTORS:

$$
\begin{array}{rcl}
\textit{Giles} & \rightsquigarrow & \textbf{giles} : \\
& & E(gi) \\
\textit{gave} & \rightsquigarrow & \lambda z.\lambda y.\lambda x.\textbf{gave}(x, y, z) : \\
& & E(e) \multimap [E(b) \multimap [E(gi) \multimap T(ga)]] \\
\textit{everyone} & \rightsquigarrow & \lambda P.\forall x.\textbf{person}(x) \rightarrow P(x) : \\
& & [E(e) \multimap T(ga)] \multimap T(ga) \\
\textit{a} & \rightsquigarrow & \lambda P.\lambda Q.\exists x.P(x) \wedge Q(x) : \\
& & [E(b) \multimap T(b)] \multimap [[E(b) \multimap T(ga)] \multimap T(ga)] \\
\textit{book} & \rightsquigarrow & \lambda x.\textbf{book}(x) : \\
& & E(b) \multimap T(b)
\end{array}
$$

Exercise 4: Quantificational determiners (7/10)

4. Common part:

$$
\dfrac{
  \dfrac{
    \dfrac{
      \begin{array}{l}\textbf{[}\boldsymbol{gave}\textbf{]}\\ \lambda z.\lambda y.\lambda x.\mathbf{gave}(x,y,z) :\\ E(e) \multimap [E(b) \multimap [E(gi) \multimap T(ga)]]\end{array}
      \quad
      \left[\begin{array}{l} z :\\ E(e)\end{array}\right]^{1}
    }{
      \begin{array}{l}\lambda y.\lambda x.\mathbf{gave}(x,y,z) :\\ E(b) \multimap [E(gi) \multimap T(ga)]\end{array}
    }
    \quad
    \left[\begin{array}{l} y :\\ E(b)\end{array}\right]^{2}
  }{
    \begin{array}{l}\lambda x.\mathbf{gave}(x,y,z) :\\ E(gi) \multimap T(ga)\end{array}
  }
  \quad
  \begin{array}{l}\textbf{[}\boldsymbol{Giles}\textbf{]}\\ \mathbf{giles} :\\ E(gi)\end{array}
}{
  \begin{array}{l}\mathbf{gave}(\mathbf{giles},y,z) :\\ T(ga)\\ \quad\vdots\end{array}
}
$$

Exercise 4: Quantificational determiners (8/10)

4. Surface scope (*everyone* $\gg$ *a book*), continuing from above:

$$
\dfrac{
\dfrac{
\dfrac{
\dfrac{
\begin{array}{l}\vdots \\ \textbf{gave}(\textbf{giles}, y, z) : \\ T(ga)\end{array}
}{
\begin{array}{l}\lambda y.\textbf{gave}(\textbf{giles}, y, z) : \\ E(b) \multimap T(ga)\end{array}
}\multimap_{\mathcal{I},2}
\quad
\begin{array}{l}\textbf{[}\textbf{\textit{a book}}\textbf{]} \\ \lambda Q.\exists y.\textbf{book}(y) \wedge Q(y) : \\ [E(b) \multimap T(ga)] \multimap T(ga)\end{array}
}{
\begin{array}{l}\exists y.\textbf{book}(y) \wedge \textbf{gave}(\textbf{giles}, y, z) : \\ T(ga)\end{array}
}
}{
\begin{array}{l}\lambda z.\exists y.\textbf{book}(y) \wedge \textbf{gave}(\textbf{giles}, y, z) : \\ E(e) \multimap T(ga)\end{array}
}\multimap_{\mathcal{I},1}
\quad
\begin{array}{l}\textbf{[}\textbf{\textit{everyone}}\textbf{]} \\ \lambda P.\forall z.\textbf{person}(z) \to P(z) : \\ [E(e) \multimap T(ga)] \multimap T(ga)\end{array}
}{
\begin{array}{l}\forall z.\textbf{person}(z) \to \exists y.\textbf{book}(y) \wedge \textbf{gave}(\textbf{giles}, y, z) : \\ T(ga)\end{array}
}
$$

Exercise 4: Quantificational determiners (9/10)

4. Inverse scope (*a book* $\gg$ *everyone*), continuing from above:

$$
\dfrac{
  \dfrac{
    \dfrac{\vdots}{\begin{array}{l}\mathbf{gave}(\mathbf{giles},y,z) : T(ga)\end{array}}
  }{
    \lambda z.\mathbf{gave}(\mathbf{giles},y,z) : E(e) \multimap T(ga)
  }\multimap_{\mathcal{I},1}
  \dfrac{
    \begin{array}{l}\mathbf{[\textit{everyone}]}\\ \lambda P.\forall z.\mathbf{person}(z) \rightarrow P(z) : [E(e) \multimap T(ga)] \multimap T(ga)\end{array}
  }{
    \dfrac{\forall z.\mathbf{person}(z) \rightarrow \mathbf{gave}(\mathbf{giles},y,z) : T(ga)}{\lambda y.\forall z.\mathbf{person}(z) \rightarrow \mathbf{gave}(\mathbf{giles},y,z) : E(b) \multimap T(ga)}\multimap_{\mathcal{I},2}
  }
  \quad
  \begin{array}{l}\mathbf{[\textit{a book}]}\\ \lambda Q.\exists y.\mathbf{book}(y) \wedge Q(y) : [E(b) \multimap T(ga)] \multimap T(ga)\end{array}
}{
  \exists y.\mathbf{book}(y) \wedge \forall z.\mathbf{person}(z) \rightarrow \mathbf{gave}(\mathbf{giles},y,z) : T(ga)
}
$$

Exercise 4: Quantificational determiners (10/10)

We get two proofs – as ever, when there are two scope-taking elements, both orders are attested.

This, it seems, is not empirically correct, since the double object construction in English is a scope-freezing construction: only the surface scope reading is licit.

The problem of constraining scope in Glue has receieved some recent attention. See Findlay & Haug (2022) and Zymla (2024) for references, discussion, and a suggested solution (albeit originally couched in terms of LFG syntax). We will also talk about this from a computational perspective later in the week.

Exercise 5: Latin

1.
$$\begin{array}{rcl} \textit{amat} & \rightsquigarrow & \lambda y.\lambda x.\mathbf{loves}(x,y) : E(a) \multimap [E(d) \multimap T(am)] \\ \textit{Angelum} & \rightsquigarrow & \mathbf{angel} : E(a) \\ \textit{Drusilla} & \rightsquigarrow & \mathbf{drusilla} : E(d) \end{array}$$

2.
$$\dfrac{\dfrac{\begin{array}{c}[\textit{amat}] \\ \lambda y.\lambda x.\mathbf{loves}(x,y) : E(a) \multimap [E(d) \multimap T(am)]\end{array} \quad \begin{array}{c}[\textit{Angelum}] \\ \mathbf{angel} : E(a)\end{array}}{\lambda x.\mathbf{loves}(x,\mathbf{angel}) : E(d) \multimap T(am)} \quad \begin{array}{c}[\textit{Drusilla}] \\ \mathbf{drusilla} : E(d)\end{array}}{\mathbf{loves}(\mathbf{drusilla},\mathbf{angel}) : T(am)}$$

Exercise 6: Reordering arguments

$$
\dfrac{
 \dfrac{
  \dfrac{
   \dfrac{
     \begin{array}{l}\textbf{[\textit{loves}]}\\ \lambda x.\lambda y.\mathbf{loves}(x,y):\\ E(s)\multimap[E(e)\multimap T(l)]\end{array}
     \qquad
     \left[\begin{array}{l}x:\\ E(s)\end{array}\right]^1
   }{
     \begin{array}{l}\lambda y.\mathbf{loves}(x,y):\\ E(e)\multimap T(l)\end{array}
   }
   \quad
   \begin{array}{l}\textbf{[\textit{everyone}]}\\ \lambda P.\forall y.\mathbf{person}(y)\to P(y):\\ [E(e)\multimap T(l)]\multimap T(l)\end{array}
  }{
   \begin{array}{l}\forall y.\mathbf{person}(y)\to\mathbf{loves}(x,y):\\ T(l)\end{array}
  }
 }{
  \begin{array}{l}\lambda x.\forall y.\mathbf{person}(y)\to\mathbf{loves}(x,y):\\ E(s)\multimap T(l)\end{array}
 }\multimap_{\mathcal{I},1}
 \quad
 \begin{array}{l}\textbf{[\textit{someone}]}\\ \lambda P.\exists x.\mathbf{person}(x)\wedge P(x):\\ [E(s)\multimap T(l)]\multimap T(l)\end{array}
}{
 \exists x.\mathbf{person}(x)\wedge\forall y.\mathbf{person}(y)\to\mathbf{loves}(x,y):T(l)
}
$$

Exercise 7: Further practice (1/5)

1

$$\dfrac{\dfrac{\begin{array}{l}\textbf{[}\textit{Snyder}\textbf{]}\\ \mathbf{snyder}: \\ E(s)\end{array} \quad \begin{array}{l}\textbf{[}\textit{frightens}\textbf{]}\\ \lambda x.\lambda y.\mathbf{frightens}(x,y): \\ E(sn) \multimap [E(st) \multimap T(f)]\end{array}}{\begin{array}{l}\lambda y.\mathbf{frightens}(x,y): \\ E(st) \multimap T(f)\end{array}} \quad \dfrac{\begin{array}{l}\textbf{[}\textit{every}\textbf{]}\\ \lambda P.\lambda Q.\forall y.P(y) \rightarrow Q(y): \\ [E(st) \multimap T(st)] \multimap [[E(st) \multimap T(f)] \multimap T(f)]\end{array} \quad \begin{array}{l}\textbf{[}\textit{student}\textbf{]}\\ \lambda x.\mathbf{student}(x): \\ E(st) \multimap T(st)\end{array}}{\begin{array}{l}\lambda Q.\forall y.\mathbf{student}(y) \rightarrow Q(y): \\ [E(st) \multimap T(f)] \multimap T(f)\end{array}}}{\forall y.\mathbf{student}(y) \rightarrow \mathbf{frightens}(\mathbf{snyder}, y) : T(f)}$$

Exercise 7: Further practice (2/5)

2. **[*every*]**, **[*vampire*]** $\vdash$ $\lambda Q.\forall x.\textbf{vampire}(x) \rightarrow Q(x) :$
$\forall\alpha.[E(v) \multimap T(\alpha)] \multimap T(\alpha)$

[*a*], **[*teenager*]** $\vdash$ $\lambda Q.\exists x.\textbf{teenager}(x) \wedge Q(x) :$
$\forall\beta.[E(t) \multimap T(\beta)] \multimap T(\beta)$

Exercise 7: Further practice (3/5)

2. Surface scope:

$$
\dfrac{
  \dfrac{
    \dfrac{
      \dfrac{
        \begin{array}{l}\textbf{[}\textit{\textbf{bit}}\textbf{]}\\ \lambda x.\lambda y.\mathbf{bit}(x,y):\\ E(v) \multimap [E(t) \multimap T(k)]\end{array}
        \quad
        \left[\begin{array}{l} x:\\ E(v)\end{array}\right]^{1}
      }{
        \begin{array}{l}\lambda y.\mathbf{bit}(x,y):\\ E(t) \multimap T(k)\end{array}
      }
      \quad
      \begin{array}{l}\textbf{[}\textit{\textbf{a teenager}}\textbf{]}\\ \lambda Q.\exists y.\mathbf{teenager}(y) \wedge Q(y):\\ [E(t) \multimap T(k)] \multimap T(k)\end{array}
    }{
      \begin{array}{l}\exists y.\mathbf{teenager}(y) \wedge \mathbf{bit}(x,y):\\ T(k)\end{array}
    }
  }{
    \begin{array}{l}\lambda x.\exists y.\mathbf{teenager}(y) \wedge \mathbf{bit}(x,y):\\ E(v) \multimap T(k)\end{array}
  }{\multimap_{\mathcal{I},1}}
  \quad
  \begin{array}{l}\textbf{[}\textit{\textbf{every vampire}}\textbf{]}\\ \lambda Q.\forall x.\mathbf{vampire}(x) \rightarrow Q(x):\\ [E(v) \multimap T(k)] \multimap T(k)\end{array}
}{
  \forall x.\mathbf{vampire}(\rightarrow)\exists y.\mathbf{teenager}(y) \wedge \mathbf{bit}(x,y) : T(k)
}
$$

Exercise 7: Further practice (4/5)

2. Inverse scope:

$$
\dfrac{
  \dfrac{
    \dfrac{
      \dfrac{
        \begin{array}{l}\textbf{[}\textit{\textbf{bit}}\textbf{]}\\ \lambda y.\lambda x.\mathbf{bit}(x,y) :\\ E(t) \multimap [E(v) \multimap T(k)]\end{array}
        \qquad
        \left[\begin{array}{l} y :\\ E(t)\end{array}\right]^{1}
      }{
        \begin{array}{l}\lambda x.\mathbf{bit}(x,y) :\\ E(v) \multimap T(k)\end{array}
      }
      \qquad
      \begin{array}{l}\textbf{[}\textit{\textbf{every vampire}}\textbf{]}\\ \lambda Q.\forall x.\mathbf{vampire}(x) \rightarrow Q(x) :\\ [E(v) \multimap T(k)] \multimap T(k)\end{array}
    }{
      \begin{array}{l}\forall x.\mathbf{vampire}(x) \rightarrow \mathbf{bit}(x,y) :\\ T(k)\end{array}
    }
  }{
    \begin{array}{l}\lambda y.\forall x.\mathbf{vampire}(x) \rightarrow \mathbf{bit}(x,y) :\\ E(t) \multimap T(k)\end{array}
  }\multimap_{\mathcal{I},1}
  \qquad
  \begin{array}{l}\textbf{[}\textit{\textbf{a teenager}}\textbf{]}\\ \lambda Q.\exists y.\mathbf{teenager}(y) \wedge Q(y) :\\ [E(t) \multimap T(k)] \multimap T(k)\end{array}
}{
  \exists y.\mathbf{teenager}(y) \wedge \forall x.\mathbf{vampire}(x) \rightarrow \mathbf{bit}(x,y) : T(k)
}
$$

Exercise 7: Further practice (5/5)

3

$$
\dfrac{
\dfrac{
\begin{array}{l}[\textit{Xander}]\\ \mathbf{xander}:\\ E(xa)\end{array}
\quad
\begin{array}{l}[\textit{thinks}]\\ \lambda x.\lambda p.\mathbf{thinks}(x,p):\\ E(xa)\multimap[T(l)\multimap T(t)]\end{array}
}{
\begin{array}{l}\lambda p.\mathbf{thinks}(\mathbf{xander},p):\\ T(l)\multimap T(t)\end{array}
}
\quad
\dfrac{
\dfrac{
\begin{array}{l}[\textit{Buffy}]\\ \mathbf{buffy}:\\ E(b)\end{array}
\quad
\begin{array}{l}[\textit{loves}]\\ \lambda x.\lambda y.\mathbf{loves}(x,y):\\ E(b)\multimap[E(v)\multimap T(l)]\end{array}
}{
\begin{array}{l}\lambda y.\mathbf{loves}(\mathbf{buffy},y):\\ E(v)\multimap T(l)\end{array}
}
\quad
\dfrac{
\begin{array}{l}[\textit{some}]\\ \lambda P.\lambda Q.\exists x.P(x)\wedge Q(x):\\ [E(v)\multimap T(v)]\multimap[[E(v)\multimap T(l)]\multimap T(l)]\end{array}
\quad
\begin{array}{l}[\textit{vampire}]\\ \lambda x.\mathbf{vampire}(x):\\ E(v)\multimap T(v)\end{array}
}{
\begin{array}{l}\lambda Q.\exists x.\mathbf{vampire}(x)\wedge Q(x):\\ [E(v)\multimap T(l)]\multimap T(l)\end{array}
}
}{
\begin{array}{l}\exists x.\mathbf{vampire}(x)\wedge\mathbf{loves}(\mathbf{buffy},x):\\ T(l)\end{array}
}
}{
\mathbf{thinks}(\mathbf{xander},\exists x.\mathbf{vampire}(x)\wedge\mathbf{loves}(\mathbf{buffy},x))
}
$$