University of Toronto

Department of Electrical and Computer Engineering

ECE410F – Control Systems Fall 2016

Experiment 6

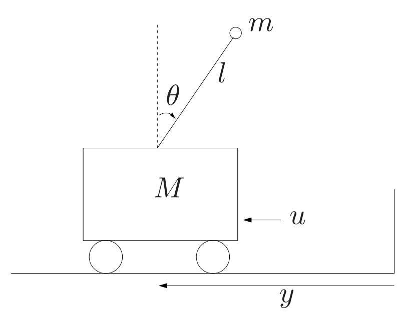
CONTROL OF AN INVERTED PENDULUM ON A CART, Part II

1 Purpose

The purpose of this experiment is to apply controller and observer design techniques to a physical "difficult-to-control" system, namely, the inverted pendulum on a cart system. The first control objective is to balance the pendulum in its vertical position. The second, more challenging objective is to design a controller for the cart to track a square wave while the pendulum remains upright. The third objective is to learn to use online tuning to improve the closed loop system's robustness to external disturbances.

2 Introduction

An inverted pendulum on a cart experiment is very often used as an illustrative model for control system design. The following figure provides an illustration of the physical system.



Ignoring inertia and friction effects, the simplified dynamics of the inverted pendulum is given by the following differential equations:

$$\ddot{y} = \frac{u - ml\dot{\theta}^2 \sin\theta + mg\cos\theta\sin\theta}{M + m\sin^2\theta} \tag{1}$$

$$\ddot{\theta} = \frac{(M+m)g\sin\theta + u\cos\theta - ml\dot{\theta}^2\sin\theta\cos\theta}{l(M + m\sin^2\theta)}. \tag{2}$$

(Note that in the above equations, the sign of θ is opposite to that in the Lab 4 description. This is a consequence of the hardware setup for the inverted pendulum in the lab.)

The above system assumes a force input that horizontally pushes the cart. The pendulum system in our lab, however, takes a voltage input that drives the motor, which in turn drives the cart. So, to convert the former to the latter, we use the following relationship between the driving force generated by the DC motor and the force acting on the cart through the motor pinion:

$$u = \alpha_1 \left(\frac{dy}{dt} \right) + \alpha_2 V \, , \tag{3}$$

where α_1 and α_2 are characteristics related to the motor, and V is the voltage input into the motor. Hence, substituting (3) into (1) and (2), we get the following nonlinear dynamic equations of the inverted pendulum system with a voltage input:

$$\ddot{y} = \frac{(\alpha_1 \dot{y} + \alpha_2 V) - ml\dot{\theta}^2 \sin\theta + mg \cos\theta \sin\theta}{M + m \sin^2\theta} \tag{4}$$

$$\ddot{\theta} = \frac{(M+m) g \sin\theta + (\alpha_1 \dot{y} + \alpha_2 V) \cos\theta - ml\dot{\theta}^2 \sin\theta \cos\theta}{l(M + m \sin^2\theta)} \, , \tag{5}$$

where V is a voltage and is now our input to the system.

For the actual inverted pendulum in the lab we have the following physical parameters:

- $M = 1.0731$ Kg
- $m = 0.2300$ Kg
- $l = 0.3302$ m
- $g = 9.8 \; m/s^2$
- $\alpha_1 = -7.7443 \frac{Ns}{m}$
- $\alpha_2 = 1.7265 \frac{N}{V}$

The system has two sensors which can measure the position of the cart and the angle of the pendulum.

Previously in Lab 4, you studied the control of the inverted pendulum on a cart using Matlab simulation. In Lab 5, you studied the control of a real cart to familiarize yourself with the hardware and laboratory setup. In this experiment, you put everything together to develop a controller for the real inverted pendulum.

2.1 Control Design Objectives:

The control design objectives of this experiment are:

(a) Balance the inverted pendulum about its vertical position;

(b) Control the cart to track a square wave of amplitude ± 20 mm with a frequency of 0.025 Hz;

(c) Achieve good transient response (Note: this is deliberately vague, and better transient response is often obtained by tuning control parameters); and

(d) Achieve a certain level of robustness to physical disturbances.

3 Preparation

In the preparation you will linearize the nonlinear model of the pendulum and cart. You will use this linear model to design a state feedback pole placement controller, the full order observer, and finally the output feedback controller using the separation principle.

First, you design a controller which balances the pendulum (the stabilization and state regulation problem). Second, you design a controller which stabilizes the pendulum and also causes the cart to track a square wave (the stabilization and output tracking problem). Since the second problem involves a reference input, normally you will need 2 models. The physical nature of the cart-pendulum system shows that the tracking problem can be transformed into a regulation problem. This transformation was used in Lab 5. The situation is similar in this lab and you can use just one model for both problems but with slightly different definitions of the variables. Understanding how this is done is part of your preparation. You will also simulate the performance of your controller on the nonlinear model.

Carry out the following steps in your preparation.

(a) Assume that the state vector is $x = [y\ \dot{y}\ \theta\ \dot{\theta}]^T$. Derive a nonlinear state equation for x with input V (compare with Lab 4 prep). Determine the equilibrium point(s) of the system, $(y_e, \dot{y}_e, \theta_e, \dot{\theta}_e)$ and V_e. This will be different for the 2 control problems. Next, compute the linearized model

$$\begin{aligned} \tilde{x} &= A\tilde{x} + B\tilde{V} \\ \tilde{y} &= C\tilde{x} \end{aligned}$$

where $\tilde{x} = x - x_e$, $\tilde{y} = y - y_e$, $\tilde{V} = V - V_e$. In this step of the prep, leave the model parameters M, m, l, and g as variables, i.e., do not plug in the numerical values.

(b) Write a Matlab script that sets the proper values of the model parameters and the equilibrium point, and defines the (A, B, C, D) matrices in terms of these parameters. Use this script to carry out the following steps, which should be done using MATLAB and SIMULINK.

(c) **Stabilization and State Regulation Design:**

(i) Assume that the state of the pendulum system is fully observed. Design a state feedback law, $u = -Kx$, so that the closed loop poles are located at $\{-2, -4, -8, -10\}$.

(ii) Now assume that only the cart position, y, and the pendulum angle, θ, are measured. Design a full-order observer to estimate the states by determining the full-order observer gain, L, so that observer poles are located at $\{-40, -40, -50, -50\}$.

(iii) Now open the SIMULINK model `lab6_prelab.slx` and modify the compensator block to include your controller and observer to give the output feedback control law, $u = -K\hat{x}$. First run the `lab6.m` file in MATLAB to define the system parameters, and then simulate the response of the nonlinear pendulum system in SIMULINK. The initial conditions of the plant are, $(y_o, \dot{y}_o, \theta_o, \dot{\theta}_o) = (0, 0, 10^\circ \times \frac{\pi}{180^\circ}, 0)$ (i.e. the inverted pendulum starts with an initial angle of 10°). These have already been set in the model. For the observer, the initial conditions are chosen to be $(\hat{y}_o, \dot{\hat{y}}_o, \hat{\theta}_o, \dot{\hat{\theta}}_o) = (0, 0, 0, 0)$. Plot your results on 4 plots, each plot containing one state component and its observer estimate.

(d) **Stabilization and Output Tracking Design:**

Modify the controller and your Matlab script in part (c) to ensure that the cart position y tracks the square wave reference y_d (recall the control design in Lab 5 to achieve a similar objective). Plot your result for y, $\hat{y}$, and the square wave input, y_d in one plot.

Note that the nonlinear model used in this file actually includes inertia and friction terms excluded from our modeling in (4) and (5). This nonlinear model better approximates the real physical system. Try to make sense of the results you obtain, i.e. does the cart track a square wave, why or why not?

(e) Experiment with several sets of choices of controller poles for both parts (c) and (d), and see how the choice affects the response. You don't need to plot these results, but they will be useful when you do online tuning.

Each student must hand in their preparation at the beginning of the lab. The preparation should include:

1. The nonlinear state equations, the equilibrium point(s), and the linearized model as a function of the model parameters.
2. 2 MATLAB scripts for the pre-lab, one for stabilization and state regulation, the other for stabilization and cart position tracking.
3. The controller gain (K) and observer gain (L) obtained in part (c).
4. Four plots for the response with the pole placement controller and observer for the stabilization and state regulation design.
5. One plot for the response with the output tracking controller and observer.

4 Experiment

Assuming that in your pre-lab you have tested out your controller in simulation and obtained a satisfactory response, it is now time to try the real thing.

CAUTION: Before running your experiment

- Please ensure that the cart is positioned at the middle of the track.
- Do not change any of the calibration gains in the SIMULINK model, as they are placed there for safety.
- Do not change the frequency and gain of the square input, unless you have permission from your TA.
- When running the experiment ensure that one student from the group is firmly holding the track.
- If for any reason your model goes unstable, turn off the experiment right away.

I. Stabilization and State Regulation Experiment:

(a) First, copy all the files from the drive that the TA specifies in your lab session into your own directory.

(b) Open the SIMULINK file `lab6_part1.slx`. Study the diagram. Modify the compensator subsystem to implement the controller and observer you designed in the pre-lab for stabilization and state regulation. Get a TA to verify your diagram and controller/observer design.

(c) Now it's time to build.

1. Run the initialization files (i.e. `lab6.m` and your pre-lab script) to define the system parameters, the controller and observer gain.

2. In SIMULINK, select the QUARC menu→Clean All.

   (Answer YES if it asks if it is OK to remove any directories or files; this clears any previous compiler files.)

   In SIMULINK, select the QUARC menu→Build.

   (This will compile your file - Matlab will display its progress.)

3. Wait for the model to finish compiling. Then go back to the Simulink model window and click the CONNECT TO TARGET button that is just below the ANALYSIS menu. The RUN button on its right should turn green.

(d) To start the system

1. First let the pendulum stick hang down (i.e. in the 'gantry' position).
2. Then press the green **Run** button.
3. Then slowly bring the pendulum stick to the upright inverted position. Once the stick reaches 180° from the hanging down position, the controller will 'kick' in, at that point gently release the stick. The system should now be stable.

   Caution: the transition of when the controller 'kicks' in, may be violent if you don't let go of the stick in time or continue to grip it when the controller is working, so beware.

Testing the Robustness of the system to Physical Perturbations

Now we tune our controller to improve the system's robustness to external disturbances.

(a) Gently tap the pendulum stick and observe the response. Save the data to a file and plot the position and angle response of the 'tap'. Briefly explain how that system reacts to the tap to recover stability.

(b) Change the controller poles to another set you tried in your preparation. Repeat the 'tap' experiment. Try a few sets until you feel you get the set of controller poles that give you the most robust response. Do a final 'tap' experiment and plot the resulting position and angle response.

II. Stabilization and Cart Position Tracking

(a) Open the SIMULINK file `lab6_part2.slx`. Study the diagram. Modify the compensator subsystem to implement the controller and observer you designed in the pre-lab for stabilization and output tracking. Get a TA to verify your diagram and controller/observer design.

(b) Build the real-time controller as you did in Part I. The system should be stable, the inverted pendulum balanced in the upright position, and the cart position following the square wave input. Does the cart track the square wave well?

Tuning the Controller for better Tracking

In this part of the experiment, we tune the controller to try to get a better response.

(a) Firstly, save the real-time data for the current controller to a file and plot on two graphs the actual and the simulated position and angle responses. For the plot with position response, also include the reference square wave. Note the square wave tracking performance of the cart.

Although there is no direct measurement of the cart velocity or pendulum velocity, an approximation of the velocities is generated in your SIMULINK model through position data differentiation. Plot on two graphs the generated cart velocity, $\dot{y}$, and the pendulum velocity, $\dot{\theta}$, along with their estimated states from the observer, $\dot{\hat{y}}$ and $\dot{\hat{\theta}}$. How well is the observer doing in estimating the states?

(b) Change the controller poles to a different set and observe how the resulting response varies. Give a brief description (i.e. tracking, oscillations, overshoot, setting time, etc.).

(c) Try to find a set of controller poles that achieves better tracking than the controller obtained in the prelab. When you feel you have found a good design, provide a plot of the position and angle response, along with the reference signal.

5 Report

Please follow the report format instructions for Lab 6 as described in format6.pdf.