

# Dynamics Jan 2010 Mark Scheme

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Q1a.

N1L. Every object continues in its state of rest or uniform motion in a straight line unless acted upon by a resultant force.

N2L. The rate of change of momentum of a body is proportional to the resultant force acting on the body and is in the direction of that force

N3L. Action and reaction are equal and opposite, and act on different bodies.

[By, Week 4: Newton's Laws, Week 9: Momentum and Collisions]

bi.

$$\underline{r} = \underline{r}_2 - \underline{r}_1 = (2, 10, 5)^T - (4, 3, 8)^T = (-2, 7, 3)^T \text{ m}$$

ii.

$$\underline{r} \cdot \underline{F} = (-2, 7, 3)^T \cdot (5, 3, -8)^T = 35J$$

(Note: bii is actually  $\int_{(2,10,5)^T}^{(4,3,8)^T} (5, 3, -8)^T \cdot d\underline{r} = 5x + 3y - 8z + c|_{(2,10,5)^T}^{(4,3,8)^T} = 35J$  because  $\nabla(5x + 3y - 8z + c) = \underline{F}$  hence no need to specify the path.)

c.

$$-\nabla U(\underline{r}) = F(\underline{r}) \implies F(x) - \frac{\partial U}{\partial x} = \frac{k}{x^2}$$

Hence the force is away from the origin

(Note: x is distance from origin not displacement hence cannot be negative and a positive values of force correspond to a force away from the origin.)

d.

$$L = \omega_0 \mathcal{I}_0 = 30 \cdot 6 = 180 \implies \omega_f \mathcal{I}_f = 180 \implies \omega_f(6 + 9) = 180 \implies \omega_f = 30 \text{ rads}^{-1}$$

ei.

$$\begin{aligned} R_E &= 6.4 \cdot 10^6 \text{ m} & h &= 630 \cdot 10^3 \text{ m} \\ r &= (R_E + h) = 7.03 \cdot 10^6 \\ F_g &= \frac{GM_E m}{r^2} & \& & F_c &= \frac{mv^2}{r} \\ \implies \frac{mv^2}{r} &= \frac{GM_E m}{r^2} \\ \implies v &= \sqrt{\frac{GM_E}{r}} = 7530 \text{ ms}^{-1} \end{aligned}$$

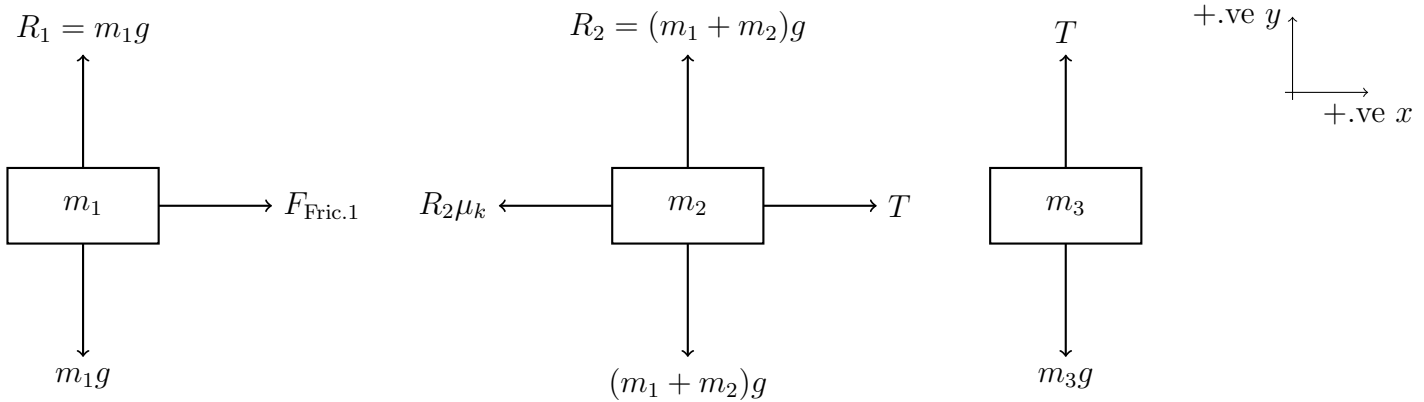
ii.

$$\frac{2\pi}{T} = \omega = \frac{v}{r} \implies T = \frac{2\pi r}{v} = 5867 \text{ s}$$

Q2a.

Because the pulley and chord itself have negligible mass they require negligible force to accelerate, and along with the chord being inextensible the tension is applied equally at the chords ends hence each  $dl$  of the chord also must experience this tension equally.

b.



c.

$$a_1 = \frac{F_{\text{Fric.1}}}{m_1} = a \text{ (i)} \quad a_2 = \frac{T - (m_1 + m_2)g\mu_k}{m_1 + m_2} = a \text{ (ii)} \quad a_3 = \frac{T - m_3g}{m_3} = -a \text{ (iii)}$$

d. as  $F_{\text{fric. max}} = R\mu_s$  then by (i):

$$a_{\text{max}} = \frac{F_{\text{fric.1 max}}}{m_1} = \frac{R_1\mu_s}{m_1} = g\mu_s \text{ (iv)}$$

e.

from (ii), (iii) and substituting in  $a_{\text{max}} = g\mu_s$  from (iv):

$$\begin{aligned} \frac{T - (m_1 + m_2)g\mu_k}{m_1 + m_2} &= g\mu_s & \frac{T - m_3g}{m_3} &= -g\mu_s \\ \Rightarrow T &= g(m_1 + m_2)(\mu_s + \mu_k) & T &= m_3g(1 - \mu_s) \\ \Rightarrow (m_1 + m_2)(\mu_s + \mu_k) &= m_3(1 - \mu_s) \\ \Rightarrow \frac{(m_1 + m_2)(\mu_s + \mu_k)}{1 - \mu_s} &= m_3 \quad \text{As required.} \end{aligned}$$

Q3a.

Conservation of Linear Momentum. If no external forces act on a system of interacting bodies, then the total momentum of the system is conserved.

[by: Week 9: Momentum and Collisions]

bi.

$$\begin{aligned} KE_f &= \Delta KE + KE_0 \quad \& \quad \Delta KE = -\Delta GPE = mg\Delta h \quad \& \quad KE = \frac{mv^2}{2} \\ \Rightarrow KE_{cf} &= \frac{m_c v_{c0}^2}{2} + m_c g \Delta h = 3050 \text{ J} \end{aligned}$$

ii.

$$\begin{aligned} KE_f &= \frac{m_c v_{cf}^2}{2} \\ \Rightarrow v_{cf} &= \sqrt{\frac{2KE_f}{m_c}} = 7.81 \text{ ms}^{-1} \end{aligned}$$

iii.

$$\begin{aligned}\hat{\underline{i}} \cdot \sum_{i=1,2,c} m_i \underline{v}_i &= \hat{\underline{i}} \cdot \underline{v} \sum_{i=1,2,c} m_i \\ \Rightarrow v_{aft} &= \frac{\hat{\underline{i}} \cdot \sum_{i=1,2,c} m_i \underline{v}_i}{\hat{\underline{i}} \cdot \sum_{i=1,2,c} m_i}\end{aligned}$$

$$\text{As: } \hat{\underline{i}} \cdot \underline{v}_1 = v_1, \quad \hat{\underline{i}} \cdot \underline{v}_2 = v_2, \quad \hat{\underline{i}} \cdot \underline{v}_c = 1.5 \cos 30^\circ = \frac{3\sqrt{3}}{4}, \quad \hat{\underline{i}} \cdot \underline{v}_{aft} = v_{aft}.$$

$$\Rightarrow v_{aft} = 0.245 \text{ ms}^{-1}$$

iv.

$$\begin{aligned}KE_{bef.} &= KE_f + \frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2} = 3240 \text{ J} & KE_{aft.} &= \frac{(\sum_{i=1,2,c} m_i) v_{aft.}^2}{2} = 33.1 \text{ J} \\ \Rightarrow \Delta KE &= 3209 \text{ J}\end{aligned}$$

(Note: given answer is 3213 J probably due to differing constants used)

c.

$$F_{Ret.} = 1.7t \Rightarrow a = -\frac{1.7t}{(\sum_{i=1,2,c} m_i)} \Rightarrow \Delta v = \int_0^{\Delta t} -\frac{1.7\tau}{(\sum_{i=1,2,c} m_i)} d\tau = -\frac{1.7\Delta t}{2(\sum_{i=1,2,c} m_i)}$$

$$\text{As: } \Delta v = -v_{aft.} \Rightarrow -v_{aft.} = -\frac{1.7t}{(2\sum_{i=1,2,c} m_i)} \Rightarrow \Delta t = \sqrt{\frac{2v_{aft.}(\sum_{i=1,2,c} m_i)}{1.7}} = 17.8 \text{ s}$$

4a.

In case (a) the mechanical energy of the sphere is conserved as all GPE is converted to KE as the friction force is over no distance so does no work, whilst in case (b) there is mechanical energy loss due to friction, hence mechanical energy isn't conserved in case (b).

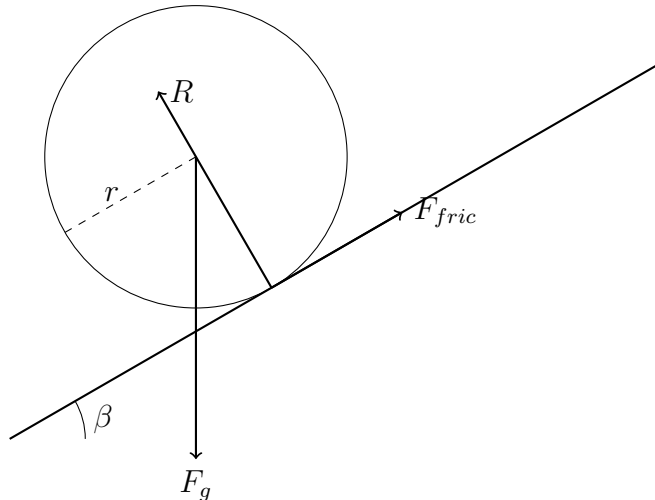
b.

As there is no slipping the constraint is  $v = \omega r$

c.

$$\begin{aligned}\Delta GPE &= \Delta KE_{rot.} + \Delta KE_{lin.} \\ \Rightarrow mgh &= \frac{mv_f^2}{2} + \frac{\mathcal{I}\omega^2}{2} = \frac{mv_f^2}{2} + \frac{mv_f^2}{5} \\ \Rightarrow v &= \sqrt{\frac{10}{7}gh}\end{aligned}$$

d.



From,  $F = ma$  and  $\tau = \mathcal{I}\alpha = \frac{\mathcal{I}a}{r}$ :

$$\begin{aligned} F = ma &= F_{fric} - mg \sin \beta & \tau &= \frac{\mathcal{I}(-a)}{r} = rF_{fric} \\ \implies F_{fric} &= \frac{\mathcal{I}(-a)}{r^2} = -\frac{2ma}{5} \\ \implies ma &= -\frac{2ma}{5} - mg \sin \beta \implies a = -\frac{5}{7}g \sin \beta \end{aligned}$$

Hence a constant acceleration down the slope with a magnitude of  $\frac{5}{7}g \sin \beta$ . As required.

(Note: minus sign in  $\frac{\mathcal{I}(-a)}{r}$  comes from a positive angular acceleration resulting in a negative linear acceleration given the slope being under the sphere)

(Note: If it was not specified that you must use forces and torques, differentiating the velocity equation with respect to time would be quickest)