Dynamics Jan 2010 Mark Scheme

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Q1a.
N1L. Every object continues in its state of rest or uniform motion in a straight line unless acted upon by a resultant force.
N2L. The rate of change of momentum of a body is proportional to the resultant force acting on the body and is in the direction of that force
N3L. Action and reaction are equal and opposite, and act on different bodies.
[By, Week 4: Newton's Laws, Week 9: Momentum and Collisions]
bi.

$$\underline{r} = \underline{r}_2 - \underline{r}_1 = (2, 10, 5)^T - (4, 3, 8)^T = (-2, 7, 3)^T \text{ m}$$

ii.

$$\underline{r} \cdot \underline{F} = (-2, 7, 3)^T \cdot (5, 3, -8)^T = 35J$$

(Note: bii is actually $\int_{(2,10,5)^T}^{(4,3,8)^T} (5, 3, -8)^T \cdot \mathrm{d}\underline{r} = 5x + 3y - 8z + c|_{(2,10,5)^T}^{(4,3,8)^T} = 35J$ because $\nabla(5x + 3y - 8z + c) = \underline{F}$ hence no need to specify the path.)
c.

$$-\nabla U(\underline{r}) = F(\underline{r}) \implies F(x) - \frac{\partial U}{\partial x} = \frac{k}{x^2}$$

Hence the force is away from the orgin
(Note: x is distance from orgin not displacement hence cannot be negative and a postive values of force corrispond to a force away from the orgin.)
d.

$$L = \omega_0 \mathcal{I}_0 = 30 \cdot 6 = 180 \implies \omega_f \mathcal{I}_f = 180 \implies \omega_f (6 + 9) = 180 \implies \omega_f = 30 \text{ rads}^{-1}$$

ei.

$$\begin{aligned} R_E &= 6.4 \cdot 10^6 \text{ m} \qquad h = 630 \cdot 10^3 \text{ m} \\ r = (R_E + r) &= 7.03 \cdot 10^6 \\ F_g &= \frac{GM_E m}{r^2} \quad \& \quad F_c = \frac{mv^2}{r} \\ \implies \frac{mv^2}{r} &= \frac{GM_E m}{r^2} \\ \implies v &= \sqrt{\frac{GM_e}{r}} = 7530 \text{ ms}^{-1} \end{aligned}$$

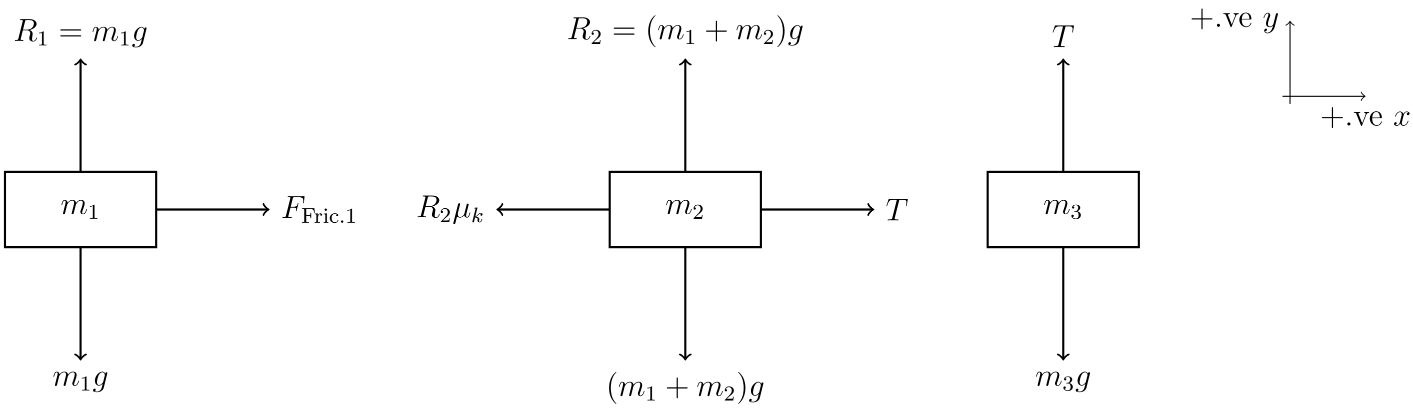
ii.

$$\frac{2\pi}{T} = \omega = \frac{v}{r} \implies T = \frac{2\pi r}{\omega} = 5867 \text{ s}$$

Q2a.

Because the pulley and chord itself have negligable mass they require negligable force to accelerate, and along with the chord being inextensable the tension is applied equally at the chords ends hence each dl of the chord also must experience this tension equally.

b.



c.

$$a_1 = \frac{F_{\text{Fric.1}}}{m_1} = a \text{ (i)} \quad a_2 = \frac{T - (m_1 + m_2)g\mu_k}{m_1 + m_2} = a \text{ (ii)} \quad a_3 = \frac{T - m_3 g}{m_3} = -a \text{ (iii)}$$

d. as $F_{fric.\ \max} = R\mu_s$ then by (i):

$$a_{\max} = \frac{F_{fric.1\ \max}}{m_1} = \frac{R_1\mu_s}{m_1} = g\mu_s \text{ (iv)}$$

e.

from (ii), (iii) and substituting in $a_{\max} = g\mu_s$ from (iv):

$$\frac{T - (m_1 + m_2)g\mu_k}{m_1 + m_2} = g\mu_s \qquad \frac{T - m_3 g}{m_3} = -g\mu_s$$

$$\implies T = g(m_1 + m_2)(\mu_s + \mu_k) \qquad T = m_3 g(1 - \mu_s)$$

$$\implies (m_1 + m_2)(\mu_s + \mu_k) = m_3(1 - \mu_s)$$

$$\implies \frac{(m_1 + m_2)(\mu_s + \mu_k)}{1 - \mu_s} = m_3 \qquad \text{As required.}$$

Q3a.

Conservation of Linear Momentum. If no external forces act on a system of interacting bodies, then the total momentum of the system is conserved.

[by: Week 9: Momentum and Collisions]

bi.

$$KE_f = \Delta KE + KE_0 \quad \& \qquad \Delta KE = -\Delta GPE = mg\Delta h \quad \& \quad KE = \frac{mv^2}{2}$$

$$\implies KE_{c\ f} = \frac{m_c v_{c\ 0}^2}{2} + m_c g\Delta h = 3050 \text{ J}$$

ii.

$$KE_f = \frac{m_c v_{c\ f}^2}{2}$$

$$\implies v_{c\ f} = \sqrt{\frac{2KE_f}{m_c}} = 7.81 \text{ ms}^{-1}$$

iii.

$$\underline{\hat{i}} \cdot \sum_{i=1,2,c} m_i \underline{v}_i = \underline{\hat{i}} \cdot \underline{v} \sum_{i=1,2,c} m_i$$

$$\implies v_{aft} = \frac{\underline{\hat{i}} \cdot \sum_{i=1,2,c} m_i \underline{v}_i}{\underline{\hat{i}} \cdot \sum_{i=1,2,c} m_i}$$

As: $\underline{\hat{i}} \cdot \underline{v}_1 = v_1, \quad \underline{\hat{i}} \cdot \underline{v}_2 = v_2, \quad \underline{\hat{i}} \cdot \underline{v}_c = 1.5\cos 30^\circ = \frac{3\sqrt{3}}{4}, \quad \underline{\hat{i}} \cdot \underline{v}_{aft.} = v_{aft.}$

$$\implies v_{aft.} = 0.245 \text{ ms}^{-1}$$

iv.

$$KE_{bef.} = KE_f + \frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2} = 3240 \text{ J} \qquad KE_{aft.} = \frac{\left(\sum_{i=1,2,c} m_i\right) v_{aft.}^2}{2} = 33.1 \text{ J}$$

$$\implies \Delta KE = 3209 \text{ J}$$

(Note: given answer is 3213 J probably due to differing constants used)

c.

$$F_{Ret.} = 1.7t \implies a = -\frac{1.7t}{\left(\sum_{i=1,2,c} m_i\right)} \implies \Delta v = \int_0^{\Delta t} -\frac{1.7\tau}{\left(\sum_{i=1,2,c} m_i\right)} \mathrm{d}\tau = -\frac{1.7\Delta t}{2\left(\sum_{i=1,2,c} m_i\right)}$$

$$\text{As: } \Delta v = -v_{aft.} \implies -v_{aft.} = -\frac{1.7t}{\left(2\sum_{i=1,2,c} m_i\right)} \implies \Delta t = \sqrt{\frac{2v_{aft.}\left(\sum_{i=1,2,c} m_i\right)}{1.7}} = 17.8 \text{ s}$$

4a.

In case (a) the mechanical energy of the sphere is conserved as all GPE is converted to KE as the friction force is over no distance so does no work, whilst in case (b) there is mechanical energy loss due to friction, hence mechanical energy isn't conserved in case (b).

b.

As there is no slipping the constraint is $v = \omega r$

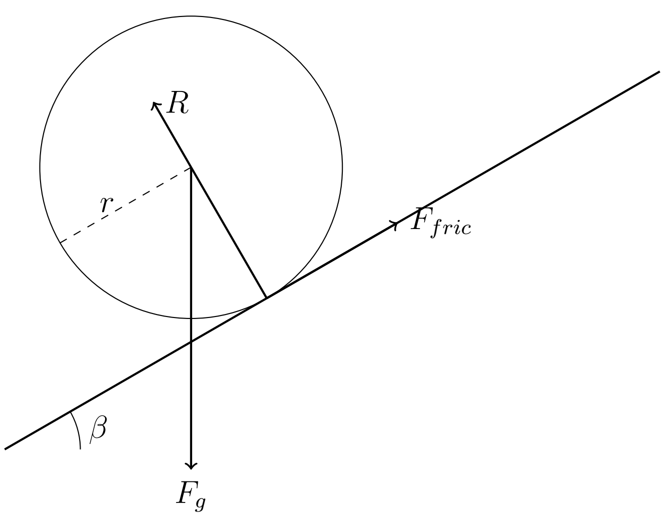
c.

$$\Delta GPE = \Delta KE_{rot.} + \Delta KE_{lin.}$$

$$\implies mgh = \frac{mv_f^2}{2} + \frac{\mathcal{I}\omega^2}{2} = \frac{mv_f^2}{2} + \frac{mv_f^2}{5}$$

$$\implies v = \sqrt{\frac{10}{7}gh}$$

d.



From, $F = ma$ and $\tau = \mathcal{I}\omega = \frac{\mathcal{I}a}{r}$:

$$F = ma = F_{fric} - mg\sin\beta \qquad\qquad \tau = \frac{\mathcal{I}(-a)}{r} = rF_{fric}$$

$$\implies F_{fric} = \frac{\mathcal{I}(-a)}{r^2} = -\frac{2ma}{5}$$

$$\implies ma = -\frac{2ma}{5} - mg\sin\beta \implies a = -\frac{5}{7}g\sin\beta$$

Hence a constant acceleration down the slope with a magnitude of $\frac{5}{7}g\sin\beta$. As required.
(Note: minus sign in $\frac{\mathcal{I}(-a)}{r}$ comes from a positive angular acceleration resulting in a negative linear accleration given the slope being under the sphere)
(Note: If it was not specified that you must use forces and torques, differentiating the velocity equation with respect to time would be quickest)