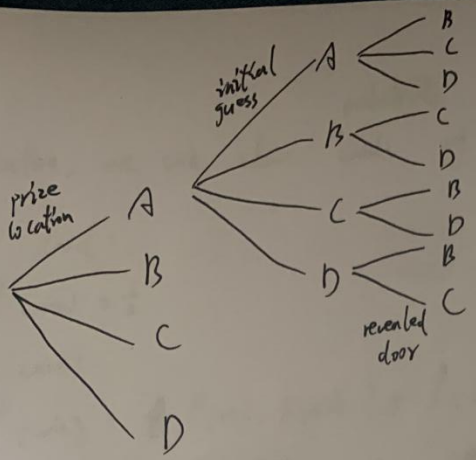
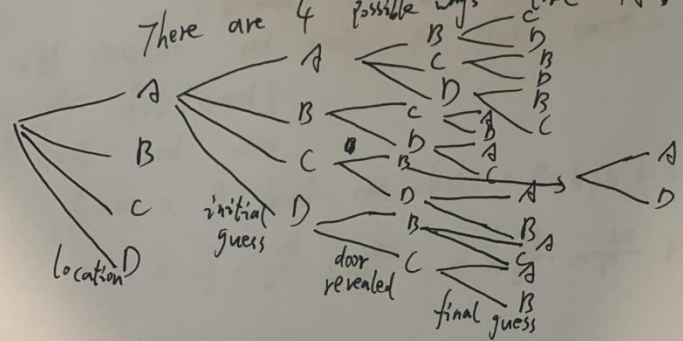
Yinfeng Cong
CS206
Hw4
1. (a).



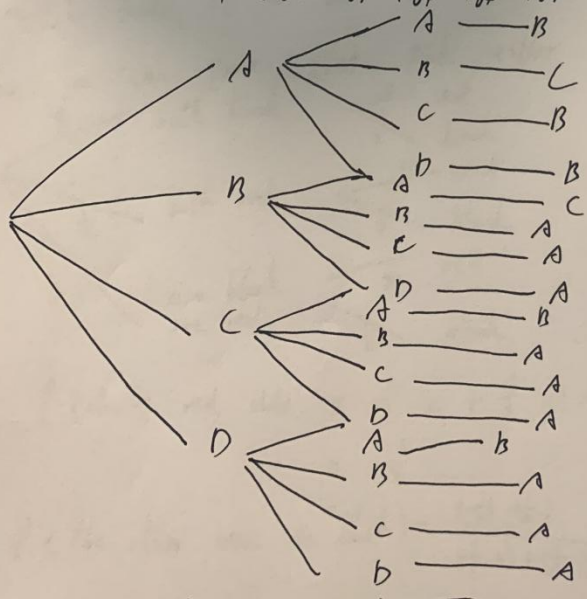
$P(\text{win}) = 4 \cdot (\frac{1}{48} + \frac{1}{48} + \frac{1}{48}) = \boxed{\frac{1}{4}}$ so It's multiply 4.

b).



$P(\text{win}) = 4 \cdot (\frac{1}{64} + \frac{1}{64} + \frac{1}{64} + \frac{1}{64} + \frac{1}{64} + \frac{1}{64}) = \boxed{\frac{3}{8}}$

c).



$P(\text{win}) = 8 \cdot \frac{1}{16} = \boxed{\frac{1}{2}}$

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2. By induction, we can show probability of winning cards are all $\frac{1}{2}$.

For 1st card:

$P(\text{win}) = \frac{1}{2}$

For 2nd card:

$P(\text{win}) = P(\text{red, black}) + P(\text{black, black})$

$= \frac{1}{2} \times \frac{26}{51} + \frac{1}{2} \times \frac{25}{51} = \frac{1}{2}$

(because 51 cards left, if 2nd card black, there are 26 possible red cards for 1st card, and 25 possible black cards for 1st card)

For 3rd card wins

$P(\text{win}) = P(\text{black, red, black}) + P(b,b,b) + P(r,r,b) + P(r,b,b)$

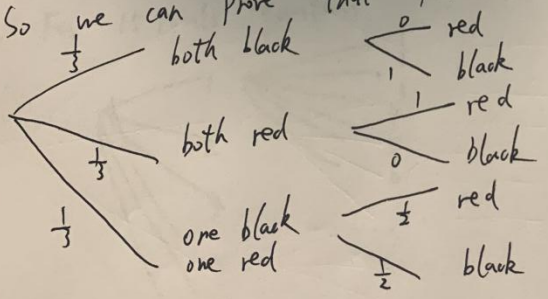
$= \frac{1}{2} \times \frac{26}{51} \times \frac{25}{50} + \frac{1}{2} \times \frac{25}{51} \times \frac{24}{50} + \frac{1}{2} \times \frac{25}{51} \times \frac{26}{50} + \frac{1}{2} \times \frac{26}{51} \times \frac{25}{50}$

$= \frac{1}{2}$

For 4~52 cards, it is all the same for the answer, which is $\frac{1}{2}$.

So we can prove that pick either card, the winning probability $= \frac{1}{2}$

3.



$P(\text{showing red side}) = \frac{1}{3} \cdot 0 + \frac{1}{3} \times 1 + \frac{1}{3} \times \frac{1}{2}$

$= \frac{1}{2}$

$P(\text{the other side is black}) = \frac{(\frac{1}{3}) \cdot (\frac{1}{2})}{(\frac{1}{3} \cdot 0) + (\frac{1}{3} \cdot 1) + (\frac{1}{3}) \cdot (\frac{1}{2})} = \frac{1}{3}$

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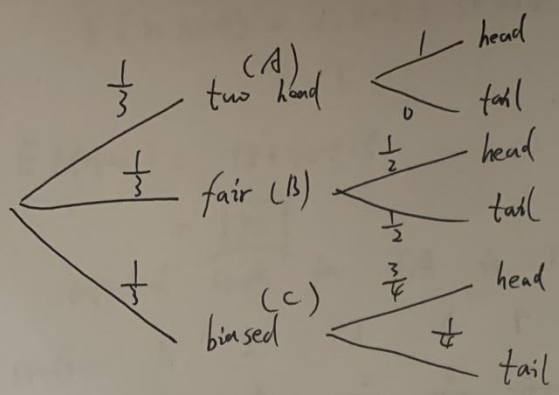
4.



$P(A) = P(B) = P(C) = \frac{1}{3}$

$P(H|A) = 1$, $P(H|B) = \frac{1}{2}$, $P(H|C) = \frac{3}{4}$

$$P(A|H) = \frac{P(A) \cdot P(H|A)}{P(A) \cdot P(H|A) + P(B) \cdot P(H|B) + P(C) \cdot P(H|C)}$$

$$= \frac{(\frac{1}{3}) \cdot 1}{(\frac{1}{3}) \cdot 1 + (\frac{1}{3}) \cdot (\frac{1}{2}) + (\frac{1}{3}) \cdot (\frac{3}{4})}$$

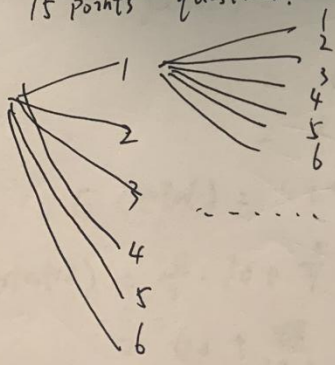
$$= \boxed{\frac{4}{9}}$$

5. (a). For T/F problem.

$E(T/F) = 10 \times \frac{3}{4} = 7.5$.

$7.5 \times 2 = \boxed{15}$

For 15 points questions.



points	2	3	4	5	6	7	8	9	10	11	12
probability	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{4}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

$$E(15 \text{ points}) = \sum_{x=2}^{12} x \cdot P(x = x) = 7$$

$7 + 3 = 10$

$10 \times 4 = \boxed{40}$

For 20 point question

$E(\text{20 point}) = 12 \times 0.5 + 18 \times 0.5$

$= 15.$

$E(\text{total}) = 15 + 40 + 15$

$= \boxed{70}$

expected score for TA is 70.

(b).

points	1	2	3	4	5	6	8	9	10	12
probability	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{4}{36}$	$\frac{2}{36}$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{4}{36}$

15	16	18	20	24	30	36
$\frac{2}{36}$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

$E(\text{point}) = \Sigma x\, p(x_i - x)$ ← as the data multiply above

$= \frac{1+4+6+12+10+24+16+9+\cdots\cdots}{36}$

$= \frac{416}{36}$

≈ 11.56

For general impression.

$E(\text{general}) = 40 \times \frac{4}{10} + 50 \times \frac{3}{10} + 60 \times \frac{3}{10}$

$= 16 + 15 + 18$

$= 49$

$E(\text{total}) = 49 + 11.56 = \boxed{60.56}$

(c), $E(\text{expected}) = \frac{4}{7} \cdot 70 + \frac{2}{7} \cdot 60.56 + \frac{1}{7} \times 84$

$= 40 + 17.3 + 12$

$= \boxed{69.3}$