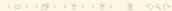
An introduction to the π -calculus

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First the λ ...

 \dots then the π

Conclusions

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First the λ ...

...then the π

Conclusions

Objectives

- Give an informal idea of what is a calculus for concurrency, what it looks like, and why it is useful
- Support the idea that in twenty years (perhaps less) we will have a PiCon conference dedicated to concurrency languages, as we will have tomorrow (28 March 2015) a LambdaCon dedicated to functional languages.

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First the λ ...

What is the λ -calculus ?

It is a formal system in mathematical logic for expressing computation based on function abstraction and application using variable binding and substitution. (Wikipedia)

- It was born (Alonzo Church) to study the definability of functions.
- It was proved that it is Turing complete, so it became a model of computation (like *Turing Machines* etcetera).
- It entered computer science in the sixties and it is now fruitfully used alongside type theory to provide ideas and test beds for the theory of programming languages.

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Remember:

It is a formal system in mathematical logic for expressing computation based on function abstraction and application using variable binding and substitution. (Wikipedia)

Variables A variable x is a valid λ -term.

Abstraction If t is a λ -term, and x is a variable, then $(\lambda x.t)$ is a λ -term. Here x is the *bound* variable.

Application If t and s are λ -terms then (ts) is a λ -term. If all the variables are bound, the term is closed. Usually λ -theories refer only to closed terms.

 $\Lambda ::= x | \lambda x. \Lambda | \Lambda \Lambda$ for x in some set of variable names

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The *dynamics* of the calculus is captured by three reductions (*i.e.* relations) among terms.

Alpha two terms are α convertible if their differ in the names of their bound variables: $\lambda x \lambda y.x \xrightarrow{\alpha} \lambda t \lambda x.t$

Beta $(\lambda x.M)N \xrightarrow{\beta} M[N/x]$ Informally: M[N/x] means substituting N for x, in all of its occurrences in M. The definition can be made rigorous.

Eta If x is not free in f then $\lambda x.(fx) \xrightarrow{\eta} i$

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- If a term M has a chain of β reductions to a term N than cannot be reduced anymore, N is said to be the normal form of M.
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- ▶ The identity $I = \lambda x.x$ is a term, which is already normal.
- ▶ Let T be the term $(\lambda x.\lambda y.xy)I$. Which is its normal form (if any ?) $(\lambda x.\lambda y.xy)I \xrightarrow{\beta} \lambda y.Iy \xrightarrow{\beta} \lambda y.y = I$
- ▶ What about the term $\Omega = (\lambda x.xx)\lambda x.xx$? It reduces to itself ... but Ω is not in normal form, since there is a redex that can be fired ... and on and on and on ...

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Examples

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- There are functions that cannot be expressed in the λ-calculus, the typical example is the parallel-or (POR):
 - 1. POR(x,1) = 1,
 - 2. POR(1,x) = 1,
 - 3. POR(0,0) = 0.
- If we give to POR two functions f and g that may either return 0 or 1, or that may diverge . . .
- ...what should we do? Should we compute first f or g?
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Proof (informal)

- ▶ If POR could be expressed in the λ -calculus there should be a context $C[\cdot, \cdot]$ such that that $C[\Omega, \Omega]$ has no normal form, but $C[\Omega, I]$ and $C[I, \Omega]$ have normal forms, where $\Delta = \lambda x.xx$, $\Omega = \Delta \Delta$ and $I = \lambda x.x$.
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So, we have three alternatives:

- 1. Either the normal reduction of C[M, N] ignores M and N then $C[\Omega, \Omega]$ has a normal form we do not want that.
- 2. Or the normal reduction starts with M, but this means that $C[\Omega, I]$ cannot have a normal form.
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- ► Concurrency ≠ Parallelism.
- ▶ In an informal sense a system that manages concurrency also manages parallelism, the converse is not true.

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Names and actions

Names There exists an infinite set \mathcal{N} of *names*. Lower letters $x, y \dots$ range over \mathcal{N} .

Actions an *action* π is one of the following:

- 1. x(y) receive y along x,
- 2. $\bar{x}\langle y\rangle$ send y along x,
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The set P^{π} of π -calculus process expressions is defined by the following syntax:

$$P ::= \sum_{i \in I} \pi_i . P_i \mid P_1 \mid P_2 \mid (va)P \mid !P$$

where I is any finite indexing set.

Sums The processes $\sum_{i \in I} \pi_i P_i$ are called *summations* or *sums*.

Parallel The operation $\cdot \mid \cdot$ is the *parallel composition*.

Restriction The process (va)P has restricted the usage of a inside P.

Replication The! (bang) operator is the replicator operator.

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- ► And rules for α -conversion, rearrangement of terms etcetera.

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- ► And rules for α -conversion, rearrangement of terms, etcetera.

▶ The basic idea is that x(y).P and $\bar{x}\langle z\rangle$ react in a way similar to β -reduction:

$$x(y).P \mid \bar{x}\langle z \rangle.Q \longrightarrow \{z/y\}P \mid Q$$

- ► There is a precise notion of substitution, to avoid name capture.
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- Research started in the sixties (CCS, a precursor of the π-calculus (Milner), CSP (Hoare), the actor model (Hewitt), etcetera).
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Robin Milner, 13 January 1934 – 20 March 2010



Picture taken at CONCUR 09, the 20th International Conference on Concurrency Theory, in Bologna, 4 September 2009.

Questions

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IDI2015 Incontro DevOps Italia 2015:
http://incontrodevops.it/idi2015/
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Thanks for paying attention.
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See you tomorrow at Λ CON or ...

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