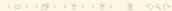
### An introduction to the $\pi$ -calculus

Michele Finelli m@biodec.com BioDec





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Outline of the talk

First the  $\lambda$  ...

 $\dots$ then the  $\pi$ 

**Conclusions** 

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Conclusions

# **Objectives**

- Give an informal idea of what is a calculus for concurrency, what it looks like, and why it is useful
- Support the idea that in twenty years (perhaps less) we will have a PiCon conference dedicated to concurrency languages, as we will have tomorrow (28 March 2015) a LambdaCon dedicated to functional languages.

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First the  $\lambda$  ...

# What is the $\lambda$ -calculus ?

It is a formal system in mathematical logic for expressing computation based on function abstraction and application using variable binding and substitution. (Wikipedia)

- It was born (Alonzo Church) to study the definability of functions.
- It was proved that it is Turing complete, so it became a model of computation (like *Turing Machines* etcetera).
- It entered computer science in the sixties and it is now fruitfully used alongside type theory to provide ideas and test beds for the theory of programming languages.

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#### Remember:

It is a formal system in mathematical logic for expressing computation based on function abstraction and application using variable binding and substitution. (Wikipedia)

Variables A variable x is a valid  $\lambda$ -term.

Abstraction If t is a  $\lambda$ -term, and x is a variable, then  $(\lambda x.t)$  is a  $\lambda$ -term. Here x is the *bound* variable.

Application If t and s are  $\lambda$ -terms then (ts) is a  $\lambda$ -term. If all the variables are bound, the term is closed. Usually  $\lambda$ -theories refer only to closed terms.

 $\Lambda ::= x | \lambda x. \Lambda | \Lambda \Lambda$  for x in some set of variable names

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The *dynamics* of the calculus is captured by three reductions (*i.e.* relations) among terms.

Alpha two terms are  $\alpha$  convertible if their differ in the names of their bound variables:  $\lambda x \lambda y.x \xrightarrow{\alpha} \lambda t \lambda x.t$ 

Beta  $(\lambda x.M)N \xrightarrow{\beta} M[N/x]$  Informally: M[N/x] means substituting N for x, in all of its occurrences in M. The definition can be made rigorous.

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- ▶ The identity  $I = \lambda x.x$  is a term, which is already normal.
- ▶ Let T be the term  $(\lambda x.\lambda y.xy)I$ . Which is its normal form (if any ?)  $(\lambda x.\lambda y.xy)I \xrightarrow{\beta} \lambda y.Iy \xrightarrow{\beta} \lambda y.y = I$
- ▶ What about the term  $\Omega = (\lambda x.xx)\lambda x.xx$  ? It reduces to itself ... but  $\Omega$  is not in normal form, since there is a redex that can be fired ... and on and on and on ...

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### Examples

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- There are functions that cannot be expressed in the λ-calculus, the typical example is the parallel-or (POR):
  - 1. POR(x,1) = 1,
  - 2. POR(1,x) = 1,
  - 3. POR(0,0) = 0.
- If we give to POR two functions f and g that may either return 0 or 1, or that may diverge . . .
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# Proof (informal)

- ▶ If POR could be expressed in the  $\lambda$ -calculus there should be a context  $C[\cdot, \cdot]$  such that that  $C[\Omega, \Omega]$  has no normal form, but  $C[\Omega, I]$  and  $C[I, \Omega]$  have normal forms, where  $\Delta = \lambda x.xx$ ,  $\Omega = \Delta \Delta$  and  $I = \lambda x.x$ .
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#### So, we have three alternatives:

- 1. Either the normal reduction of C[M, N] ignores M and N then  $C[\Omega, \Omega]$  has a normal form we do not want that.
- 2. Or the normal reduction starts with M, but this means that  $C[\Omega, I]$  cannot have a normal form.
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Conclusions

- ▶ To express parallelism we need something different from the  $\lambda$ -calculus. We need *concurrency*.
- ► Concurrency ≠ Parallelism.
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#### Names and actions

Names There exists an infinite set  $\mathcal{N}$  of *names*. Lower letters  $x, y \dots$  range over  $\mathcal{N}$ .

Actions an *action*  $\pi$  is one of the following:

- 1. x(y) receive y along x,
- 2.  $\bar{x}\langle y\rangle$  send y along x,
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The set  $P^{\pi}$  of  $\pi$ -calculus process expressions is defined by the following syntax:

$$P ::= \sum_{i \in I} \pi_i . P_i \mid P_1 \mid P_2 \mid (va)P \mid !P$$

where I is any finite indexing set.

Sums The processes  $\sum_{i \in I} \pi_i P_i$  are called *summations* or *sums*.

Parallel The operation  $\cdot \mid \cdot$  is the *parallel composition*.

Restriction The process (va)P has restricted the usage of a inside P.

Replication The! (bang) operator is the replicator operator.

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- ► And rules for  $\alpha$ -conversion, rearrangement of terms etcetera.

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▶ The basic idea is that x(y).P and  $\bar{x}\langle z\rangle$  react in a way similar to  $\beta$ -reduction:

$$x(y).P \mid \bar{x}\langle z \rangle.Q \longrightarrow \{z/y\}P \mid Q$$

- ► There is a precise notion of substitution, to avoid name capture.
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**Conclusions** 

- There are many calculi for concurrency, and concurrency is way less understood that sequentiality.
- Research started in the sixties (CSS, a precursor of the π-calculus (Milner), CSP (Hoare), the actor model (Hewitt), etcetera).
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- ▶ Research started in the sixties (CSS, a precursor of the  $\pi$ -calculus (Milner), CSP (Hoare), the actor model (Hewitt), etcetera).
- ► The links with logical theories are less understood (*i.e.* there is no Curry-Howard Isomorphism, nor a good type system).

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### Robin Milner, 13 January 1934 – 20 March 2010



Picture taken at CONCUR 09, the 20th International Conference on Concurrency Theory, in Bologna, 4 September 2009.

#### Questions

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Thanks for paying attention.
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See you tomorrow at ACON or ....

The biggest Italian DevOps meeting is in Bologna, April 10<sup>th</sup>, 2015.

IDI2015 Incontro DevOps Italia 2015:

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