CoxIter

Computations of invariants of hyperbolic Coxeter groups

1.0b

17/08/2016

Rafael Guglielmetti

Rafael Guglielmetti
Email: rafael@rgug.ch
Homepage: https://rgug.ch
Address: Chemin du Musee, CH-1700 Fribourg

Contents

Chapter 1

Introduction

1.1 The standalone program

This chapter gives general information about the GAP package CoxIter. CoxIter was first developped as a standalone C++ program whose goal is to compute invariant of hyperbolic Coxeter groups. We consider a hyperbolic Coxeter group $\Gamma \leq \mathrm{Isom}(\mathbb{H}^n)$ and the associated polyhedron P. The input of the program consists a file containing the description of the Coxeter graph of Γ. Then, the output consists of the following information:

- Euler characteristic (and thus volume of P if n is even)
- f-vector $(f_0, f_1, \ldots, f_n)$: P has f_0 vertices, f_1 edges, f_2 2-faces, ...
- Cofiniteness test: test whether P has finite volume or not
- Cocompactness test: test whether P is compact or not
- Growh series
- Growth rate and some of its algebraic properties (note: this is not available in the GAP package)

A description of the mathematical results behind CoxIter can be found in the article [Gug15].

Chapter 2

CoxIter automatic generated documentation

2.1 CoxIter automatic generated documentation of methods

2.1.1 CreateCoxIterFromCoxeterGraph (for IsList, IsInt)

▷ `CreateCoxIterFromCoxeterGraph(`*`gr, dimension`*`)` (operation)

Returns: a CoxIter object

Creates a CoxIter object from the Coxeter graph *gr*. If the dimension *dim* is unknown, 0 can be given.

2.1.2 CreateCoxIterFromCoxeterMatrix (for IsMatrix, IsInt)

▷ `CreateCoxIterFromCoxeterMatrix(`*`mat, dimension`*`)` (operation)

Returns: a CoxIter object

Creates a CoxIter object from the Coxeter matrix *mat*. If the dimension *dim* is unknown, 0 can be given.

2.1.3 CoxIterCompute (for IsCoxIter)

▷ `CoxIterCompute(`*`ci`*`)` (operation)

Returns:

Compute the invariants of the Coxiter object *ci*

Chapter 3

Some examples

First, we consider the 8 dimensional cocompact group found by Bugaenko.

Example

```
gap> LoadPackage( "CoxIter" );
gap> buga8 := CreateCoxIterFromCoxeterGraph([[1,[2,5]],[2,[3,3]],[3,[4,3]],[4,
> [5,3],[10,3]],[5,[6,3]],[6,[7,3],[11,3]],[7,[8,3]],[8,[9,5]],[10,[11,1]]],8);
CoxIter : Coxeter group with 11 generators in dimension 8
gap> Cofinite(buga8);
1
gap> Cocompact(buga8);
1
gap> FVector(buga8);
[ 41, 164, 316, 374, 294, 156, 54, 11, 1 ]
gap> EulerCharacteristic(buga8);
24187/8709120000
```

References

[Gug15] Rafael Guglielmetti. CoxIter - Computing invariants of hyperbolic Coxeter groups. *LMS Journal of Computation and Mathematics*, 18(1):754–773, December 2015. 3

Index