Dynamics Jan 2024 Mark Scheme

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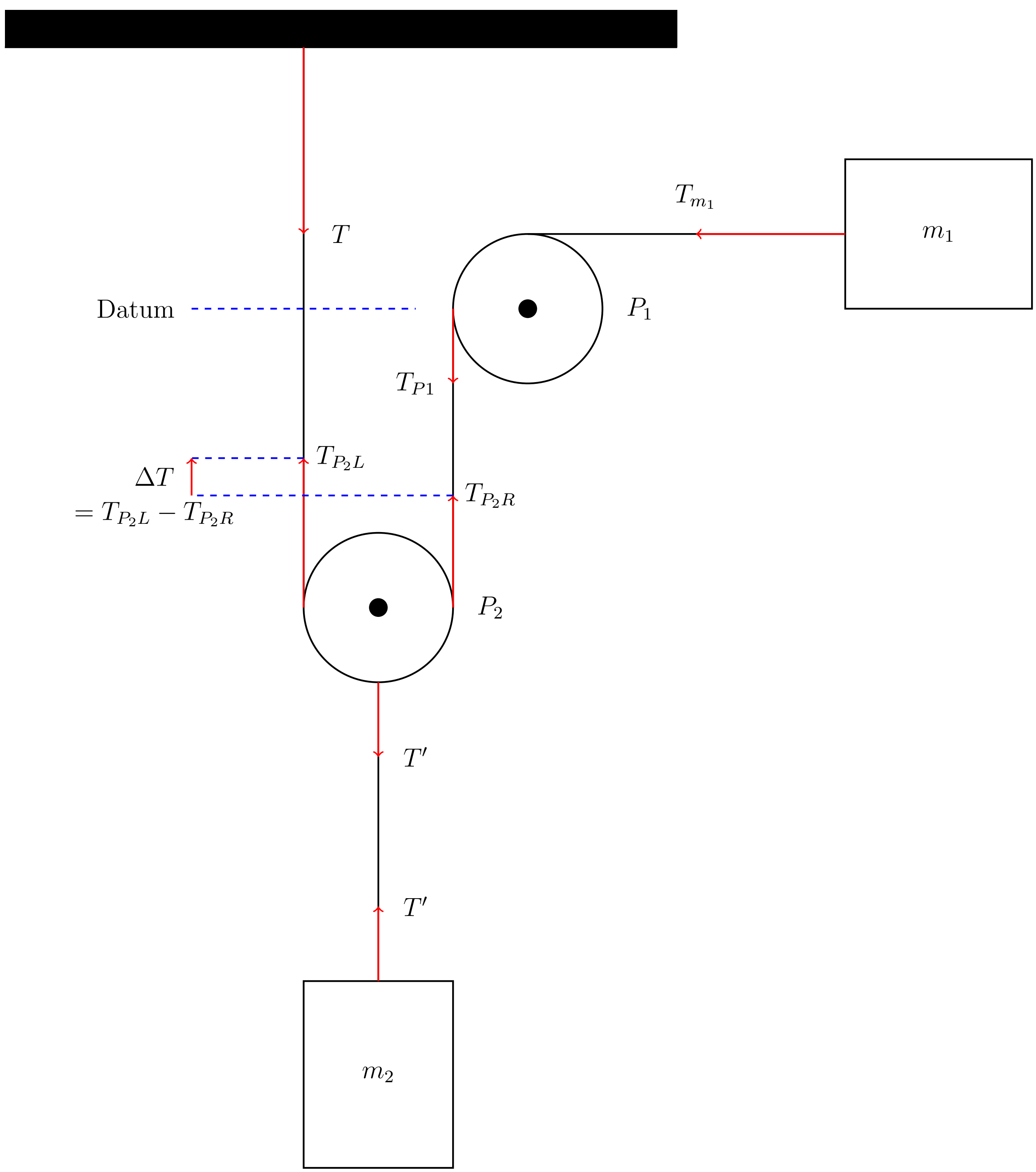
Q2

Diagram with features labled:



ai.

This can be intuited by the fact if P_2 drops by δx then to "fill" the dropped distance with string it would require $2\delta x$ of string whilst if m_1 were to move to the left by δx then the "lost" string would only be δx.

Here is the mathematical argument:

Let the total length of the string from a datum be l then we can express l as:

$$l = 2x_{p_2} + x_{m_1} + \frac{3}{4}\pi r,$$

where x_{p_2} is the distance from the datum to P_2's left most touch with the string and x_{m_1} is the distance from the P_1's top most touch with the string to m_1 hence by differentiating we get:

$$0 = 2v_{p_2} + v_{m_1},$$

and again:

$$0 = 2a_{p_2} + a_{m_1},$$

where each symbol has its obvious meaning.
Hence it can be concluded:

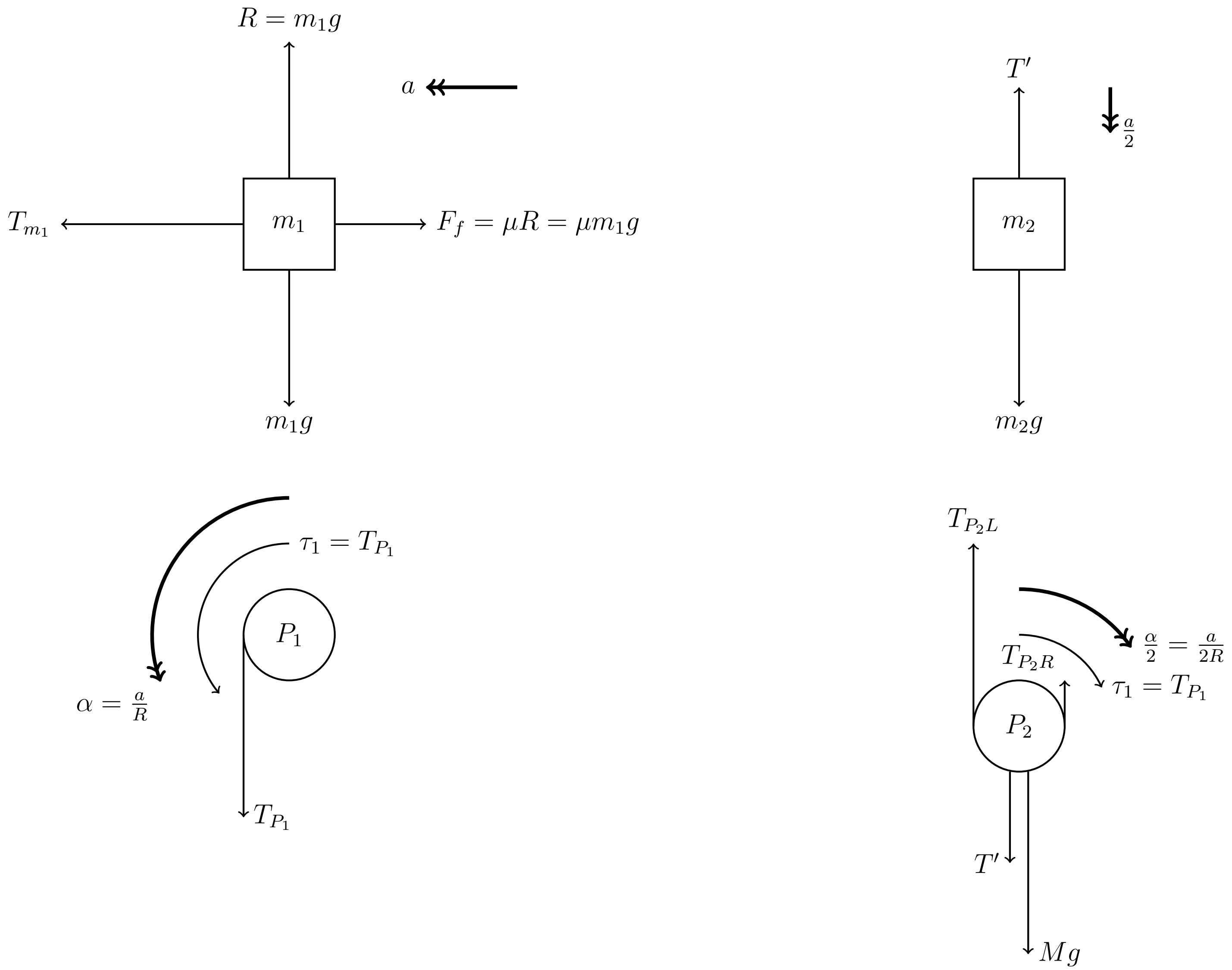
$$2|a_{m_2}| = |a_{m_1}|.$$

where a_{m_2} is the acceleration of m_2 as m_2 is a constant displacement from P_2.

ii.

Since P_2 doesn't slip on the left vertical string and is accellerating downwards like m_2 then it must have a rotational acceleration clockwise (the right veritcal string moves so this doesn't apply to the right string), also as P_2 doesn't slip then the tangential acceleration that P_2 falls with at its edge (a distance of R from its centre) is equal to $a/2$ where $a/2$ is it's downwards acceleration and is half a, the magnitude of the acceleration of m_1, by part i. Now for P_1, the string which runs over P_1 is accelearting to the left with an acceleartion a which is given by the fact that the string is in tension and m_1 is accelearting at a hence the tangential acceleration at the edge of P_1 is a a and its rotational acceleration is anticlockwise, thus opposite to the rotational acceleration of P_2. Hence it can be concluded that magnitudes of P_1's and P_2's rotational acceleration: α_{P_1}, α_{P_2}, respectively, have the relation $\alpha_{P_1} = 2\alpha_{P_2}$ by the fact that their radii are the same and their tangential acceleration are of the same ratio.

b.



c.

	m_1	P_1	m_2	P_2
Linear	$a = \frac{T_{m_1}}{m_1} - \mu g$	NA	$2a_{m_2} = a = 2g - \frac{2T'}{m_2}$	$2a_{P_2} = a = g + \frac{T' - T_{P2L} - T_{P2R}}{M}$
Angular	NA	$\alpha_{P_1} R = a = \frac{2T_{P_1}}{M}$	NA	$\alpha_{P_2} R = \alpha R = a = \frac{4(T_{P2L} - T_{P2R})}{M}$

d.

This is a more useful formulation of what we know:

$$T_{m_1} = am_1 + \mu m_1 g$$

$$T_{P_1} = \frac{Ma}{2}$$

$$T_{P_2L} + T_{P_2R} = (g - \frac{a}{2})(m_2 + M) \qquad \text{this is the linear equation of the } P_2 \text{ and } m_2 \text{ system}$$

$$T_{P_2L} - T_{P_2R} = \frac{Ma}{4} \qquad \text{this is the angular equation of the } P_2 \text{ and } m_2 \text{ system}$$

Now lets look at the tensions in the string

(Note: We know that the acceleartion of the string is 0 as the string is stationary at the ceiling and as it is inextensible and isn't in compression anywhere it hance cannot accelerate into itself.)

From this illistration it is clear that given the forces are ballenced due to the zero acceleration that:

$$T + T_{P_2R} = T_{P2L} + T_{P_1} + T_{m_1}$$

Hence:

$$T = T_{P2L} - T_{P_2R} + T_{P_1} + T_{m_1} = \frac{3}{4}Ma + m_1a + \mu m_1 g \tag{1}$$

(Note: This is equation is making use of the angular equation of the P_2 and m_1 system so now our aim is going to be to find a way to make use of the linear equation of the same system.)

Now to get our final equation note that the implosive and explosive forces on the string must balence everywhere, as the string is inextensible so cannot be streching. Thus if you look at the ends of the string you get the above equation, however if you look at P_2 the implosive forces are: T_{P_2L}, and T_{P_2R}; whilst the explosive forces are: T, T_{P_1}, and T_{m_1}. Hence we are left with the equations. This allows us to make use of our knowledge of $T_{P_2L} + T_{P_2R} = (g - \frac{a}{2})(m_2 + M)$, and allow us to solve for a. Hence:

$$T + T_{P_1} + T_{m_1} = T_{P_2L} + T_{P_2R} = (g - \frac{a}{2})(m_2 + M) \tag{2}$$

And thus by (1) and (2) we get:

$$\frac{3}{4}Ma + m_1a + \mu m_1 g = (g - \frac{a}{2})(m_2 + M)$$

which once rearranged gives the given answer of:

$$a = \left(\frac{m_2 + M - 2\mu m_1}{8m_1 + 2m_2 + 7M}\right)4g$$