CS 206

HW 2

1. (a). To get x^8y^9 in $(3x+2y)^{17}$

we can set $a=3x$, $b=2y$,

so $(a+b)^{17} = \sum_{k=0}^{n} \binom{17}{9} a^8 b^9$

$= \sum_{k=0}^{n} \binom{17}{9} (3x)^8 (2y)^9$

$= \boxed{81662929920\, x^8 y^9}$

(b). To get a^6b^6 in $(a^2+b^3)^5$

it must be $(a^2)^3(b^3)^2$ for it.

then $n-k=3$

$k=2$

$\sum_{k=0}^{n} \binom{5}{2} (a^2)^3 (b^3)^2 = \boxed{10\, a^6 b^6}$

2. (a). The number of triangles is:

$\binom{6}{3} = \frac{6!}{3!\,3!} = \boxed{20}$

(b). we set one vertice as the original one, and all the pairs based on this vertices are incident pair (There are five possible pairs), so we pick 2 from these 5

$\binom{5}{2} = 10$

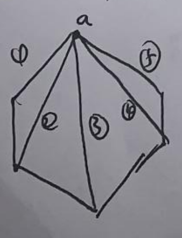
And there are 6 edges, so it's $10 \times 6 = \boxed{60}$

(c). we set $\{(u-v), (v-w)\}$ as the original multicolored t.p.

so one of them must be red, and the other must be blue

when adding $u-w$ to make it a triangle, no matter it's red / blue, it will be multicolored. So it's 2-to-1.

(d).



For a K_6, there can only be five edges from a center.

To make it multicolored t.p. It can have ~~4 red/blue~~ (4, 1) and (3, 2) two choices.

[color is not the problem]

If it's (4, 1) it will be 4 multicolored t.p.s

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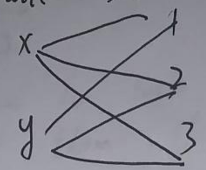
If it's (3, 2) it will be



$\boxed{6 \text{ ways.}}$ (That's the reason)

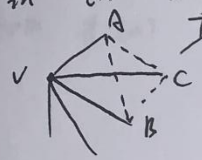
(e). from #d. we know from a center there can have at most 6 multicolored i.p.s.

From the picture in #d. we see there are 6 vertex

so in total is $6 \times 6 = 36$.

(f). so it means in a K_6, there are 2 triangles that the edges of it are in the same color.

~~From lecture,~~



From V, there are 5 edges

$|A| > k\,|B|$
$\;5 \quad\;\; 2 \;\; 2$ $\Longrightarrow$ 3 edges of same color (same as lecture)

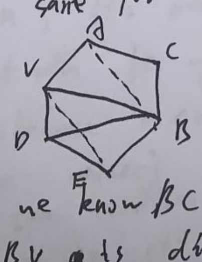
we set VA, VC, VB has same color

we can find that AC, AB, BC can't be the same color as them to make multicolored triangles.

Then AC, BC, AB would have same color.

They will form a triangle in the same color.

It's the same for vertex B.



From above, we know BC, AB have same color

BV is different from them.

To make 3 edges have same color,

we set BD. BE and BV have same color

Then $\triangle VDE$ will form another triangle in same color

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3. (a). $\underbrace{A\ 2\ 3\ 4\ 5}_{1}$, $\underbrace{2\ 3\ 4\ 5\ 6}_{2}$ - - - - -

There are 10 such ways

And for all of them, they might have 4 suits

$\underline{a}\ \underline{b}\ \underline{c}\ \underline{d}\ \underline{e}$

$4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$

total: $4^5 \cdot 10 = 10240$

(b). First we need to pick 5 cards

$\binom{13}{5} = \frac{13!}{5!\ 8!} = 1287$

And there are 4 suits. (to choose one from four)

$\binom{4}{1} = 4 \quad \therefore 1287 \times 4 = \boxed{5148}$

(c). from # (a) we know there are 10 such ways.

and from #(b) we know there are 4 ways to pick suit.

$\therefore 10 \times 4 = \boxed{40}$

(d). for straight is just the sequence ~~times~~ minus the times of straight flush (To make suits not same)

$10240 - 40 = \boxed{10200}$

4. First we set $|A| = m$, $|B| = n$

(a). To make it bijection

it's $|A| = |B| = m$

so the number is $m!$

(b). There are two conditions.

First, if $m > n$, it's not injection.

if $m \leq n$, it will be $\frac{n!}{(n-m)!}$

(because $m < n$ and we only count to $n-m$ to get the answer)

(c). Again, there are two conditions.

First, if $m < n$, it's not surjection.

if $m \geq n$, it will be

$n^m - [\binom{n}{1}(n-1)^m - \binom{n}{2}(n-2)^m + \cdots (-1)^{(k-1)}\binom{n}{k}(n-k)^m]$

(from the knowledge of week 4 "inclusion - exclusion")

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5. (a). $\cancel{\binom{n+m}{r}}$ $\sum_{k=0}^{r} \binom{n}{k}\binom{m}{r-k} = \binom{n+m}{r}$

because we see 0 to r and r to 0, so I think about use k to sign it.

(b). because we see number changes from 0 to r in the $\binom{n}{\dots}$ part, and number changes from r to 0 on $\binom{m}{\dots}$ part. so I set k to express the change of it. (From 0 to r for k)

(c). We put $m=n$, $r=n$ in.

$$\sum_{k=0}^{r} \binom{n}{k}\binom{m}{r-k} = \binom{n}{k}\binom{n}{n-k} = \binom{n+n}{n} = \binom{2n}{n}$$

From symmetry we know

$$\binom{n}{n-k} = \binom{n}{k}$$

so it's $\binom{n}{k}^2 = \binom{2n}{n}$