Formal Semantics for Dependency Grammar

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- Typically adopts strong lexical integrity (Bresnan & Mchombo, 1995): words are the *only* atoms of syntax
- In principle agnostic about semantics, but with an affinity for graph-based formalisms
    - Tectogrammatical layer of FGD (Sgall et al., 1986)
    - Meaning-Text Theory (Mel'cuk et al., 1988; Kahane, 2003)
    - Abstract Meaning Representation (Banarescu et al., 2013)

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  - Proof theory: answer questions like "if a set of sentences P is true, does it follow that sentence h is true?"
- Theoretical desiderata rather than necessarily the most practical way of creating semantic representation or tackle inference tasks
- Also, practical benefit of connecting to a rich branch of ongoing work

Frege's principle

- Logic trivially solves the proof theory requirement, but what about compositionality?
- Frege's principle: the meaning of a (syntactically complex) whole is a function only of the meanings of its (syntactic) parts together with the manner in which these parts were combined

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- Frege's principle: the meaning of a (syntactically complex) whole is a function only of the meanings of its (syntactic) parts together with the manner in which these parts were combined
- This is a much stronger version of compositionality than we required!
- Not trivial to build logical representations from natural language compositionality

The basic idea

(1) a. Every man loves Chris

b. $\forall x.man(x) \rightarrow love(x, c)$

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- Lexical integrity:
  - single lexical items might provide different meanings that interact with other elements of the sentence in complex ways
  - finite verbs might introduce predicate-argument structure, temporal meaning *and* modal meaning
  - $\rightarrow$ meanings might need to be scattered around the composition tree (cf. abstract heads in Chomskyan syntax)

Glue semantics

We will solve these problems using ideas from Glue Semantics (Dalrymple et al., 1993; Asudeh, 2022), which was developed within the tradition of another lexicalist theory of syntax that also does not enforce binary syntax, namely Lexical Functional Grammar (Kaplan & Bresnan, 1982; Dalrymple et al., 2019)

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  A crude characterisation would be that glue semantics is like categorial grammar and its semantics, but without the categorial grammar.

(Crouch & van Genabith, 2000, 91)

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- But decoupled from surface syntax, which is good for universality
- By making *love* type $A \multimap B \multimap C$, did we just stipulate a particular composition order? No!

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- Writing $(\hat{*})$ for the current node
  - $E(\hat{*})$ is a type e meaning for that node
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  - $E(\hat{*}\ \textsc{subj})$ is a type e meaning for the current node's subject
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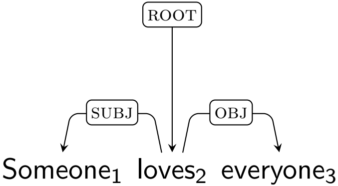
  - $E(\hat{*}\ \textsc{subj}) \multimap E(\hat{*}\ \textsc{obj}) \multimap T(\hat{*})$ is the type we need for *love*

Examples

Simple transitive



- We type
  - *someone* as $(E(\hat{*}) \multimap T(\uparrow)) \multimap T(\uparrow)$, i.e. $(E(1) \multimap T(2)) \multimap T(2)$

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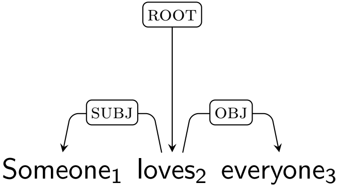

- We type
  - *someone* as $(E(\hat{\ast}) \multimap T(\uparrow)) \multimap T(\uparrow)$, i.e. $(E(1) \multimap T(2)) \multimap T(2)$
  - *love* $E(\hat{\ast}\ \textsc{subj}) \multimap E(\hat{\ast}\ \textsc{obj}) \multimap T(\hat{\ast})$, i.e. $E(1) \multimap E(3) \multimap T(2)$

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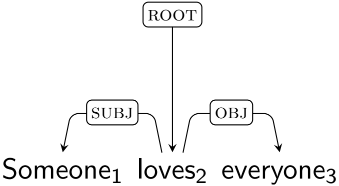

- We type
  - *someone* as $(E(\hat{\circledast}) \multimap T(\uparrow)) \multimap T(\uparrow)$, i.e. $(E(1) \multimap T(2)) \multimap T(2)$
  - *love* $E(\hat{\circledast}\ \textsc{subj}) \multimap E(\hat{\circledast}\ \textsc{obj}) \multimap T(\hat{\circledast})$, i.e. $E(1) \multimap E(3) \multimap T(2)$
  - *everyone* as $(E(\hat{\circledast}) \multimap T(\uparrow)) \multimap T(\uparrow)$, i.e. $(E(3) \multimap T(2)) \multimap T(2)$

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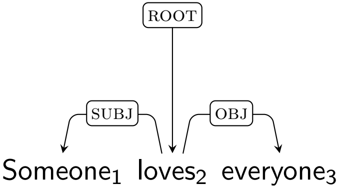

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  - *everyone* as $(E(\hat{*}) \multimap T(\uparrow)) \multimap T(\uparrow)$, i.e. $(E(3) \multimap T(2)) \multimap T(2)$
- This is isomorphic to the atomic types we used: $E(1) \mapsto A$, $E(3) \mapsto B$, $T(2) \mapsto C$ and so we get the same proofs

Prodrop

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- So if a verb type refers to its subject with $E(\hat{*}\ \textsc{subj})$ it *constructs* that semantic type

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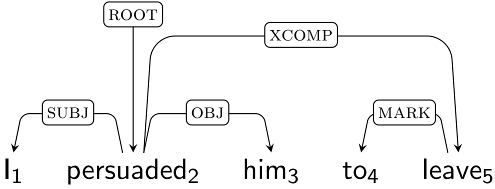
- Sometimes we neeed assume that positions that are not filled syntactically are nevertheless active in the semantics
- For that, we adopt a *constructive* interpretation of Glue types
- So if a verb type refers to its subject with $E(\hat{*}\ \mathrm{SUBJ})$ it *constructs* that semantic type
- We can then deal with prodrop using optional meaning constructors

Control infinitive



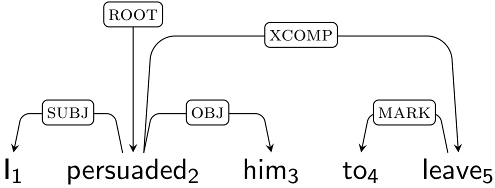
I $s : E(\hat{*})$

persuaded $\lambda x.\lambda y.\lambda P.persuade(x, y, P(y)) :$
$E(\hat{*}\ \textsc{subj}) \multimap E(\hat{*}\ \textsc{obj}) \multimap (E(\hat{*}\ \textsc{xcomp}\ \textsc{subj}) \multimap T(\hat{*}\ \textsc{xcomp})) \multimap T(\hat{*})$

him $a_1 : E(\hat{*})$

leave $\lambda x.admire(x) : E(\hat{*}\ \textsc{subj}) \multimap T(\hat{*})$

Control infinitive



I $s : E(1)$

persuaded $\lambda x.\lambda y.\lambda P.persuade(x, y, P(y)) :$
$E(1) \multimap E(3) \multimap (E(6) \multimap T(5)) \multimap T(2)$

him $a_1 : E(3)$

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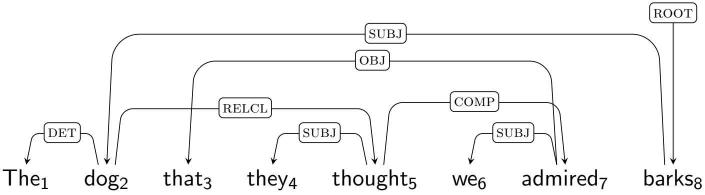
$$\frac{\lambda P.persuade(s, a_1, P(a_1)) : (E(6) \multimap T(5)) \multimap T(2) \qquad \lambda x.leave(x) : E(6) \multimap T(5)}{persuade(s, a_1, leave(a_1)) : T(2)}$$

Relative clauses



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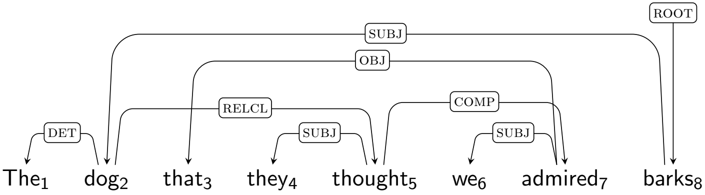
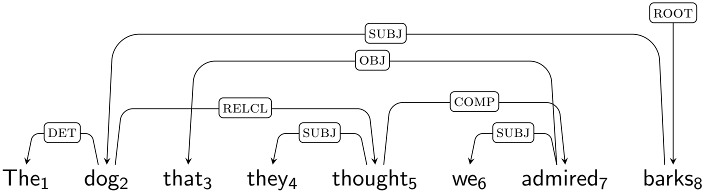


they	a_1	:	$E(\hat{*})$
thought	$\lambda x.\lambda P.think(x, P)$	:	$E(\hat{*}\ \textsc{subj}) \multimap T(\hat{*}\ \textsc{comp}) \multimap T(\hat{*})$
we	s_+	:	$E(\hat{*})$
admired	$\lambda x.\lambda y.admire(x, y)$	:	$E(\hat{*}\ \textsc{subj}) \multimap E(\hat{*}\ \textsc{obj}) \multimap T(\hat{*})$
thought-RELCL	$\lambda P.\lambda Q.\lambda x.P(x) \wedge Q(x)$	:	$\forall\xi.(E(\xi) \multimap T(\hat{*})) \multimap (E(\uparrow) \multimap T(\uparrow)) \multimap E(\uparrow) \multimap T(\uparrow)$

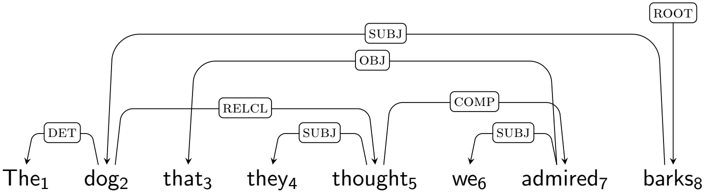
Relative clauses



they	a_1	:	$E(4)$
thought	$\lambda x.\lambda P.think(x, P)$	:	$E(4) \multimap T(7) \multimap T(5)$
we	s_+	:	$E(6)$
admired	$\lambda x.\lambda y.admire(x, y)$	:	$E(6) \multimap E(3) \multimap T(7)$
thought-RELCL	$\lambda P.\lambda Q.\lambda x.P(x) \wedge Q(x)$	:	$(E(3) \multimap T(5)) \multimap (E(2) \multimap T(2)) \multimap E(2) \multimap T(2)$

- The gap is identified with the object position of *admire*

Relative clauses



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thought	$\lambda x.\lambda P.think(x, P)$	:
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thought-RELCL	$\lambda P.\lambda Q.\lambda x.P(x) \wedge Q(x)$	:

- The gap is identified with the object position of *admire*
- **thought-RELCL** allows for *any* gap position ($\forall \xi$)
- We can restrict the gap position at the syntax-semantics interface

Restricted indeterminacy

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- Sets of paths through the dependency tree can be expressed through regular expressions over the alphabet $\mathcal{L} \cup \{\uparrow\}$, where $\mathcal{L}$ is the set of syntactic labels and $\uparrow$ refers to the mother node.
- Node-references can be non-deterministic:
  - $\hat{*}$ $\textsc{obj}$ = the object daughter
  - $\hat{*}$ $(\textsc{subj}|\textsc{obj})$ = the set of the subject and object daughter,
  - $\hat{*}$ $\textsc{comp}^{*}$ $\textsc{subj}$ = the set of $\textsc{subj}$ daughters embedded under zero or more $\textsc{comp}$ daughters.
- Useful if you don't think gaps exist in the syntax!

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- The syntax can underspecify the semantics, making it easier to "offload" work to the interface
- Opens an avenue for connecting to a rich strand of work in semantics
- For a concrete implementation based on this system, listen to our talk tomorrow!

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Inference rules

Application : implication elimination

$$\frac{f : A \multimap B \qquad a : A}{f(a) : B} \multimap_{\mathcal{E}}$$

Inference rules

Application : implication elimination

$$\frac{f : A \multimap B \qquad a : A}{f(a) : B} \multimap_{\mathcal{E}}$$

Abstraction : implication introduction

$$\frac{\begin{array}{c}[x_1 : A]^1 \\ \vdots \\ f : B\end{array}}{\lambda x.f : A \multimap B} \multimap_{\mathcal{I},1}$$

Switching argument orders

$$\dfrac{\dfrac{\dfrac{\lambda x.\lambda y.love(x,y) : A \multimap B \multimap C \qquad [\mathrm{n} : A]^1}{\lambda y.love(\mathrm{n},y) : B \multimap C} \qquad [\mathrm{j} : B]^2}{\dfrac{love(n,j) : C}{\lambda x.love(x,\mathrm{n}) : A \multimap C} \multimap_{\mathcal{I},1}}}{\lambda y.\lambda x.love(x,y) : B \multimap A \multimap C} \multimap_{\mathcal{I},2}$$

- $(A \multimap B \multimap C) \multimap (B \multimap A \multimap C)$ is a theorem of linear logic

Switching argument orders

$$
\dfrac{\dfrac{\dfrac{\dfrac{\lambda x.\lambda y.\mathit{love}(x,y) : A \multimap B \multimap C \qquad [\mathrm{n} : A]^1}{\lambda y.\mathit{love}(\mathrm{n},y) : B \multimap C} \qquad [\mathrm{j} : B]^2}{\mathit{love}(n,j) : C}}{\lambda x.\mathit{love}(x,\mathrm{n}) : A \multimap C}\multimap_{\mathcal{I},1}}{\lambda y.\lambda x.\mathit{love}(x,y) : B \multimap A \multimap C}\multimap_{\mathcal{I},2}
$$

- $(A \multimap B \multimap C) \multimap (B \multimap A \multimap C)$ is a theorem of linear logic
- There is also other stuff we get for free, like type raising operators $(A \multimap ((A \multimap B) \multimap B))$

Switching argument orders

$$\dfrac{\dfrac{\dfrac{\dfrac{\lambda x.\lambda y.love(x,y) : A \multimap B \multimap C \qquad [\mathrm{n} : A]^1}{\lambda y.love(\mathrm{n},y) : B \multimap C} \qquad [\mathrm{j} : B]^2}{love(n,j) : C}}{\lambda x.love(x,\mathrm{n}) : A \multimap C}\multimap_{\mathcal{I},1}}{\lambda y.\lambda x.love(x,y) : B \multimap A \multimap C}\multimap_{\mathcal{I},2}$$

- $(A \multimap B \multimap C) \multimap (B \multimap A \multimap C)$ is a theorem of linear logic
- There is also other stuff we get for free, like type raising operators $(A \multimap ((A \multimap B) \multimap B))$
- Moreover, there is an efficient proof algorithm (based on CYK) and a good implementation (Glue semantics workbench, (**?**))

Quantifier scope ambiguity: surface scope

$$
\dfrac{\dfrac{\dfrac{\dfrac{\boxed{\begin{array}{l}\textbf{loves}\\ \lambda x.\lambda y.love(x,y) :\\ A \multimap B \multimap C\end{array}} \qquad [\mathrm{x}_1 : A]^1}{\begin{array}{l}\lambda y.love(\mathrm{x}_1,y) :\\ B \multimap C\end{array}} \qquad \boxed{\begin{array}{l}\textbf{everyone}\\ \lambda P.\forall z.person(z) \to P(z) :\\ (B \multimap C) \multimap C\end{array}}}{\begin{array}{l}\forall z.person(z) \to love(\mathrm{x}_1,z) :\\ C\end{array}}}{\begin{array}{l}\lambda x.\forall z.person(z) \to love(x,z) :\\ A \multimap C\end{array}} \multimap_{\mathcal{I},1} \qquad \boxed{\begin{array}{l}\textbf{someone}\\ \lambda P.\exists x.person(x) \wedge P(x) :\\ (A \multimap C) \multimap C\end{array}}}{\begin{array}{l}\exists x.person(x) \wedge (\forall z.person(z) \to love(z,x)) :\\ C\end{array}}
$$

Quantifier scope ambiguity: inverse scope

$$\dfrac{\boxed{\begin{array}{l}\textbf{loves}\\ \lambda x.\lambda y.love(x,y):\\ A \multimap B \multimap C\end{array}} \qquad [x_1 : A]^1}{\lambda y.love(a,y): B \multimap C}$$

$$\dfrac{\lambda y.love(a,y): B \multimap C \qquad [x_2 : B]^2}{\dfrac{love(x_1,x_2): C}{\lambda x.love(x,x_2): A \multimap C}\multimap_{\mathcal{I},1}}$$

$$\dfrac{\lambda x.love(x,x_2): A \multimap C \qquad \boxed{\begin{array}{l}\textbf{someone}\\ \lambda P.\exists x.person(x) \wedge P(x):\\ (A \multimap C) \multimap C\end{array}}}{\dfrac{\exists x.person(x) \wedge love(x,x_2): C}{\lambda y.\exists x.person(x) \wedge love(x,y): B \multimap C}\multimap_{\mathcal{I},2}}$$

$$\dfrac{\lambda y.\exists x.person(x) \wedge love(x,y): B \multimap C \qquad \boxed{\begin{array}{l}\textbf{everyone}\\ \lambda P.\forall z.person(z) \rightarrow P(z):\\ (B \multimap C) \multimap C\end{array}}}{\forall z.person(z) \rightarrow (\exists x.person(x) \wedge love(x,z)): C}$$