

Вариант 1.

Задача А.

$$x^2 \frac{\partial^2}{\partial x^2} u(x, y) + y^2 \frac{\partial^2}{\partial y^2} u(x, y) = 0.$$

$$x, y > 0$$

Задача В.

$$-(2x + \ln y) \frac{\partial^2}{\partial x^2} u(x, y) + 2(2xy + y \ln y) \frac{\partial^2}{\partial x \partial y} u(x, y) - \frac{\partial}{\partial x} u(x, y) + 2y \frac{\partial}{\partial y} u(x, y) = 0.$$

$$y > 1$$

$$\text{Начальные данные: } u|_{x=0} = y\sqrt{\ln y}, \quad \frac{\partial}{\partial x} u|_{x=0} = y\sqrt{\ln y},$$

Вариант 2.

Задача А.

$$x \frac{\partial^2}{\partial x^2} u(x, y) + x y \frac{\partial^2}{\partial x \partial y} u(x, y) + y u(x, y) = 0$$

Задача В.

$$x \frac{\partial^2}{\partial x^2} u(x, y) + 2(x - \sqrt{x}) \frac{\partial^2}{\partial x \partial y} u(x, y) + (x - 2\sqrt{x} + 1) \frac{\partial^2}{\partial y^2} u(x, y) + \frac{1}{2} \frac{\partial}{\partial x} u(x, y) + \frac{1}{2} \frac{\partial}{\partial y} u(x, y) = 0.$$

$$x > 0$$

$$\text{Начальные данные: } u|_{x=9} = 1, \quad \frac{\partial}{\partial x} u|_{x=9} = y^2,$$

Вариант 3.

Задача А.

$$y \frac{\partial^2}{\partial x^2} u(x, y) + x y \frac{\partial^2}{\partial x \partial y} u(x, y) + x u(x, y) = 0.$$

Задача В.

$$2 \frac{\partial^2}{\partial x^2} u(x, y) + 4\sqrt{-y} \frac{\partial^2}{\partial x \partial y} u(x, y) - 2y \frac{\partial^2}{\partial y^2} u(x, y) - \frac{\partial}{\partial y} u(x, y) = 0.$$

$$y < 0,$$

$$\text{Начальные данные: } u|_{y=-1} = x^3; \quad \frac{\partial}{\partial y} u|_{y=-1} = 3x$$

Вариант 4.

Задача А.

$$\frac{\partial^2}{\partial x^2} u(x, y) + 2 \frac{y \frac{\partial^2}{\partial x \partial y} u(x, y)}{x} + \frac{(1 + y^2) y^2 \frac{\partial^2}{\partial y^2} u(x, y)}{x^2} + \frac{y^3 \frac{\partial}{\partial y} u(x, y)}{x^2} = 0$$

Задача В.

$$\frac{\partial^2}{\partial x \partial y} u(x, y) + x \frac{\partial^2}{\partial y^2} u(x, y) = 0.$$

$$x < 0, y > 0,$$

$$\text{Начальные данные: } u|_{y=2} = x^2; \frac{\partial}{\partial y} u|_{y=2} = e^x$$

Вариант 5.

Задача А.

$$\frac{\partial^2}{\partial x^2} u(x, y) - 2 \frac{y \frac{\partial^2}{\partial x \partial y} u(x, y)}{x} + \frac{y^3 \frac{\partial}{\partial y} u(x, y)}{x^2} = 0.$$

Задача В.

$$-x^2 \frac{\partial^2}{\partial x^2} u(x, y) + 2xy \frac{\partial^2}{\partial x \partial y} u(x, y) - y^2 \frac{\partial^2}{\partial y^2} u(x, y) - x \frac{\partial}{\partial x} u(x, y) - y \frac{\partial}{\partial y} u(x, y) = 0.$$

$$x > 0, y > 0$$

$$\text{Начальные данные: } u|_{x=1} = y, \frac{\partial}{\partial x} u|_{x=1} = 1,$$

Вариант 6.

Задача А.

$$(x + y) \frac{\partial^2}{\partial x^2} u(x, y) + \frac{\partial^2}{\partial x \partial y} u(x, y) + (x - y) \frac{\partial}{\partial y} u(x, y) = 0.$$

Задача В.

$$(1 + x^2) \frac{\partial^2}{\partial x^2} u(x, y) - 2x\sqrt{1 + x^2} \frac{\partial^2}{\partial x \partial y} u(x, y) + x^2 \frac{\partial^2}{\partial y^2} u(x, y) - \frac{1}{x} \frac{\partial}{\partial x} u(x, y) = 0.$$

$$x > 1$$

$$\text{Начальные данные: } u|_{y=0} = x^3; \frac{\partial}{\partial y} u|_{y=0} = x^2,$$

Вариант 7.

Задача А.

$$(x-y) \frac{\partial^2}{\partial x^2} u(x,y) + \frac{\partial^2}{\partial x \partial y} u(x,y) + (x+y) \frac{\partial}{\partial y} u(x,y) = 0$$

Задача В.

$$y^2 \frac{\partial^2}{\partial x^2} u(x,y) - 2y \frac{\partial^2}{\partial x \partial y} u(x,y) + \frac{\partial^2}{\partial y^2} u(x,y) - \frac{\partial}{\partial x} u(x,y) = 0.$$

$$y > 0$$

Начальные данные: $u|_{y=1} = x$; $\frac{\partial}{\partial y} u|_{y=1} = x$,

Вариант 8.

Задача А.

$$\frac{x \frac{\partial^2}{\partial x^2} u(x,y)}{y} + \frac{y \frac{\partial^2}{\partial x \partial y} u(x,y)}{x} + \frac{\partial}{\partial x} u(x,y) = 0.$$

Задача В.

$$2x \frac{\partial^2}{\partial x^2} u(x,y) + 4x\sqrt{-x} \frac{\partial^2}{\partial x \partial y} u(x,y) - 2x^2 \frac{\partial^2}{\partial y^2} u(x,y) - \frac{\partial}{\partial x} u(x,y) = 0.$$

$$x < 0$$

Начальные данные: $u|_{x=-1} = y^2$, $\frac{\partial}{\partial x} u|_{x=-1} = y$,

Вариант 9.

Задача А.

$$\frac{x \frac{\partial^2}{\partial x^2} u(x,y)}{y} + 2 \frac{\partial^2}{\partial x \partial y} u(x,y) + \frac{y \frac{\partial^2}{\partial y^2} u(x,y)}{x} + \frac{\partial}{\partial y} u(x,y) = 0.$$

Задача В.

$$xy \frac{\partial^2}{\partial x^2} u(x,y) - 2x^2 y \frac{\partial^2}{\partial x \partial y} u(x,y) - (x^2 + y) \frac{\partial}{\partial x} u(x,y) = 0.$$

$$x > 0, y > 0$$

Начальные данные: $u|_{x=1} = y^2$, $\frac{\partial}{\partial x} u|_{x=1} = 5y$,

Вариант 10.**Задача А.**

$$\frac{x}{y} \frac{\partial^2 u}{\partial x^2}(x, y) + 2 \frac{\partial^2 u}{\partial x \partial y}(x, y) + \frac{y}{x} \frac{\partial^2 u}{\partial y^2}(x, y) + (1+x) x^2 \frac{\partial u}{\partial y}(x, y) + u(x, y) = 0.$$

Задача В.

$$x^2 \frac{\partial^2 u}{\partial x^2}(x, y) - xy \frac{\partial^2 u}{\partial x \partial y}(x, y) + \frac{x}{2} \frac{\partial u}{\partial x}(x, y) + \frac{y}{2} \frac{\partial u}{\partial y}(x, y) = 0.$$

$$y > 0,$$

$$\text{Начальные данные: } u|_{x=1} = y\sqrt{y}, \frac{\partial u}{\partial x}|_{x=1} = y^2,$$

Вариант 11.**Задача А.**

$$(1+x^2)^2 \frac{\partial^2 u}{\partial x^2}(x, y) + \frac{\partial^2 u}{\partial y^2}(x, y) + 3y^2 x \frac{\partial u}{\partial y}(x, y) = 0$$

Задача В.

$$y \frac{\partial^2 u}{\partial x^2}(x, y) + 2\sqrt{-xy} \frac{\partial^2 u}{\partial x \partial y}(x, y) - x \frac{\partial^2 u}{\partial y^2}(x, y) + \frac{x}{2y} \frac{\partial u}{\partial y}(x, y) - \frac{y}{2x} \frac{\partial u}{\partial x}(x, y) = 0.$$

$$x < 0, y > 0,$$

$$\text{Начальные данные: } u|_{x=-4} = y, \frac{\partial u}{\partial x}|_{x=-4} = 1,$$

Вариант 12.**Задача А.**

$$x^2 \frac{\partial^2 u}{\partial x^2}(x, y) - y^2 \frac{\partial^2 u}{\partial y^2}(x, y) + e^{xy} \frac{\partial u}{\partial y}(x, y) = 0.$$

Задача В.

$$y \frac{\partial^2 u}{\partial x^2}(x, y) + 2x\sqrt{-y} \frac{\partial^2 u}{\partial x \partial y}(x, y) - x^2 \frac{\partial^2 u}{\partial y^2}(x, y) - \frac{y}{x} \frac{\partial u}{\partial x}(x, y) + \frac{x^2}{2y} \frac{\partial u}{\partial y}(x, y) = 0.$$

$$x > 0, y < 0,$$

$$\text{Начальные данные: } u|_{y=-1} = x^2, \frac{\partial u}{\partial y}|_{y=-1} = x,$$

Вариант 13.**Задача А.**

$$\frac{\partial^2}{\partial x^2} u(x, y) - 2 \sin x \frac{\partial^2}{\partial x \partial y} u(x, y) - \cos^2 x \frac{\partial^2}{\partial y^2} u(x, y) - \cos x \frac{\partial}{\partial y} u(x, y) = 0$$

Задача В.

$$x \frac{\partial^2}{\partial x^2} u(x, y) + 2y\sqrt{-x} \frac{\partial^2}{\partial x \partial y} u(x, y) - y^2 \frac{\partial^2}{\partial y^2} u(x, y) + \frac{1}{2} \frac{\partial}{\partial x} u(x, y) - y \frac{\partial}{\partial y} u(x, y) = 0.$$

$$x < 0, y > 0,$$

$$\text{Начальные данные: } u|_{y=1} = x, \frac{\partial}{\partial y} u|_{y=1} = x^2,$$

Вариант 14.**Задача А.**

$$y \frac{\partial^2}{\partial x^2} u(x, y) - xy \frac{\partial^2}{\partial x \partial y} u(x, y) + y \frac{\partial}{\partial y} u(x, y) = 0.$$

Задача В.

$$x^2 \frac{\partial^2}{\partial x^2} u(x, y) - 2x \frac{\partial^2}{\partial x \partial y} u(x, y) + \frac{\partial^2}{\partial y^2} u(x, y) + 2 \frac{\partial}{\partial x} u(x, y) + \left(1 - \frac{2}{x}\right) \frac{\partial}{\partial y} u(x, y) = 0.$$

$$x > 0$$

$$\text{Начальные данные: } u|_{y=0} = \ln x, \frac{\partial}{\partial y} u|_{y=0} = 1,$$

Вариант 15.**Задача А.**

$$y^2 \frac{\partial^2}{\partial x^2} u(x, y) + x^2 \frac{\partial^2}{\partial y^2} u(x, y) + (1 - x^2) \frac{\partial}{\partial y} u(x, y) = 0$$

Задача В.

$$x(\sqrt{-y} + \sqrt{x}) \frac{\partial^2}{\partial x^2} u(x, y) + y(\sqrt{x} + \sqrt{-y}) \frac{\partial^2}{\partial y^2} u(x, y) + \frac{\sqrt{-y}}{2} \frac{\partial}{\partial x} u(x, y) + \frac{\sqrt{x}}{2} \frac{\partial}{\partial y} u(x, y) = 0.$$

$$x > 0, y < 0,$$

$$\text{Начальные данные: } u|_{y=-1} = \frac{2}{3} x\sqrt{x} + x, \frac{\partial}{\partial y} u|_{y=-1} = \sqrt{x} + 1,$$

Вариант 16.**Задача А.**

$$\frac{\partial^2}{\partial x^2} u(x, y) - 2 \cos x \frac{\partial^2}{\partial x \partial y} u(x, y) - \sin^2 x \frac{\partial^2}{\partial y^2} u(x, y) - \cos x \frac{\partial}{\partial y} u(x, y) = 0.$$

Задача В.

$$x \frac{\partial^2}{\partial x^2} u(x, y) - 4x^2 \frac{\partial^2}{\partial x \partial y} u(x, y) + 4x^3 \frac{\partial^2}{\partial y^2} u(x, y) - \frac{\partial}{\partial x} u(x, y) = 0.$$

$$x > 0$$

Начальные данные: $u|_{y=0} = e^x$, $\frac{\partial}{\partial y} u|_{y=0} = x^2$,

Вариант 17.**Задача А.**

$$\frac{\frac{\partial^2}{\partial x^2} u(x, y)}{\cos x} - 2 \tan x \frac{\partial^2}{\partial x \partial y} u(x, y) - \cos x \frac{\partial^2}{\partial y^2} u(x, y) - \frac{\partial}{\partial x} u(x, y) = 0.$$

Задача В.

$$x^2 \frac{\partial^2}{\partial x^2} u(x, y) - 2xy \frac{\partial^2}{\partial x \partial y} u(x, y) + y^2 \frac{\partial^2}{\partial y^2} u(x, y) + 3x \frac{\partial}{\partial x} u(x, y) - y \frac{\partial}{\partial y} u(x, y) = 0.$$

$$x > 0, y > 0$$

Начальные данные: $u|_{x=1} = y^3$, $\frac{\partial}{\partial x} u|_{x=1} = y$,

Вариант 18.**Задача А.**

$$|x| \frac{\partial^2}{\partial x^2} u(x, y) - y \frac{\partial^2}{\partial x \partial y} u(x, y) - \frac{\partial}{\partial x} u(x, y) = 0,$$

$$x < 0.$$

Задача В.

$$y^3 \frac{\partial^2}{\partial x^2} u(x, y) - 2y^2 \frac{\partial^2}{\partial x \partial y} u(x, y) + y \frac{\partial^2}{\partial y^2} u(x, y) - (y+2) \frac{\partial}{\partial x} u(x, y) + \frac{2}{y} \frac{\partial}{\partial y} u(x, y) = 0.$$

$$y > 0,$$

Начальные данные: $u|_{x=0} = y$, $\frac{\partial}{\partial x} u|_{x=0} = \frac{1}{y}$,

Вариант 19.**Задача А.**

$$|x| \frac{\partial^2}{\partial x^2} u(x, y) - y \frac{\partial^2}{\partial x \partial y} u(x, y) - \frac{\partial}{\partial x} u(x, y) + |y - 1| u(x, y) = 0,$$

$$x < 0, y < 1.$$

Задача В.

$$x^2 \frac{\partial^2}{\partial x^2} u(x, y) - 2xy \frac{\partial^2}{\partial x \partial y} u(x, y) + y^2 \frac{\partial^2}{\partial y^2} u(x, y) - 7x \frac{\partial}{\partial x} u(x, y) + 9y \frac{\partial}{\partial y} u(x, y) = 0.$$

$$x > 0, y > 0,$$

$$\text{Начальные данные: } u|_{y=1} = x^2, \frac{\partial}{\partial y} u|_{y=1} = x^2,$$

Вариант 20.**Задача А.**

$$(1 + x^2) \frac{\partial^2}{\partial x^2} u(x, y) + 2x \frac{\partial^2}{\partial x \partial y} u(x, y) + |y| \frac{\partial}{\partial x} u(x, y) = 0,$$

$$y > 0.$$

Задача В.

$$x^2 \frac{\partial^2}{\partial x^2} u(x, y) - 2x \frac{\partial^2}{\partial x \partial y} u(x, y) + \frac{\partial^2}{\partial y^2} u(x, y) + \frac{\partial}{\partial y} u(x, y) = 0.$$

$$x > 0$$

$$\text{Начальные данные: } u|_{y=0} = x \ln x - x, \frac{\partial}{\partial y} u|_{y=0} = x^2,$$

Вариант 21.**Задача А.**

$$\frac{\partial^2}{\partial x^2} u(x, y) + 2y \frac{\partial^2}{\partial x \partial y} u(x, y) + |x| \frac{\partial}{\partial y} u(x, y) + e^x u(x, y) = 0,$$

$$x < 0.$$

Задача В.

$$\frac{\partial^2}{\partial x^2} u(x, y) + 2x \frac{\partial^2}{\partial x \partial y} u(x, y) + x^2 \frac{\partial^2}{\partial y^2} u(x, y) + \frac{1}{x} \frac{\partial}{\partial x} u(x, y) + 2 \frac{\partial}{\partial y} u(x, y) = 0.$$

$$\text{Начальные данные: } u|_{y=0} = x^4, \frac{\partial}{\partial y} u|_{y=0} = x^2,$$

Вариант 22.**Задача А.**

$$\frac{\frac{\partial^2}{\partial x^2} u(x, y)}{y} + \frac{\partial^2}{\partial x \partial y} u(x, y) + |x| y \frac{\partial}{\partial x} u(x, y) = 0,$$

$$x < 0.$$

Задача В.

$$y^2 \frac{\partial^2}{\partial x^2} u(x, y) - 2y \frac{\partial^2}{\partial x \partial y} u(x, y) + \frac{\partial^2}{\partial y^2} u(x, y) - 2 \frac{\partial}{\partial x} u(x, y) + \frac{1}{y} \frac{\partial}{\partial y} u(x, y) = 0.$$

$$y > 0,$$

$$\text{Начальные данные: } u|_{x=0} = y^2, \frac{\partial}{\partial x} u|_{x=0} = y^2,$$

Вариант 23.**Задача А.**

$$x^2 \frac{\partial^2}{\partial x^2} u(x, y) + 2xy \frac{\partial^2}{\partial x \partial y} u(x, y) + y^2 \frac{\partial^2}{\partial y^2} u(x, y) + |x| y \frac{\partial}{\partial y} u(x, y) = 0,$$

$$x < 0.$$

Задача В.

$$\frac{\partial^2}{\partial x^2} u(x, y) - 2y \frac{\partial^2}{\partial x \partial y} u(x, y) = 0.$$

$$\text{Начальные данные: } u|_{x=0} = y, \frac{\partial}{\partial x} u|_{x=0} = ye^y,$$

Вариант 24.**Задача А.**

$$\frac{\partial^2}{\partial x^2} u(x, y) + 2xy \frac{\partial^2}{\partial x \partial y} u(x, y) + x^2 u(x, y) = 0.$$

Задача В.

$$x^2 \frac{\partial^2}{\partial x^2} u(x, y) - 2xy \frac{\partial^2}{\partial x \partial y} u(x, y) + y^2 \frac{\partial^2}{\partial y^2} u(x, y) + 2 \left(x - \frac{x}{y} \right) \frac{\partial}{\partial x} u(x, y) + 2 \frac{\partial}{\partial y} u(x, y) = 0.$$

$$\text{Начальные данные: } u|_{x=1} = e^y, \frac{\partial}{\partial x} u|_{x=1} = ye^y,$$

Вариант 25.**Задача А.**

$$(x - 2y) \frac{\partial^2}{\partial x^2} u(x, y) + \frac{\partial^2}{\partial x \partial y} u(x, y) + x \frac{\partial}{\partial x} u(x, y) = 0.$$

Задача В.

$$x \frac{\partial^2}{\partial x^2} u(x, y) + 2\sqrt{-xy} \frac{\partial^2}{\partial x \partial y} u(x, y) - y \frac{\partial^2}{\partial y^2} u(x, y) + \frac{1}{2} \frac{\partial}{\partial x} u(x, y) - \frac{1}{2} \frac{\partial}{\partial y} u(x, y) = 0.$$

$$x > 0, y < 0,$$

$$\text{Начальные данные: } u|_{x=4} = y, \frac{\partial}{\partial x} u|_{x=4} = 1,$$

Вариант 26.**Задача В.**

$$x^2 \frac{\partial^2}{\partial x^2} u(x, y) + 2xy \frac{\partial^2}{\partial x \partial y} u(x, y) - 3y^2 \frac{\partial^2}{\partial y^2} u(x, y) + 3x \frac{\partial}{\partial x} u(x, y) + 3y \frac{\partial}{\partial y} u(x, y) = 0.$$

$$x > 0, y > 0,$$

$$\text{Начальные данные: } u|_{y=1} = x^2, \frac{\partial}{\partial y} u|_{y=1} = x,$$

Вариант 27.**Задача В.**

$$2 \frac{\partial^2}{\partial x \partial y} u(x, y) + 2x \frac{\partial^2}{\partial y^2} u(x, y) + \frac{1}{x} \frac{\partial}{\partial y} u(x, y) = 0.$$

$$x > 0$$

$$\text{Начальные данные: } u|_{y=0} = x^2, \frac{\partial}{\partial y} u|_{y=0} = -2,$$

Вариант 28.**Задача В.**

$$(4\sqrt{-y} + 2x) \frac{\partial^2}{\partial x^2} u(x, y) + 2(xy + 2y\sqrt{-y}) \frac{\partial^2}{\partial y^2} u(x, y) - 2 \frac{\partial}{\partial x} u(x, y) + x \frac{\partial}{\partial y} u(x, y) = 0.$$

$$y < 0$$

$$\text{Начальные данные: } u|_{x=0} = (-y)^{5/4}, \frac{\partial}{\partial x} u|_{x=0} = (-y)^{1/4},$$

Вариант 29.**Задача В.**

$$x \frac{\partial^2}{\partial x^2} u(x, y) - y \frac{\partial^2}{\partial x \partial y} u(x, y) + \frac{1}{2} \frac{\partial}{\partial x} u(x, y) = 0.$$

$$\text{Начальные данные: } u|_{x=1} = y^2, \frac{\partial}{\partial x} u|_{x=1} = 2y^2,$$

Вариант 30.

Задача В.

$$x^2 \frac{\partial^2}{\partial x^2} u(x, y) + 2xy \frac{\partial^2}{\partial x \partial y} u(x, y) - 3y^2 \frac{\partial^2}{\partial y^2} u(x, y) - x \frac{\partial}{\partial x} u(x, y) - y \frac{\partial}{\partial y} u(x, y) = 0.$$

$$x > 0, y > 0,$$

$$\text{Начальные данные: } u|_{x=1} = \ln y, \frac{\partial}{\partial x} u|_{x=1} = \frac{1}{y},$$