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CS 206
Hw 7

1. Let x be a relation of D, we can find $x \subseteq D \times D$

$\therefore$ No. relation in D = No. subsets of $n \times n$.

we can set $(a_i, a_i) \in R$

By symmetric, we know $(a_i, a_i) \in R$

By antisymmetric, we know $a_i = a_i$

(symmetric: if $(x, y) \in R \Rightarrow (y, x) \in R$

antisymmetric: if $(x, y) \in R$ and $(y, x) \in R \Longrightarrow x = y$)

Then we find all the common entries for both relations are the diagonal entries.

$\therefore$ For n elements of diagonal entries,

we have 2 choices: related / not related to itself

$\therefore$ Total number of relation of symmetric and antisymmetric is $\boxed{2^n}$

2. (a). Increasing sequences: (maximum)

(1, 2, 3, 8) ~~(6, 7, 9)~~ ~~(4, 7, 9)~~ (1, 2, 5, 8)

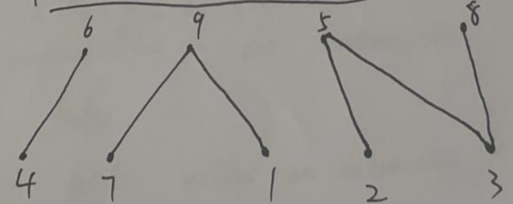
$\boxed{\text{maximum length} = 4}$

decreasing sequences: (maximum)

(6, 4, 1) (6, 4, 2) ~~(9, 1)~~ ~~(5, 3)~~ (6, 4, 3) (9, 5, 3) (6, 5, 3) (7, 5, 3)

$\boxed{\text{maximum length} = 3}$

(b).



maximal elements: 6, 9, 5, 8

minimal elements: 4, 7, 1, 2, 3

(c). The connection between increasing and decreasing sequences is the minimum and maximal elements.

chain: a totally ordered subset of a poset S

here, a subset $C \subseteq S$ is a chain if $(C, \prec)$ is linearly ordered

antichain: A subset of S in which any two elements are incomparable.

here, a subset $B \subseteq S$ is a antichain if no two distinct elements are comparable.

(d)

Sequence S : $a_1, a_2, a_3 \cdots a_n$

Suppose b_i is the length of longest increasing subsequence start at a_i, C_i is the length of a longest decreasing subsequence at a_i.

For a_1, we have orded pair (b_i, c_i), for a_n, ordered pair (b_n, C_n)

we show all these n ordered pairs are distinct.

Suppose on the contrary $(b_i, c_i) = (b_j, c_j)$. let $i < j$.

Set $a_i \cdots a_k$, $a_j \cdots a_m$ have same number of elements and both are increasing subsequences.

Set $a_i \cdots a_l$, $a_j \cdots a_n$ have same number of elements and both are decreasing subbquences.

we know $i < j$, then $a_i \neq a_j$ ① Suppose $a_i < a_j$.

Then $a_i \cdots a_m$ (a increasing subsequence), its length will be $b_i + 1$

It's contradiction

② suppose $a_i > a_j$,

then $a_i \cdots a_n$ (a decreasing subsequence), its length will be $b_i + 1$

It's contradiction.

∴ Our assumption $(b_i, c_i) = (b_j, c_j)$ is false. All the orders are distinct.

From 1 to n, we have n ordered pairs. (All ordered pairs are distinct)

suppose every decreasing / increasing subsequence $< \sqrt{n}$

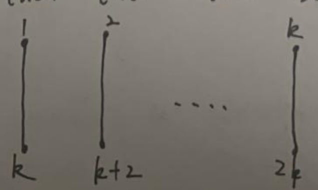
Then $1 \leq b_i \leq \sqrt{n}$, $1 \leq c_i \leq \sqrt{n}$

(b_i, c_i) will $< n$, we have known there are n distinct orded pairs.

∴ A contradiction, our 'assumption' every decreasing / increasing subsequence $< \sqrt{n}$ is false.

∴ There exists either an increasing / decreasing subsequence $\geq \sqrt{n}$

3. we set $j=1$, then there will be only one block



we can find it's perfect matching.

we have 2k vertices, so we have even number vertices.

Then because it's a regular graph, so there're no isolated vertices.

At last it's bipartite, so it has same number left / right vertices.

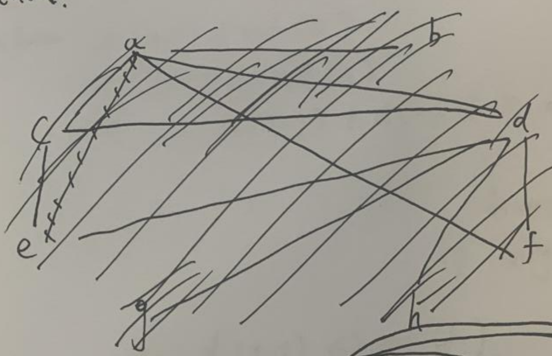
(which means it has even number of components)

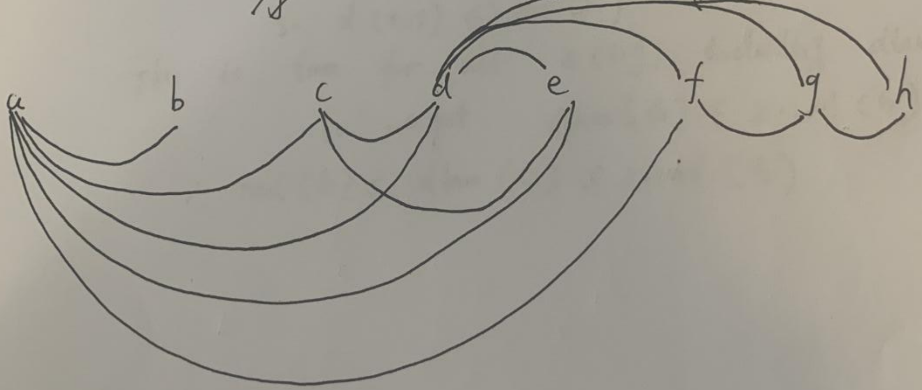
we know that every components are block with even number of vertices

∴ every block is perfect matching. (based on Tutte theorem)

4. (a). vertices correspond to the variables

If two variables can't be stored together, then there's an edge between them.





(b). we need 4 registers.

a、g : R1

b、c、f、h : R2

d : R3

e : R4

(c). When a variable is reassigned, we see it as a new variable

For instance,

t = r + s

u = t × 3

t_1 = m − k

→ v = t_1 + u

. . . .

Then we can go on with graph construction and coloring as we did in (a). (b).

5. First we prove $rad(G) \leq diam(G)$

$rad(G) = \min_{x \in V} \max_{y \in V} d(x,y)$

$diam(G) = \max_{x \in V} \max_{y \in V} d(x,y)$

so minimum of $\max_{y \in V} d(x,y) \leq$ maximum of $\max_{y \in V} d(x,y)$

(it's always true)

so $rad(G) \leq diam(G)$

Then we prove $diam(G) \leq 2 \cdot rad(G)$

set m as a central vertex

we know $d(m,y) \leq rad(G)$

$d(m,x) \leq rad(G)$

and $d(m,y) + d(m,x) \geq d(x,y)$

$\Downarrow$

because it's the minimum length of x-y path.

so $d(x,y) \leq 2 \cdot rad(G)$

This is true for all $d(x,y)$ including $diam(G)$

so we get $diam(G) \leq 2 \cdot rad(G)$

$\therefore rad(G) \leq diam(G) \leq 2 \cdot rad(G)$