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CS 206 HW1

1. The 1st password contains 16 characters
There are $\begin{cases} 52 & \text{for the amount of letter (upper or lower)} \\ 10 & \text{numbers} \\ 5 & \text{special characters} \end{cases}$

$$52 + 10 + 5 = 67$$

$$\text{so it's } 67^{16} = 1.65 \times 10^{29}$$

The 2st has 24 characters

26 letter for lowercase

$$\text{it's } 26^{24} = 9.11 \times 10^{33}$$

$$26^{24} > 67^{16}$$

So the second one is more secure.

2. we know there are 10 numbers between 0 or 9
to better calculate this, we can set $1 \sim 1000000$ to
 $0 \sim 999999$ (to make it from 1 digit to 6 digits)
because there's only one 7 digits number 1000000
(we can add it in later)

among 0 or 9, there are 8 numbers aren't 1 or 5

and there's 1 or 6 digits.

\therefore Among $0 \sim 999999$, there are 8^6 numbers doesn't contain 1 or 5.
In the question, ~~there~~ 0 is not in the list, so it's $(8^6 - 1)$ numbers.

And there's 1000000 which contains 1 (in the list)

so from $1 \sim 1000000$, it's $1000000 - (8^6 - 1)$ contains 1 or 5.

$$\therefore \text{It's } \boxed{737857}$$

3. (a). 8 seats

$$\left(\frac{8}{s_1} \frac{7}{s_2} \frac{6}{s_3} \frac{5}{s_4} \frac{4}{s_5} \frac{3}{s_6} \frac{2}{s_7} \frac{1}{s_8} \right) = 8! = \boxed{40320}$$

(b). According to the lecture.

$$|S| = 8 \cdot |A|$$

$$8! = 8 \cdot |A|$$

$$|A| = \frac{8!}{8}$$

$$= 7!$$

$$= \boxed{5040}$$

(c). It's said Anna sits next to Brian, so they are always together
 \therefore we only need to consider the position of other 6 students

$$\text{so it's } 6! = 720$$

However, there can be two seating ways for Anna and Brian

Anna Brian — — — — — or

Brian Anna — — — — —

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so it's $720 \times 2 = 1440$ arrangements

(d), similar to part (c).

The three students can seem as one part, they always sit together.

so it's $5! = 120$

And there are again two seating way

... — $\frac{A}{C}$ $\frac{B}{B}$ $\frac{C}{A}$ — ...

... — $\frac{C}{C}$ $\frac{B}{B}$ $\frac{A}{A}$ — ...

so it's $120 \times 2 = 240$ arrangements.

(e). This questions there are situations

① A sits with B

② B sits with C

③ B sits with both A and C

we have calculated sitting together for 2 people and 3 people.

So we just add it

$$1440 + 240 + 1440 = 3120.$$

However, for ①, ②, we included ③ in both of them, so we need to delete the repeat of ③.

$$\text{it's } 3120 - 2 \times 240 = 2640.$$

4. First we calculate $\binom{15}{5}$ to get how many ways we can choose the 5 members from 15 people.

And then we use $\binom{15}{5}$ minus the choices that we chose no women to get the choices we choose at least one woman.

$$\text{It's } \binom{15}{5} - \binom{6}{0} \cdot \binom{9}{5}$$

$$= 2877$$

5. First we use mod 3 for all the numbers from 1 to 15. Then I get

$$(1, 4, 7, 10, 13 \text{ mod } 3 \text{ is } 1)$$

$$(2, 5, 8, 11, 14 \text{ mod } 3 \text{ is } 2)$$

$$(3, 6, 9, 12, 15 \text{ mod } 3 \text{ is } 0)$$

For 2, 1, 0; there are all 5 numbers,

To make it divisible for 3, there are four possibilities for combination.

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① "0", "0", "0"

② "1", "1", "1"

③ "2", "2", "2"

④ "0", "1", "2"

There's no order need, and ~~the~~ ~~the~~ ~~the~~ number we choose can't be repeated, so we use element subsets.

for ①: It's $\binom{5}{3}$

for ②: It's also $\binom{5}{3}$

for ③: It's also $\binom{5}{3}$

for ④: It's $\binom{5}{1} \cdot \binom{5}{1} \cdot \binom{5}{1}$

Answer: $\binom{5}{3} + \binom{5}{3} + \binom{5}{3} + \binom{5}{1} \cdot \binom{5}{1} \cdot \binom{5}{1}$

$$= 10 + 10 + 10 + 5 \cdot 5 \cdot 5$$

$$= 30 + 125$$

$$= \boxed{155}$$