1.
(a) i, iii
(b) i, iv
(c) i, iii
(d) ii
2.
(a).(1).
For the same matrix A, and a different right side b'
Ax = b'
However, L*U = A. So we do not need to solve L and U again
So the only system we need to solve is LUx = b'
(2).
For a different matrix A
A'x = b
This time, A' = L' * U', so we need to find L' and U' first.
Then, we also need to solve the system L'U'x = b
(b)

$$M_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{1}{3} & 0 & 1 & 0 \\ -\frac{4}{3} & 0 & 0 & 1 \end{bmatrix}$$

Then $M_2a_2 = \begin{bmatrix} 3 \\ 2 \\ 0 \\ 0 \end{bmatrix}$ which satisties the requirement of the question

(c)
We have n*n matrix A and B, A is non-singular, and $c\epsilon R^n$, then we can know $A^{-1}Bc$ is an n*1 matrix
we can set $A^{-1}Bc = x$
Then $Bc = Ax$
So the first thing we need to do is compute Bc, we can make Bc = y
Then we use $Ax = y$ (LU factorization) to get the answer of x
3.
(a)
det A = 1*(4-3) + 1*(1-2) = 0
so A is a singular matrix
(b)

b = $\begin{bmatrix} 2 & 4 & 6 \end{bmatrix}^T = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$

In Ax=b,

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 1 & 3 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} \Longrightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$$

After calculating, we get $x_1 + x_2 = 2, x_2 + x_3 = 2$
Rank(A)=Rank(A|B) = 2 <3

So it has infinitely many solutions

4.

$\|Ax-b\|_2^2 = (Ax-b)^T(Ax-b) = x^TA^TAx - x^TA^Tb - b^TAx + b^Tb$

$\|A(x-x_0)\|_2^2 + \|Ax_0-b\|_2^2 = (Ax-Ax_0)^T(Ax-Ax_0) + (Ax_0-b)^T(Ax_0-b)$

After calculating about it, we get

$\|A(x-x_0)\|_2^2 + \|Ax_0-b\|_2^2 = x^TA^TAx - x^TA^Tb - b^TAx + b^Tb$

which is the same as $\|Ax-b\|_2^2$

Then it is proved

5.

$\begin{bmatrix} l_{11} & 0 \\ l_{21} & l_{22} \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} \\ 0 & u_{22} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$\begin{bmatrix} l_{11}u_{11} & l_{11}u_{12} \\ l_{21}u_{11} & l_{21}u_{12} + l_{22}u_{22} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$l_{11}u_{11} = 0, l_{11}u_{12} = 1, l_{21}u_{11} = 1, l_{21}u_{12} + l_{22}u_{22} = 0$

from the 1st equation, we know one of $l_{11}, u_{11}$is 0

from the 2nd equation, we know l_{11}can not be 0

Then $u_{11} = 0$

but in 3rd equation, we find u_{11}can not be 0

It is a contradiction.

So A has no LU factorization