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CS 323
1. (a) ii, iii
(b) iii
(c) i
(d) ii
2. (a)

The problem with the Vandermonde matrix is that when the dimensionality of the matrix gets higher and higher, the problem will become more and more ill-conditioned when solving linear equations, and the condition number will become larger and larger. And it is much simpler for Newton interpolation to calculate than the Vandermonde matrix method. For an incremental problem, Newton interpolation is also quicker and simpler.

When computing higher-order interpolation, the error of Newton interpolation is bigger than Lagrange interpolation

(b) NO, bisection search is like binary search in another class. Its mission is to find a root of it, and it is recursive to find the value between two numbers (a bigger one and a smaller one), and finally find the value we want to get. So it is not a fixed point method.

(c) 1. because the basis matrix is a lower triangular, and it is much easier to list the triangular matrix than using Vandermonde matrix. And when we plug in the data and calculate the triangular matrix, we can directly get the answer easily. (We can calculate the answer easily, but for Vandermonde matrix it is much more complex)

2. Also triangular matrix is easier to understand and write, we can devide it to lower triangular, x values, y values three parts, it is clearly easy to understand what we have here. However, we need to m*(n-1) numbers together in the matrix for Vandermonde matrix, it is much harder to understand from so many numbers.

3.

We know that

$$x_{k+1} = g(x_k)h(x_k)$$

and we also know that $g(x_k), h(x_k)$are both guaranteed to converge in the same interval

so there is a number x = m that makes $g'(m) = 0$ and $h'(m) = 0$

From the hint we can get

$$[g(m)h(m)]' = g(m)h'(m) + g'(m)h(m) = 0 + 0 = 0$$

so it also exhibit quadratic convergence

4.

Using Lagrange interpolation

$$f(x) = \frac{(x+1)(x-2)(x-4)}{(-2+1)(-2-2)(-2-4)} * (-5) + \frac{(x+2)(x-2)(x-4)}{(-1+2)(-1-2)(-1-4)} * (-6)$$

$$+\frac{(x+2)(x+1)(x-4)}{(2+2)(2+1)(2-4)}*(-8)+\frac{(x+1)(x+2)(x-2)}{(4+1)(4+2)(4-2)}*(-4)=$$

$$\frac{3x^3}{40}+\frac{19x^2}{120}-\frac{21x}{20}-\frac{107}{15}$$

5.

The iteration function g(x) for Newton's method is:

$$g(x)=x-\frac{f(x)}{f'(x)}=x-\frac{x^2-5}{2x}=x-\frac{1}{2}x+\frac{5}{2x}$$

$$g'(x)=\frac{1}{2}-\frac{5}{2x^2}$$

The condition |g'(x)| < 1 is equivalently written as

$$|\frac{1}{2}-\frac{5}{2x^2}|<1$$

$$\Longrightarrow -1<\frac{1}{2}-\frac{5}{2x^2}<1$$

$$\Longrightarrow -\frac{1}{2}<\frac{5}{2x^2}<\frac{3}{2}$$

($\frac{5}{2x^2}$ is always bigger than $-\frac{1}{2}$, so we do not need to calculate about it any more)

$$\Longrightarrow \frac{5}{x^2}<3 \Longrightarrow 3x^2>5 \Longrightarrow x^2>\frac{5}{3}$$

From the problem we know that we have the initial guess $x_0 \geq 2$, so x^2 always ≥ 4

which is also true for all x > $\frac{5}{3}$, indicating that g is a contraction in this case

To make $f(a)=0$, we need to make $\frac{1}{2}-\frac{5}{2x^2}=0 \Longrightarrow x^2=5$ (bigger than 4 so it is ok) , then

$$g'(a)=\frac{1}{2}-\frac{5}{2x^2}=\frac{1}{2}-\frac{5}{2*5}=0$$

As a result, convergence has to be quadratic.