Dynamics Jan 2009 Mark Scheme

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Q1a.

$$\underline{v_0} = v_0(\cos\theta, \sin\theta)^T \qquad \underline{a} = g(0,1)^T$$
$$\implies \underline{v}(t) = (v_0 cos\theta, v_0\sin\theta - gt)^T$$
$$\text{At max height: } \underline{v}(t)\cdot\hat{\underline{i}} = 0 \qquad \implies v_0\sin\theta - gt = 0$$
$$\implies t = \frac{v_0\sin\theta}{g} \qquad \text{As required.}$$

As parabolas are symetric then:

$$T = 2t = \frac{v_0\sin\theta}{g}$$
$$\underline{x}(t) = (v_0 t cos\theta, v_0 t\sin\theta - \frac{gt^2}{2})^T$$
$$\implies \underline{x}(T)\cdot\hat{\underline{i}} = \frac{2v_0^2\sin\theta\cos\theta}{g} \qquad \text{As required.}$$

b.

$$\underline{\tau} = \underline{r}\times\underline{F} = \begin{vmatrix} \hat{\underline{i}} & \hat{\underline{j}} & \hat{\underline{k}} \\ 7 & 3 & 1 \\ -3 & 1 & 5 \end{vmatrix} = (14, -38, 16)^T \text{ Nm}$$

c.

$$\Delta KE = -\Delta GPE \implies \frac{mv_{esc.}^2}{2} = \frac{GM_{murc.}m}{R_{murc.}} \implies v_{esc.} = \sqrt{\frac{2GM_{murc.}}{R_{murc.}}} = 4255 \text{ ms}^{-1}$$

(Note: Bottomline answer is 4254 ms^{-1} probably due to differeing G values)

di.

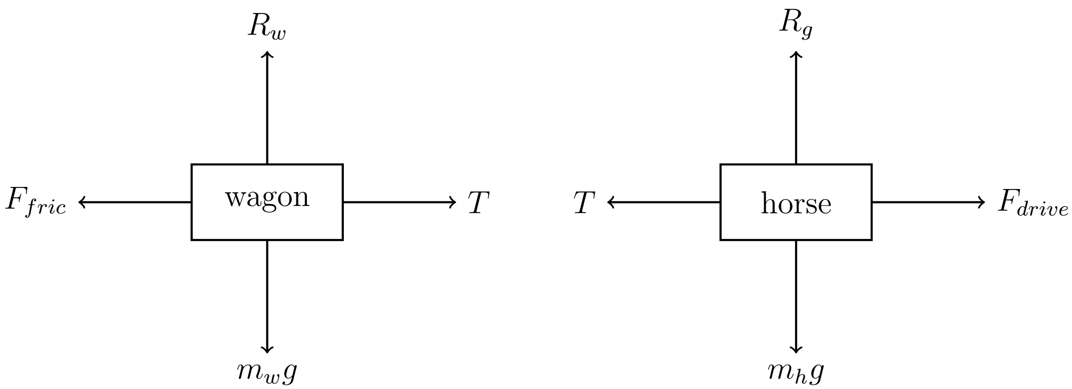
No, as given a moving object the linear momentum of the system is non-zero and due to the conservation of linear momentum if one of the objects becomes stationary due to the collision then the other must be moving in order to have a non-zero final momentum.

ii.

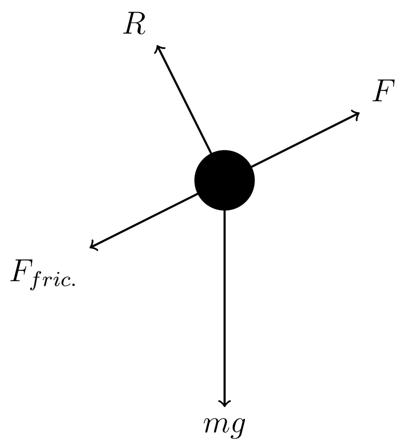
Yes, if $e = 1$ and masses are equal and the masses are balls and the collision is not oblique this necessitates the initially moving mass to be stationary after the collision.

$$p = v_{1i}m \text{ (i)} \qquad \Delta v_0 = v_{1i} \text{ (ii)}$$
$$\text{(i)} \implies mv_{1i} = mv_{1f} + mv_{2f} \qquad \implies v_{1i} = v_{1f} + v_{2f} \qquad \text{by conservation of linear momentum}$$
$$\text{(ii)} \implies -\Delta v = -v_{1i} = v_{1f} - v_{2f} \qquad \text{As collision is elastic}$$
$$\implies v_{1f} = 0 \qquad v_{2f} = v_{1i}$$

e.



The foreces labelled T on both diagrams are N3L pairs.
The groups with equal magnitudes are: T, F_{fric}, and F_{drive}; R_w, and $m_w g$; and R_h, and $m_h g$.
2a.



b.

$$F_{res.} = \alpha t^2 + \beta - mg(\sin 30^\circ - \mu_k \cos 30^\circ)$$

c.

$$I =$$