

Software engineering during the Italian Renaissance

And some other stuff. It will not be a rant, I promise.

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BioDec

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Some mathematics

The Renaissance

Not enough mathematics

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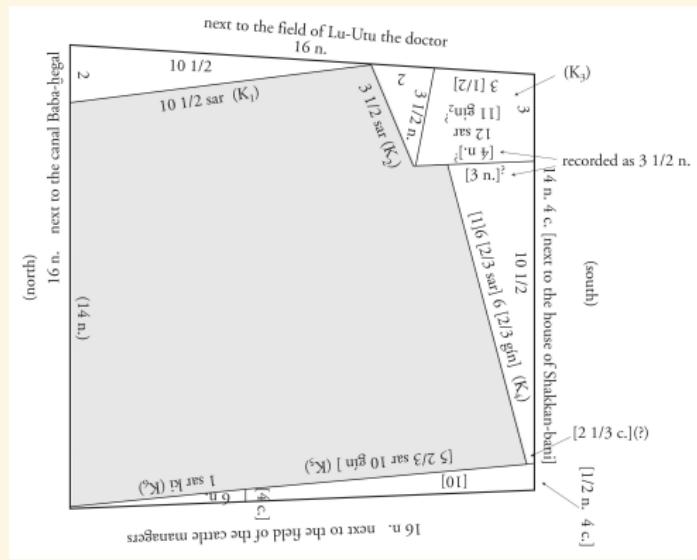
The Renaissance

Not enough mathematics

Once upon a time in Babylonia . . .

... they had problems like measuring the size of fields, after a flood. As a consequence, they developed geometry (in Egypt too).





“A Geometric Algorithm with Solutions to Quadratic Equations in a Sumerian Juridical Document from Ur III Umma” by Jörn Friberg, Cuneiform Digital Library Journal 2009:3, ISSN 1540-8779. 

The quadratic equation

- ▶ To solve their problems they asked for solution to equations, in particular to instances of the **quadratic equation**.
- ▶ You know, the stuff you studied in high school.
- ▶ Like that:

$$ax^2 + bx + c = 0$$

with $a \neq 0$.

- ▶ The curve $y = ax^2 + bx + c$ is a **parabola** with the axis of symmetry parallel to the ordinate axis.

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But . . .

That formula is not what was used in Babylonia, since they had trouble acknowledging the existence of **negative numbers**.

Instead . . .

They distinguished three **different** cases, where $b, c \geq 0$:

1. $x^2 + bx = c$
2. $x^2 + c = bx$
3. $x^2 = bx + c$

(notice also that they put $a = 1$ and divided the other coefficients accordingly).

So ...

In a sense, they had **different algorithms** because of the different representations of the **same** equation.

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Fast forward few millennia

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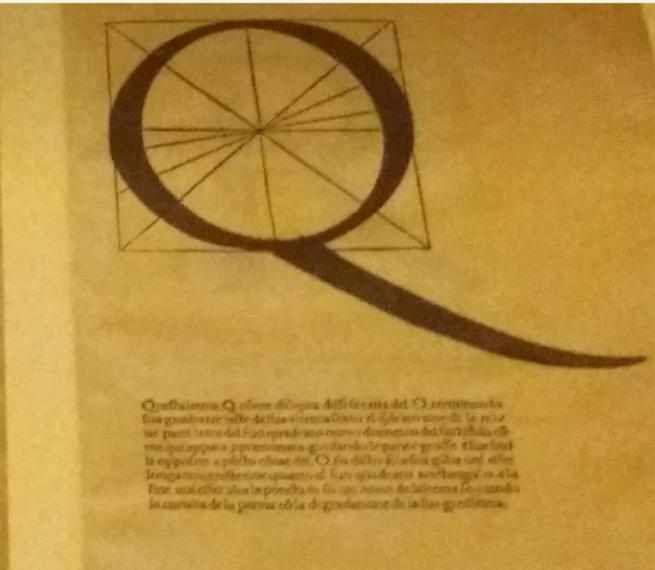
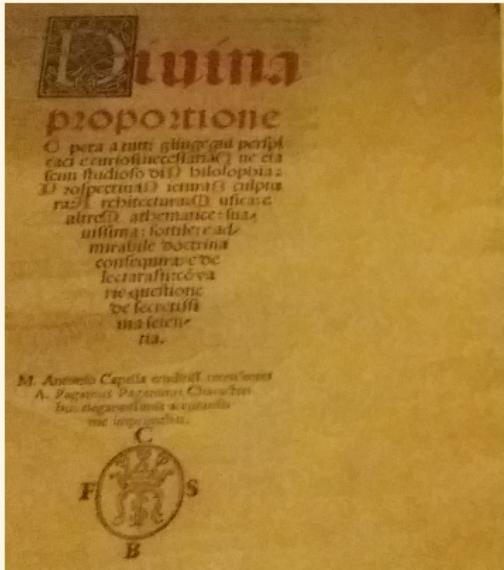
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Luca Pacioli

- ▶ But even great mathematicians make mistakes.
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The cubic equation

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The cubic function

This is the plot of a cubic function with three real roots:



Some Python code !

And since we are at a Plone conference, this is the Wikipedia Python code to plot the function:

```
# Author: Ika, 2013-07-24
from pylab import *
import pylab as pl
import numpy as np
pl.figure(figsize=(8,8), dpi=80)
# Create a plot of the cubic function y=(x+4)(x+1)(x-2)/4
x = np.linspace(-5.0,4.0,256, endpoint=True)
y = (x+4)*(x+1)*(x-2)/4
pl.plot(x,y,color="magenta", linewidth=3.0, linestyle="--")
# Set labels
pl.xlim(-5,4)
pl.ylim(-4,7)
# Move the spines
ax = pl.gca()
ax.set_xlabel('x')
ax.set_ylabel('y')
ax.text(-5, 3, r'$y=\frac{1}{4}(x+4)(x+1)(x-2)$', fontsize=20)
# Set up the grid
ax.spines['right'].set_color('none')
ax.spines['top'].set_color('none')
ax.xaxis.set_ticks_position('bottom')
ax.spines['bottom'].set_position(('data',0))
ax.yaxis.set_ticks_position('left')
ax.spines['left'].set_position(('data',0))
ax.grid(color="grey", linestyle='--', linewidth=1)
```

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- ▶ His wage increased six folds (25 lire in 1496, 150 lire after 1511) because of his mathematical skills.
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Solving an equation is software, isn't it ?

There should be no surprise that finding the values of $x_{1,2}$ in

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

given the values of a , b and c , requires the creation and the execution of a (simple) algorithm.

A software with strange rules

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Where:

- ▶ **finding the specification is extremely difficult** (any small change to the formula means that you get **wrong** results);
- ▶ building the algorithm is **obvious** (once you know some basic mathematics);
- ▶ executing the algorithm can be worked in your head in a few seconds, if you train a little (*i.e.* **running the software is not the hard part**).

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Back to the sixteenth century

- ▶ Enter Gerolamo Cardano (1501-1576), Niccolò Tartaglia (1499-1557) and Lodovico Ferrari (1522-1565).
- ▶ Antonio Maria Del Fiore learned that Tartaglia was able to solve a cubic equation of the form $x^3 + x^2 = r$ and not believing him, challenged Tartaglia to a "disfida matematica" (a *mathematical fight*) in the year 1535.
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- ▶ Ferrari insisted on having oral expositions, while Tartaglia preferred written ones. The last *disfida* was lost by Tartaglia in Milan, the 10th of August of 1548, against Ferrari, mainly because of his stuttering problems (since the “Tartaglia” nickname).

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The Dal Ferro-Tartaglia-Ferrari-Cardano formula

The formula for the cubic equation is the following:

$$x = \sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}$$

where the equation $ax^3 + bx^2 + cx + d = 0$ has been changed to the form

$$x^3 + px + q = 0$$

through the change $x \rightarrow x - b/3a$ (and a division by a).

The Dal Ferro-Tartaglia-Ferrari-Cardano formula

In the formula you first compute the cube root:

$$u = \sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}$$

and then choose the value of the other cube root:

$$v = \sqrt[3]{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}$$

for which the relation $uv = -p/3$ holds. Since a cubic root has three solutions over the complex field, you get three solutions for the original formula — the naïve application of the formula would end in *nine* values.

Strange phenomena

Take for example the equation $x^3 - 15x - 4 = 0$.

- Once you apply the formula you get

$$x = \sqrt[3]{2 + 11\sqrt{-1}} + \sqrt[3]{2 - 11\sqrt{-1}}$$

- Remember that at the time $\sqrt{-1}$ was literally garbage, a writing with no meaning at all, an obvious mistake.
- How could it be that the formula was right but you need other formulas to get the solutions $x_1 = 4$, $x_2 = \sqrt{3} - 2$ and $x_3 = -\sqrt{3} - 2$?

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- ▶ Can we solve equations of any power order (*i.e.* quintic equations, and more) ?
- ▶ If not, can we prove the impossibility result ?

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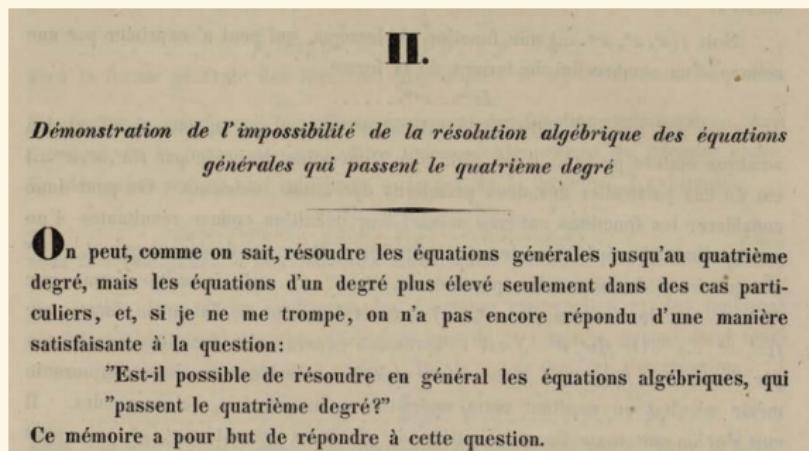
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- ▶ Can we solve equations of any power order (*i.e.* quintic equations, and more) ?
- ▶ If not, can we prove the impossibility result ?

Quintic equation

These, and other questions were definitely settled **centuries** later: Lagrange (1770), the conjecture by Gauss (1798) and finally the first (incomplete) proof by Ruffini (1799) and then Abel (1824 and 1826).



Why did it take so long ?

In my opinion, because they lacked the **abstractions** needed to move forward.

- ▶ Babylonians did not acknowledge the existence of **negative numbers**.
- ▶ Dal Ferro and company were comfortable with irrational numbers (like $\sqrt{3}$) but did not know how to deal with **imaginary numbers**.
- ▶ Gauss and Abel were well acquainted with complex numbers, but it required **Galois theory**, by Évariste Galois (1830), to develop a more general theory of algebraic equations.

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A theory of software

└ Not enough mathematics

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Some mathematics

The Renaissance

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Similarities

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Don't get me wrong

- ▶ My position is not that we should resort to the kind of mathematics that was used to solve algebraic equations.
- ▶ The history of the discovery of their solutions was a tale to tell that in science progress usually stalls until we are able to go **from one language to another**.
- ▶ In the story I told, the new languages needed **new symbols** ($-$, $\sqrt{\cdot}$, i) and a **new understanding**.
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The agile movement

- ▶ The <http://agilemanifesto.org/> (2001).
 1. *Individuals and Interactions* over processes and tools.
 2. *Working Software* over comprehensive documentation.
 3. *Customer Collaboration* over contract negotiation.
 4. *Responding to Change* over following a plan.
- ▶ It is a tale of improving the software development by tackling other issues (communication, organization, contracts, people).
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Unreasonably effective

COMMUNICATIONS ON PURE AND APPLIED MATHEMATICS. VOL. XIII. 001-14 (1960)

The Unreasonable Effectiveness of Mathematics in the Natural Sciences

Richard Courant Lecture in Mathematical Sciences delivered at New York University,
May 11, 1959

EUGENE P. WIGNER

Princeton University

"and it is probable that there is some secret here which remains to be discovered." (C. S. Peirce)

There is a story about two friends, who were classmates in high school, talking about their jobs. One of them became a statistician and was working on population trends. He showed a reprint to his former classmate. The reprint started, as usual, with the Gaussian distribution and the statistician explained to his former classmate the meaning of the symbols for the actual population, for the average population, and so on. His classmate was a bit incredulous and was not quite sure whether the statistician was pulling his leg. "How can you know that?" was his query. "And what is this symbol here?" "Oh," said the statistician, "this is π ." "What is that?" "The ratio of the circumference of the circle to its diameter." "Well, now you are pushing your joke too far," said the classmate, "surely the population has nothing to do with the circumference of the circle."

Not enough π ... so what ?

<insert the picture of
some random venture
capitalist from the
Silicon Valley>

Yes, we do not have enough π and other mathematical stuff, but we did a pretty decent job: we built the Internet ! We disrupted and created whole markets ! The world IS different because of OUR software !

Be a better citizen of the world

- ▶ Yes, software built a **different** world.
- ▶ It should also be a **better** world.
- ▶ **Safety**, for example, is often left behind: our personal data are not safe, our privacy is not guaranteed, our digital rights are more often than not waived for a mistaken sense of security.
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Build better software (for the world)

- ▶ We need to **better understand** the systems we contribute to build.
- ▶ We need to be better in guaranteeing that these systems **will be safe to use**, that they will not be misused.
- ▶ We need to use better tools to design and build our software, to do **less mistakes**.
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Yesterday !



Thanks

Email m@biodec.com

Useless social @gaunilone

Incontro DevOps Italia See you in Bologna, March 5-6, 2020.

<https://2020.incontrodevops.it/>

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