Algorithm 1 CAPO: Cost-Aware Prompt Optimization

Require: Dataset $\mathcal{D} = \{(X_i, y_i)\}_{i=1}^{n}$, Meta-LLM $\Phi(x)$, Downstream LLM $\phi(x)$, Cost function $\ell(y, \hat{y})$, Initial instructions $\Lambda = \lambda_1, \ldots, \lambda_p$, Population size p, Block size b, Number of iterations n, Number of crossovers per iteration c

for $\lambda \in \Lambda$ **do**
  $\text{num\_shots} \sim \text{U}(\text{lower}, \text{upper})$
  $\xi \leftarrow \text{sample}(X, \text{num\_shots})$ ▷ Sample few shot examples
  $\theta \leftarrow \phi(\lambda || \xi)$ ▷ Generate few shots with reasoning
  $\pi \leftarrow (\lambda, \theta)$
  $\Pi \leftarrow \Pi.\text{append}(\pi)$
end for
Divide dataset $\mathcal{D}$ into blocks $\mathcal{B} = \{B_1, ..., B_k\}$ where $|B_i| = b$
for $i = 1$ to n **do**
  $\Pi_{\text{off}} \leftarrow \text{cross\_over}(\Pi, c)$
  $\Pi_{\text{off}} \leftarrow \text{mutate}(\Pi_{\text{off}})$
  $\Pi \leftarrow \text{do\_racing}(\Pi \cup \Pi_{\text{off}}, k = p)$
end for
$\text{best\_prompt} \leftarrow \text{do\_racing}(\Pi, k = 1)$
return best_prompt

Is reasoning always necessary? Maybe this is not important for simple tasks and costs a lot of tokens. (Idea: some few shot examples can be included without reasoning)

Algorithm 2 cross_over

Require: Population Π, Meta-LLM $\Phi(x)$, Cross-Over-Meta-Prompt λ_{C}, Number of crossovers c

$\Pi_{\text{off}} \leftarrow []$
for $j = 1$ to c **do**
  $P_1 \leftarrow \text{sample}(\Pi, 1)$ ▷ $P_1 = (\lambda_1, \theta_1)$
  $P_2 \leftarrow \text{sample}(\Pi, 1)$ ▷ $P_2 = (\lambda_2, \theta_2)$
  $\lambda_{\text{off}} \leftarrow \Phi(\lambda_{\text{C}} || \lambda_1 || \lambda_2)$ ▷ Let Meta-LLM cross over the prompts
  $\theta_{\text{off},j} \leftarrow \text{sample}(\theta_1 \cup \theta_2, \left\lfloor \frac{|\theta_1| + |\theta_2|}{2} \right\rfloor)$ ▷ Sample from all few-shot examples
  $\pi_{\text{off},j} \leftarrow (\lambda_{\text{off},j}, \theta_{\text{off},j})$
  $\Pi_{\text{off}} \leftarrow \Pi_{\text{off}}.\text{append}(\pi_{\text{off},j})$
end for
return Π_{off}

Currently we do a random parent selection. In EvoPrompt they do a roulette wheel selection based on the fitness scores. This would require us to find a way of already having scores here.
We have to clarify where the new few-shot examples are coming from.
Extra Split:
+ no data leakage (fair, comparable assessment of prompts)

Algorithm 3 mutate

Require: Population of offsprings Π_{off}, Meta-LLM $\Phi(x)$, Mutation-Meta-Prompt λ_{M}, Dataset samples X

for $\pi_{\text{off}} \in \Pi_{\text{off}}$ **do**
  $\lambda_{\text{off}} \leftarrow \Phi(\lambda_{\text{M}} || \lambda_{\text{off}})$
  $\text{num\_shots} \sim \text{U}(\text{lower}, \text{upper})$
  $\text{num\_new\_shots} \sim \text{U}(\text{lower}, \text{num\_shots})$
  $\xi \leftarrow \text{sample}(X, \text{num\_new\_shots})$ ▷ Sample new few shot examples
  $\theta_{\text{new}} \leftarrow \phi(\lambda_{\text{off}} || \xi)$ ▷ Generate few shots with reasoning
  $\theta_{\text{old}} \leftarrow \text{sample}(\theta_{\text{off}}, \text{num\_shots} - \text{num\_new\_shots})$
  $\theta \leftarrow \theta_{\text{old}} \cup \theta_{\text{new}}$
  $\theta \leftarrow \text{shuffle}(\theta)$
  $\pi_{\text{off}} \leftarrow (\lambda_{\text{off}}, \theta)$
end for
return Π_{off}

- constrained pool of few-shot examples (how to get this pool?) - potentially smaller dev Split

From Train Split:
- data leakage (prompts that contain eval data point already as few-shot examples which are already confirmed as correct have advantages)
+ we can use the full train set

Algorithm 4 do_racing

Require: Prompts Π, Top-k k, cost function $\ell(y, \hat{y})$, blocks $\mathcal{B}$, Downstream LLM $\phi(x)$, number of evaluations a

$\text{survivors} \leftarrow \Pi$
$i \leftarrow 0$
$\text{scores} \leftarrow [0] * \text{len}(\Pi)$
$\text{shuffle}(\mathcal{B})$
while $\text{len}(\text{survivors}) > k \ \wedge \ i < \text{len}(\mathcal{B})$ **do**
  $i \leftarrow i + 1$
  $\text{scores} \leftarrow \frac{1}{i} \left(\text{evaluate}(\Pi, B_i) + (i - 1) * \text{scores}\right)$
  $\text{survivors} \leftarrow \text{racing\_elimination}(\Pi, \text{scores}, i * b, \alpha, k)$
end while
return survivors

Algorithm 5 racing_elimination

Require: Prompts Π, scores S, confidence level α, top-k k

$z_\alpha \leftarrow \Phi^{-1}(1 - \alpha/2)$ $\triangleright$ Critical value

survivors $\leftarrow \Pi$

for $\pi_i \in \Pi$ **do**

  n_subst_better $\leftarrow \sum_{j \neq i} \mathbf{1}_{\left[(s_j - s_i)\sqrt{a} > z_\alpha\right]}$

  if n_subst_better $\geq k$ **then**

    survivors $\leftarrow$ survivors $\setminus \{\pi_i\}$ $\triangleright$ Eliminate π_i

  end if

end for

return survivors