# Physics 160 Pset 1

Vassilios Kaxiras

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**Solution** (1)**.** Consider a 3-bit gate $G$ with inputs $A, B, C$ and outputs $A', B', C'$.

| A | B | C | A' | B' | C' |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 1 | 0 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |

This gate is reversible, since all the outputs are simply a permutation of all the inputs. And, we can get NAND from this:

$$\text{NAND}(x, y) = G(0, x, y)_{(1)} \tag{1}$$

where $G_{(1)} = A'$.

**Solution** (2)**.** (a) i. Yes

$$C^\dagger = (A + B)^\dagger = A^\dagger + B^\dagger = A + B = C \tag{2}$$

ii. No, unless $A, B$ commute.

$$C^\dagger = (AB)^\dagger = B^\dagger A^\dagger = BA \tag{3}$$

which is not necessarily equal to $AB = C$.

iii. No, unless $C = 0$.

$$C^\dagger = (i(A + B))^\dagger = -i(A + B)^\dagger = -i(A^\dagger + B^\dagger) = -i(A + B) = -C \tag{4}$$

iv. Yes

$$C^\dagger = (AB + BA)^\dagger = B^\dagger A^\dagger + A^\dagger B^\dagger = BA + AB = C \tag{5}$$

v. Yes

$$C^\dagger = (i(AB - BA))^\dagger = -i(B^\dagger A^\dagger - A^\dagger B^\dagger) = i(AB - BA) = C \tag{6}$$

(b) TODO

(c)